

# Total Dominator Color Class Total Dominating Sets In Star Relaed Graphs

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**Abstract** - Let  $G = (V, E)$  be a finite, undirected and connected graph. A proper coloring  $\mathcal{C}$  of  $G$  is said to be a total dominator color class total dominating set of  $G$  if each vertex properly dominates a color class in  $\mathcal{C}$  and each color class in  $\mathcal{C}$  is properly dominated by a vertex in  $V(G)$ . A total dominator color class total dominating set  $D$  of  $G$  is a minimal total dominator color class total dominating set if no proper subset of  $D$  is a total dominator color class total dominating set of  $G$ . The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in  $G$  and is denoted by  $\gamma_{\chi}^{td}(G)$ . Here we obtain total dominator color class total domination number of modular star graph and kalami modular star graph.

**Keywords:** Chromatic number, Total Domination number, Dominator Color Class Domination number, Total Dominator Color Class Total Domination number.

**Mathematics Subject Classification:** 05C15, 05C69

## 1. Introduction

All graphs considered in this paper are finite, undirected and connected graphs without loops and multiple edges. We follow standard definitions of graph theory as found in [19].

A subset  $S$  of  $V(G)$  is called a total dominating set if every vertex in  $V(G)$  is adjacent to some vertex in  $S$ . A total dominating set  $S$  is called a minimal total dominating set if no proper subset of  $S$  is a total dominating set of  $G$ . The total domination number  $\gamma_t(G)$  is the minimum cardinality taken over all minimal total dominating sets of  $G$ . A proper coloring of  $G$  is an assignment of colors to the vertices of  $G$  such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of  $G$  is called chromatic number of  $G$  and is denoted by  $\chi(G)$ . A total dominator coloring of  $G$  is a proper coloring of  $G$  with the extra property that every vertex in  $G$  properly dominates a color class. The total dominator chromatic number is denoted by  $\chi_{td}(G)$ . This notion was introduced by A.Vijayalekshmi et al [20]. A color class dominating set of  $G$  is a proper coloring  $\mathcal{C}$  of  $G$  with the extra property that every color class in  $\mathcal{C}$  is dominated by a vertex in  $G$ . A color class

dominating set is said to be a minimal color class dominating set if no proper subset of  $\mathcal{C}$  is a color class dominating set of  $G$ . The color class domination number of  $G$  is the minimum cardinality taken over all minimal color class dominating sets of  $G$  and is denoted by  $\gamma_{\chi}(G)$ . This notion was introduced by A.Vijayalekshmi et al [1].

A dominator color class dominating set of  $G$  is a proper coloring  $\mathcal{C}$  of  $G$  with the extra property that each vertex  $v$  in  $G$  is dominated by a color class in  $\mathcal{C}$  and each color class in  $\mathcal{C}$  is dominated by a vertex in  $G$ . The dominator color class domination number of  $G$  is the minimum cardinality taken over all minimal dominator color class dominating sets in  $G$  and is denoted by  $\gamma_{\chi}^d(G)$ . This notion was introduced by A.Vijayalekshmi et al [3]. A proper coloring  $\mathcal{C}$  of  $G$  is said to be a total dominator color class total dominating set of  $G$  if each vertex properly dominates a color class in  $\mathcal{C}$  and each color class in  $\mathcal{C}$  is properly dominated by a vertex in  $V(G)$ . A total dominator color class total dominating set  $D$  of  $G$  is a minimal total dominator color class total dominating set if no proper subset of  $D$  is a total dominator color class total dominating set of  $G$ . The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in  $G$  and is denoted by  $\gamma_{\chi}^{td}(G)$ . This notion was introduced by A.Vijayalekshmi et al [7].

