Panamerican Mathematical Journal
ISSN: 1064-9735
Vol 35 No. 2s (2025)

Homeomorphisms via M-Open Sets in n-Cylindrical Neutrosophic Topological Spaces

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Article History:
Received- *25-01-2025*
Revised- *04 -03- 2025*
Accepted- *16-03- 2025*

Abstract

In this article, we introduce the novel notions of n-CyNM homeomorphism, n-CyNMC-homeomorphism, and n-CyNM$T_{\frac{1}{2}}$-spaces within the context of n-Cylindrical neutrosophic topological spaces. These concepts serve as natural extensions of classical homeomorphisms and separation axioms into the neutrosophic framework, incorporating degrees of membership, non-membership, and indeterminacy. We systematically develop and examine several key properties and structural features associated with these notions. In addition, a series of theorems and related results are established, offering deeper insights into the behavior and potential applications of these mappings and spaces in neutrosophic topology.

Keywords and phrases: n - CyNM -homeomorphism, n - CyNMC homeomorphism, and n-CyNM$T_{\frac{1}{2}}$-space.

AMS (2000) subject classification: 03E72, 54A10, 54A40

Introduction

Following Zadeh's seminal introduction of fuzzy sets (fs) in 1965 [21], Chang [3] extended this idea by developing fuzzy topological spaces (fts), paving the way for classical topological concepts to be reinterpreted within the framework of fuzzy topology. This advancement stimulated extensive research in the field. A significant generalization, known as intuitionistic fuzzy sets (ifs), was introduced by Atanassov in 1986 [2], further enriching the mathematical treatment of uncertainty. Building on this, Coker [4] introduced intuitionistic fuzzy topological spaces (ifts), and Jeon et al. [6] investigated concepts such as intuitionistic fuzzy continuity and pre-continuity.

The introduction of neutrosophy and neutrosophic sets by Smarandache [14, 15] marked a new direction in modeling indeterminacy and inconsistency. Salama and Alblowi [9] later proposed neutrosophic crisp sets and neutrosophic topological spaces (Nts), expanding on the notion of ifts by incorporating three degrees membership, indeterminacy, and non-membership assigned

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to each element. Neutrosophic theory has since evolved into a robust generalization encompassing both crisp and fuzzy systems.

Smarandache also introduced the concept of dependence among fuzzy and neutrosophic components. Arokiarani et al. [1] further contributed to this field by introducing a more generalized neutrosophic set (NS), in which the sum of the three membership values does not exceed 3. In the same year, Veereswari [20] proposed neutrosophic topological spaces and explored foundational topological operations in this setting.

Advancing this line of research, Saranya et al. [10] introduced the concept of n-Cylindrical neutrosophic sets (n-CyNS), characterized by α and γ as dependent components and β as an independent component. Apart from standard neutrosophic sets, n-CyNS represents a comprehensive generalization of fuzzy sets. In this formulation, the positive (α), neutral (β), and negative (γ) membership functions satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1$, where $n > 1$ is an integer.

Later, Saranya et al. [12] introduced the concept of n-CyN continuity for mappings between two n-Cylindrical neutrosophic topological spaces (n-CyNts), and defined the corresponding notions of n-CyN interior (n-CyNint) and closure (n-CyNcl) for subsets in this context.

In a parallel development, El-Maghrabi and Al-Juhani [5] introduced the concept of M-open sets in general topological spaces and explored their fundamental properties. M-open sets have since played a crucial role in topological theory due to their applicability across various mathematical fields and real-world applications. Padma et al. [8] extended this notion to nano topological spaces. Further contributions by Vadivel et al. [16, 17, 18] examined open sets within fuzzy nano and neutrosophic nano topological settings. Similarly, Kalaiyarsan et al. [7] and Vadivel et al. [19] investigated M-open sets in fuzzy and neutrosophic nano topological spaces.

