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Strongly Nowhere Dense set via Generalized Intuitionistic Topological Space

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Abstract:

Our scope of this article is to create a different operators such as strongly μ_I g-nowhere dense set, strongly μ_I g-first category set, strongly μ_I g-second category set and strongly μ_I g-residual set on GITS. Also we discuss their natures of Strongly μ_I g-Baire space and explain how to correlate the above operators with the other Kuratowski operators in GITS with clear cut examples.

Keywords: μ_Ig-SNWDS, μ_Ig-SFCS, μ_Ig-SSCS, μ_Ig-SRS, μ_Ig-SBS, E, O, Ứ.

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1.Introduction: In 1899, Rene Louis Baire introduced the terms of first and second category sets. In classical Topology Baire space named in honor of Rene Louis Baire. The concept of Fuzzy Baire Space was derivated by G.Thangaraj&S.Anjalmose and also they were discuss the characterizations of strongly Fuzzy nowhere dense sets. In Intuitionistic set $A \cap \bar{A} \neq \phi_{\sim}$, so we using that result and derived the definition of strongly μ_Ig-nowhere dense set from that condition as well as talk about strongly μ_Ig-baire space in GITS. In this paper we demonstrate their basic properties.

2.Primary Needs: On the whole paper, we discussed the non-void set X and mentioned GITS (X, μ_I) as X.

Definition:2.1[6]Let $\mu_I$be the collection of intuitionistic subsets of X. Then X is said to be GITS if $\phi_{\sim} \in \mu_I$ and μ_I is closed under arbitrary unions. Then the elements of μ_I are called μ_I-open and their complements are named as μ_I-closed sets.

Definition:2.2[6] The μ_I-closure and μ_I-interior are defined as follows: $c_{\mu_I}(A) = \cap\{F: F$ is μ_I-closed set and $A \subseteq F\}$ and $i_{\mu_I}(A) = \cup\{G: G$ is μ_I-open set, $G \subseteq A\}$.

Definition:2.3[6]If $c_{\mu_I}(A) \subseteq U$ whenever $A \subseteq U$ where U is μ_I- open set in X then $A \subseteq X$ is called $\mu_I g$-closed set (μ_Ig-CSGITS). Also $c^*_{\mu_I}(A)$ and $i^*_{\mu_I}(A)$ are defined as follows, $c^*_{\mu_I}(A) = \cap\{F: F$ is $\mu_I g$-CSGITS and $A \subseteq F\}$ and $i^*_{\mu_I}(A) = \cup\{G: G$ is $\mu_I g$-open set (μ_Ig-OSGITS), $G \subseteq A\}$.

Definition:2.4[3] The $\mu_I g$-Frontier, $\mu_I g$-Exterior and $\mu_I g$-border is defined as follows: $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A)$, $E^*_{\mu_I}(A) = i^*_{\mu_I}(\bar{A})$ and $b^*_{\mu_I}(A) = A - i^*_{\mu_I}(A)$.

Definition:2.5[4] If$c^*_{\mu_I}(A) = X_\sim$ (resp.$c^*_{\mu_I}(\bar{A}) = X_\sim$) then A is named as μ_Ig-DGITS (resp.μ_Ig-CDGITS).

Definition:2.6[4] $A \subset X$ is said to be μ_I-nowhere dense set in GITS if $i_{\mu_I}(c_{\mu_I}(A)) = \phi_\sim$.

Definition:2.7[4] A subset A of X is said to be μ_Ig-NDGITS if the μ_Ig-closure of A contains no μ_Ig- interior points or $i^*_{\mu_I}(c^*_{\mu_I}(A)) = \phi_\sim$.

Definition:2.8[4]An intuitionistic set (IS)A in X is called $\mu_I g$-FCGITS if A=$\bigcup_{i=1}^{\infty} B_i$,where $B_i \in Nd^*(\mu_I)$. Remaining sets in X are said to be of $\mu_I g$-SCGITS. The complement of $\mu_I g$-FCGITS is called a $\mu_I g$-residual set in X. The pair(X, μ_I) is said to be a $\mu_I g$-Baire space if$i^*_{\mu_I}(\bigcup_{i=1}^{\infty} A_i) = \phi_\sim$, where $A_i \in Nd^*(\mu_I)$.

