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About K-Dependent Isolate Inclusive Sets In Graphs

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Abstract:

In this paper, we introduce the concept of k-dependent isolate inclusive sets in graphs and establish the necessary and sufficient conditions for a set $S$to be a maximal k-dependent set. We then characterize 1-maximal k-dependent inclusive sets and prove that if $k \geq 1$and $S \subseteq V(G)$is a 1-maximal k-dependent inclusive set of G, then Sforms a k-dominating set of G. Furthermore, we show that if $u$and $v$are k-dependent vertices within S, then the degree of $u$equals the degree of v. In addition, we examine the impact of vertex removal and edge removal on k-dependent isolate inclusive sets. Finally, we introduce the notion of the k-dependent isolate inclusive bondage number of a graph, defined as the minimum number of edges whose removal increases the k-dependent isolate inclusive number of the graph.

Keywords: k-dependent set, maximum k-dependent set, maximal k-dependent set, k-dependent vertex, k-dependent inclusive set, 1-maximal k-dependent inclusive set, k-dependent inclusive bondage number.

1. Introduction:

In this paper, we have defined a new concept called k-dependent isolate inclusive sets in graphs. we have also defined maximum and 1-maximal k-dependent isolate inclusive sets in graphs. We have proved a characterization of a 1-maximal k-dependent isolate inclusive sets of a graph can have degree of each k-dependent vertex of S is $k-1$ in the induced subgraph S. we have also proved several results related to the effect of vertex removal, edge removal and edge addition on k-dependent inclusive number of a graph. In addition, we introduce a new graph invariant called the k-dependent isolate inclusive bondage number. This parameter is defined as the minimum number of edges whose removal increases the k-dependent isolate inclusive number of the graph.

2. Preliminaries and Notations:

If G is a graph, then $V(G)$ denotes the vertex set of the graph G and $E(G)$ denote the edge set of the graph G. If v is vertex of the graph G then $G-v$ is the subgraph of G induced by all the vertices different from v.
We will consider only simple undirected graphs with finite vertex set.

3. Definitions and examples:

Definition 3.1 **(k-dependent set):**

Let G be a graph and $S \subset V(G)$ and suppose $k \geq 1$. Then S is said to be a k-dependent set if every vertex of S is adjacent to atmost $k-1$ vertices of S.

If S is a non dominating set and T ⊂ S. Then T is a non dominating set. Therefore, to be non dominating set is a hereditary property.

Definition 3.2 **(maximum k-dependent set):**

A k-dependent set with maximum cardinality is called a maximum k-dependent set and its cardinality is called k-dependent number of the graph and it is denoted as $\beta_k(G)$.

Note that, every k-dependent set is also a $k+1$ dependent set.
Let G be a graph and $e = \{u, v\}$ be any edge of G. Let $k = 2$. Let $S = \{u, v\}$ then S is a 2-dependent set of G.
Example 1: Consider the graph G mention below

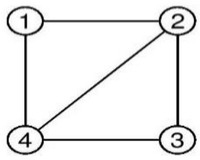

Figure 1: Graph *G*

Let $\mathrm{k} = 3$ and $S = \{1, 2, 4\}$ then S is a 3-dependent set of G.
Let $T = \{1, 2, 3, 4\}$ then T is not a 3-dependent set because 2 is adjacent to 3 vertices of T.
Also, note that $\beta_k(G) = 3$.
Note that to be k-dependent is hereditary property but it is not a super hereditary property.

Definition 3.3 **(maximal k-dependent set):**

Let G be a graph and $S \subset V(G)$ and $k \geq v$. Suppose S is said to be a k-dependent set if every $v \in V(G) - S,\ S \cup \{v\}$ is not a k-dependent set.

Definition 3.4 **(k-dependent vertex):**

Let G be a graph and $S \subset V(G)$ and $k \geq 1$. Then v is said to be a k-dependent vertex of S if v is adjacent to at most $k-1$ vertices of S.

If $S = V(G)$ then we say that v is a k-dependent vertex of G.
Note that, if $k = 1$ then v isolated vertex of G. Also, note that v is a k-dependent vertex of G iff $d(v) \leq k - 1$.

