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On Some Characteristics Of Pentapartitioned Neutrosophic Topological Spaces

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Abstract - Some important algorithms are being adopted to determine the permanence of decisions on uncertainty in this modern world. Using Pentapartitioned Neutrosophic sets in particular has improved the accuracy of the solutions for uncertainty problems. Some of the characteristics of the Pentapartitioned Neutrosophic functions like continuity, open mapping and closed mappings in Pentapartitioned Neutrosophic Topological Spaces are to be introduced and discussed some of their properties, in this paper.

Keywords: Pentapartitioned Neutrosophic Continuity, Pentapartitioned Neutrosophic Open Map, Pentapartitioned Neutrosophic Closed Map, Pentapartitioned Neutrosophic Topology.

1. INTRODUCTION

Neutrosophic sets the extension of Zadeh's Fuzzy set theory[1] was grounded by Florintin Smarandache [2] in 1998. In 2020, Rama Mallick and Surapati Pramanik [3] introduced the concept of Pentapartitioned Neutrosophic sets and study their basic properties. By using the concept, Neutrosophic Topology of the authors Karatas[4], Serkan and Cemil Kuru, in the year 2020, Suman Das and Binod Chandra Tripathy [5] were introduced $Pentapartitioned\ Neutrosophic$ Topological Spaces. Arockiarani [6], Martina Jency were discussed the $F_N\ continuity$ in 2014. Just as neutrosophic logic extends classical binary logic to accommodate the concept of indeterminacy, pentapartitioned neutrosophic topology expands conventional topological structures to embrace five distinct membership grades: the absolute certainty, the contradiction certainty, the ignorance, the unknown falsity, and the absolute falsity. This innovative approach not only enriches the mathematical toolkit but also finds profound applications in diverse fields such as decision-making, artificial intelligence, data analysis, and more.

Within this intriguing domain, sets and spaces take on a whole new level of abstraction, providing a versatile framework for handling vague and uncertain data. By incorporating the five membership degrees into the topological principles, $Pentapartitioned\ Neutrosophic$ Topology opens the gateway to a more nuanced and flexible representation of information, enabling us to navigate the intricacies of the real world with greater precision.

2. PRELIMINARIES

Definition.2.1. [3] Let X be a universal set. A $Pentapartitioned\ Neutrosophic$ Set (simply PN set) A over X is an object of the form, $K = \{(x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x))/x \in X\}$ where $T, C, G, U, F : X \to [0,1]$, represents the degree of the truth membership, contradiction membership, ignorance membership, unknown membership and falsity membership functions respectively and $0 \leq T_K(x) + C_K(x) + G_K(x) + U_K(x) + F_K(x) \leq 5$ for each $x \in X$.

Definition.2.2. [3] Suppose K, L be two $Pentapartitioned\ Neutrosophic$ sets over X.

(i) $K \subseteq L$ if and only if $T_K(x) \leq T_L(x)$, $C_K(x) \leq C_L(x)$, $G_K(x) \geq G_L(x)$, $U_K(x) \geq U_L(x)$, $F_K(x) \geq F_L(x)$ for each $x \in X$.

(ii) $K \cup L = \{(x, max\ (T_K(x), T_L(x)), max\ (C_K(x), C_L(x)), min\ (G_K(x), G_L(x)), min(U_K(x), U_L(x)), min(F_K(x), F_L(x)))/x \in X\}$.

(iii) $K \cap L = \{(x, \min\ (T_K(x), T_L(x)), \min\ (C_K(x), C_L(x)), max\big(G_K(x), G_L(x)\big),$

$max\ (U_K(x), U_L(x)), max\ (F_K(x), F_L(x)))/x \in X\}$.

(iv) If $L = \{(x, T_L(x), C_L(x), G_L(x), U_L(x), F_L(x))/x \in X\}$, then $L^C = \{(x, F_L(x), U_L(x), 1 - (G_L(x)), C_L(x), T_L(x))/x \in X\}$.

(v) $K \nsubseteq L$ if atleast one of the following occurs $T_K(x) \geq T_L(x), C_K(x) \geq C_L(x), G_K(x) \leq G_L(x), U_K(x) \leq U_L(x), F_K(x) \leq F_L(x)\ for\ any\ x \in X$.

