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Oscillating Polar Anti Fuzzy Graph

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Abstract

The newly developed Oscillating Polar Anti Fuzzy Graph framework is established in this paper. We define fundamental operations including anti-union, anti-composition, anti-join and anti-cartesian Product. Furthermore, we have proved some theorems that validate these operations.

1. Introduction

Kauffmann [2] established fuzzy graphs using fuzzy relations on fuzzy sets in 1973, laying the groundwork for graph theory. This theory was subsequently extended by Rosenfeld [3] in 1975. Later, John N. Mordeson, Chang-shyh Peng [10] introduced operations on fuzzy graphs in 1994.The progression of fuzzy graph theory was significant with several contributions. R. Seethalakshmi R and Gnanajothi R.B. [4] presented the new idea of anti-fuzzy graphs in 2016. which makes a significant advancement in the study of fuzzy structures. Building upon this work, R. Muthuraj & A. Sasirekha [6] introduced the conception of characterisation on operations of anti-fuzzy graph.

Zhang [6] introduced the concept of bipolar fuzzy sets in 1994. Later, it was extended to bipolar fuzzy graphs by Muhammed Akram [9]. In 2019, S. Meena Devi, S. Selvi.[7] introduced operational frameworks on bipolar anti fuzzy graphs. As a further development of bipolar fuzzy sets, Chen et al. [11] proposed the concept of m-polar fuzzy sets. Building on this theoretical foundation, Ghorai and Pal [13] explored few operations and density measures in m-polar fuzzy graphs. Their subsequent work [12] is on few m-polar fuzzy graph traits.

Most recently, in 2023, A. Anthoni Amali and J. Jesintha Rosline [8] suggested the idea of Oscillating polar Fuzzy Graphs in addition to their real-life applications. Building on this foundation, the present work introduces the principle of Oscillating Polar Anti Fuzzy Graph inspite of fundamental operations involving anti-union, anti-composition, anti-join and anti-cartesian product. Additionally, we established few theorems to provide the structural and operational properties of these graphs.

2. Preliminaries

Definition: 2.1 [3]

A fuzzy graph $G = (V, \sigma, \mu)$ consists of a non-empty vertex set V, along with two functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\forall\, x, y \in V, \mu(x,y) \le \sigma(x) \wedge \sigma(y)$.

Definition: 2.2 [5]

A fuzzy graph $G = (\sigma, \mu)$ is called an anti-fuzzy graph if it is characterized by a pair of functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\forall\, x, y \in V, \mu(x,y) \ge \sigma(x) \vee \sigma(y)$.It is symbolised as $G_A(\sigma, \mu)$.

Definition: 2.3 [9]

A pair $G = (A, B)$ is defined as a bipolar fuzzy graph with an underlying V, where $A = ({\mu_A}^P, {\mu_A}^N)$ is a bipolar fuzzy set in V and $B = ({\mu_B}^P, {\mu_B}^N)$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that, $\mu_{B^P}(\{x,y\}) \le min(\mu_{A^P}(x), \mu_{A^P}(y))$ and $\mu_{B^N}(\{x,y\}) \ge max(\mu_{A^N}(x), \mu_{A^N}(y))$, $\forall \{x,y\} \in E$
We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is a symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus $G = (A, B)$ is a bipolar graph of $G^* = (V, E)$ if

$$ {\mu_B}^P(xy) \le min({\mu_A}^P(x),\ {\mu_A}^P(y)) \text{ and} $$
$$ {\mu_B}^N(xy) \ge max({\mu_A}^N(x),\ {\mu_A}^N(y)) \qquad \forall\, xy \in E $$

Definition:2.4 [7]

If a bipolar fuzzy graph has an underlying set V, it has been identified as a bipolar anti fuzzy graph. V is defined to be a pair $G = (A, B)$ where $A = ({\mu_A}^P\ , {\mu_A}^N)$is a bipolar fuzzy set in V and $B = (\mu_{B^P}\ , \mu_{B^N})$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that $\mu_{B^P}(\{x, y\}) \geq max(\mu_{A^P}(x), \mu_{A^P}(y))$ and $\mu_{B^N}(\{x, y\}) \leq min(\mu_{A^N}(x), \mu_{A^N}(y))$, $\forall\{x, y\} \in E$

Definition:2.5 [8]

An Oscillating Polar Fuzzy Graph G_{OP}: $(V, \sigma_{OP}, \mu_{OP})$ is defined to be a pair of functions with vertex membership $\sigma_{OP} = ({\sigma_{OP}}^N, {\sigma_{OP}}^P)$, σ_{OP}: $v \to [-1, 0]$, [0, 1]and with edge membership is denoted with notation $\mu_{OP} = ({\mu_{OP}}^N, {\mu_{OP}}^P)$, μ_{OP}: $v \times v \to [-1, 0]$, [0,1] for all values of $a, b \in V$, which lies between two equal opposite extremes of [-1, 1] Such that G_{OP}: $(V, \sigma_{OP}, \mu_{OP})$ is an OPFG with underlying crisp graph, ${G_{OP}}^*$: $(V, {\sigma_{OP}}^*, {\mu_{OP}}^*)$

$$i.e)\ {\mu_{OP}}^N(a, b) \geq max({\mu_{OP}}^N(a), {\mu_{OP}}^N(b))$$
$${\mu_{OP}}^P(a, b) \leq min({\mu_{OP}}^P(a),\ {\mu_{OP}}^P(b)) \qquad \forall\ a, b\ \in V$$