Theorem:2.9[6] If A is $\mu_I g$-CSGITS (resp.$\mu_I g$-OSGITS) then $c^*_{\mu_I}(A) = A$ (resp.$i^*_{\mu_I}(A) = A$).

Proposition:2.10[4] Every subset of a μ_Ig-NDGITS is a μ_Ig-NDGITS.

Proposition:2.11[6](a)$c^*_{\mu_I}(\bar{A}) = \overline{i^*_{\mu_I}(A)}$; (b)$\overline{c^*_{\mu_I}(A)} = i^*_{\mu_I}(\bar{A})$; (c)$\overline{c^*_{\mu_I}(\bar{A})} = i^*_{\mu_I}(A)$; (d) $c^*_{\mu_I}(A) = \overline{i^*_{\mu_I}(\bar{A})}$.

Proposition:2.12[4] Let$A$ be an ISs of X. If $A \in Nd^*(\mu_I)$ in X, then $i^*_{\mu_I}(A) = \mathfrak{E}$.

Proposition:2.13[6] For $A_{GI}, B_{GI} \subset X$.Then

(i)$i^*_{\mu_I}(A_{GI}) \cup i^*_{\mu_I}(B_{GI}) \subseteq i^*_{\mu_I}(A_{GI} \cup B_{GI})$,

(ii)$c^*_{\mu_I}(A_{GI}) \cup c^*_{\mu_I}(B_{GI}) \subseteq c^*_{\mu_I}(A_{GI} \cup B_{GI})$,

(iii)$c^*_{\mu_I}(A_{GI} \cap B_{GI}) \subseteq c^*_{\mu_I}(A_{GI}) \cap c^*_{\mu_I}(B_{GI})$,

(iv)$i^*_{\mu_I}(A_{GI} \cap B_{GI}) \subseteq i^*_{\mu_I}(A_{GI}) \cap i^*_{\mu_I}(B_{GI})$.

Corallary:2.14[4] Let$A \subseteq X$. If A is $\mu_I g$-CSGITS with $i^*_{\mu_I}(A) = \mathfrak{E}$ then A is $\mu_I g$-NDGITS.

Proposition:2.15[4] Let(X, μ_I) be a GITS. Then the following are equivalent

(i)$(X, \mu_I)$is a $\mu_I g$-Baire space.

(ii)$i^*_{\mu_I}(A) = \mathfrak{E}$, $\forall A \in \mathcal{F}^*(\mu_I)$.

(iii) $c^*_{\mu_I}(B) =$ Ứ, for every $\mu_I g$-residual set B in X.

Throughout this paper, we call $\langle X, \phi, X\rangle$as$\mathfrak{E}$, $\langle X, \phi, \phi\rangle$ as $\mathcal{O}$ and $\langle X, X, \phi\rangle$ as Ứ.

3. $\mu_I g$- Strongly Nowhere Dense Set in GITS

Definition:3.1 An ISs A is said to be μ_Ig-Strongly Nowhere dense set (in short,μ_Ig-SNWDS) if

$i^*_{\mu_I}(c^*_{\mu_I}(A \cap \bar{A})) = \mathfrak{E}$. The collection of μ_Ig-SNWDS is denoted by $SNd^*(\mu_I)$.

Example:3.2 Let $X = \{\vartheta_X, \hbar_X, \varpi_X\}$ with μ_I={$\mathfrak{E}$,$\langle X, \{\vartheta_X\}, \{\hbar_X\}\rangle$,$\langle X, \phi, \{\hbar_X\}\rangle$,$\langle X, \{\vartheta_X, \hbar_X\}, \phi\rangle$, $\langle X, \{\hbar_X\}, \{\varpi_X, \vartheta_X\}\rangle$,$\langle X, \{\hbar_X\}, \phi\rangle$,$\langle X, \{\vartheta_X\}, \{\varpi_X\}\rangle$,$\langle X, \{\vartheta_X\}, \phi\rangle$,$\langle X, \{\vartheta_X, \hbar_X\}, \{\varpi_X\}\rangle$, $\langle X, \{\hbar_X\}, \{\vartheta_X\}\rangle$}. Then $SNd^*(\mu_I) =$ {$\mathfrak{E}$,Ú,$\langle X, \phi, \{\vartheta_X, \hbar_X\}\rangle$,$\langle X, \{\vartheta_X, \hbar_X\}, \phi\rangle$,$\langle X, \{\vartheta_X\}, \{\hbar_X\}\rangle$,$\langle X, \{\hbar_X\}, \{\vartheta_X\}\rangle$, $\langle X, \{\vartheta_X\}, \{\hbar_X, \varpi_X\}\rangle$,$\langle X, \{\hbar_X\}, \{\vartheta_X, \varpi_X\}\rangle$,$\langle X, \{\varpi_X\}, \{\hbar_X, \vartheta_X\}\rangle$,$\langle X, \{\vartheta_X, \hbar_X\}, \{\varpi_X\}\rangle$, $\langle X, \{\varpi_X, \hbar_X\}, \{\vartheta_X\}\rangle$, $\langle X, \{\vartheta_X, \varpi_X\}, \{\hbar_X\}\rangle$}.