(vi) $K \neq L\ if\ K \nsubseteq L\ and\ L \nsubseteq K$.

Definition.2.3. [3] A $Pentapartitioned\ Neutrosophic$ set over X is said to be $\boldsymbol{null\ Pentapartitioned\ Neutrosophic}$ set if, $T_K(a) = 0, C_K(a) = 0, G_K(a) = 1, U_K(a) = 1, F_K(a) = 1$. We denote the $null\ Pentapartitioned\ Neutrosophic$ set by 0_{PN}. That is $0_{PN} = \{(a, 0, 0, 1, 1, 1)/a \in X\}$.

Definition.2.4. [3] A $Pentapartitioned\ Neutrosophic$ set over X is said to be $\boldsymbol{absolute\ Pentapartitioned\ Neutrosophic}$ set if, $T_K(a) = 1, C_K(a) = 1, G_K(a) = 0, U_K(a) = 0, F_K(a) = 0$. We denote the $absolute\ Pentapartitioned\ Neutrosophic$ set by 1_{PN}. That is $1_{PN} = \{(a, 1, 1, 0, 0, 0)/a \in X\}$.

Definition.2.5. [1] Let X be the universal set. Then the collection τ_{PN} of $Pentapartitioned\ Neutrosophic$ sets over X is said to be a $Pentapartitioned\ Neutrosophic$ Topology on X, if the following three conditions are hold.

1) $0_{PN}, 1_{PN} \in \tau_{PN}$

2) If $K, L \in \tau_{PN}\ then\ K \cap L \in \tau_{PN}$

3) If $\{L_i, where\ i \in I\} \in \tau_{PN}\ then \cup_{i \in I} L_i \in \tau_{PN}$

The pair (X, τ_{PN}) is called $Pentapartitioned\ Neutrosophic$ Topological space. Each element of τ_{PN} is called a $Pentapartitioned\ Neutrosophic\ open$ sets. If $L \in \tau_{PN}$, then L^C is called a $Pentapartitioned\ Neutrosophic\ closed$ set.

Definition.2.6. [1] Suppose (X, τ_{PN}) be a $Pentapartitioned\ Neutrosophic$ Topological space. Let W be a $Pentapartitioned\ Neutrosophic$ set over X. Then $PN - interior$ of W is the union of all PN-$open$ sets of (X, τ_{PN}) contained in W and is denoted by PN-int (W). That is PN-int (W) = ∪ $\{U/U \subseteq\ W\ and\ U\ is\ a\ PN - open\ set\ in\ (X, \tau)\}$.

Definition.2.7. [1] Suppose (X, τ_{PN}) be a $Pentapartitioned\ Neutrosophic$ Topological space. Let W be $Pentapartitioned\ Neutrosophic$ set over X. Then $PN - closure$ of W is the intersection of all PN-$closed$ sets of (X, τ_{PN}) containing W and is denoted by PN-cl (W). That is PN-cl (W) = ∩ $\{U/W \subseteq\ U\ and\ U\ is\ a\ PN - closed\ set\ in\ (X, \tau)\}$.

Remark.2.8. If W is $Pentapartitioned\ Neutrosophic\ open$ set in a PN Topological space (X, τ_{PN}), then PN-int(W)=W. If W is $Pentapartitioned\ Neutrosophic\ closed$ set in a PN Topological space (X, τ_{PN}), then PN-cl(W)=W

Definition.2.9. [3] Suppose (X, τ_{PN}) be a $Pentapartitioned\ Neutrosophic$ Topological space. A $Pentapartitioned\ Neutrosophic$ set P over X is said to be

1) PN $regular\ open$ if P = PN-int [PN-cl (P)]

2) PN $regular\ closed$ if P = PN-cl [PN-int (P)].

Definition.2.10. [1] Suppose (X, τ_{PN}) be a $Pentapartitioned\ Neutrosophic$ Topological space. Then a $Pentapartitioned\ Neutrosophic$ set K over X is called a

1) PN-$semi\ open$ set if and only if $K \subseteq PN - cl\ [PN - int\ (K)]$.