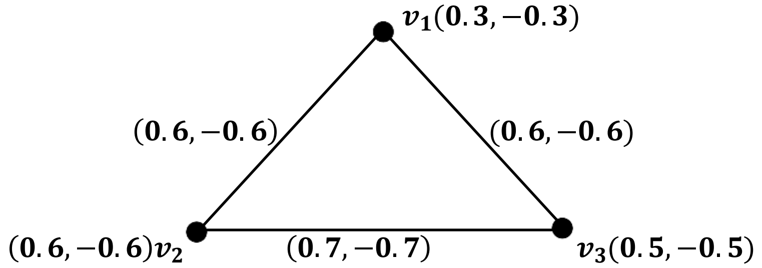
With an equal opposite extreme of the ranges $[-1, 1]$.

3. Oscillating Polar Anti Fuzzy Graph

An Oscillating Polar Anti Fuzzy Graph G_{OPA}: $(V, \sigma_{OPA}, \mu_{OPA})$ is characterized by two functions with vertex membership, $\sigma_{OPA} = ({\sigma_{OPA}}^P, {\sigma_{OPA}}^N)$, where, σ_{OPA}: $v \to [-1,0]$, [0, 1]and with edge membership, $\mu_{OPA} = ({\mu_{OPA}}^P, {\mu_{OPA}}^N)$, μ_{OPA}: $v \times v \to [-1,0]$, [0, 1] for all values of $a, b \in V$, the membership values of $\sigma_{OPA}(a)$ and $\mu_{OPA}(a, b)$ lie within the oscillating range of two equal and opposite extremes in the interval $[-1, 1]$ such that,

$${\mu_{OPA}}^P(a, b) \geq max({\sigma_{OPA}}^P(a), {\sigma_{OPA}}^P(b))$$
$${\mu_{OPA}}^N(a, b) \leq min({\sigma_{OPA}}^N(a),\ {\sigma_{OPA}}^N(b)),\ \forall\ a, b \in V$$

Example

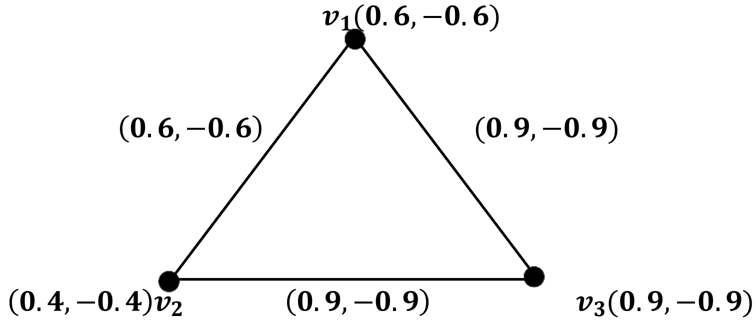


Oscillating Polar Anti Fuzzy Graph

Complete Oscillating Polar Anti Fuzzy Graph

If an OPAFG meets the following requirements, it is considered complete.

$${\mu_{OPA}}^P(a, b) = \max\ ({\sigma_{OPA}}^P(a), {\sigma_{OPA}}^P(b))$$
$${\mu_{OPA}}^N(a, b) = min\ ({\sigma_{OPA}}^N(a), {\sigma_{OPA}}^N(b)) \qquad \forall a, b\ \in V$$



Complete Oscillating polar Anti Fuzzy Graph

Anti-union of two OPAFG

Let $\sigma_{OPA_1} = (\sigma_{OPA_1}^{P}, \sigma_{OPA_1}^{N})$ and $\sigma_{OPA_2} = (\sigma_{OPA_2}^{P}, \sigma_{OPA_2}^{N})$ be an OPAF subsets and the edge set is represented as $\mu_{OPA_1} = (\mu_{OPA_1}^{P}, \mu_{OPA_1}^{N})$ and $\mu_{OPA_2} = (\mu_{OPA_2}^{P}, \mu_{OPA_2}^{N})$. Then the anti-union of two Oscillating Polar anti Fuzzy Graphs G_{OPA_1} & G_{OPA_2} is denoted by $G_{OPA_1} \cup G_{OPA_2}$ and it is defined as follows:

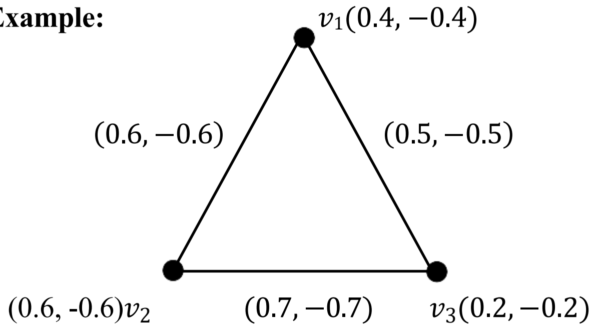
i) $(\sigma_{OPA_1}^{P} \cup \sigma_{OPA_2}^{P})(x) = \sigma_{OPA_1}^{P}(x)$, if $x \in \sigma_{v1} - \sigma_{v2}$
$(\sigma_{OPA_1}^{P} \cup \sigma_{OPA_2}^{P})(x) = \sigma_{OPA_2}^{P}(x)$, if $x \in \sigma_{v2} - \sigma_{v1}$
$(\sigma_{OPA_1}^{P} \cup \sigma_{OPA_2}^{P})(x) = \min(\sigma_{OPA_1}^{P}(x), \sigma_{OPA_2}^{P}(x))$, if $x \in \sigma_{v1} \cap \sigma_{v2}$

ii) $(\sigma_{OPA_1}^{N} \cap \sigma_{OPA_2}^{N})(x) = \sigma_{OPA_1}^{N}(x)$, if $x \in \sigma_{v1} - \sigma_{v2}$
$(\sigma_{OPA_1}^{N} \cap \sigma_{OPA_2}^{N})(x) = \sigma_{OPA_2}^{N}(x)$, if $x \in \sigma_{v2} - \sigma_{v1}$
$(\sigma_{OPA_1}^{N} \cap \sigma_{OPA_2}^{N})(x) = \max(\sigma_{OPA_1}^{N}(x), \sigma_{OPA_2}^{N}(x))$, if $x \in \sigma_{v1} \cap \sigma_{v2}$

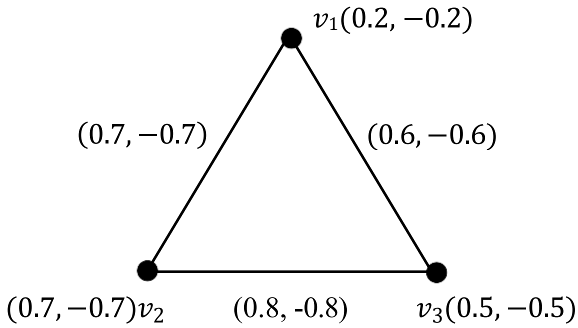
iii) $(\mu_{OPA_1}^{P} \cup \mu_{OPA_2}^{P})(x,y) = \mu_{OPA_1}^{P}(x,y)$, if $(x,y) \in \mu_{v1} - \mu_{v2}$
$(\mu_{OPA_1}^{P} \cup \mu_{OPA_2}^{P})(x,y) = \mu_{OPA_2}^{P}(x,y)$, if $(x,y) \in \mu_{v2} - \sigma_{v1}$
$(\mu_{OPA_1}^{P} \cup \mu_{OPA_2}^{P})(x,y) = \min(\mu_{OPA_1}^{P}(x,y), \mu_{OPA_2}^{P}(x,y))$, if $(x,y) \in \mu_{v1} \cap \mu_{v2}$

iv) $(\mu_{OPA_1}^{N} \cup \mu_{OPA_2}^{N})(x,y) = \mu_{OPA_1}^{N}(x,y)$, if $(x,y) \in \mu_{v1} - \mu_{v2}$
$(\mu_{OPA_1}^{N} \cup \mu_{OPA_2}^{N})(x,y) = \mu_{OPA_2}^{N}(x,y)$, if $(x,y) \in \mu_{v2} - \sigma_{v1}$
$(\mu_{OPA_1}^{N} \cup \mu_{OPA_2}^{N})(x,y) = \max(\mu_{OPA_1}^{N}(x,y), \mu_{OPA_2}^{N}(x,y))$, if $(x,y) \in \mu_{v1} \cap \mu_{v2}$

Example:

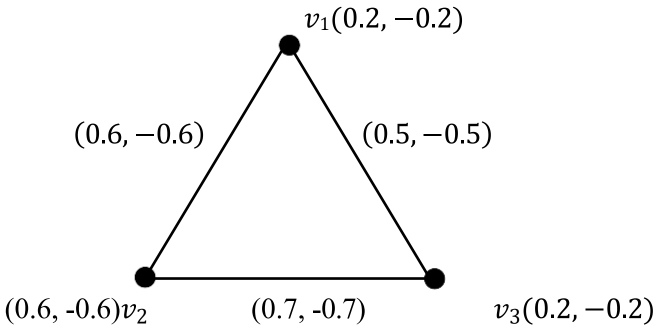


G_{OPA_1}



G_{OPA_2}

The Union of these two is



$G_{OPA_1} \cup G_{OPA_2}$

Theorem:

If G_{OPA_1} and G_{OPA_2} are Oscillating Polar Anti Fuzzy Graphs then $G_{OPA_1} \cup G_{OPA_2}$ is also an OPAFG.