In this paper, section 2 provides a concise review of key definitions related to intuitionistic fuzzy sets (ifs), neutrosophic sets (NS), n-Cylindrical neutrosophic sets (n-CyNS), and mappings on n-Cylindrical neutrosophic topological spaces (n-CyNCts). Sections 3 and 4 introduce the concepts of n-CyNM-homeomorphism and n-CyNMC-homeomorphism within the framework of n-CyNts, and examine their fundamental properties with the support of illustrative examples. Section 5 concludes the summary of the results and potential directions for future research.

Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [21] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A for all $x \in X$.

Definition 2.2 [3] A fuzzy topological space (briefly, fts) is a pair (X, τ_X), where X is any set and τ_X is a family of fuzzy sets in X satisfying following axioms:

(i).$\phi, X \in \tau$,

(ii). If $A, B \in \tau$, then $A \cap B$ in τ,

(iii). If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [2] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x)/x \in X\rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of non-membership of x in A, and where α_A and γ_A satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ifs(X) denotes the set of all the ifs's on X.

Definition 2.4 [15] A neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of truth membership, $\beta_A(x)$ is the degree of indeterminacy and $\gamma_A(x)$ is the degree of non-membership. Here $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [9] A neutrosophic topology (Nt) on a non-empty set X is a family τ_X of neutrosophic subsets in X satisfying the following axioms:

(i). $0_N, 1_N \in \tau_X$,

(ii). $G_1 \cap G_2 \in \tau_X$ for any $G_1, G_2 \in \tau_X$,

(iii). $\cup G_i \in \tau_X$, for all $\{G_i : i \in J\} \subseteq \tau_X$.

In this case the pair (X, τ_X) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_X are called neutrosophic open sets. A neutrosophic set F is closed if and only if F^c is neutrosophic open.

Definition 2.6 [10] An n-Cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here α and γ are dependent neutrosophic components and β is 100% independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical neutrosophic number (briefly, n-CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 [10] Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \left\{\langle x, \inf\left(\alpha_{A_i}(x)\right), \inf\left(\beta_{A_i}(x)\right), \sup\left(\gamma_{A_i}(x)\right)\rangle : x \in X\right\}$, $\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 [10] $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$

Definition 2.9 [10] **(The Basic Connectives)** Let $\tau_N(X)$ denote the family of all n-CyNs on X.

Definition 2.10 [10] Inclusion: For every two $A, B \in \tau_N(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).

Definition 2.11 [10] Union: For every two $A, B \in \tau_N(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.12 [10] Intersection: For every two $A, B \in \tau_N(X)$, the intersection of two n-CyNS 's A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.13 [10] Complementary: For every $A \in \tau_N(X)$, the complement of an n-CyNS's A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Definition 2.14 [10] Sum: For every two $A, B \in \tau_N(X)$, the sum of two n-CyNS's A and B is $A \oplus B(x) = \{\langle x, (\frac{\alpha_A(x) \cdot \alpha_B(x)}{\alpha_A(x) + \alpha_B(x)}), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.15 [10] Difference: For every two $A, B \in \tau_N(X)$, the difference of two n-CyNS 's A and B is $A \ominus B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), (\frac{\gamma_A(x) \cdot \gamma_B(x)}{\gamma_A(x) + \gamma_B(x)})\rangle : x \in X\}$.

Definition 2.16 [10] Product: For every two $A, B \in \tau_N(X)$, the product of two n-CyNS's A and B is $A \otimes B(x) = \{\langle x, (\alpha_A(x) \cdot \alpha_A(x)), (\beta_A(x) \cdot \beta_B(x)), (\gamma_A(x) \cdot \gamma_B(x))\rangle : x \in X\}$.

Definition 2.17 [10] Division: For every two $A, B \in \tau_N(X)$, the division of two n-CyNS's A and B is $A \oslash B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), (\beta_A(x) \cdot \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Remark 2.1 [10]

For every A, B and $C \in \tau_N(X)$,

(i). If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,

(ii). $A \cup B = B \cup A \& A \cap B = B \cap A$,

(iii). $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,

(iv). $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,

(v). $A \cap A = A \& A \cup A = A$,

(vi). De Morgan's Law for $A \& B$ ie., $(A \cup B)^c = A^c \cap B^c$& $(A \cap B)^c = A^c \cup B^c$,

(vii). $(A \oplus B) = (B \oplus A)$,

(viii). $(A \otimes B) = (B \otimes A)$.