2) PN-$\alpha\ open$ set if and only if $K \subseteq PN - int\ (PN - cl\ (PN - int\ (K)))$.

3) PN-$\beta\ open$ set if and only if $K \subseteq PN - cl\ (PN - int\ (PN - cl\ (K)))$.

Definition.2.11. [1] Suppose (X, τ_{PN}) be a Pentapartitioned Neutrosophic Topological space.

Then a Pentapartitioned Neutrosophic set K over X is called a

1) PN-$semi\ closed$ set if and only if $PN - int\ [PN - cl\ (K)] \subseteq K$.

2) PN-$\alpha\ closed$ set if and only if $PN - cl\ (PN - int\ (PN - cl\ (K))) \subseteq K$.

3) PN-$\beta\ closed$ set if and only if $PN - int\ (PN - cl\ (PN - int\ (K))) \subseteq K$.

Definition.2.12: [1]A $PN\ set\ K$ is taken from PN Topological space (X, τ_{PN}). Then K is called PN $Pre\ closed$ set in (X, τ_{PN}) if and only if $PN - cl\ [PN - int\ (K)] \subseteq K$.

Definition.2.13: [1]A $PN\ set\ K$ is taken from PN Topological space (X, τ_{PN}). Then K is called PN $Pre\ Open$ set in (X, τ_{PN}) if and only if $K \subseteq PN - int\ [PN - cl\ (K)]$.

3. MAPPING AND CONTINUITY IN PN TOPOLOGICAL SPACES

Definition.3.1: Let $a, b, c \in [0,1]$ and $0 \leq a + b + c + d + e \leq 5$. A $Pentapartitioned\ Neutrosophic$ point [briefly PNP] $k_{(a,b,c,d,e)}$of X is a $Pentapartitioned\ Neutrosophic$ Set of X which is defined by

$$k_{(a,b,c,d,e)} = \begin{cases}(a,b,c,d,e), & y = k\\(0,0,1,1,1), & y \neq k\end{cases}$$

Here, k is said to be the $support$ of $k_{(a,b,c,d,e)}$ and a,b,c,d,e are called the truth value, contradiction value, ignorance value, unknown value, false value of $k_{(a,b,c,d,e)}$ respectively. So we can say a PNP $k_{(a,b,c,d,e)}$ is in a PN set $K = \{(x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x))/x \in X\}$ if $a \leq T_K(\mathrm{x})$, $b \leq C_K(\mathrm{x})$, $c \geq G_K(\mathrm{x})$, $d \geq U_K(\mathrm{x})$ and $e \geq F_K(\mathrm{x})$ and it is denoted by $k_{(a,b,c,d,e)} \in K$. Thus, we can represent a PNP by an ordered 5-tuple of PNP as $k_{(a,b,c,d,e)} = (k_a, k_b, C(k_{C(c)}), C(k_{C(d)}), C(k_{C(e)}))$ where $C(k_{C(c)}) = 1 - k_{1-c}, C(k_{C(d)}) = 1 - k_{1-d}$ and $C(k_{C(e)}) = 1 - k_{1-e}$.

Definition.3.2: Given $f_{PN}: X \to Y$ be a function for two non-empty sets X and Y.

Then for the PN set $M = \{(y, T_M(y), C_M(y), G_M(y), U_M(y), F_M(y))/y \in Y\}$ in Y.

We define the inverse image of M in X as $\{\left(x, {f_{PN}}^{-1}(T_M)(x), {f_{PN}}^{-1}(C_M)(x), {f_{PN}}^{-1}(G_M)(x), {f_{PN}}^{-1}(U_M)(x), {f_{PN}}^{-1}(F_M)(x)\right) : x \in X\}$ and it is denoted by ${f_{PN}}^{-1}(M)$.

Definition.3.3: Given $f_{PN}: X \to Y$ be a function for two non-empty sets X and Y.