Theorem:3.3 Every μ_Ig-NDGITS is a μ_Ig-SNWDS.

Proof: Suppose $\xi_X \in Nd^*(\mu_I)$ then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X})) = \mathfrak{E} \Rightarrow \xi_X \in SNd^*(\mu_I)$.

Remark:3.4 The μ_Ig-SNWDS need not be a μ_Ig-NDGITS. For example:3.2, Ú, $\langle X, \{\vartheta_X, \hbar_X\}, \phi\rangle$,$\langle X, \{\vartheta_X\}, \{\hbar_X\}\rangle$,$\langle X, \{\hbar_X\}, \{\vartheta_X\}\rangle$,$\langle X, \{\vartheta_X\}, \{\hbar_X, \varpi_X\}\rangle$,$\langle X, \{\hbar_X\}, \{\vartheta_X, \varpi_X\}\rangle$, $\langle X, \{\vartheta_X, \hbar_X\}, \{\varpi_X\}\rangle$,$\langle X, \{\varpi_X, \hbar_X\}, \{\vartheta_X\}\rangle$ and $\langle X, \{\vartheta_X, \varpi_X\}, \{\hbar_X\}\rangle$ are μ_Ig-SNWDS but not a μ_Ig-NDGITS.

Remark:3.5 The μ_Ig-SNWDS and μ_I-nowhere dense set are not related to each other. In Example:3.2, μ_I-nowhere dense set = {$\langle X, \phi, \{\vartheta_X\}\rangle$,$\langle X, \phi, \{\vartheta_X, \varpi_X\}\rangle$,$\mathfrak{E}$, $\langle X, \phi, \{\vartheta_X, \hbar_X\}\rangle$, $\langle X, \{\varpi_X\}, \{\vartheta_X\}\rangle$,$\langle X, \{\varpi_X\}, \{\hbar_X, \vartheta_X\}\rangle$}. So both are independent to each other.

Theorem:3.6 If $i^*_{\mu_I}(\xi_X)$ is a μ_Ig-DGITS, for an ISs ξ_X defined on μ_I, then ξ_X is μ_Ig-SNWDS.

Proof: Suppose that $i^*_{\mu_I}(\xi_X)$ is μ_Ig-DGITS then $c^*_{\mu_I}(i^*_{\mu_I}(\xi_X)) =$ Ú. Now $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap i^*_{\mu_I}(\overline{i^*_{\mu_I}(\xi_X)}) = i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap \mathfrak{E} = \mathfrak{E} \Rightarrow \xi_X$ is a μ_Ig-SNWDS.

Theorem:3.7 If $\overline{\xi_X}$is μ_Ig-NDGITS, then ξ_X is μ_Ig-SNWDS but the converse need not be true.

Proof: Assume that $\overline{\xi_X}$is μ_Ig-NDGITS. Then $i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X})) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X})) \subseteq \mathfrak{E}$. But $\mathfrak{E} \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X}))$ and hence we get ξ_X is a μ_Ig-SNWDS.

But the reverse yields to false. From Example:3.2, $\xi_X = \langle X, \{\varpi_X\}, \{\hbar_X, \vartheta_X\}\rangle$ is a μ_Ig-SNWDS but $\overline{\xi_X} = \langle X, \{\vartheta_X, \hbar_X\}, \{\varpi_X\}\rangle$ is not a μ_Ig-NDGITS.