Proof:

Let $(x,y) \in \mu_{v_1} \cap \mu_{v_2}$

Then, $(\mu_{OPA_1}{}^{P} \cup \mu_{OPA_2}{}^{P})(x,y) = \min(\mu_{OPA_1}{}^{P}(x,y), \mu_{OPA_2}{}^{P}(x,y))$

$\geq \min(\max(\sigma_{OPA_1}{}^{P}(x), \sigma_{OPA_1}{}^{P}(y)), \max(\sigma_{OPA_2}{}^{P}(x), \sigma_{OPA_2}{}^{P}(y)))$

$= max((\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(x), (\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(y))$

$(\mu_{OPA_1}{}^{N} \cap \mu_{OPA_2}{}^{N})(x,y) = \max(\mu_{OPA_1}{}^{N}(x,y), \mu_{OPA_2}{}^{N}(x,y))$

$\leq \max(\min(\sigma_{OPA_1}{}^{N}(x), \sigma_{OPA_1}{}^{N}(y)), \min(\sigma_{OPA_2}{}^{N}(x), \sigma_{OPA_2}{}^{N}(y)))$

$= min((\sigma_{OPA_1}{}^{N} \cup \sigma_{OPA_2}{}^{N})(x), (\sigma_{OPA_1}{}^{N} \cup \sigma_{OPA_2}{}^{N})(y))$

Similarly, we can prove, if $(x,y) \in \mu_{v_1} - \mu_{v_2}$ then,

$(\mu_{OPA_1}{}^{P} \cup \mu_{OPA_2}{}^{P})(x,y) \geq max((\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(x), (\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(y))$

$(\mu_{OPA_1}{}^{N} \cap \mu_{OPA_2}{}^{N})(x,y) \leq min((\sigma_{OPA_1}{}^{N} \cap \sigma_{OPA_2}{}^{N})(x), (\sigma_{OPA_1}{}^{N} \cap \sigma_{OPA_2}{}^{N})(y))$

If $(x,y) \in \mu_{v_2} - \mu_{v_1}$, then we get,

$(\mu_{OPA_1}{}^{P} \cup \mu_{OPA_2}{}^{P})(x,y) \geq max((\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(x), (\sigma_{OPA_1}{}^{P} \cup \sigma_{OPA_2}{}^{P})(y))$

$\mu_{OPA_1}{}^{N} \cap \mu_{OPA_2}{}^{N})(x,y) \leq min((\sigma_{OPA_1}{}^{N} \cap \sigma_{OPA_2}{}^{N})(x), (\sigma_{OPA_1}{}^{N} \cap \sigma_{OPA_2}{}^{N})(y))$

Hence the proof

Anti-Composition of two OPAFG

Let $\sigma_{OPA_1} = (\sigma_{OPA_1}{}^{P}, \sigma_{OPA_1}{}^{N})$ and $\sigma_{OPA_2} = (\sigma_{OPA_2}{}^{P}, \sigma_{OPA_2}{}^{N})$ be an OPAF subsets and the edge set is represented as $\mu_{OPA_1} = (\mu_{OPA_1}{}^{P}, \mu_{OPA_1}{}^{N})$ and $\mu_{OPA_2} = (\mu_{OPA_2}{}^{P}, \mu_{OPA_2}{}^{N})$. Subsequently, two Oscillating Polar Anti Fuzzy Graphs G_{OPA_1} & G_{OPA_2} possess the anti-composition shown by $G_{OPA_1}[G_{OPA_2}] = \{\sigma_{OPA_1} \, O \, \sigma_{OPA_2}, \mu_{OPA_1} \, O \, \mu_{OPA_2}\}$ and it is defined as follows:

i) $(\sigma_{OPA_1}{}^{P} \, O \, \sigma_{OPA_2}{}^{P})(x_1,x_2) = max(\sigma_{OPA_1}{}^{P}(x_1), \sigma_{OPA_2}{}^{P}(x_2)), \forall (x_1,x_2) \in V$

$(\sigma_{OPA_1}{}^{N} \, O \, \sigma_{OPA_2}{}^{N})(x_1,x_2) = min(\sigma_{OPA_1}{}^{N}(x_1), \sigma_{OPA_2}{}^{N}(x_2)), \forall (x_1,x_2) \in V$

ii) $(\mu_{OPA_1}{}^{P} \, O \, \mu_{OPA_2}{}^{P})((x,x_2)(x,y_2)) = max(\sigma_{OPA_1}{}^{P}(x), \mu_{OPA_2}{}^{P}(x_2,y_2)), \; x \in \sigma_{v_1}, (x_2,y_2) \in \mu_{v_2}$

$(\mu_{OPA_1}{}^{N} \, O \, \mu_{OPA_2}{}^{N})((x,x_2)(x,y_2)) = min(\sigma_{OPA_1}{}^{N}(x), \mu_{OPA_2}{}^{N}(x_2,y_2)), x \in \sigma_{v_1}, (x_2,y_2) \in \mu_{v_2}$

iii) $(\mu_{OPA_1}{}^{P} \, O \, \mu_{OPA_2}{}^{P})((x_1,z)(y_1,z)) = max(\mu_{OPA_1}{}^{P}(x_1,y_1), \sigma_{OPA_2}{}^{P}(z)), \forall z \in \sigma_{v_2}, (x_1,y_1) \in \mu_{v_1}$