Example 2: Consider the cycle graph C_5 with 5 vertices $\{1,2,3,4,5\}$

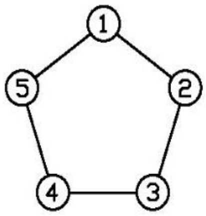

Figure 2: Cycle graph C_5

Let $k=2$ and $S=\{1,2,3\}$. Then 1 and 3 are 2-dependent vertices of S, while 2 is not 2-dependent vertex of S.

Definition 3.5 **(k-dependent inclusive set):**

Let G be a graph, $S\subset V(G)$ and $k\geq 1$. Then S is said to be k-dependent inclusive set, if it contains k-dependent vertex.

Example 3: Consider the cycle graph C_5 with 5 vertices $\{1,2,3,4,5\}$

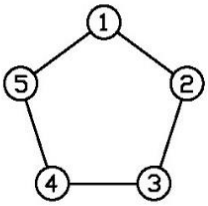

Figure 3: Cycle graph C_5

Let $k=1$ and $S=\{1,2,3\}$ then S is said to be 1-dependent inclusive set, but it is not 2-dependent inclusive set.

Also, note that every vertex of k-dependent set is a k-dependent inclusive set of that set.

Example 4: Consider the cycle graph C_5 with 5 vertices $\{1,2,3,4,5\}$

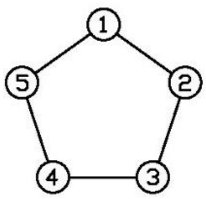

Figure 4: Cycle graph C_5

Let $S=\{1,2,4\}$ and let $k=1$.

Obviously, S is a 1-dependent inclusive set.

Let $T=\{1,2,3,4\}$.

Then $S \subset T$ but T is not a 1-dependent inclusive set.

Let $T_1 = \{1, 2\}$.
Then $T_1 \subset S$ but $T_1$is not a 1-dependent inclusive set.
Example 5: Consider the graph with 4 vertices $\{1, 2, 3, 4\}$

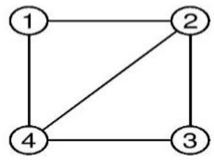

Figure 5: Graph G

Let $k = 2$ and $S = \{1, 2, 3, 4\}$ then S is said to be 2-dependent inclusive set.
Let $S_1 = \{1, 2, 3\}$. Then S_1 is not a 2-dependent inclusive set of G because $< S_1 >$ does not 2-dependent vertex.
Also S_1 is subset of S.
Thus, 2-dependent inclusive set is not hereditary property.

Example 6: Consider the cycle graph C_4 with 4 vertices $\{1, 2, 3, 4\}$

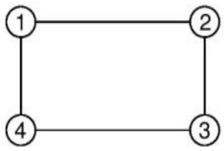

Figure 6: Cycle graph C_4

Let $k = 2$ and $S = \{4, 1, 2\}$ then S is said to be 2-dependent inclusive set.
Let $S_1 = \{1, 2, 3, 4\}$. Then S_1 is not a 2-dependent inclusive set.

Thus, to be 2-dependent inclusive set is not super hereditary property.

***Definition 3.6* (1-maximal k-dependent inclusive set):**
Let G be a graph, $k \geq 1$ and $S \subset V(G)$ be a k-dependent inclusive set of G. Then S is said to be 1-maximal k-dependent inclusive set of G if $S \cup \{v\}$ is not a k −dependent inclusive set of G, for every $v \in V(G) - S$.

4. Main Results:

Theorem 4.1: Let G be a graph and $S \subset V(G)$ be a k-dependent set and $k \geq 1$. Then S is a maximal k-dependent set if and only if for each $V(G) - S$, at least one of the following two condition is holds:

(1) There is a vertex u in S which is adjacent to v and which is adjacent to exactly $k-1$ vertices of S.
(2) v is adjacent to at least k vertices of S.

Proof: Suppose S is a 1-maximal k-dependent set.
Let $v \in \mathrm{V(G)} - \mathrm{S}$.
Since $\mathrm{S} \cup \{v\}$ is not a k-dependent set. There is vertex x in $\mathrm{S} \cup \{v\}$ such that x is adjacent to at least k vertices of $\mathrm{S} \cup \{v\}$.