Theorem:3.8 If $c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X}))$ = Ú, for a μ_Ig-OSGITS ξ_X, then ξ_X is μ_Ig-SNWDS.

Proof: Given that $c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X}))$ = Ú, for an ISs ξ_X defined on μ_I. Taking complements on both side, $\overline{c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X}))} = \mathfrak{E} \Longrightarrow \overline{c^*_{\mu_I}(\overline{c^*_{\mu_I}(\xi_X)})} = \mathfrak{E} \Longrightarrow i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) = \mathfrak{E} \Longrightarrow \xi_X \in Nd*(\mu_I)$ and using Theorem 3.3, we have, ξ_X is a μ_Ig-SNWDS.

Theorem:3.9 If ξ_X is a μ_Ig-SNWDS then $\overline{\xi_X}$ is also a μ_Ig-SNWDS.

Proof: Let $\xi_X \in SNd^*(\mu_I)$. Then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X} \cap \overline{\overline{\xi_X}})) = i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X} \cap \xi_X)) = \mathfrak{E}$. Hence $\overline{\xi_X}$ is also a μ_Ig-SNWDS.

Theorem:3.10 If ξ_X is μ_Ig-NDGITS then $\overline{\xi_X}$ is μ_Ig-SNWDS but the $\Longleftarrow$ tends to fails.

Proof: Using Theorem 3.3 and 3.9 we get $\overline{\xi_X} \in SNd*(\mu_I)$ but the converse is not true. In Example 3.2, $\xi_X = \langle X, \{\vartheta_X, \varpi_X\}, \{\hbar_X\}\rangle \Longrightarrow \overline{\xi_X} = \langle X, \{\hbar_X\}, \{\vartheta_X, \varpi_X\}\rangle$ is a μ_Ig-SNWDS but ξ_X is not a μ_Ig-NDGITS.

Theorem:3.11 If ξ_X is μ_Ig-SNWDS then $c^*_{\mu_I}(\xi_X \cup \overline{\xi_X}) = Ú$.

Proof: Suppose that $\xi_X \in \mu_I$g-SNWDS then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Taking complements on both sides we have $c^*_{\mu_I}\left(\overline{c^*_{\mu_I}\,(\xi_X \cap \overline{\xi_X})}\right) = Ú \Longrightarrow c^*_{\mu_I}\,\left(i^*_{\mu_I}\left(\overline{\xi_X \cap \overline{\xi_X}}\right)\right) = Ú$. But $c^*_{\mu_I}(i^*_{\mu_I}(\xi_X \cup \overline{\xi_X})) \subseteq c^*_{\mu_I}(\xi_X \cup \overline{\xi_X}) \Longrightarrow Ú \subseteq c^*_{\mu_I}(\xi_X \cup \overline{\xi_X})$. Therefore $c^*_{\mu_I}(\xi_X \cup \overline{\xi_X}) = Ú$.

Theorem:3.12 If $i^*_{\mu_I}(Fr^*_{\mu_I}(\xi_X)) = \mathfrak{E}$, for a μ_Ig-OSGITS ξ_X, then ξ_X is μ_Ig-SNWDS.

Proof: Now $i^*_{\mu_I}(\, c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(Fr^*_{\mu_I}(\xi_X)) = \mathfrak{E}$.Therefore ξ_X is μ_Ig-SNWDS.

Theorem:3.13 If ξ_X is μ_Ig-CSGITS with $i^*_{\mu_I}(\xi_X) = \mathfrak{E}$, then $\xi_X \in \mu_I$g-SNWDS.

Proof: $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(\xi_X \cap Ú) = i^*_{\mu_I}(\xi_X) = \mathfrak{E}$. Therefore $\xi_X \in \mu_I$g-SNWDS.

Theorem:3.14 If ξ_X is μ_Ig- OSGITS and $\xi_X \in \mu_I$g-DSGITS then $\xi_X \in \mu_I$g-SNWDS.

Proof: $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(Ú \cap \overline{\xi_X}) \subseteq i^*_{\mu_I}(\overline{\xi_X}) = \overline{Ú} = \mathfrak{E}$ and hence we have $\xi_X \in \mu_I$g-SNWDS.