A Wheel graph  $W_n$  consists of a cycle  $C_n$  whose vertices are  $\{w_i: 1 \leq i \leq n\}$  and central vertex  $\{u\}$  connected to each vertex of the cycle. The Triangulated Wheel graph  $TW_n$  is constructed from a wheel graph  $W_n$  in which new vertex  $v_i$  and the edges  $uv_i, v_iw_i, v_jw_{j+1}, v_nw_1$  and  $v_1w_n$  are added ( $1 \leq i \leq n, 2 \leq j \leq n-1$ ). The Modular Star graph  $MS_n$  is obtained from triangulated wheel graph  $TW_n$  by connecting  $w_i$  to a new vertex  $x_i$  ( $1 \leq i \leq n$ ). The Helm graph is a graph obtained from a wheel graph by attaching a pendant edge at each vertex of  $n$ -cycle. The Web graph  $Wb_n$  is the graph obtained from a helm graph by joining the pendant vertices to form an  $n$ -cycle. The Kalami Modular Star graph  $KMS_n$  is the graph obtained from the web graph  $Wb_n$  by attaching a pendant edge at each vertex of the outer cycle in  $Wb_n$ , then joining each pendant vertex to form a cycle and then by subdividing each path of the outer cycle with the vertex.

## 2.1 Main Results

### Theorem 2.1

Let  $MS_n$  be a Modular Star graph where  $n \geq 4$ . Then

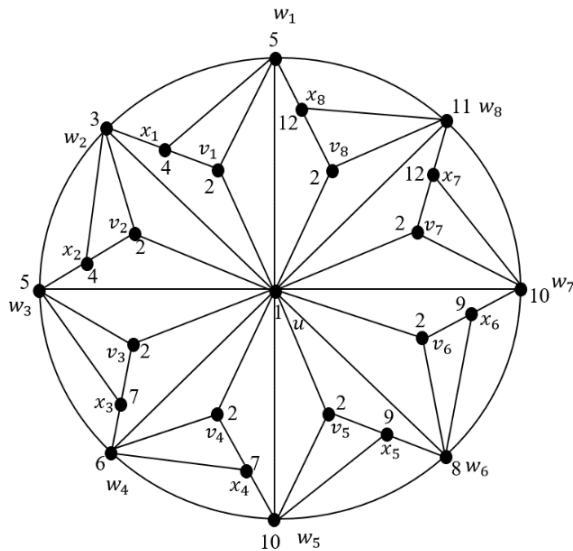
$$\gamma_{\chi}^{td}(MS_n) = \begin{cases} 5 \left\lfloor \frac{n}{4} \right\rfloor + 2 & \text{if } n \equiv 0,3 \pmod{4} \\ 5 \left\lfloor \frac{n}{4} \right\rfloor - 1 & \text{if } n \equiv 1 \pmod{4} \\ 5 \left\lfloor \frac{n}{4} \right\rfloor & \text{orelse} \end{cases}$$

### Proof

Let  $V(MS_n) = \{u, v_i, w_i, x_i: 1 \leq i \leq n\}$  with  $\deg u = 2n, \deg w_i = 6$  and  $\deg v_i = \deg x_i = 3$  for all  $1 \leq i \leq n$ . The vertices  $\{u\}$  and  $\{v_i: 1 \leq i \leq n\}$  are assigned colors 1 and 2 respectively. We consider four cases.