Then for the PN set $K = \{(x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x))/x \in X\}$ in X we define the image of K in Y as $\{\left(y, f_{PN}(T_K)(y), f_{PN}(C_K)(y), f_{PN_\sim}(G_K)(y), f_{PN_\sim}(U_K)(y), f_{PN_\sim}(F_K)(y)\right) : y \in Y\}$ and it is denoted by $f_{PN}(K)$ where

$$f_{PN}(T_K)(y) = \begin{cases}\underset{x \in {f_{PN}}^{-1}(y)}{Sup} \; T_K(x) & if\ {f_{PN}}^{-1}(y) \neq 0_{PN}\\ 0 & otherwise\end{cases}$$

$$f_{PN}(C_K)(y) = \begin{cases}\underset{x \in {f_{PN}}^{-1}(y)}{Sup} \; C_K(x) & if\ {f_{PN}}^{-1}(y) \neq 0_{PN}\\ 0 & otherwise\end{cases}$$

$$\left(1 - f_{PN}(1 - G_K)\right)(y) = f_{PN_\sim}(G_K)(y) = \begin{cases}\underset{x \in {f_{PN}}^{-1}(y)}{Inf} \; G_K(x) & if\ {f_{PN}}^{-1}(y) \neq 0_{PN}\\ 1 & otherwise\end{cases}$$

$$\left(1 - f_{PN}(1 - U_K)\right)(y) = f_{PN_\sim}(U_K)(y) = \begin{cases}\underset{x \in {f_{PN}}^{-1}(y)}{Inf} \; U_K(x) & if\ {f_{PN}}^{-1}(y) \neq 0_{PN}\\ 1 & otherwise\end{cases}$$

$$\left(1 - f_{PN}(1 - F_K)\right)(y) = f_{PN_\sim}(F_K)(y) = \begin{cases}\underset{x \in {f_{PN}}^{-1}(y)}{Inf} \; F_K(x) & if\ {f_{PN}}^{-1}(y) \neq 0_{PN}\\ 1 & otherwise\end{cases}$$

Remark.3.4: Given $f_{PN}: X \to Y$ be a function for two non-empty PN sets X and Y. Then,

i. ${f_{PN}}^{-1}(1_{PN}) = 1_{PN}$ and ${f_{PN}}^{-1}(0_{PN}) = 0_{PN}$

ii. $f_{PN}(1_{PN}) = 1_{PN}$

iii. If f_{PN} is onto then $f_{PN}(0_{PN}) = 0_{PN}$

Definition.3.5: A function $f_{PN}: X \to Y$ is said to be $Pentapartitioned\ Neutrosophic\ Continuous$ if for every PN- $open$ set L in Y we have ${f_{PN}}^{-1}(L)$ is PN – $open$ in X.

Example.3.6: Let (X, τ_{PN}) and (Y, τ'_{PN}) be two PN Topological spaces over $X = \{u, v\}$ and $Y = \{m, n\}$ respectively where $\tau_{PN} = \{0_{PN}, 1_{PN}, P, Q\}$ and $\tau'_{PN} = \{0_{PN}, 1_{PN}, K\}$.

Also, $P = \{(u,\ 0.8, 0.5, 0.6, 0.7, 0.6), (v,\ 0.4, 0.6, 0.6, 0.3, 0.7)\}$,

$Q = \{(u,\ 0.7, 0.3, 0.8, 0.7, 0.9), (v,\ 0.1, 0.4, 0.6, 0.7, 0.8)\}$ and

$K = \{(m,\ 0.7, 0.5, 0.4, 0.8, 0.4), (n,\ 0.4, 0.3, 0.8, 0.4, 0.6)\}$. Let $f_{PN}: X \to Y$ defined as $f_{PN}(u) = m$ and $f_{PN}(v) = n$.

Then ${f_{PN}}^{-1}(K) = \{(u, \sup[0.8, 0.7], \sup[0.5,0.3], \inf[0.6,0.8], \inf[0.7,0.7, \inf[0.6,0.9]]), (v,\ \sup[0.4, 0.1], \sup[0.6.0.4], \inf[0.6, 0.6], \inf[0.3,0.7], \inf[0.7,0.8])\}$

$= \{(u, \quad 0.8, 0.5, 0.6, 0.7, 0.6), (v, \quad 0.4, 0.6, 0.6, 0.3, 0.7)\} = P$

Hence f_{PN} is a $Pentapartitioned\ Neutrosophic\ Continuous$ function.