$(\mu_{OPA_1}{}^{N} \, O \, \mu_{OPA_2}{}^{N})((x_1,z)(y_1,z)) = min(\mu_{OPA_1}{}^{N}(x_1,y_1), \sigma_{OPA_2}{}^{N}(z)), \forall z \in \sigma_{v_2}, (x_1,y_1) \in \mu_{v_1}$

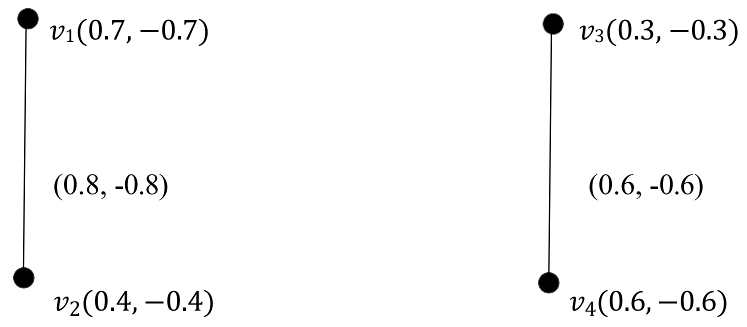
iv) $(\mu_{OPA_1}{}^{P} \, O \, \mu_{OPA_2}{}^{P})((x_1,x_2)(y_1,y_2)) = max(, \sigma_{OPA_2}{}^{P}(x_2),, \sigma_{OPA_2}{}^{P}(y_2), \; \mu_{OPA_1}{}^{P}(x_1,y_1)),$

$\forall (x_1,x_2)(y_1,y_2) \in \mu^0 - \mu$

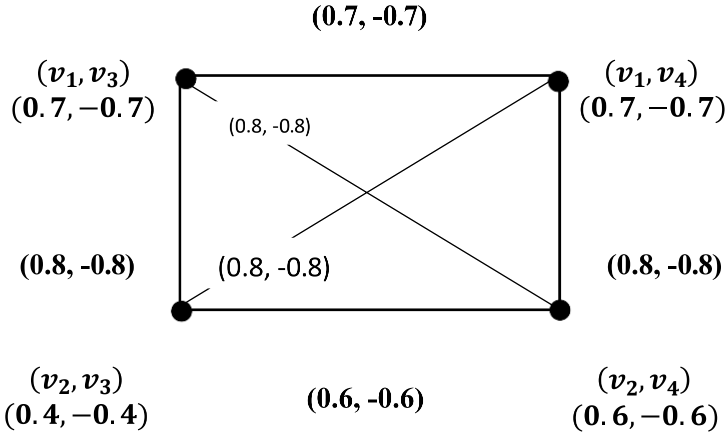
$(\mu_{OPA_1}{}^{N} \, O \, \mu_{OPA_2}{}^{N})((x_1,x_2)(y_1,y_2)) = min(, \sigma_{OPA_2}{}^{N}(x_2),, \sigma_{OPA_2}{}^{N}(y_2), \; \mu_{OPA_1}{}^{N}(x_1,y_1)),$

$\forall (x_1,x_2)(y_1,y_2) \in \mu^0 - \mu$

Example:



G_{OPA_1} $\qquad$ G_{OPA_2}



$G_{OPA_1}[G_{OPA_2}]$

Theorem:

If G_{OPA_1}& G_{OPA_2} are two Oscillating Polar Anti Fuzzy Graphs, then their composition $G_{OPA_1}[G_{OPA_2}]$ results in a Oscillating Polar Anti Fuzzy Graph.

Proof:

Let $x \in \sigma_{v_1}$ and $(x_2, y_2) \in \mu_{v_2}$Then, we have,

$$({\mu_{OPA_1}}^P \text{O}\ {\mu_{OPA_2}}^P)((x, x_2), (x, y_2)) = max\ ({\sigma_{OPA_1}}^P(x), {\mu_{OPA_2}}^P(x_2, y_2))$$
$$\geq max\ ({\sigma_{OPA_1}}^P(x), max\ ({\sigma_{OPA_2}}^P(x_2), {\sigma_{OPA_2}}^P(y_2)))$$
$$= max\ (max\ ({\sigma_{OPA_1}}^P(x), {\sigma_{OPA_2}}^P(x_2))\ , max\ ({\sigma_{OPA_1}}^P(x), {\sigma_{OPA_2}}^P(y_2)))$$
$$= max\ (({\sigma_{OPA_1}}^P \text{O}\ {\sigma_{OPA_2}}^P)(x, x_2), ({\sigma_{OPA_1}}^P \text{O}\ {\sigma_{OPA_2}}^P)(x, y_2))$$
$$({\mu_{OPA_1}}^N \text{O}\ {\mu_{OPA_2}}^N)((x, x_2), (x, y_2)) = min\ ({\sigma_{OPA_1}}^N(x), {\mu_{OPA_2}}^N(x_2, y_2))$$
$$\leq min\ ({\sigma_{OPA_1}}^N(x), min\ ({\sigma_{OPA_2}}^N(x_2), {\sigma_{OPA_2}}^N(y_2)))$$
$$= min\ (min\ ({\sigma_{OPA_1}}^N(x), {\sigma_{OPA_2}}^N(x_2))\ , min\ ({\sigma_{OPA_1}}^N(x), {\sigma_{OPA_2}}^N(y_2)))$$
$$= min\ (({\sigma_{OPA_1}}^N \text{O}\ {\sigma_{OPA_2}}^N)(x, x_2), ({\sigma_{OPA_1}}^N \text{O}\ {\sigma_{OPA_2}}^N)(x, y_2))$$