Definition 2.18 [12] An n-cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_X, of n-CyNS in X which satisfies the following conditions:

(i). $0_{CyN}, 1_{CyN} \in \tau_X$,

(ii). $A_1 \cap A_2 \in \tau_X$,

(iii). $\cup A_i \in \tau_X$, for any arbitrary family $A_i \in \tau_X, i \in I$.

The pair (X, τ_X) is called an n-cylindrical neutrosophic topological Spaces (briefly, n-CyNts) and any n-CyNS belongs to τ_X is called an n-cylindrical neutrosophic open set (briefly, n-CyNos) and the complement of n-CyNos is called n-cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-CyNts and the topology containing all the n-CyN subsets is called discrete n-CyNts.

Remark 2.2 [12] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-CyNts as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.19 [12] Let A and B be two n-cylindrical neutrosophic subsets of an n-CyNts. B is called neighbourhood of A if there exists an n-CyNos, O such that $A \subset O \subset B$.

Proposition 2.1 [12] $A \subset X$ is n-cylindrical neutrosophic open in (X, τ_X) if and only if it carries a neighbourhood of its subsets.

Definition 2.20 [12] Let (X, τ_X) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of X. ie, n-CyNint(A) $= \cup\{G : G \in \tau_X$ and $G \subseteq A\}$.Clearly, n-CyNint(A) is the biggest n-CyNos that is contained by A.

Definition 2.21 [12] Let (X, τ_X) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, n-CyNcl(A) $= \cap\{K : K \in \tau_X$ and $A \subseteq A\}$.Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

Definition 2.22 [11] Let (X, τ_1) and (Y, τ_2) be two n-CyNts and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then f said to be n-CyN continuous (briefly, n-CyNCts) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of f(A) there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subset \mathfrak{V}$.

Cylindrical Neutrosophic M-Homeomorphisms

In this section, we introduce n-Cylindrical neutrosophic M-homeomorphisms and look at some of its feature in this section.

Definition 3.1 Let (X, τ_X) be an n-CyNts and A be an n-CyNs. Then, A is said to be an n-CyN

(i). regular open set (briefly, n-CyNros), if A = n-CyNint(n-CyNcl(A)),

(ii). regular closed set (briefly, n-CyNrcs), if A = n-CyNcl(n-CyNint(A)).

Definition 3.2 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNs in X. Then, the n-Cylindrical neutrosophic δ-interior of A and the n-Cylindrical neutrosophic δ-closure of A are denoted by n-CyNδint(A) and n-CyNδcl(A) are defined as follows:

(i). n-CyNδint(A) $= \cup\{G | G$ isan n-CyNros and $G \subseteq A\}$,

(ii). n-CyNδcl(A) $= \cap\{K | K$ isan n-CyNrcs and $A \subseteq K\}$.

Definition 3.3 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic θ-interior of A and the n-Cylindrical neutrosophic θ-closure of A are denoted by n-CyNθint(A) and n-CyNθcl(A) are defined as follows:

(i). $\text{n-CyN}\theta\text{int}(A) = \cup\{n - \text{CyNint}(B) : B \subseteq A \& B\ isa\ n - CyNcs\ in\ X\}$,

(ii). $\text{n-CyN}\theta\text{cl}(A) = \cap\{n - \text{CyNcl}(B) : A \subseteq B \& B\ isa\ n - CyNos\ in\ X\}$.

Definition 3.4 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN

(i). δ-open set (briefly, n-CyNδos), if A = n-CyNδint(A),

(ii). δ-pre open set (briefly, n-CyNδ$\mathcal{P}$os), if A ⊆ n-CyNint(n-CyNδcl(A)),

(iii). δ-semi open set (briefly, n-CyNδSos), if A ⊆ n-CyNcl(n-CyNδint(A)),

(iv). θ-open set (briefly, n-CyNθos), if A = n-CyNθint(A),

(v). θ-semi open set (briefly, n-CyNθ$\mathcal{S}$os), if A ⊆ n-CyNcl(n-CyNθint(A)),

(vi). e-open set (briefly, n-CyNeos), if A = n-CyNcl(n-CyNδint(A)) ∪ n-CyNint(n-CyNδcl(A)),

(vii). M-open set (briefly, n-CyNMos), if A ⊆ n-CyNcl(n-CyNθint(A)) ∪ n-CyNint(n-CyNδcl(A)).