Figure:3.15

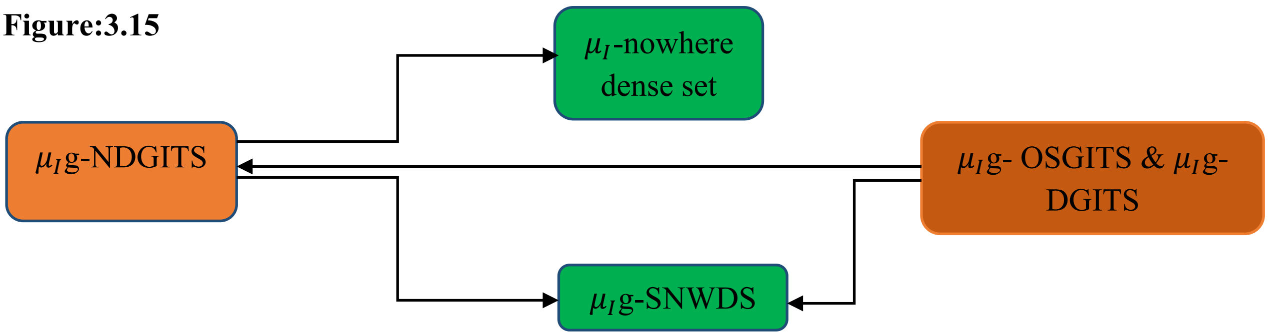

Fig:3.15, Represents the relations between μ_Ig-SNWDS,μ_Ig-NDGITS, μ_Ig-OSGITS,μ_Ig-DGITS and μ_I-nowhere dense.

Theorem:3.16 If $\xi_X \in Nd^*(\mu_I)$ after that $c^*_{\mu_I}(\xi_X)$ is μ_Ig-SNWDS.

Proof: Suppose $\xi_X \in Nd^*(\mu_I)$ then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(c^*_{\mu_I}(\xi_X) \cap \overline{c^*_{\mu_I}(\xi_X)})) =$

$i^*_{\mu_I}(c^*_{\mu_I}(\xi_X) \cap \overline{i^*_{\mu_I}(c^*_{\mu_I}(\xi_X))}) = i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap Ú = i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) = \mathfrak{E}$. Therefore $c^*_{\mu_I}(\xi_X) \in \mu_I$g-

Theorem:3.17 Every subset of μ_Ig-SNWDS is a μ_Ig-SNWDS.

Proof: Let ξ_X is μ_Ig-SNWDS then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Suppose $\S_X \subseteq \xi_X$, we have $\S_X \cap \overline{\xi_X} \subseteq \xi_X \cap \overline{\xi_X} \Longrightarrow i^*_{\mu_I}(c^*_{\mu_I}(\S_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Therefore $\S_X$ is μ_Ig-SNWDS.

Theorem:3.18 An ISs ξ_X is μ_Ig-SNWDS iff $c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X \cap \overline{\xi_X}})) = Ứ$.

Proof: Suppose $\xi_X \in SNd^*(\mu_I)$ then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Now $c^*_{\mu_I}\left(i^*_{\mu_I}\left(\overline{\xi_X \cap \overline{\xi_X}}\right)\right) = \overline{c^*_{\mu_I}(\overline{c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})})} = \overline{i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X}))}$= Ứ. Conversely assume that $c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X \cap \overline{\xi_X}})) = Ứ$. After that $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \overline{c^*_{\mu_I}(\overline{c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})})} = \overline{c^*_{\mu_I}(i^*_{\mu_I}(\overline{\xi_X \cap \overline{\xi_X}}))} = \mathfrak{E}$.

Theorem:3.19 If ξ_X is μ_Ig-SNWDS then $i^*_{\mu_I}(\xi_X \cap \overline{\xi_X}) = \mathfrak{E}$.

Proof: Suppose ξ_X is μ_Ig-SNWDS then $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$. Now $i^*_{\mu_I}(\xi_X \cap \overline{\xi_X}) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) = \mathfrak{E}$.

Theorem:3.20 If $c^*_{\mu_I}(\xi_X)$ is μ_Ig-CDGITS, for an ISs ξ_X defined on μ_I, then ξ_X is μ_Ig-SNWDS.