Proposition.3.7: A function $f_{PN}: X \to Y$ is $Pentapartitioned\ Neutrosophic\ Continuous$ iff for every PN- $closed$ set M in Y we have ${f_{PN}}^{-1}(M)$ is PN – $closed$ in X.

Proof.

Suppose M be a PN – $closed$ set in Y. Then M^c is PN – $open$ in Y. Therefore, by Definition 3.5 we have ${f_{PN}}^{-1}(M^c) = X - {f_{PN}}^{-1}(M)$ is PN – $open$ in X. Hence ${f_{PN}}^{-1}(M)$ is PN – $closed$ in X.

Conversely, let N be any PN $closed$ set in Y such that ${f_{PN}}^{-1}(N)$ is PN $closed$ in X. Then $X - {f_{PN}}^{-1}(N) = {f_{PN}}^{-1}(Y - N)$ is PN $open$ in X where $Y - N$ is PN $open$ in Y. If we choose $K = Y - N$ then by Definition 3.5 we can conclude that $f_{PN}: X \to Y$ is $Pentapartitioned\ Neutrosophic\ Continuous$.

Proposition.3.8: Given $f_{PN}: X \to Y$ be a function for two non-empty sets X and Y. Suppose K be a PNS in X. Then,

1. $K \leq {f_{PN}}^{-1}(f_{PN}(K))$

2. If f_{PN} is one to one then $K = {f_{PN}}^{-1}\big(f_{PN}(K)\big)$

Proof:

1. Let $K = \{(x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x))/x \in\ X\}$

Then ${f_{PN}}^{-1}\big(f_{PN}(K)\big) = {f_{PN}}^{-1}\big(f_{PN}((x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x)))\big)$

$$= {f_{PN}}^{-1}\Big(\big(y, f_{PN}(T_K)(y), f_{PN}(C_K)(y), f_{PN\sim}(G_K)(y), f_{PN\sim}(U_K)(y), f_{PN\sim}(F_K)(y)\big)\Big)$$

$= \left(x, {f_{PN}}^{-1}(f_{PN}(T_K)(y)), {f_{PN}}^{-1}(f_{PN}(C_K)(y)), {f_{PN}}^{-1}(f_{PN\sim}(G_K)(y)), {f_{PN}}^{-1}(f_{PN\sim}(U_K)(y)), {f_{PN}}^{-1}(f_{PN\sim}(F_K)(y)\right)$

Since ${f_{PN}}^{-1}(f_{PN}(T_K)) \geq T_K$; ${f_{PN}}^{-1}(f_{PN}(C_K)) \geq C_K$; ${f_{PN}}^{-1}\left(f_{PN\sim}(G_K)\right) = {f_{PN}}^{-1}(1 - f_{PN}(1 - G_K)) = 1 - {f_{PN}}^{-1}(f_{PN}(1 - G_K)) \leq 1 - (1 - G_K) = G_K$, that is, ${f_{PN}}^{-1}\left(f_{PN\sim}(G_K)\right) \leq G_K$; similarly, ${f_{PN}}^{-1}\left(f_{PN\sim}(U_K)\right) \leq U_K$; ${f_{PN}}^{-1}\left(f_{PN\sim}(F_K)\right) \leq F_K$

Then, ${f_{PN}}^{-1}(f_{PN}(K)) \geq \{x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x): x \in X\} = K$

2. If f_{PN} is one-one function. Then ${f_{PN}}^{-1}(f_{PN}(T_K)) = T_K$; ${f_{PN}}^{-1}(f_{PN}(C_K)) = C_K$; ${f_{PN}}^{-1}\left(f_{PN\sim}(G_K)\right) = {f_{PN}}^{-1}(1 - f_{PN}(1 - G_K)) = 1 - {f_{PN}}^{-1}(f_{PN}(1 - G_K)) = 1 - (1 - G_K) = G_K$, that is, ${f_{PN}}^{-1}\left(f_{PN\sim}(G_K)\right) = G_K$; similarly, ${f_{PN}}^{-1}\left(f_{PN\sim}(U_K)\right) = U_K$; ${f_{PN}}^{-1}\left(f_{PN\sim}(F_K)\right) = F_K$

Hence, ${f_{PN}}^{-1}(f_{PN}(K)) = \{x, T_K(x), C_K(x), G_K(x), U_K(x), F_K(x): x \in X\} = K$.