Case (1): suppose $x = v$.
Then v is adjacent to at least k vertices of S.
Thus, the condition (2) is satisfied.

Case (2): suppose $x \neq v$.
Let $x = u$.
Then $u \in S$ and u is adjacent to at least k vertices of $S \cup \{v\}$.

Now u is adjacent to at least $k-1$ vertices of S and by the above statement u is adjacent to at least k vertices of $S \cup \{v\}$.

Therefore, u is adjacent to v and u is adjacent to exactly $k-1$ vertices of S.
Thus, one of the conditions (1) and (2) holds.

Conversely, suppose $v \in \mathrm{V(G)} - \mathrm{S}$.
Suppose conditions (2) holds.
Then v is adjacent to at least k vertices of S and therefore $S \cup \{v\}$ is not a k-dependent set.
Suppose conditions (1) holds.
Then u is a vertex of $S \cup \{v\}$ such that u is adjacent to excatly k vertices of $S \cup \{v\}$ and therefore $S \cup \{v\}$ is not a k-dependent set.

Thus, S is a maximal k-dependent set. ▌

Corollary 4.2: Let G be a graph, $k \geq 1$ and $\mathrm{S} \subset \mathrm{V(G)}$ be a maximal k-dependent set. Then S is a dominating set.

Proof: Obvious.

Now we characterize 1-maximal k-dependent inclusive set in graphs.

Theorem 4.3: Let G be a graph, $k \geq 1$ and $\mathrm{S} \subset \mathrm{V(G)}$ be a k-dependent inclusive set of G. Then S is a 1-maximal k-dependent inclusive set of G if and only if for each $v \in \mathrm{V(G)} - \mathrm{S}$. The following two conditions are satisfied:
(1) v is adjacent to at least k vertices of S.

(2) Every k-dependent vertex of S has degree exactly $k-1$ in the $< S >$ and it is adjacent to v.

Proof: Suppose S is a 1-maximal k-dependent inclusive set of G. Let $v \in \mathrm{V(G)} - \mathrm{S}$.

(1) Now $\mathrm{S} \cup \{v\}$ is not a k-dependent inclusive set of G.
Therefore, $d(v) \geq k$ in the $< S \cup \{v\} >$.
Therefore, v is adjacent to at least k vertices of S.
Thus, the condition (1) is satisfied.

(2) Suppose there is a vertex z in S, which is a k-dependent vertex of S and $d(z)$ in the $< S >$ is less than $k-1$.

Now z is not a k-dependent vertex of $S \cup \{v\}$.
Therefore $d(z)$ in the $S \cup \{v\} \geq k$.

However, $d(z)$ in the $< S \cup \{v\} >$ is less than or equal to $k-1$.
Which is contradiction.
Therefore, degree of each k-dependent vertex of S is $k-1$ in the $< S >$.

Now, z is not a k-dependent vertex of $S \cup \{v\}$.
Therefore z is adjacent to at least k vertices of $S \cup \{v\}$.

However, $d(z)$ in the $< S >$ is equal to $k-1$.
Therefore, z must be adjacent to v.

Conversely, suppose for each $v \in \mathrm{V(G)} - \mathrm{S}$, conditions (1) and (2) are satisfied.

Let $v \in \mathrm{V(G)} - \mathrm{S}$.

Since condition (1) is satisfied, v is not a k-dependent vertex of $S \cup \{v\}$.

Therefore v is adjacent to at least k vertices of S.

Let $z \in \mathrm{S} \cup \{v\}$ such that $z \neq v$.
If z is not a k-dependent vertex of S, then z is also not a k-dependent vertex of $S \cup \{v\}$.

Suppose z is a k-dependent vertex of S.
By condition (2), $d(z) = k-1$ and v is adjacent to z.
Therefore, $d(z)$ in the $S \cup \{v\}$ is equal to k.
$\therefore z$ is not a k-dependent vertex of $S \cup \{v\}$.
Thus, $S \cup \{v\}$ does not contain any k-dependent vertex.