**Case 1:** When  $n \equiv 0 \pmod{4}$

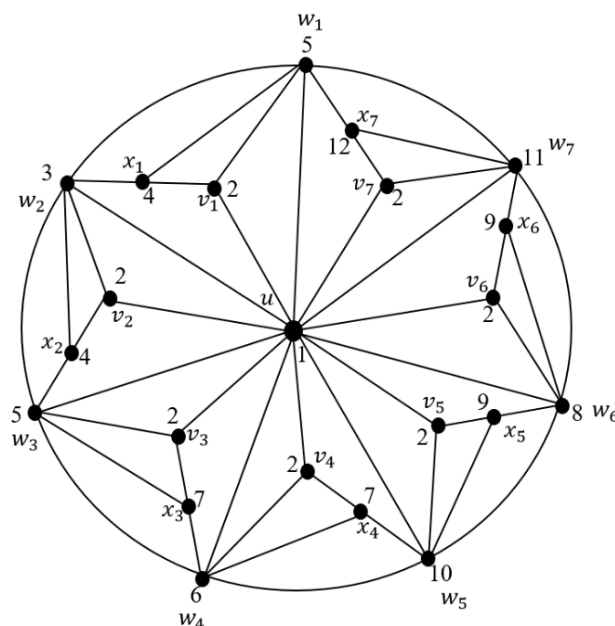
Assign color  $5i-2, 5i-1, 5i, 5i+1$  and  $5i+2$  to the vertices  $\{w_{4i-2}\}, \{x_{4i-3}, x_{4i-2}\}, \{w_{4i-3}, w_{4i-1}\}, \{w_{4i}\}$  and  $\{x_{4i-1}, x_{4i}\}$  respectively ( $1 \leq i \leq \lfloor \frac{n}{4} \rfloor$ ). Hence we get  $\gamma_{\chi}^{td}$ - coloring of  $MS_n$ . Thus  $\gamma_{\chi}^{td}(MS_n) = 5 \lfloor \frac{n}{4} \rfloor + 2$ .



**Figure 2.1.**  $\gamma_{\chi}^{td}(MS_8) = 12$

**Case 2:** When  $n \equiv 3 \pmod{4}$

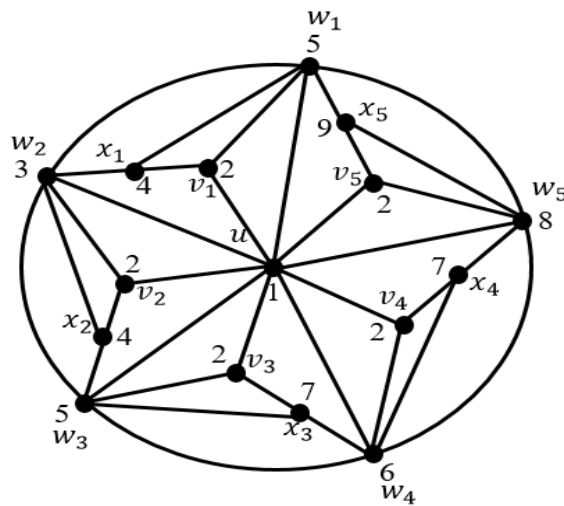
Since  $n - 3 \equiv 0 \pmod{4}$  by case 1,  $\gamma_{\chi}^{td}(MS_{n-3}) = 5 \lfloor \frac{n-3}{4} \rfloor + 2$ . On assigning color  $5 \lfloor \frac{n}{4} \rfloor - 2, 5 \lfloor \frac{n}{4} \rfloor - 1, 5 \lfloor \frac{n}{4} \rfloor, 5 \lfloor \frac{n}{4} \rfloor + 1$  and  $5 \lfloor \frac{n}{4} \rfloor + 2$  to the vertices  $\{w_{n-1}\}, \{x_{n-1}, x_{n-2}\}, \{w_{n-2}\}, \{w_n\}$  and  $\{x_n\}$  respectively we obtain  $\gamma_{\chi}^{td}$ - coloring of  $MS_n$ . Thus  $\gamma_{\chi}^{td}(MS_n) = 5 \lfloor \frac{n}{4} \rfloor + 2$ .