Proposition.3.9: Given $f_{PN}: X \to Y$ be a function for two non-empty sets X and Y. Suppose L be a PNS in Y. Then,

1. $f_{PN}\left({f_{PN}}^{-1}(L)\right) \leq L$

2. If f_{PN} is onto then $f_{PN}\left({f_{PN}}^{-1}(L)\right) = L$

Proof:

1. Suppose $L = \{(y, T_L(y), C_L(y), G_L(y), U_L(y), F_L(y))/y \in Y\}$

Then $f_{PN}\left({f_{PN}}^{-1}(L)\right) = f_{PN}\left({f_{PN}}^{-1}((y, T_L(y), C_L(y), G_L(y), U_L(y), F_L(y)))\right)$

$= f_{PN}\left((x, {f_{PN}}^{-1}(T_L(y)), {f_{PN}}^{-1}(C_L(y)), {f_{PN}}^{-1}(G_L(y)), {f_{PN}}^{-1}(U_L(y)), {f_{PN}}^{-1}(F_L(y))\right)$

$= (y, f_{PN}({f_{PN}}^{-1}(T_L(y))), f_{PN}({f_{PN}}^{-1}(C_L(y))), f_{PN\sim}({f_{PN}}^{-1}(G_L(y))), f_{PN\sim}({f_{PN}}^{-1}(U_L(y))),$
$f_{PN\sim}({f_{PN}}^{-1}(F_L(y)))$

Since, $f_{PN}\left({f_{PN}}^{-1}(T_L(y))\right) \leq T_L(y)$; $f_{PN}\left({f_{PN}}^{-1}(C_L(y))\right) \leq C_L(y)$; $f_{PN\sim}\left({f_{PN}}^{-1}(G_L(y))\right) = 1 - f_{PN}\left(1 - {f_{PN}}^{-1}(G_L(y))\right) = 1 - f_{PN}\left({f_{PN}}^{-1}(1 - G_L(y))\right) \geq 1 - (1 - G_L(y)) = G_L(y)$ that is, $f_{PN\sim}\left({f_{PN}}^{-1}(G_L(y))\right) \geq G_L(y)$; similarly $f_{PN\sim}\left({f_{PN}}^{-1}(U_L(y))\right) \geq U_L(y)$ and $f_{PN\sim}\left({f_{PN}}^{-1}(F_L(y))\right) \geq F_L(y)$.

Then, $f_{PN}\left({f_{PN}}^{-1}(L)\right) \leq (y, T_L(y), C_L(y), G_L(y), U_L(y), F_L(y)) = L$

Hence, $f_{PN}\left({f_{PN}}^{-1}(L)\right) \leq L$.

2. Suppose f_{PN} is onto function. Then $f_{PN}\left({f_{PN}}^{-1}(T_L(y))\right) = T_L(y)$; $f_{PN}\left({f_{PN}}^{-1}(C_L(y))\right) = C_L(y)$; $f_{PN\sim}\left({f_{PN}}^{-1}(G_L(y))\right) = G_L(y)$; $f_{PN\sim}\left({f_{PN}}^{-1}(U_L(y))\right) = U_L(y)$ and $f_{PN\sim}\left({f_{PN}}^{-1}(F_L(y))\right) = F_L(y)$.

Hence, $f_{PN}\left({f_{PN}}^{-1}(L)\right) = \left(y, T_L(y), C_L(y), G_L(y), U_L(y), F_L(y)\right) = L$

Thus, $f_{PN}\left({f_{PN}}^{-1}(L)\right) = L$.