Therefore, S is a 1-maximal k-dependent inclusive set of G.
Thus, the theorem is proved. ▌

Corollary 4.4: Let G be a graph, $k \geq 1$ and $\mathrm{S} \subset \mathrm{V(G)}$ be a 1-maximal k-dependent inclusive set of G. Then S is a k-dominating set of G.

Proof: Let $v \in \mathrm{V(G)} - \mathrm{S}$, by condition (1) of the above theorem-3, v is adjacent to at least k vertices of S.
Thus, S is a k-dominating set of G. ▌

Corollary 4.5: Let G be a graph, $k \geq 1$ and $\mathrm{S} \subset \mathrm{V(G)}$ be a 1-maximal k-dependent inclusive set of G. Then S is a k-dominating set of G. If u and v are k-dependent vertices of S then $d(u) = d(v)$.

Proof: Note that since $v \in S$ is a k-dependent vertices of S, v is adjacent to at least $k - 1$ vertices of S.
Also, by condition (1) of the above theorem-3, v is adjacent to every vertex of $V(G) - S$.
Therefore, $d(v) = |N(v)| = k - 1 + |\mathrm{V(G)} - \mathrm{S}|$.

Similarly, for u also, $d(u) = |N(u)| = k - 1 + |\mathrm{V(G)} - \mathrm{S}|$.
Therefore, $d(u) = d(v) = k - 1 + |\mathrm{V(G)} - \mathrm{S}|$. ▌

Corollary 4.6: Let G be a graph, $k \geq 1$ and S be a 1-maximal k-dependent inclusive set of G. Then for each $x \in \mathrm{V(G)} - \mathrm{S}, d(x) \geq$ the number of k-dependent vertices of S.

Proof: x is adjacent to every k-dependent vertex of S.
Therefore, $d(x) \geq$ the number of k-dependent vertices of S. ▌

Corollary 4.7: Let G be a graph, $k \geq 1$ and $\mathrm{S} \subset \mathrm{V(G)}$ be a proper 1-maximal k-dependent inclusive set of G. Then the number of k-dependent vertices of $S \leq \Delta(G)$.

Proof: Let $x \in \mathrm{V(G)} - \mathrm{S}$.
Then the number of k-dependent vertices of $S \leq d(x) \leq \Delta(G)$. ▌

Remarks:

(1) The above corollary says that a proper 1-maximal k-dependent inclusive set cannot contain more than $\Delta(G)$, k-depedent vertices.

(2) If S is not a proper 1-maximal k-dependent set then above statement may not be true.

Vertex removal and k-dependent inclusive set

Proposition 4.8: Let G be a graph, $k \geq 1$ and $v \in \mathrm{V(G)}$. Then $\beta_{ik}(G - v) \leq \beta_{ik}(G)$.

Proof: Let S be a maximum k-dependent inclusive set of $G-v$.
It is obvious that S is a k-dependent inclusive set of G also.
Therefore, $\beta_{ik}(G-v)=|S|\leq\beta_{ik}(G)$.

Thus, $\beta_{ik}(G-v)\leq\beta_{ik}(G)$. ▌

Proposition 4.9: Let G be a graph, $k\geq 1$ and $v\in$ V(G) be such that $d(v)<k$. Then $\beta_{ik}(G-v)<\beta_{ik}(G)$.

Proof: Let S be a maximum k-dependent inclusive set of $G-v$.
Since v is adjacent to at most $k-1$ vertices of $S, S\cup\{v\}=T$ is a k-dependent inclusive set of G.
Therefore, $\beta_{ik}(G)\geq T>S=\beta_{ik}(G-v)$.

Thus, $\beta_{ik}(G)>\beta_{ik}(G-v)$. ▌

Theorem 4.10: Let G be a graph, $k\geq 1$ and $v\in$ V(G). Then $\beta_{ik}(G-v)<\beta_{ik}(G)$ if and only if $v\in$ S, for every maximum k-dependent inclusive set of G.