**Figure 2.2.**  $\gamma_{\chi}^{td}(MS_7) = 12$

**Case 3:** When  $n \equiv 1 \pmod{4}$

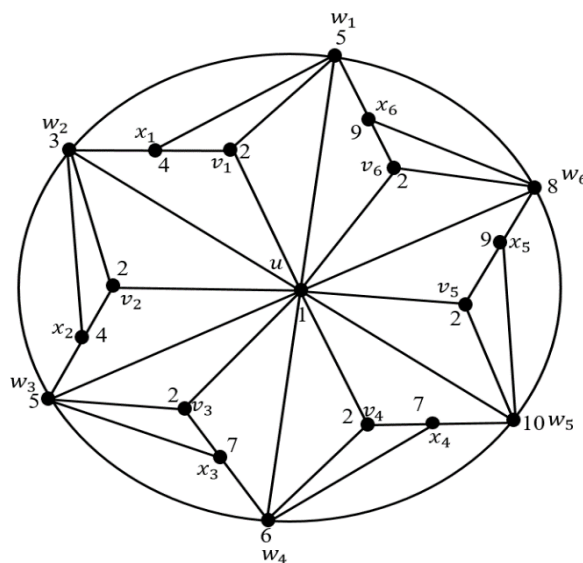
Since  $n - 1 \equiv 0 \pmod{4}$  by case 1,  $\gamma_{\chi}^{td}(MS_{n-1}) = 5 \left\lfloor \frac{n-1}{4} \right\rfloor + 2$ . Assign color  $5 \left\lfloor \frac{n}{4} \right\rfloor - 2$  and  $5 \left\lfloor \frac{n}{4} \right\rfloor - 1$  to the vertices  $\{w_n\}$  and  $\{x_n\}$  respectively. Hence we get  $\gamma_{\chi}^{td}$ - coloring of  $MS_n$ . Thus  $\gamma_{\chi}^{td}(MS_n) = 5 \left\lfloor \frac{n}{4} \right\rfloor - 1$ .



**Figure 2.3.**  $\gamma_{\chi}^{td}(MS_5) = 9$

**Case 4:** When  $n \equiv 2 \pmod{4}$

Since  $n - 2 \equiv 0 \pmod{4}$  by case 1,  $\gamma_{\chi}^{td}(MS_{n-2}) = 5 \left\lfloor \frac{n-2}{4} \right\rfloor + 2$ . Assign color  $5 \left\lfloor \frac{n}{4} \right\rfloor - 2$ ,  $5 \left\lfloor \frac{n}{4} \right\rfloor - 1$  and  $5 \left\lfloor \frac{n}{4} \right\rfloor$  to the vertices  $\{w_n\}$ ,  $\{x_{n-1}, x_n\}$  and  $\{w_{n-1}\}$  respectively. Therefore we get  $\gamma_{\chi}^{td}$ - coloring of  $MS_n$ . So  $\gamma_{\chi}^{td}(MS_n) = 5 \left\lfloor \frac{n}{4} \right\rfloor$ .



**Figure 2.4.**  $\gamma_{\chi}^{td}(MS_6) = 10$

**Theorem 2.2**

Let  $KMS_n$  be a Kalami Modular Star graph with  $n \geq 4$ . Then

$$\gamma_{\chi}^{td}(KMS_n) = \begin{cases} 2n + 2 & \text{if } n \text{ is even} \\ 2n + 3 & \text{if } n \text{ is odd} \end{cases}$$