Proposition.3.10: Given $f_{PN}: X \to Y$ be a function for two non-empty sets X and Y. Suppose $K, K_i (i \in Index)$ be PNS in X and $L, L_i (i \in Index)$ be PNS in Y. Then,

1. $K_1 \leq K_2 \Rightarrow f_{PN}(K_1) \leq f_{PN}(K_2)$
2. $L_1 \leq L_2 \Rightarrow {f_{PN}}^{-1}(L_1) \leq {f_{PN}}^{-1}(L_2)$

Proof:

1. Suppose $K_1 \leq K_2$

Which implies $T_{k_1}(x) \leq T_{k_2}(x)$; $C_{k_1}(x) \leq C_{k_2}(x)$; $G_{k_1}(x) \geq G_{k_2}(x)$; $U_{k_1}(x) \geq U_{k_2}(x)$ and $F_{k_1}(x) \geq F_{k_2}(x)$.

Then $f_{PN}(T_{k_1}(x)) \leq f_{PN}(T_{k_2}(x))$; $f_{PN}(C_{k_1}(x)) \leq f_{PN}(C_{k_2}(x))$

And $G_{k_1}(x) \geq G_{k_2}(x)$ implies $-G_{k_1}(x) \leq -G_{k_2}(x)$ which implies $1 - G_{k_1}(x) \leq 1 - G_{k_2}(x)$

Therefore, we get $f_{PN}(1 - G_{k_1}(x)) \leq f_{PN}(1 - G_{k_2}(x))$ which implies $1 - f_{PN}\left(1 - G_{k_1}(x)\right) \geq 1 - f_{PN}(1 - G_{k_2}(x))$

Hence, $f_{PN\sim}\left(G_{k_1}(x)\right) \geq f_{PN\sim}\left(G_{k_2}(x)\right)$ similarly, $f_{PN\sim}\left(U_{k_1}(x)\right) \geq f_{PN\sim}\left(U_{k_2}(x)\right)$ and $f_{PN\sim}\left(F_{k_1}(x)\right) \geq f_{PN\sim}\left(F_{k_2}(x)\right)$.

From this we can conclude that $f_{PN}(K_1) \leq f_{PN}(K_2)$.

2. Suppose $L_1 \leq L_2$

Then, ${f_{PN}}^{-1}(L_1) \quad = {f_{PN}}^{-1}\left((y, T_{L_1}(y), C_{L_1}(y), G_{L_1}(y), U_{L_1}(y), F_{L_1}(y))\right)$

$= \left((x, {f_{PN}}^{-1}(T_{L_1}(y)), {f_{PN}}^{-1}(C_{L_1}(y)), {f_{PN}}^{-1}(G_{L_1}(y)), {f_{PN}}^{-1}(U_{L_1}(y)), {f_{PN}}^{-1}(F_{L_1}(y))\right)$

$$f_{PN}\left({f_{PN}}^{-1}(L_1)\right)$$

$= f_{PN}\left((x, {f_{PN}}^{-1}(T_{L_1}(y)), {f_{PN}}^{-1}(C_{L_1}(y)), {f_{PN}}^{-1}(G_{L_1}(y)), {f_{PN}}^{-1}(U_{L_1}(y)), {f_{PN}}^{-1}(F_{L_1}(y))\right)$

$= (y, f_{PN}\left({f_{PN}}^{-1}\left(T_{L_1}(y)\right)\right), f_{PN}\left({f_{PN}}^{-1}\left(C_{L_1}(y)\right)\right), f_{PN\sim}\left({f_{PN}}^{-1}\left(G_{L_1}(y)\right)\right), f_{PN\sim}\left({f_{PN}}^{-1}\left(U_{L_1}(y)\right)\right),$

$$f_{PN\sim}\left({f_{PN}}^{-1}\left(F_{L_1}(y)\right)\right)$$

$\leq L_1 \leq L_2$

Therefore, $f_{PN}\left({f_{PN}}^{-1}(L_1)\right) \leq L_2$

Hence, ${f_{PN}}^{-1}(L_1) \leq {f_{PN}}^{-1}(L_2)$.

Theorem.3.11: Composition of two *Pentapartitioned Neutrosophic Continuous* functions is again *Pentapartitioned Neutrosophic Continuous* function.