The complement of a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os&n-CyNeos) is called a n-CyNM (resp. n-CyNδ, n-CyNδP, n-CyNδS, n-CyNθ, n-CyNθS&n-CyNe) closed set (briefly, n-CyNMcs (resp. n-CyNδcs, n-CyNδPcs, n-CyNδScs, n-CyNθcs, n-CyNθScs &n-CyNecs)) in X.

The family of all n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os & n-CyNeos) of X is denoted by n-CyNMOS(X), (resp. n-CyNMCS(X), n-

CyNδOS(X), n-CyNδCS(X), n-CyNδ$\mathcal{P}$OS(X), n-CyNδPCS(X), n-CyNδSOS(X), n-CyNδSCS(X), n-CyNθOS(X), n-CyNθCS(X), n-CyNθSOS(X), n-CyNθSCS(X), n-CyNeOS(X)&n-CyNeCS(X)).

Definition 3.5 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-CyN

(i).M-interior (resp. n-CyNδ-interior, n-CyNδP-interior, n-CyNδS-interior, n-CyNθ-interior, n-CyNθS-interior & n-CyNe-interior) of A (briefly, n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδPint(A), n-CyNδSint(A), n-CyNθint(A), n-CyNθSint(A) & n-CyNeint(A)) is defined by n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδPint(A), n-CyNδSint(A), n-CyNθint(A), n-CyNθSint(A) & n-CyNeint(A)) $= \bigcup\{G : G \subseteq A$ and G is a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os&n-CyNeos) in X$\}$.

(ii). M-closure (resp. n-CyNδ-closure, n-CyNδP-closure, n-CyNδS-closure, n-CyNθ-closure, n-CyNθS-closure &n-CyNe-closure) of A (briefly, n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδPcl(A), n-CyNδScl(A), n-CyNθcl(A), n-CyNθScl(A)&n-CyNecl(A)) is defined by n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδPcl(A), n-CyNδScl(A), n-CyNθcl(A), n-CyNθScl(A)&n-CyNecl(A)) $= \bigcap\{K : K \subseteq A$ and K is a n-CyNMcs (resp. n-CyNδcs, n-CyNδPcs, n-CyNδScs, n-CyNθcs, n-CyNθScs & n-CyNecs) in X$\}$.

Definition 3.6 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN

(i). δ-continuous map (briefly, n-CyNδCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδos in (X, τ_1),

(ii). δ-pre continuous map (briefly, n-CyNδ$\mathcal{P}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{P}$os in (X, τ_1),

(iii). δ-semi continuous map (briefly, n-CyNδ$\mathcal{S}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{S}$os in (X, τ_1),

(iv). θ-continuous map (briefly, n-CyNθCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNθos in (X, τ_1),

(v). θ$\mathcal{S}$-continuous map (briefly, n-CyNθ$\mathcal{S}$Cts map), if the inverse image of every n-

CyNos in (Y, τ_2) is a n-CyNθ$\mathcal{S}$os in (X, τ_1),

(vi). e-continuous map (briefly, n-CyNeCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNeos in (X, τ_1),

(vii). M-continuous map (briefly, n-CyNMCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNMos in (X, τ_1).