Proof: Given that $c^*_{\mu_I}(\xi_X)$is μ_Ig-CDGITS which implies $c^*_{\mu_I}(\overline{c^*_{\mu_I}(\xi_X)}) = Ứ \Rightarrow \overline{i^*_{\mu_I}(c^*_{\mu_I}(\xi_X))} = Ứ \Rightarrow i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(\xi_X \cap \overline{\xi_X})) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(\xi_X)) \cap i^*_{\mu_I}(c^*_{\mu_I}(\overline{\xi_X})) = \mathfrak{E} \Rightarrow \xi_X$ isaμ_Ig-SNWDS.

4. Strongly μ_Ig-First category set in GITS

Definition:4.1 An ISs $§_X$ is said to be Strongly μ_Ig-First Category Set in GITS (μ_Ig-SFCS) if $§_X = \bigcup_{i=1}^{\infty} §_{X_i}$ where $§_{X_i}$'s are μ_Ig-SNWDS. Remaining sets are called strongly μ_Ig-Second Category Set (μ_Ig-SSCS). The complement of μ_Ig-SFCS is named as a strongly μ_Ig-Residual set (μ_Ig-SRS).

Example:4.2 Let $X = \{ɕ_X, ɗ_X, ɚ_X, ɤ_X\}$ with $\mu_I = \{\mathfrak{E}, \langle X, \{ɕ_X, ɗ_X, ɚ_X\}, \phi\rangle, \langle X, \phi, \{ɕ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X\}, \{ɗ_X, ɤ_X\}\rangle, \langle X, \{ɕ_X\}, \phi\rangle, \langle X, \{ɗ_X, ɚ_X\}, \{ɤ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X\}, \phi\rangle, \langle X, \{ɕ_X, ɗ_X, ɚ_X\}, \{ɤ_X\}\rangle\}$.
Then μ_Ig-SFCS$=\{Ứ, \langle X, \phi, \{ɕ_X, ɗ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X\}, \{ɗ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X\}, \{ɗ_X, ɚ_X, ɤ_X\}\rangle, \langle X, \{ɗ_X\}, \{ɕ_X, ɚ_X\}\rangle, \langle X, \{ɗ_X\}, \{ɕ_X, ɚ_X, ɤ_X\}\rangle, \langle X, \{ɚ_X\}, \{ɗ_X, ɕ_X\}\rangle, \langle X, \{ɚ_X\}, \{ɗ_X, ɕ_X, ɤ_X\}\rangle, \langle X, \{ɤ_X\}, \{ɗ_X, ɚ_X, ɕ_X\}\rangle, \langle X, \{ɗ_X, ɕ_X\}, \{ɚ_X\}\rangle, \langle X, \{ɗ_X, ɕ_X\}, \{ɚ_X, ɤ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X\}, \{ɕ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X\}, \{ɕ_X, ɤ_X\}\rangle, \langle X, \{ɤ_X, ɚ_X\}, \{ɕ_X, ɗ_X\}\rangle, \langle X, \{ɤ_X, ɕ_X\}, \{ɚ_X, ɗ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X\}, \{ɗ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X\}, \{ɗ_X, ɤ_X\}\rangle, \langle X, \{ɗ_X, ɤ_X\}, \{ɕ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X, ɗ_X, ɚ_X\}, \phi\rangle, \langle X, \{ɕ_X, ɗ_X, ɚ_X\}, \{ɤ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X, ɤ_X\}, \{ɕ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X, ɤ_X\}, \{ɗ_X\}\rangle, \langle X, \{ɕ_X, ɗ_X, ɤ_X\}, \{ɚ_X\}\rangle\}$. μ_Ig-SSCS = { $\mathfrak{E}$} and