Let $z \in \sigma_{v_2}$ and $(x_1, y_1) \in \mu_{v_1}$, then the proof is obvious as above.
Let $(x_1, x_2), (x_1, y_2) \in \mu^0 - \mu$,
So, $(x_1, y_1) \in \mu_{v_1}$, $x_2 \neq y_2$, we get,

$$({\mu_{OPA_1}}^P \text{O}\ {\mu_{OPA_2}}^P)((x_1, x_2), (y_1, y_2)) = max\ ({\sigma_{OPA_2}}^P(x_2), {\sigma_{OPA_2}}^P(y_2), {\mu_{OPA_1}}^P(x_1, y_1))$$
$$\geq max\ ({\sigma_{OPA_2}}^P(x_2), {\sigma_{OPA_2}}^P(y_2), max\ ({\sigma_{OPA_1}}^P(x_1), {\sigma_{OPA_1}}^P(y_1)))$$
$$= max\ (max\ ({\sigma_{OPA_1}}^P(x_1), {\sigma_{OPA_2}}^P(x_2))\ , max\ ({\sigma_{OPA_1}}^P(y_1), {\sigma_{OPA_2}}^P(y_2)))$$
$$= max\ (({\sigma_{OPA_1}}^P \text{O}\ {\sigma_{OPA_2}}^P)(x_1, x_2), ({\sigma_{OPA_1}}^P\ \text{O}\ {\sigma_{OPA_2}}^P)(y_1, y_2))$$

$$({\mu_{OPA_1}}^N \text{O}\ {\mu_{OPA_2}}^N)((x_1, x_2), (y_1, y_2)) = min\ ({\sigma_{OPA_2}}^N(x_2), {\sigma_{OPA_2}}^N(y_2), {\mu_{OPA_1}}^N(x_1, y_1))$$
$$\leq min\ ({\sigma_{OPA_2}}^N(x_2), {\sigma_{OPA_2}}^N(y_2), min\ ({\sigma_{OPA_1}}^N(x_1), {\sigma_{OPA_1}}^N(y_1)))$$
$$= min\ (min\ ({\sigma_{OPA_1}}^N(x_1), {\sigma_{OPA_2}}^N(x_2))\ , min\ ({\sigma_{OPA_1}}^N(y_1), {\sigma_{OPA_2}}^N(y_2)))$$
$$= min\ (({\sigma_{OPA_1}}^N \text{O}\ {\sigma_{OPA_2}}^N)(x_1, x_2), ({\sigma_{OPA_1}}^N\ \text{O}\ {\sigma_{OPA_2}}^N)(y_1, y_2))$$

Hence the Proof

Anti-cartesian product of two OPAFG

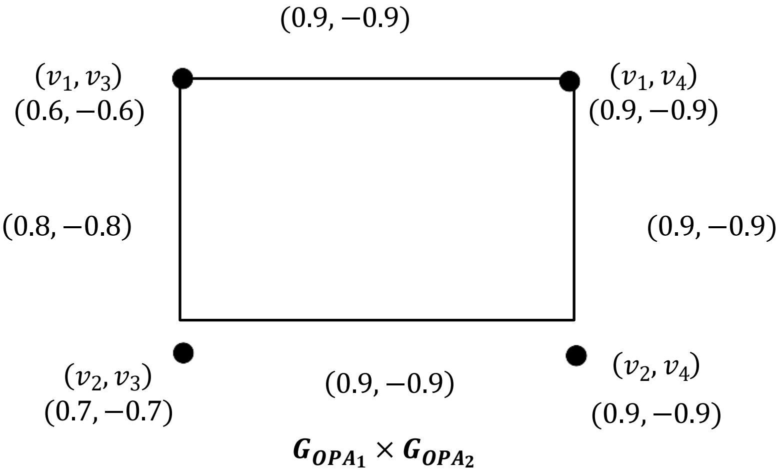
Let $\sigma_{OPA_1} = ({\sigma_{OPA_1}}^P, {\sigma_{OPA_1}}^N)$ and $\sigma_{OPA_2} = ({\sigma_{OPA_2}}^P, {\sigma_{OPA_2}}^N)$ be an Oscillating Polar Anti fuzzy subsets and the edge set is represented as $\mu_{OPA_1} = ({\mu_{OPA_1}}^P, {\mu_{OPA_1}}^N)$ and $\mu_{OPA_2} = ({\mu_{OPA_2}}^P, {\mu_{OPA_2}}^N)$. The anti cartesian product of two OPAFG G_{OPA_1} and G_{OPA_2} is indicated by $G_{OPA_1} \times G_{OPA_2}$ and it is defined as follows:

i) $({\sigma_{OPA_1}}^P \times {\sigma_{OPA_2}}^P)(x_1, x_2) = max({\sigma_{OPA_1}}^P(x_1), {\sigma_{OPA_2}}^P(x_2))$
$({\sigma_{OPA_1}}^N \times {\sigma_{OPA_2}}^N)(x_1, x_2) = min({\sigma_{OPA_1}}^N(x_1), {\sigma_{OPA_2}}^N(x_2))$, $\forall (x_1, x_2) \in V$

ii) $({\mu_{OPA_1}}^P \times {\mu_{OPA_2}}^P)((x, x_2), (x, y_2)) = max({\sigma_{OPA_1}}^P(x_1), {\mu_{OPA_2}}^P(x_2, y_2))$
$({\mu_{OPA_1}}^N \times {\mu_{OPA_2}}^N)((x, x_2), (x, y_2)) = min\,({\sigma_{OPA_1}}^N(x_1), {\mu_{OPA_2}}^N(x_2, y_2))$,
$\forall\, x \in \sigma_{v_1}, (x_2, y_2) \in \mu_{v_2}$

iii) $({\mu_{OPA_1}}^P \times {\mu_{OPA_2}}^P)((x_1, z), (y_1, z)) = max({\mu_{OPA_1}}^P(x_1, y_1), {\sigma_{OPA_2}}^P(z))$
$({\mu_{OPA_1}}^N \times {\mu_{OPA_2}}^N)((x_1, z), (y_1, z)) = min\,({\mu_{OPA_1}}^N(x_1, y_1), {\sigma_{OPA_2}}^N(z))$,
$\forall\, z \in \sigma_{v_2}, (x_1, y_1) \in \mu_{v_1}$

Example



Theorem

If G_{OPA_1} and G_{OPA_2} are Oscillating Polar Anti Fuzzy Graphs, then $G_{OPA_1} \times G_{OPA_2}$ is an Oscillating Polar Anti Fuzzy Graph.

Proof:

Let's $x \in \sigma_{v_1}$ and $(x_2, y_2) \in \mu_{v_2}$ then, we have,

$$({\mu_{OPA_1}}^P \times {\mu_{OPA_2}}^P)((x, x_2), (x, y_2)) = max\,({\sigma_{OPA_1}}^P(x), {\mu_{OPA_2}}^P(x_2, y_2))$$

$$\geq max\,({\sigma_{OPA_1}}^P(x), max\,({\sigma_{OPA_2}}^P(x_2), {\sigma_{OPA_2}}^P(y_2)))$$

$$= max\,(max\,({\sigma_{OPA_1}}^P(x), {\sigma_{OPA_2}}^P(x_2)), max\,({\sigma_{OPA_1}}^P(x), {\sigma_{OPA_2}}^P(y_2)))$$

$$= max\,(({\sigma_{OPA_1}}^P \times {\sigma_{OPA_2}}^P)(x, x_2), ({\sigma_{OPA_1}}^P \times {\sigma_{OPA_2}}^P)(x, y_2))$$

$(\mu_{OPA_1}{}^N \times \mu_{OPA_2}{}^N)((x, x_2), (x, y_2)) = min\,(\sigma_{OPA_1}{}^N(x), \mu_{OPA_2}{}^N(x_2, y_2))$

$\leq min\,(\sigma_{OPA_1}{}^N(x), min\,(\sigma_{OPA_2}{}^N(x_2), \sigma_{OPA_2}{}^N(y_2)))$

$= min\,(min\,(\sigma_{OPA_1}{}^N(x), \sigma_{OPA_2}{}^N(x_2)), min\,(\sigma_{OPA_1}{}^N(x), \sigma_{OPA_2}{}^N(y_2)))$

$= min\,((\sigma_{OPA_1}{}^N \times \sigma_{OPA_2}{}^N)(x, x_2), (\sigma_{OPA_1}{}^N \times \sigma_{OPA_2}{}^N)(x, y_2))$

Let $z \in \sigma_{v_2}$ and $(x_1, y_1) \in \mu_{v_1}$, then we have,

$(\mu_{OPA_1}{}^P \times \mu_{OPA_2}{}^P)((x_1, z), (y_1, z)) = max\,(\mu_{OPA_1}{}^P(x_1, y_1), \sigma_{OPA_2}{}^P(z))$

$\geq max\,(max\,(\sigma_{OPA_1}{}^P(x_1), \sigma_{OPA_1}{}^P(y_1)), \sigma_{OPA_2}{}^P(z))$

$= max\,(max\,(\sigma_{OPA_1}{}^P(x_1), \sigma_{OPA_2}{}^P(z)), max\,(\sigma_{OPA_1}{}^P(y_1), \sigma_{OPA_2}{}^P(z)))$

$= max\,((\sigma_{OPA_1}{}^P \times \sigma_{OPA_2}{}^P)(x_1, z), (\sigma_{OPA_1}{}^P \times \sigma_{OPA_2}{}^P)(y_1, z))$

$(\mu_{OPA_1}{}^N \times \mu_{OPA_2}{}^N)((x_1, z), (y_1, z)) = min\,(\mu_{OPA_1}{}^N(x_1, y_1), \sigma_{OPA_2}{}^N(z))$

