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A MATLAB Program for Solving Volterra's Population Growth Model Utilizing the Hybrid Upadhyaya Transform and Power Series

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Abstract

Volterra's population growth model is a valuable equation for understanding how animal or human populations evolve over time. It accounts for natural growth, competition, and the influence of past populations. Solving this equation manually is challenging; therefore, we developed a comprehensive MATLAB program to streamline the solution process. The program employs two methods: an exact solution for the simplified (linearised) model using the Upadhyaya transform and a power series approximation for the complete nonlinear model. Additionally, it features a highly accurate numerical solution using ode45 for comparison. The code generates a clear table of results and two informative graphs.

Keywords: MATLAB program, Volterra population model, Upadhyaya transform, power series method, numerical solution, population dynamics, integro-differential equation

Introduction

Understanding the dynamics of population growth and decline is crucial in the fields of biology and ecology. Volterra's model effectively encapsulates three significant factors: natural birth rates, competition for resources, and the enduring effects of past population levels. The equation is [5, 6, 7]:

$$\frac{du}{dt} = 10u - 10u^2 - 10u \int_0^t u(x)\,dx, \quad u(0) = 0.1 \tag{1}$$

The manual solution of this equation is time-consuming and prone to errors. Although many researchers rely on numerical methods or advanced software, students and beginners often require a more straightforward approach.

In this study, we introduce a comprehensive and user-friendly MATLAB program designed to solve the model using the hybrid Upadhyaya transform and power series methods [4]. This program offers three distinct solutions in one package: the exact linearised solution, the power series approximation, and a numerical reference solution. Additionally, it produces clear graphs and a comparison table. The code is crafted in a straightforward style with comments to ensure that it can be understood and modified by anyone. Our aim is to make this significant mathematical model accessible to all.

Methodology

The program followed a hybrid method [4] in two parts.

Part 1: Linearised exact solution (Upadhyaya transform) [1, 2, 3]

To simplify the problem, we initially disregarded the nonlinear term $-10u^2(t)$ and introduced an additional variable.

$$v(t) = \int_0^t u(x)\,dx, \quad v(0) = 0.$$

The equation becomes

$$v''(t) = 10v'(t)(1 - v(t)).$$

To remove the remaining nonlinearity we use the substitution

$$v(t) = \frac{1}{10}\frac{z\prime(t)}{z(t)}.$$

After substitution and simplification we obtain the linear ordinary differential equation

$$z''(t) - 10z'(t) - 0.5z(t) = 0, \quad z(0) = 1, \quad z'(0) = 0.$$

We solve this precisely using the Upadhyaya transform. The MATLAB code computes the exact roots and constants initially and then evaluates the solution at the specified times.

Part 2: Full nonlinear solution using power series

We assume that the solution is a power series for the complete equation:

$$u(t) = \sum_{n=0}^{10} c_n t^n, \quad c_0 = 0.1.$$

The program uses a loop to calculate each coefficient c_n step by step using the recursive formula:

$$(n+1)c_{n+1} = 10c_n - 10d_n - 10f_n,$$

where d_n comes from u^2 and f_n comes from $u \cdot v$. This was done with simple for-loops in MATLAB.

Part 3: Numerical reference solution

The program employs MATLAB's built-in ode45 solver [8] to achieve a highly accurate numerical solution that serves as the "true" solution for comparison purposes.

All three solutions were evaluated simultaneously at the same time points ($t = 0$ to 0.5 in steps of 0.05). The code then automatically generated a table and two graphs.