μ_Ig-SRS $=\{\mathfrak{E}, \langle X, \{ɕ_X, ɗ_X, ɚ_X\}, \phi\rangle, \langle X, \{ɗ_X, ɚ_X\}, \{ɕ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X, ɤ_X\}, \{ɕ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X\}, \{ɗ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X, ɤ_X\}, \{ɗ_X\}\rangle, \langle X, \{ɗ_X, ɕ_X\}, \{ɚ_X\}\rangle, \langle X, \{ɗ_X, ɕ_X, ɤ_X\}, \{ɚ_X\}\rangle, \langle X, \{ɗ_X, ɚ_X, ɕ_X\}, \{ɤ_X\}\rangle, \langle X, \{ɚ_X\}, \{ɗ_X, ɕ_X\}\rangle, \langle X, \{ɚ_X, ɤ_X\}, \{ɗ_X, ɕ_X\}\rangle, \langle X, \{ɕ_X\}, \{ɗ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X, ɤ_X\}, \{ɗ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X, ɗ_X\}, \{ɚ_X, ɤ_X\}\rangle, \langle X, \{ɚ_X, ɗ_X\}, \{ɕ_X, ɤ_X\}\rangle, \langle X, \{ɗ_X\}, \{ɕ_X, ɚ_X\}\rangle, \langle X, \{ɗ_X, ɤ_X\}, \{ɕ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X, ɚ_X\}, \{ɗ_X, ɤ_X\}\rangle, \langle X, \phi, \{ɕ_X, ɗ_X, ɚ_X\}\rangle, \langle X, \{ɤ_X\}, \{ɕ_X, ɗ_X, ɚ_X\}\rangle, \langle X, \{ɕ_X\}, \{ɗ_X, ɚ_X, ɤ_X\}\rangle, \langle X, \{ɗ_X\}, \{ɕ_X, ɚ_X, ɤ_X\}\rangle, \langle X, \{ɚ_X\}, \{ɕ_X, ɗ_X, ɤ_X\}\rangle\}$.

Theorem:4.3 If $§_X$ is μ_Ig-FCGITS then $§_X$ is μ_Ig-SFCS.

Proof: Let $\S_X$ be μ_Ig-FCGITS. Then $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-NDGITS. By theorem: 3.3, $\S_{X_i}'$s are μ_Ig-SNWDS and hence $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-SNWDS. Therefore $\S_X$ is μ_Ig-SFCS.

Theorem:4.4 If $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-CSGITS with $i_{\mu_I}^*(\S_{X_i}) = \mathfrak{E}$ then $\S_X$ is a μ_Ig-SFCS.

Proof: Suppose $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-CSGITS with $i_{\mu_I}^*(\S_{X_i}) = \mathfrak{E}$. By theorem:3.14, μ_Ig-CSGITS with $i_{\mu_I}^*(\S_{X_i}) = \mathfrak{E}$ are μ_Ig-SNWDS and then we have $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-SNWDS. Therefore $\S_X$ is a μ_Ig-SFCS.

Theorem:4.5 If $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $i_{\mu_I}^*(Fr_{\mu_I}^*(\S_{X_i})) = \mathfrak{E}$, then $\S_X$ is μ_Ig-SFCS.

Proof: Assume that $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $i_{\mu_I}^*(Fr_{\mu_I}^*(\S_{X_i}))= \mathfrak{E}$. By theorem:3.12, we have $\S_{X_i}$'s are μ_Ig-SNWDS. Therefore $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$where $\S_{X_i}$'sare μ_Ig-SNWDS and hence $\S_X$ is μ_Ig-SFCS.

Theorem:4.6 If $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-OSGITS and μ_Ig-DSGITS, then $\S_X$ is μ_Ig-SFCS.

Proof: Given that $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}'$s are μ_Ig-OSGITS and μ_Ig-DSGITS. By theorem:3.12, $\S_{X_i}$'s are μ_Ig-SNWDS and hence $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}$'sare μ_Ig-SNWDS. Therefore $\S_X$is μ_Ig-SFCS.

Theorem:4.7 Every subset of a μ_Ig-SFCS is μ_Ig-SFCS.

Proof: Let $\S_X$be $\mu_I g$-SFCS. Then $\S_X$ =$\bigcup_{i=1}^{\infty} \S_{X_i}$, where $\S_{X_i}$'s are $\mu_I g$-SNWDS. Suppose $\zeta_X \subseteq \S_X$ =$\bigcup_{i=1}^{\infty} \S_{X_i}$.Therefore $\zeta_X \subseteq \bigcup_{i=1}^{\infty} \S_{X_i} \Rightarrow \zeta_X \subseteq \S_{X_i}$, for some $\mu_I g$-SNWDS. By using Theorem 3.17, ζ_X is $\mu_I g$-SFCS.