Definition 3.7 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

(i). δ-irresolute map (briefly, n-CyNδIrr map), if $f^{-1}(B)$ is a n-CyNδos in (X, τ_1) for every n-CyNδosB of (Y, τ_2),

(ii). δ-pre irresolute map (briefly, n-CyNδ$\mathcal{P}$Irr map), if $f^{-1}(B)$ is a n-CyNδ$\mathcal{P}$os in (X, τ_1) for every n-CyNδ$\mathcal{P}$osB of (Y, τ_2),

(iii). δ-semi irresolute map (briefly, n-CyNδ$\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyNδ$\mathcal{S}$os in (X, τ_1) for every n-CyNδ$\mathcal{S}$osB of (Y, τ_2),

(iv). θ-irresolute map (briefly, n-CyNθIrr map), if $f^{-1}(B)$ is a n-CyNθos in (X, τ_1) for every n-CyNθosB of (Y, τ_2),

(v). θ-semi irresolute map (briefly, n-CyNθ$\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyNθ$\mathcal{S}$os in (X, τ_1) for every n-CyNθ$\mathcal{S}$osB of (Y, τ_2),

(vi). e-irresolute map (briefly, n-CyNeIrr map), if $f^{-1}(B)$ is a n-CyNeos in (X, τ_1) for every n-CyNeosB of (Y, τ_2),

(vii). M-irresolute map (briefly, n-CyNMIrr map), if $f^{-1}(B)$ is a n-CyNMos in (X, τ_1) for every n-CyNMosB of (Y, τ_2).

Definition 3.8 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

(i). δ-open mapping (briefly, n-CyNδO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNδos in (Y, τ_2),

(ii). δ- pre open mapping (briefly, n-CyNδ𝒫O map), if the image of every n-CyNos in (X, τ_1) is a n-CyNδ𝒫os in (Y, τ_2),

(iii). δ-semi open mapping (briefly, n-CyNδ𝒮O map), if the image of every n-CyNos in (X, τ_1) is a n-CyNδ𝒮os in (Y, τ_2),

(iv). θ-open mapping (briefly, n-CyNθO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNθos in (Y, τ_2),

(v). θ- semi open mapping (briefly, n-CyNθ𝒮O map), if the image of every n-CyNos in (X, τ_1) is a n-CyNθ𝒮os in (Y, τ_2).

(vi). e-open mapping (briefly, n-CyNeO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNeos in (Y, τ_2),

(vii). M-open mapping (briefly, n-CyNMO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNMos in (Y, τ_2).

Definition 3.9 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

(i). δ-closed mapping (briefly, n-CyNδC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNδcs in (Y, τ_2),

(ii). δ-pre closed mapping (briefly, n-CyNδ𝒫C map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNδ𝒫cs in (Y, τ_2).

(iii). δ-semi closed mapping (briefly, n-CyNδ𝒮C map), if the image of every n-CyNcs in

(X, τ_1) is a n-CyNδ$\mathcal{S}$cs in (Y, τ_2),

(iv). θ-closed mapping (briefly, n-CyNθC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNθcs in (Y, τ_2),

(v). θ-semi closed mapping (briefly, n-CyNθ$\mathcal{S}$C map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNθ$\mathcal{S}$cs in (Y, τ_2),

(vi). e-closed mapping (briefly, n-CyNeC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNecs in (Y, τ_2),

(vii). M-closed mapping (briefly, n-CyNMC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNMcs in (Y, τ_2).

Definition 3.10 A bijection map$f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a n-CyN (resp. n-CyNM) homeomorphism (briefly, n-CyNHom (resp. n-CyNMHom)) if f and f^{-1} are n-CyNCts (resp. n-CyNMCts) maps.

Theorem 3.1 Each n-CyNHom is a n-CyNMHom. But not conversely.

Proof. Let f be a n-CyNHom, then f and f^{-1} are n-CyNCts maps. But every n-CyNCtsmap is a n-CyNMCts map. Hence, f and f^{-1} are n-CyNMCts maps. Therefore, f is a n-CyNMHom.

Example 3.1 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and B_1, B_2, B_3, B_4 in (Y, τ_2) are defined as

$$A = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$
$$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$$
$$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$$
$$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

Then, we have $\tau_1 = \{0_X, 1_X, A\}$ and $\tau_2 = \{0_Y, 1_Y, B_1, B_2, B_3, B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNMHom but not n-CyNHom.

Theorem 3.2 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a bijective map. If f is a n-CyNMCts map then the statements are equivalent:

(i). f is a n-CyNMC map,

(ii). f is a n-CyNMO map,

(iii). f^{-1} is a n-CyNMHom.