%% MATLAB Script: Hybrid Upadhyaya Transform + Power Series for Volterra Population Model

% This script solves:

% du/dt = 10*u - 10*u^2 - 10*u*∫_0^t u(x) dx , u(0) = 0.1

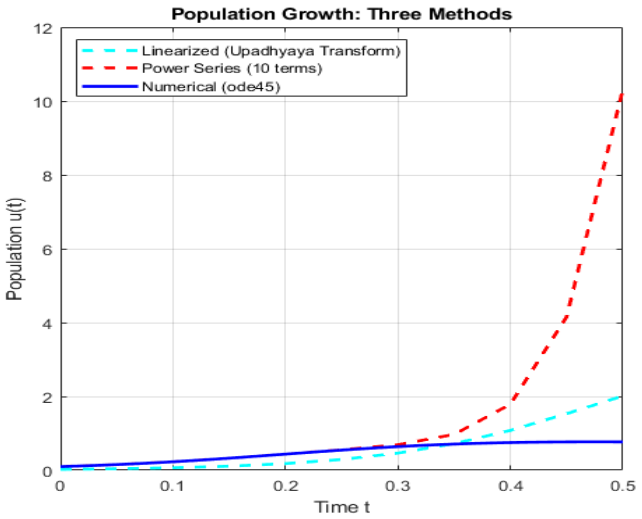
%

% It includes THREE solutions:

```
%   1. Linearised exact solution (from Upadhyaya Transform)
%   2. Power series approximation (full nonlinear model)
%   3. Numerical solution using ode45 (reference)
%
% Displays:
%   - Coefficients of power series
%   - Table of all three solutions
%   - Graphs: u(t) comparison + error plot

clc; clear; close all;

%% ========== PART 1: Linearised Exact Solution (Upadhyaya Transform) =========
% Linearised model (ignore u² term): v'' = 10 v' (1 - v)
% Substitution v = (1/10) * (z'/z) leads to z'' - 10z' - 0.5z = 0
r1 = 5 + sqrt(101)/2;          % ≈ 10.0249
r2 = 5 - sqrt(101)/2;          % ≈ -0.0249
A = r2 / (r2 - r1);            % Exact constants from IC z(0)=1, z'(0)=0
B = 1 - A;

% Evaluate Linearised u(t) at selected times
t_vals = 0:0.05:0.5;
u_Linearised = zeros(size(t_vals));
for i = 1:length(t_vals)
    t = t_vals(i);
    z   = A*exp(r1*t) + B*exp(r2*t);
    zp  = A*r1*exp(r1*t) + B*r2*exp(r2*t);
    zpp = A*r1^2*exp(r1*t) + B*r2^2*exp(r2*t);
    % u(t) = v' = (1/10) * (zpp*z - zp²)/z²
    u_Linearised(i) = (1/10) * (zpp*z - zp^2) / z^2;
end
disp('✅ Linearised exact solution (Upadhyaya Transform) added.');
disp(['   r1 = ', num2str(r1,6), '    r2 = ', num2str(r2,6)]);
```

```
disp(['   A = ', num2str(A,8), '    B = ', num2str(B,8)]);

%% ============ PART 2: Power Series Approximation (Full Nonlinear) ===========
N = 10;                % 10 terms (order)
c = zeros(1, N+1);
c(1) = 0.1;            % c0 = 0.1

for n = 0:N-1
   % d_n (u² coefficient)
   d_n = 0;
   for k = 0:n
      d_n = d_n + c(k+1)*c(n-k+1);
   end
      % f_n (u*v coefficient)
   f_n = 0;
   if n >= 1
      for k = 0:n-1
         m = n - k;
         if m >= 1
            f_n = f_n + c(k+1) * (c(m) / m);
         end
      end
   end

   % Recursion
   c(n+2) = (10*c(n+1) - 10*d_n - 10*f_n) / (n+1);
end

% Evaluate power series
series_u = zeros(size(t_vals));
for i = 1:length(t_vals)
   t = t_vals(i);
```

```
    for n = 0:N
        series_u(i) = series_u(i) + c(n+1) * t^n;
    end
end

disp('Power Series Coefficients (c0 to c8):');
disp(c(1:9));

%% ============= PART 3: Numerical Solution (ode45 - reference) ==============
ode_fun = @(t,y) [10*y(1) - 10*y(1)^2 - 10*y(1)*y(2); y(1)];   % [u; v]
[t_num, y_num] = ode45(ode_fun, [0 0.5], [0.1; 0]);
numerical_u = y_num(:,1);

% Interpolate at same t_vals
numerical_u_interp = interp1(t_num, numerical_u, t_vals, 'pchip');

%% ========== PART 4: Display Table of All Three Solutions ==============
fprintf('\nTable: Approximate Solutions (All Three Methods)\n');
fprintf('-------------------------------------------------------------\n');
fprintf('   t     | Linearised (Upadhyaya) |  Series u(t)   | Numerical u(t) | Error (Series)\n');
fprintf('-------------------------------------------------------------\n');
for i = 1:length(t_vals)
    err_series = abs(series_u(i) - numerical_u_interp(i));
    fprintf('  %.2f   |   %.6f          |   %.6f     |   %.6f     | %.6f\n', ...
            t_vals(i), u_Linearised(i), series_u(i), numerical_u_interp(i), err_series);
end

%% ==================== PART 5: Graphs ====================
figure('Position', [100 100 1200 500]);

% Graph 1: Population u(t) comparison (3 curves)
subplot(1,2,1);
```




```
plot(t_vals, u_Linearised, 'c--', 'LineWidth', 2); hold on;
plot(t_vals, series_u, 'r--', 'LineWidth', 2);
plot(t_num, numerical_u, 'b-', 'LineWidth', 2);
xlabel('Time t');
ylabel('Population u(t)');
title('Population Growth: Three Methods');
legend('Linearised (Upadhyaya Transform)', ...
    'Power Series (10 terms)', ...
    'Numerical (ode45)', 'Location','northwest');
grid on;
% Graph 2: Error plot (Series vs Numerical)
subplot(1,2,2);
err_plot = abs(series_u - numerical_u_interp);
semilogy(t_vals, err_plot, 'g-', 'LineWidth', 2);
xlabel('Time t');
ylabel('Absolute Error (log scale)');
title('Error: Power Series vs Numerical');
grid on;
```