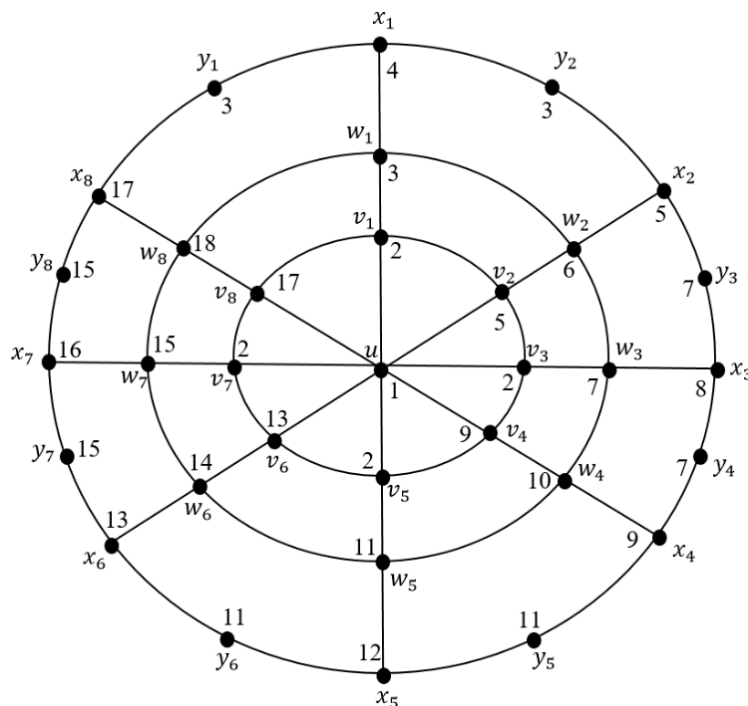
**Proof:**

Let  $V(KMS_n) = \{u, v_i, w_i, x_i, y_i : 1 \leq i \leq n\}$  with  $deg u = n, deg v_i = deg w_i = 4,$   
 $deg x_i = 3$  and  $deg y_i = 2$ . We consider two cases.

**Case 1:** When  $n$  is even. We consider two subcases.

**Subcase 1.1:** When  $n \equiv 0(mod 4)$

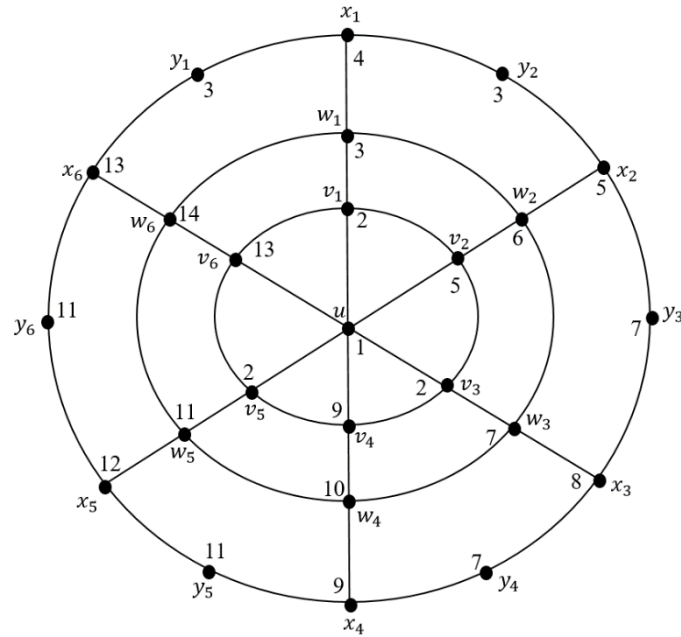
Assign color 1 and 2 to the vertices  $\{u\}$  and  $\{v_{2i-1} : 1 \leq i \leq \frac{n}{2}\}$  respectively. On assigning colors  $4i - 1, 4i, 4i + 1$  and  $4i + 2$  to the remaining vertices  $\{y_{2i-1}, y_{2i}, w_{2i-1}\},$   
 $\{x_{2i-1}\}, \{x_{2i}, v_{2i}\}$  and  $\{w_{2i}\}$  respectively  $(1 \leq i \leq \frac{n}{2})$  we get  $\gamma_{\chi}^{td}$ - coloring of  $KMS_n$ . So  
 $\gamma_{\chi}^{td}(KMS_n) = 2n + 2$ .



**Figure 2.5.**  $\gamma_{\chi}^{td}(KMS_8) = 18$

**Subcase 1.2:** When  $n \equiv 2(mod 4)$

Since  $n - 2 \equiv 0(mod 4)$  by subcase 1.1,  $\gamma_{\chi}^{td}(KMS_{n-2}) = 2n - 2$ . The vertices  $\{y_{n-1}, y_n, w_{n-1}\}, \{x_{n-1}\}, \{x_n, v_n\}$  and  $\{w_n\}$  are assign colors  $2n - 1, 2n, 2n + 1$  and  $2n + 2$  respectively. Hence we get  $\gamma_{\chi}^{td}$ - coloring of  $KMS_n$ . So  $\gamma_{\chi}^{td}(KMS_n) = 2n + 2$ .