Proof: First suppose that $\beta_{ik}(G-v)<\beta_{ik}(G)$.
Suppose for some k-dependent inclusive set of G, $v\notin S$.
Then S is also k-dependent inclusive set of $G-v$.
Since $\beta_{ik}(G-v)\leq\beta_{ik}(G)$, S must be a maximum k-dependent inclusive set of $G-v$.
Therefore, $\beta_{ik}(G-v)=|S|=\beta_{ik}(G)$ which is contradict our hypothesis.

Thus, $v\in$ S, for every maximum k-dependent inclusive set of G.

Conversely, suppose $v\in$ S, for every maximum k-dependent inclusive set of G.

Suppose $\beta_{ik}(G-v)=\beta_{ik}(G)$.
Let T is a maximum k-dependent inclusive set of $G-v$.
Since $\beta_{ik}(G-v)=\beta_{ik}(G)$, T is also a maximum k-dependent inclusive set of G.
Also note that $v\notin T$.
This is a contradiction.

Therefore $\beta_{ik}(G-v)<\beta_{ik}(G)$. ▌

Edge removal and k-dependent inclusive set

Now we consider the operation of edge removing from the graph G and its effect on k-dependent inclusive number of the graph.

Proposition 4.11: Let G be a graph, $k\geq 1$ and let e be an edge of G. Then $\beta_{ik}(G-e)\geq\beta_{ik}(G)$.

Proof: Let S is a maximum k-dependent inclusive set of G.
It is obvious that every k-dependent vertex of $< S >$ in G will remain k-dependent vertex of $< S >$ in $G - e$.

Therefore S will remain a k-dependent inclusive set of $G - e$.
$\therefore \beta_{ik}(G - e) \geq |S| = \beta_{ik}(G)$.

Thus, $\beta_{ik}(G - e) \geq \beta_{ik}(G)$. ▌

Example 7: Consider the graph G mention below

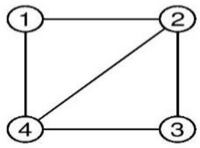

Figure 7: Graph *G*

Let k $= 2$ and $S = \{1, 2, 4\}$ then S is a maximum k-dependent inclusive set of G.
Therefore $\beta_{ik}(G) = |S| = 3$.
Let $e = \{1, 2\}$
Here, $T = \{1, 2, 3, 4\}$ then T is a maximum k-dependent inclusive set of $G - e$.
Therefore $\beta_{ik}(G) = |T| = 4$.
Thus, in this example $\beta_{ik}(G) < \beta_{ik}(G - e)$.

In this example, if $f = \{2, 4\}$ then $\beta_{ik}(G - f) = 30$

Thus, here $\beta_{ik}(G - f) = \beta_{ik}(G)$.
Now we state and prove a necessary and sufficient condition under which the k-dependent inclusive number of a graph increases when an edge remove from the graph.

Theorem 4.12: Let G be a graph, $k \geq 1$ and let $e = \{u, v\}$ be an edge of G. Then $\beta_{ik}(G - e) > \beta_{ik}(G)$ if and only if there is a subset S of $V(G)$ such that $|S| > \beta_{ik}(G), u, v \in S$ and $d(u) = k$ or $d(v) = k$ in the $< S >$.

Proof: First suppose that condition is satisfied.
Let S be a subset of $V(G)$ such that the above requirement are satisfied.
Now, S is a subset of $V(G - e)$ and $d(u) = k - 1$, if $d(u) = k$ in G or $d(v) = k - 1$, if $d(v) = k$ in G.

Therefore S is a k-dependent inclusive set of G.

Thus $\beta_{ik}(G - e) \geq |S| > \beta_{ik}(G)$.

Conversely, suppose $\beta_{ik}(G - e) > \beta_{ik}(G)$.

Let S is a maximum k-dependent inclusive set of $G - e$.
Then $|S| > \beta_{ik}(G)$.

Therefore S cannot be a k-dependent inclusive set of G.
$\therefore d(x) \geq k$ (in G) for every vertex x of S.

However, there is a vertex z of S which is a k-dependent vertex in the $< S >$ in $G - e$. Therefore $z = u$ or $z = v$ and $d(z) = k - 1$ in $G - e$.