$\leq min\,(min\,(\sigma_{OPA_1}{}^N(x_1), \sigma_{OPA_1}{}^N(y_1)), \sigma_{OPA_2}{}^N(z))$

$= min\,(min\,(\sigma_{OPA_1}{}^N(x_1), \sigma_{OPA_2}{}^N(z)), min\,(\sigma_{OPA_1}{}^N(y_1), \sigma_{OPA_2}{}^N(z)))$

$= min\,((\sigma_{OPA_1}{}^N \times \sigma_{OPA_2}{}^N)(x_1, z), (\sigma_{OPA_1}{}^N \times \sigma_{OPA_2}{}^N)(y_1, z))$

Hence the proof

Anti-Join of two OPAFG

Let $\sigma_{OPA_1} = (\sigma_{OPA_1}{}^P, \sigma_{OPA_1}{}^N)$ and $\sigma_{OPA_2} = (\sigma_{OPA_2}{}^P, \sigma_{OPA_2}{}^N)$ be an Oscillating Polar Anti fuzzy subsets and the edge set is represented as $\mu_{OPA_1} = (\mu_{OPA_1}{}^P, \mu_{OPA_1}{}^N)$ and $\mu_{OPA_2} = (\mu_{OPA_2}{}^P, \mu_{OPA_2}{}^N)$. Then the anti-join of two OPAFG G_{OPA_1} and G_{OPA_2} is denoted by $G_{OPA_1} + G_{OPA_2}$ and it carries the following definition:

i) $(\sigma_{OPA_1}{}^P + \sigma_{OPA_2}{}^P)(x) = (\sigma_{OPA_1}{}^P \cup \sigma_{OPA_2}{}^P)(x)$ if $x \in \sigma_{v_1} \cup \sigma_{v_2}$
$(\sigma_{OPA_1}{}^N + \sigma_{OPA_2}{}^N)(x) = (\sigma_{OPA_1}{}^N \cap \sigma_{OPA_2}{}^N)(x)$ if $x \in \sigma_{v_1} \cup \sigma_{v_2}$

ii) $(\mu_{OPA_1}{}^P + \mu_{OPA_2}{}^P)(x, y) = (\mu_{OPA_1}{}^P \cup \mu_{OPA_2}{}^P)(x, y) = \mu_{OPA_1}{}^P(x, y)$, if $x, y \in \mu_{v_1} \cap \mu_{v_2}$
$(\mu_{OPA_1}{}^N + \mu_{OPA_2}{}^N)(x, y) = (\mu_{OPA_1}{}^N \cap \mu_{OPA_2}{}^N)(x, y) = \mu_{OPA_1}{}^N(x, y)$, if $x, y \in \mu_{v_1} \cap \mu_{v_2}$

iii) $(\mu_{OPA_1}{}^P + \mu_{OPA_2}{}^P)(x, y) = \min(\sigma_{OPA_1}{}^P(x), \sigma_{OPA_2}{}^P(y))$

$(\mu_{OPA_1}{}^N + \mu_{OPA_2}{}^N)(x, y) = \max(\sigma_{OPA_1}{}^N(x), \sigma_{OPA_2}{}^N(y))$,

If $(x, y) \in E'$, where E' is the set of all edges joining the nodes of $V_1 and V_2$.

Theorem

If G_{OPA_1} and G_{OPA_2} are the oscillating anti polar fuzzy graphs, then $G_{OPA_1} + G_{OPA_2}$ is an oscillating polar anti fuzzy graph

Proof:

$(\mu_{OPA_1}{}^P + \mu_{OPA_2}{}^P)(x, y) = min(\sigma_{OPA_1}{}^P(x), \sigma_{OPA_2}{}^P(y))$

$\geq min\,((\sigma_{OPA_1}{}^P \cup \sigma_{OPA_2}{}^P)(x), (\sigma_{OPA_1}{}^P \cup \sigma_{OPA_2}{}^P)(y))$

$= min\,((\sigma_{OPA_1}{}^P + \sigma_{OPA_2}{}^P)(x), (\sigma_{OPA_1}{}^P + \sigma_{OPA_2}{}^P)(y))$

$(\mu_{OPA_1}{}^N + \mu_{OPA_2}{}^N)(x, y) = max(\sigma_{OPA_1}{}^N(x), \sigma_{OPA_2}{}^N(y))$

$\leq max\,((\sigma_{OPA_1}{}^N \cap \sigma_{OPA_2}{}^N)(x), (\sigma_{OPA_1}{}^N \cap \sigma_{OPA_2}{}^N)(y))$

$= max\,((\sigma_{OPA_1}{}^N + \sigma_{OPA_2}{}^N)(x), (\sigma_{OPA_1}{}^N + \sigma_{OPA_2}{}^N)(y))$

$If\ xy \in \mu_{v_1} \cup \mu_{v_2}$, Then the result is obvious.

Conclusion

In this research, we have effectively presented and advanced the idea of an Oscillating Polar Anti Fuzzy Graph. We have established some fundamental operations namely anti-union, anti-join, anti-composition and anti-cartesian product. Each operation has been rigorously proved and validated by theorems.

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