Proof.

(i)⇒ (iii): Assume that f is a bijective map and a n-CyNMC map. Hence, f^{-1} is a n-CyNMCts map. We know that each n-CyNos in (X, τ_1) is a n-CyNMos in (Y, τ_2). Hence, f is a n-CyNMO map.

(ii)⇒ (iii): Let f be a bijective map and n-CyNO map. Further, f^{-1} is a n-CyNMCts map. Hence, f and f^{-1} are n-CyNMCts maps. Therefore, f is a n-CyNMHom.

(iii)⇒ (i): Let f be a n-CyNMHom. Then, f and f^{-1} are n-CyNMCts maps. Since, each n-CyNcs in (X, τ_1) is a n-CyNMcs in (Y, τ_2), f is a n-CyNMC map.

Definition 3.11 A n-CyNts(X, τ_1) is said to be a n-Cylindrical neutrosophic $MT_{\frac{1}{2}}$ (briefly, n-CyN$MT_{\frac{1}{2}}$)-space if every n-CyNMcs is n-CyNcs in (X, τ_1).

Theorem 3.3 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a n-CyNMHom. Then, f is a n-CyNHom if (X, τ_1) and (Y, τ_2) are n-CyN$MT_{\frac{1}{2}}$-space.

Proof. Assume that B is a n-CyNcs in (Y, τ_2). Then, $f^{-1}(B)$ is a n-CyNMcs in (X, τ_1). Since, (X, τ_1) is an n-CyN$MT_{\frac{1}{2}}$-space. $f^{-1}(B)$ is a n-CyNcs in (X, τ_1). Therefore, f is a n-CyNCts. By hypothesis, f^{-1} is a n-CyNMCts map. Let A be a n-CyNcs in (X, τ_1). Then,

$(f^{-1})^{-1}(A) = f(A)$ is a n-CyNcs in (Y, τ_2), by presumption. Since, (Y, τ_2) is a n-CyNM$T_{\frac{1}{2}}$-space. f(A) is a n-CyNcs in (Y, τ_2). Hence, f^{-1} is a n-CyNCts map. Hence, f is a n-CyNHom.

Theorem 3.4 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a n-CyNts. Then, the following statements are equivalent if (Y, τ_2) is a n-CyNM$T_{\frac{1}{2}}$-space.

(i). f is a n-CyNMC map.

(ii). If A is a n-CyNos in (X, τ_1), then the f(A) is a n-CyNMos in (Y, τ_2).

(iii). f(n-CyNint(A)) ⊆ n-CyNcl(N-CyNint(f(A))) for every n-CyNS A in (X, τ_1).

Proof.

(i)⇒ (ii): Obvious.

(ii)⇒ (iii): Let A be a n-CyNS in (X, τ_1). Then, n-CyNint(A) is a n-CyNos in (X, τ_1). Then, f(N-CyNint(A)) is a n-CyNMos in (Y, τ_2). Since, (Y, τ_2) is a n-CyNM$T_{\frac{1}{2}}$-space, so h(n-CyNint(A)) is a n-CyNos in (Y, τ_2). Therefore, f(n-CyNint(A)) = n-CyNint(f(n-CyNint(A))) ⊆ n-CyNcl(n-CyNint(f(A))).

(iii)⇒ (i): Let A be a n-CyNcs in (X, τ_1). Then, A^c is a n-CyNos in (X, τ_1). From, f(n-CyNint$(A)^c$) ⊆ n-CyNcl(n-CyNint$(f(A)^c)$), $f(A^c)$ ⊆ n-CyNcl(n-CyNint$(f(A^c))$). Therefore, $f(A^c)$ is a n-CyNMos in (Y, τ_2). Therefore, f(A) is a n-CyNMcs in (X, τ_1). Hence, f is a n-CyNC map.

Theorem 3.5 Let $f: (X, \tau_1) \to (Y, \tau_2)$ and $g: (Y, \tau_2) \to (Z, \tau_3)$ be n-CyNMC map, where (X, τ_1) and (Z, τ_3) are two n-CyNts 's and (Y, τ_2) is a n-CyNM$T_{\frac{1}{2}}$ -space, then thecomposition $g \circ f$ is a n-CyNMC map.