5. Strongly μ_Ig-Baire Space in GITS

Definition:5.1 A GITS (X, μ_I) is called a Strongly μ_Ig-Baire Space (μ_Ig-SBS) if $c_{\mu_I}^*(\bigcup_{i=1}^{\infty} \S_{X_i}) = \text{Ư}$, where $\S_{X_i}$'s are μ_Ig-SNWDS.

Example:5.2 In example:3.2, take $A = \langle X, \phi, \{\vartheta_X, \hbar_X\}\rangle$ and $B = \langle X, \{\vartheta_X, \hbar_X\}, \phi\rangle$. Then $c_{\mu_I}^*(A \cup B) = c_{\mu_I}^*(\langle X, \{\vartheta_X, \hbar_X\}, \phi\rangle) = \text{Ư}$. Therefore (X, μ_I) is a Strongly μ_Ig-Baire Space (μ_Ig-SBS).

Theorem:5.3 Let(X, μ_I) be GITS. Then the subsequent are equivalent

(i)(X, μ_I) is $\mu_I g$-SBS.

(ii)$c_{\mu_I}^*(\S_X) = \text{Ư}$, for every μ_Ig-SFCS$\S_X$ in X.

(iii) $i_{\mu_I}^*(\wp_X) = \mathfrak{E}$, for every $\mu_I g$-SRS $\wp_X$ in X.

Proof: (i) $\Longrightarrow$ (ii), Let $\S_X$ be a μ_Ig-SFCS in X. Then $\S_X$ = $(\bigcup_{i=1}^{\infty} \S_X)$ where $\S_X$'s are $\mu_I g$-SNWDS. Since (X, μ_I) is $\mu_I g$-SBS, $c_{\mu_I}^*(\bigcup_{i=1}^{\infty} \S_{X_i}) = \text{Ư}$. Therefore $c_{\mu_I}^*(\S_X) = \text{Ư}$.

(ii) $\Longrightarrow$ (iii) Let $\wp_X$ be $\mu_I g$-SRS in X. Then $\overline{\wp_X}$ is μ_Ig-SFCSin X. From(ii), $c^*_{\mu_I}(\overline{\wp_X}) =$ Ư$\Longrightarrow\overline{i^*_{\mu_I}(\wp_X)}$ = Ư. Hence $i^*_{\mu_I}(\wp_X) = \mathfrak{E}$.

(iii) $\Longrightarrow$ (i) Let $\S_X$ be μ_Ig-SFCS in X. Then $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$where $\S_{X_i}$'s are $\mu_I g$-SNWDS. We have, if $\S_X$ is a μ_Ig-SFCS in X then $\overline{\S_X}$ is a $\mu_I g$-SRS. By (iii) we get $i^*_{\mu_I}(\overline{\S_X}) = \mathfrak{E}$, which gives $\overline{c^*_{\mu_I}(\S_X)} = \mathfrak{E}$. Therefore $c^*_{\mu_I}(\S_X) =$ Ư and hence $c^*_{\mu_I}(\bigcup_{i=1}^{\infty} \S_{X_i})$ = Ư, where $\S_{X_i}$'s are $\mu_I g$-SNWDS. Hence (X, μ_I) is $\mu_I g$-SBS.

Theorem:5.4 If $\{\S_{X_i}\}$, $i = 1$ to ∞, is μ_Ig-OSGITS and μ_Ig-DSGITS in X then X is μ_Ig-SBS.

Proof: Using theorem:3.14, we have$\S_{X_i}$, i=1 to $\infty\mu_I$g-SNWDS in X. Let $\S_X = (\bigcup_{i=1}^{\infty} \S_{X_i})$. Then $\S_X$ is μ_Ig-SFCS. $c^*_{\mu_I}(\S_X) = c^*_{\mu_I}\left(\bigcup_{i=1}^{\infty} \S_{X_i}\right) \supseteq \bigcup_{i=1}^{\infty} c^*_{\mu_I}\left(\S_{X_i}\right)$ = Ư. Henceforth X is a μ_Ig-SBS.

Conclusion: In this paper, first we defined μ_Ig-SNWDS and μ_Ig- SFCS, then introduce μ_Ig-SBS. Some more properties are to be discussed. In future we discuss μ_Ig σ-Baire space and μ_Ig D-Baire space.

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