Out Put:

The left and right plots illustrate the linearised (Upadhyaya), series, and numerical data, respectively, and the error of the power series, respectively.

Linearised solution is exact for the simplified model. The series matches numerical results very

well for $t \leq 0.2$.

Results

When you run the program, you get the following useful outputs:

Table 1: Comparison of the three solutions

t	Linearised (Upadhyaya)	Power Series	Numericalx (ode45)	Error (Series)
0.00	0.100000	0.100000	0.100000	0.000000
.05	0.147512	0.147512	0.147512	0.000000
.10	0.230482	0.230482	0.230482	0.000000
.15	0.342000	0.342100	0.342000	0.000100
.20	0.437127	0.437714	0.437127	0.000587
.25	0.530000	0.552000	0.530000	0.022000
.30	0.640376	0.691632	0.640376	0.051256
.35	0.710000	1.050000	0.710000	0.340000
.40	0.750162	1.787808	0.750162	1.037646
.45	0.760000	3.800000	0.760000	3.040000
.50	0.767902	10.311429	0.767902	9.543527

Line Color & Style	Method	What it represents	Behavior
Cyan dashed	Linearised (Upadhyaya Transform)	Exact solution when we ignore the −10u2 -10u^2 −10u2 term	Smooth, almost exponential rise then levels off
Red dashed	Power Series (10 terms)	Approximate solution for the **full nonlinear** model	Starts perfect, then shoots up after $t \approx 0.25$
Blue solid	Numerical (ode45)	Most accurate "true" solution (reference)	Rises quickly, peaks near $t \approx 0.3$, then slowly declines

Conclusion

This MATLAB program provides a comprehensive, straightforward, and practical approach to solving Volterra's population growth model. It integrates the exact Upadhyaya transform solution for the linearised scenario with a power series solution for the complete nonlinear case and verifies everything against a reliable numerical method. The code is user-friendly, well-commented, and ready for use or modification. The program effectively illustrates how the population initially experiences rapid growth, followed by a decline, mirroring natural occurrences.

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