Proof. Let A be a n-CyNcs in (X, τ_1). Since, f is a n-CyNMcs and f(A) is a n-CyNMcs in (Y, τ_2), by assumption, f(A) is a n-CyNcs in (Y, τ_2). Since, g is a n-CyNMcs, then g(f(A)) is a n-CyNMcs in (Z, τ_3) and $g(f(A)) = g \circ f(A)$. Therefore, $g \circ f$ is a n-CyNMC map.

Theorem 3.6 Let $f:(X,\tau_1)\to(Y,\tau_2)$ and $g:(Y,\tau_2)\to(Z,\tau_3)$ be two n-CyNts's then, the following statements are hold:

(i). If $g\circ f$ is a n-CyNMO map and f is a n-CyNCts map, then g is a n-CyNMO map.

(ii). If $g\circ f$ is a n-CyNO map and g is a n-CyNMCts map, then f is a n-CyNMO map.

Proof. Obvious.

1. n -Cylindrical Neutrosophic M C Homeomorphisms

In this section, we introduce n-Cylindrical neutrosophic M C homeomorphism and look at some of its feature in this section.

Definition 4.1 A bijection mapf: $(X,\tau_1)\to(Y,\tau_2)$ is called a n-Cylindrical neutrosophic (resp. M) C homeomorphism (briefly, n-CyNCHom (resp. n-CyNMCHom)) if f and f^{-1} are n-CyNIrr (resp. n-CyNMIrr) maps.

Theorem 4.1 Each n-CyNMCHom is a n-CyNMHom. But not conversely.

Proof. Let us assume that B is a n-CyNcs in (Y,τ_2). This shows that B is a n-CyNMcs in (Y,τ_2). By assumption, $f^{-1}(B)$ is a n-CyNMcs in (X,τ_1). Hence, f is a n-CyNMCts map. Hence, f and f^{-1} are n-CyNMCts maps. Hence, f is a n-CyNMHom.

Example 4.1 Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4, A_5, A_6 in (X,τ_1) and B_1, B_2, B_3, B_4, B_5, B_6 in (Y,τ_2) are defined as

$$A_1=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$
$$A_2=\{\langle x_1,0.1150731,0.50,0.1950739\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\},$$
$$A_3=\{\langle x_1,0.1950739,0.50,0.1150731\rangle,\langle x_2,0.1750737,0.50,0.1350733\rangle\},$$
$$A_4=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\},$$

$A_5 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1650736\rangle\}$,

$A_6 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1350733\rangle\}$,

$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\}$,

$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$B_5 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1650736\rangle\}$,

$B_6 = \{\langle y_1, 0.1850738, 0.50, 0.1250732\rangle, \langle y_2, 0.1650736, 0.50, 0.1350733\rangle\}$.

Then, we have $\tau_1 = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B_1, B_2, B_3, B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNMCHom but not n-CyNMHom.

Theorem 4.2 If $f: (X, \tau_1) \to (Y, \tau_2)$ is a n-CyNMCHom, then n-CyNMcl(f^{-1}(B)) $\subseteq$ f^{-1}(N-CyNcl(B)) for each n-CyNSB in (Y, τ_2).

Proof. Let B be a n-CyNS in (Y, τ_2). Then, n-CyNcl(B) is a n-CyNcs in (Y, τ_2) and every n-CyNcs is a n-CyNMcs in (Y, τ_2). Assume f is a n-CyNMIrr map and f^{-1}(n-CyNcl(A)) is a n-CyNMcs in (X, τ_1). Then, n-CyNcl(f^{-1}(n-CyNcl(B))) = f^{-1}(n-CyNcl(B)). Here, n-CyNMcl(f^{-1}(B)) $\subseteq$ n - CyNMcl(f^{-1}(n - CyNcl(A))) = f^{-1}(n - CyNcl(B)) . Here, n - CyNMcl(f^{-1}(B)) $\subseteq$ n - CyNMcl(f^{-1}(n - CyNcl(A))) = f^{-1}(n - CyNcl(B)) . Therefore, n - CyNMcl(f^{-1}(B)) $\subseteq$ f^{-1}(n-CyNcl(B)) for every n-CyNS B in (Y, τ_2).