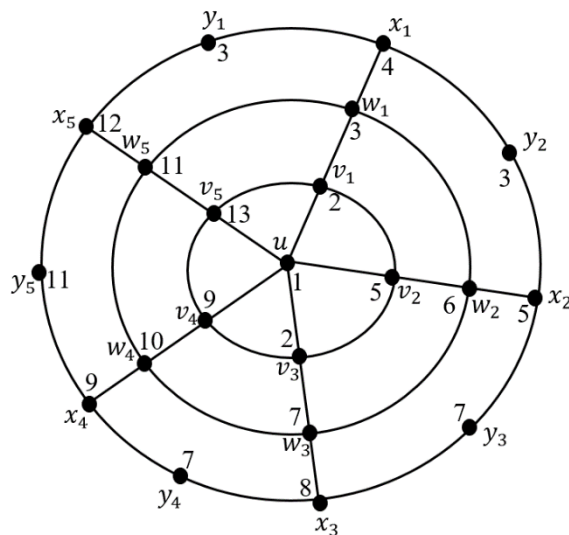


**Figure 2.6.**  $\gamma_{\chi}^{td}(KMS_6) = 14$

**Case 2:** When  $n$  is odd. We consider two subcases.

**Subcase 2.1:** When  $n \equiv 1 \pmod{4}$

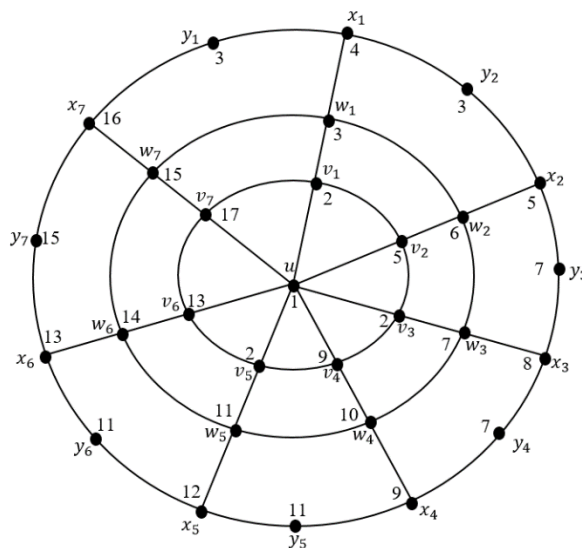
Since  $n - 1 \equiv 0 \pmod{4}$  by subcase 1.1,  $\gamma_{\chi}^{td}(KMS_{n-1}) = 2n$ . The vertices  $\{y_n, w_n\}$ ,  $\{x_n\}$  and  $\{v_n\}$  on assigning colors  $2n+1$ ,  $2n+2$  and  $2n+3$  respectively, we get  $\gamma_{\chi}^{td}$ - coloring of  $KMS_n$ . Thus  $\gamma_{\chi}^{td}(KMS_n) = 2n+3$ .



**Figure 2.7.**  $\gamma_{\chi}^{td}(KMS_5) = 13$

**Subcase 2.2:** When  $n \equiv 3 \pmod{4}$

Since  $n - 1 \equiv 2 \pmod{4}$  by subcase 1.2,  $\gamma_{\chi}^{td}(KMS_{n-1}) = 2n$ . On assigning color  $2n+1$ ,  $2n+2$  and  $2n+3$  to the vertices  $\{y_n, w_n\}$ ,  $\{x_n\}$  and  $\{v_n\}$  respectively we obtain  $\gamma_{\chi}^{td}$ - coloring of  $KMS_n$ . Hence  $\gamma_{\chi}^{td}(KMS_n) = 2n+3$ .



**Figure 2.8.**  $\gamma_{\chi}^{td}(KMS_7) = 17$

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