$\therefore d(z) = k$ in G.
Also, if $u \notin S$ or $v \notin S$ then S remains k-dependent inclusive set of G.
Which is a contradiction.

Therefore, $u \in$ S and $v \in$ S. ▌

Lemma 4.13: Let G be a graph and H is a spanning subgraph of G. Then $\beta_{ik}(G) \leq \beta_{ik}(H)$

Proof: Let S is a maximum k-dependent inclusive set of G.
Then S is also a maximum k-dependent inclusive set of H.
Therefore $\beta_{ik}(H) \geq |S| = \beta_{ik}(G)$.

Thus, $\beta_{ik}(H) \geq \beta_{ik}(H)$. ▌

We have proved in theorem 4.8 that if $v \in$ V(G) and $v \notin T$ for some maximum k-dependent inclusive set of G then $\beta_{ik}(G - v) = \beta_{ik}(G)$.

k-dependent inclusive bondage number of a graph

Definition 3.7 **(k-dependent inclusive bondage number):**

Let G be a graph and $k \geq 1$, the minimum number of edges whose removal increases the k-dependent inclusive number of the graph is called k-dependent inclusive bondage number of the graph and it is denoted as $B_{ik}(G)$.

Let G be a graph and $v \in$ V(G) be such that $\beta_{ik}(G - v) = \beta_{ik}(G)$.
Let F be the set of all edges whose end vertex is v.

Now consider the spanning subgraph G − F obtain by removing all the edges F from G.

By proposition-4.11, $\beta_{ik}(G - F) \geq \beta_{ik}(G)$. Also note that v is an isolated vertex of G − F. Therefore v lies in every maximum k-dependent inclusive set of $G - F$.
Thus $\beta_{ik}(G - F) - v < \beta_{ik}(G - F)$.

It is noted that, $\beta_{ik}(G - F) - v = G - v$.
$\therefore \beta_{ik}(G - v) < \beta_{ik}(G - F)$.

Since $\beta_{ik}(G - v) = \beta_{ik}(G), \beta_{ik}(G) < \beta_{ik}(G - F)$.
Thus, we have proved that $\beta_{ik}(G) \leq$ minimum of $\{d(v) \ni \beta_{ik}(G - v) = \beta_{ik}(G)\}$.

Now we consider the operation of edge addition and its effect on the k-dependent inclusive number of a graph.

Proposition 4.14: Let G be a graph, $k \geq 1$ and u, v be non-adjacent vertices of G. Then $\beta_{ik}(G + e) \leq \beta_{ik}(G)$.

Proof: Let F is a minimum k-dependent inclusive set of G.
Then there is a vertex x in F such that $d(x)$ in $G + e \leq k - 1$
since e is not edge of $G, d(x) \leq k - 1$ in G also and $x \in F$.

Thus, F is a k-dependent inclusive set of G.

Therefore, $\beta_{ik}(G) \geq |F| = \beta_{ik}(G + e)$.

Thus, $\beta_{ik}(G + e) \leq \beta_{ik}(G)$. ▌

Now, we state necessary and sufficient conditions under which the k-dependent inclusive number of a graph decreases when an edge added to the graph.

Necessary and sufficient conditions:

Let G be a graph, $k \geq 1$ and u, v be non-adjacent vertices of G. Then $\beta_{ik}(G + e) \leq \beta_{ik}(G)$ if and only if for every maximum k-dependent inclusive set S of G. The following two conditions are satisfied:
(1) $u, v \in S$.
(2) $d(u) = k - 1$ and $d(v) \geq k - 1$ or $d(u) \geq k - 1$ and $d(v) = k - 1$ in the $< S >$ in G.

5. Conclusion:

This investigation introduces the concept of k-dependent isolate inclusive sets in graphs, provides characterization for both maximal and 1- maximal k-dependent inclusive sets and proves that the latter are also k-dependent isolate inclusive sets under certain conditions. The study also examines the impact of vertex removal and edge removal on these sets and defines a new graph parameter called the k-dependent isolate inclusive bondage number, which is the minimum number of edges whose removal increases the k-dependent isolate inclusive number of the graph.

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