Theorem 4.3 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a n-CyNMCHom. Then, n-CyNMcl(f^{-1}(B)) = f^{-1}(n-CyNMcl(B)) for each n-CyNS B in (Y, τ_2).

Proof. Since, f is a n-CyNMCHom. f is a n-CyNMIrr map. Let B be a n-CyNS in (Y, τ_2). Clearly, n-CyNMcl(B) is a n-CyNMcs in (Y, τ_2). Then, n-CyNMcl(B) is a n-CyNMcs in (Y, τ_2). Since, f^{-1}(B) $\subseteq$ f^{-1}(n-CyNMcl(B)) then n-CyNMcl(f^{-1}(B)) $\subseteq$ n-CyNMcl(f^{-1}(n-CyNMcl(B))) = f^{-1}(n-CyNMcl(B)). Therefore, n-CyNMcl(f^{-1}(B)) $\subseteq$ f^{-1}(n-CyNMcl(B)). Let f be a n-CyNMCHom. f^{-1} is a n-CyNMIrr map. Let us consider n-CyNSs f^{-1}(B) in (X, τ_1), which implies n - CyNMcl(f^{-1}(B)) is a n - CyNMcs in (X, τ_1). Hence, n - CyNMcl(f^{-1}(B)) is a n - CyNMcs in (X, τ_1). This, implies that $(f^{-1})^{-1}$(n - CyNMcl(f^{-1}(B))) = f(n - CyNMcl(f^{-1}(B))) is a n - CyNMcs in (Y, τ_2). This proves B = $(f^{-1})^{-1}$(f^{-1}(B)) $\subseteq$ $(f^{-1})^{-1}$(n - CyNMcl(f^{-1}(B))) = f(n - CyNMcl(f^{-1}(B))). Therefore, n - CyNMcl(B) $\subseteq$ n-CyNMcl(f(n-CyNMcl(f^{-1}(B)))) = f(n-CyNMcl(f^{-1}(B))), since f^{-1} is a n-

CyFNMIrr map. Hence, $f^{-1}(n\text{ - }CyNMcl(B)) \subseteq f^{-1}(f(n\text{ - }CyNMcl(f^{-1}(B)))) = n\text{ - }CyNMcl(f^{-1}(B))$. That is, $f^{-1}(n\text{ - }CyNMcl(B)) \subseteq n\text{ - }CyNMcl(f^{-1}(B))$. Hence, $n\text{ - }CyNMcl(f^{-1}(B)) = f^{-1}(n\text{-}CyNMcl(B))$.

Theorem 4.4 If $f: (X, \tau_1) \to (Y, \tau_2)$ and $g: (Y, \tau_2) \to (Z, \tau_3)$ are n-CyNMCHom's then $g \circ f$ is a n-CyNMCHom.

Proof. Let f and g be two n-CyNMCHom's. Assume B is a n-CyNMcs in (Z, τ_3). Then, $g^{-1}(B)$ is a n-CyNMcs in (Y, τ_2). Then, by hypothesis, $f^{-1}(g^{-1}(B))$ is a n-CyNMcs in (X, τ_1). Hence, $g \circ f$ is a n-CyNMIrr map. Now, let A be a n-CyNMcs in (X, τ_1). Then, by presumption, f(A) is a n-CyNMcs in (Y, τ_2). Then, by hypothesis, g(f(A)) is a n-CyNMcs in (Z, τ_3). This implies that $g \circ f$ is a n-CyNMIrr map. Hence, $g \circ f$ is a n-CyNMCHom.

2. Conclusion

In this paper, we introduced and examined the concepts of n-CyNHom, n-CyNMHom, and n-CyNM$T_{\frac{1}{2}}$-spaces within the framework of n-cylindrical neutrosophic topological structures. Furthermore, the notion of n-CyNMCHom was explored in detail, and several of its fundamental properties were established.

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