Proof: Let $f_{PN}: X \to Y$ and $g_{PN}: Y \to Z$ be two *Pentapartitioned Neutrosophic Continuous* functions defined on the topologies $(X, \tau_{PN}), (Y, \tau'_{PN})$ and (Z, τ''_{PN}) respectively. And let K be a PN *open* set in Z.

Since g_{PN} is *Pentapartitioned Neutrosophic Continuous* function, then ${g_{PN}}^{-1}(K)$ is a PN open set in Y.

Also, since f_{PN} is *Pentapartitioned Neutrosophic Continuous* function, then ${f_{PN}}^{-1}[{g_{PN}}^{-1}(K)] = (g \circ f)^{-1}(K)$ is a PN open set in X.

Therefore, $(g \circ f)^{-1}$ is *Pentapartitioned Neutrosophic Continuous*.

Theorem.3.12: Suppose (X, τ_{PN}) and (Y, τ'_{PN}) be two PN Topological spaces. Consider the function $f_{PN}: X \to Y$. Then $f_{PN}: X \to Y$ is a *Pentapartitioned Neutrosophic Continuous* function if and only if $f_{PN}\big(PN\ cl(K)\big) \subseteq PN\ cl(f_{PN}(K))$ for every PN set K in X.

Proof: Let K be a PN set in X and $f_{PN}: X \to Y$ is a *Pentapartitioned Neutrosophic Continuous* function.

Then $f_{PN}(K) \subseteq PN\ cl(f_{PN}(K))$ -----(1)

By Proposition 3.8, $K \subseteq {f_{PN}}^{-1}\big(f_{PN}(K)\big)$

(from (1)), $K \subseteq {f_{PN}}^{-1}(PN\ cl(f_{PN}(K)))$

$PN\ cl(K) \subseteq PN\ cl[{f_{PN}}^{-1}\left(PN\ cl\big(f_{PN}(K)\big)\right)]$

Since f_{PN} is *Pentapartitioned Neutrosophic Continuous* and $PN\ cl\big(f_{PN}(K)\big)$ is a PN closed set then $PN\ cl\left[{f_{PN}}^{-1}\left(PN\ cl\big(f_{PN}(K)\big)\right)\right] = {f_{PN}}^{-1}\left(PN\ cl\big(f_{PN}(K)\big)\right)$

Therefore, $PN\ cl(K) \subseteq {f_{PN}}^{-1}\left(PN\ cl\big(f_{PN}(K)\big)\right)$

Hence, $f_{PN}\big(PN\ cl(K)\big) \subseteq PN\ cl(f_{PN}(K))$ for every PN set K in X.

Conversely, suppose $f_{PN}\big(PN\ cl(K)\big) \subseteq PN\ cl(f_{PN}(K))$ for every PN set K in X.

Consider a PN closed set L in Y. Then ${f_{PN}}^{-1}(L)$ is PN closed set in X.

By Proposition 3.9, $f_{PN}\left({f_{PN}}^{-1}(L)\right) \subseteq L$

$PN\ cl\left[f_{PN}\left({f_{PN}}^{-1}(L)\right)\right] \subseteq PN\ cl(L) = L$

Then by hypothesis, $f_{PN}\left(PN\ cl\left[{f_{PN}}^{-1}(L)\right]\right) \subseteq PN\ cl\left[f_{PN}\left({f_{PN}}^{-1}(L)\right)\right] \subseteq L$

$PN\ cl\left[{f_{PN}}^{-1}(L)\right] \subseteq {f_{PN}}^{-1}(L)$ but always, ${f_{PN}}^{-1}(L) \subseteq PN\ cl\left[{f_{PN}}^{-1}(L)\right]$

Therefore, $PN\ cl\left[{f_{PN}}^{-1}(L)\right] = {f_{PN}}^{-1}(L)$

Hence f_{PN} is a $Pentapartitioned\ Neutrosophic\ Continuous$ function.

4. CONCLUSION

In this paper, the characteristics of the Pentapartitioned Neutrosophic functions like continuity, open mapping and closed mappings in Pentapartitioned Neutrosophic Topological Spaces are introduced and discussed some of their properties.

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