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Construction of graphs from the topological spaces

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Abstract: In this article, we present a novel definition of discrete topological space, where a relationship was found between the elements of these spaces and their representation in graphs. Additionally, we compute fundamental graph parameters such as the clique number, and planar graph. Furthermore, we prove that a topological graph is a simple undirected graph.

Key words: graph theory, Topological graph, girth number, clique number, planer graph.

1. Introduction

A graph φ = (V, E) is an ordered pair of disjoint sets (V, E), where $V \neq \emptyset$ and E are a subset of unordered pairs of V. The elements V = V (φ), and E = E (φ) respectively vertices and edges of a graph (ϕ)[8]. A topological graph theory is one of the important types in mathematics which is interesting for both graph theory and specialists in the topological space, Recently some topological graphs have been presented by some researchers such as "some results of domination on the discrete topological graph with it is inverse [4]. "Out topological Digraph Space and Some Related Properties"[7]."The neighborhood topology converted from the undirected graphs" [5]. some graphs that depend on multiple topological properties have been known in [2, 3, 6]. Also "Using graphs to depict relationships among elements in various Topological Spaces"[9]. In this paper, we defined a new graph associated with discrete topology denoted by ϕ(T) which has the set vertex V(φ) = {M; $M \in \tau$ and $M \neq \emptyset$} and two vertices M and N are adjacent where $M \cap N$ is a singleton set, also we calculated Girth, Clique number of this graph and we discussed when this graph is planar or non-planar. The vertices and edges of a graph (φ) play a vital role in network science and have applications in diverse domains such as spam detection, graph analysis, graph modeling, and community detection.

2. Fundamental Concepts

This part applies a new approach to the work of the topological spaces under investigation. Many graph properties have been studied and proven in discrete topological spaces.

Definition 2.1: The girth of a φ graph refers to the length of the shortest cycle in φ. [8].

Definition 2.2: A complete graph is a graph in which every vertex is adjacent to all vertices and denoted by K_n [8].

Definition 2.3: A clique in graph φ is a fully connected subgraph of φ, and the size of the largest fully connected subgraph in φ is referred to as the clique number and denoted by $\omega(\tau)$ [8].

Definition 2.4: A planer graph is a graph whose edges intersect at the final points only. In other words, there is no intersection between the edges of the plane [8].

3. Construction of graphs using intersection of two sets in a topological space

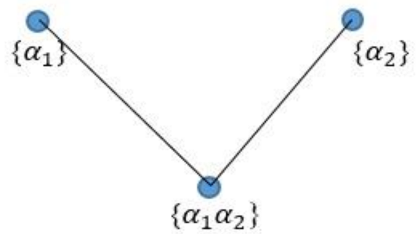

Figure 1: The graph ϕ(τ) is not complete

Definition 3.1: let (X, τ) be a discrete topology on non-empty set X, then we can define a graph on τ which is defined by φ(τ) as follows the vertex set $V(\varphi(\tau))$ = {M; M ∈ τ and M ≠ ∅,} and every two vertices M, N where N ≠ M is adjacent were M ∩N is a singleton set.

Proposition 3.2: φ(τ) is not complete

Proof : case i: if n ≤ 2, then we have $\{\alpha_1\}$ is not a subset of $\{\alpha_2\}$ then $\{\alpha_1\} \cap \{\alpha_2\}$ is a empty set for all vertices of the single element. From figure 1There is no adjacent vertices between $\{\alpha_1\}$ and $\{\alpha_2\}$. Therefore φ (τ) is not complete

Case ii: if n ≥ 2, then there is no adjacent vertices in singleton sets. Therefore φ(τ) is not complete.

Proposition 3.3: Consider a non-empty set X of size n and a topology τ defined on X. if n equals 2, then the independent number of φ(τ) = 2.

Proof: According to the definition of independent set, its vertices are not adjacent, and the proof is obtained.

Example 3.4: X = $\{\alpha_1, \alpha_2, \alpha_3\}$, then τ = { X, ϕ, $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$, $\{\alpha_2\alpha_3\}$}, and V(φ) = { $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$,$\{\alpha_2\alpha_3\}$,$\{\alpha_1, \alpha_2, \alpha_3\}$} Let X = $\{\alpha_1\}$, and Y = $\{\alpha_2\}$ be vertices of single elements. Since {a_1} is not a subset of {a_2} for all vertices of the single element, by the definition of the topological space, X ∩ Y is not a singleton set, Then X is not adjacent to Y. The groups of two vertices, each vertex is adjacent to all the vertices of the group of two vertices. According to the definition, vertex $\{\alpha_1\}$ is adjacent to vertex $\{\alpha_1, \alpha_2\}$ and $\{\alpha_1, \alpha_3\}$. For vertex $\{\alpha_1\}$, it is adjacent to vertex $\{\alpha_1, \alpha_2\}$ and $\{\alpha_2, \alpha_3\}$, finally the vertex $\{\alpha_3\}$ is adjacent to the vertices $\{\alpha_1, \alpha_3\}$ and $\{\alpha_2, \alpha_3\}$ and $\{\alpha_1, \alpha_2, \alpha_3\}$ are adjacent with only singleton set Thus we have the graph. As depicted in figure 2

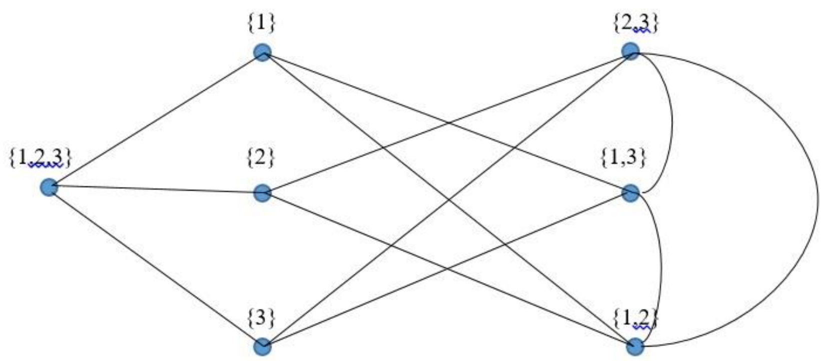

Figure 2

Example 3.5: If |X| = 4, then $\tau = \{X, \phi, \{\alpha_1\}, \{\alpha_2\}, \{\alpha_3\}, \{\alpha_4\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}, \{\alpha_1, \alpha_4\}, \{\alpha_2, \alpha_3\}, \{\alpha_2, \alpha_4\}, \{\alpha_3, \alpha_4\}, \{\alpha_1, \alpha_2, \alpha_3\}, \{\alpha_1, \alpha_2, \alpha_4\}, \{\alpha_1, \alpha_3, \alpha_4\}, \{\alpha_2, \alpha_3, \alpha_4\}, \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}\}$ we also notice that the individual vertices are not adjacent to each other because their intersection is a empty set. See figure 3

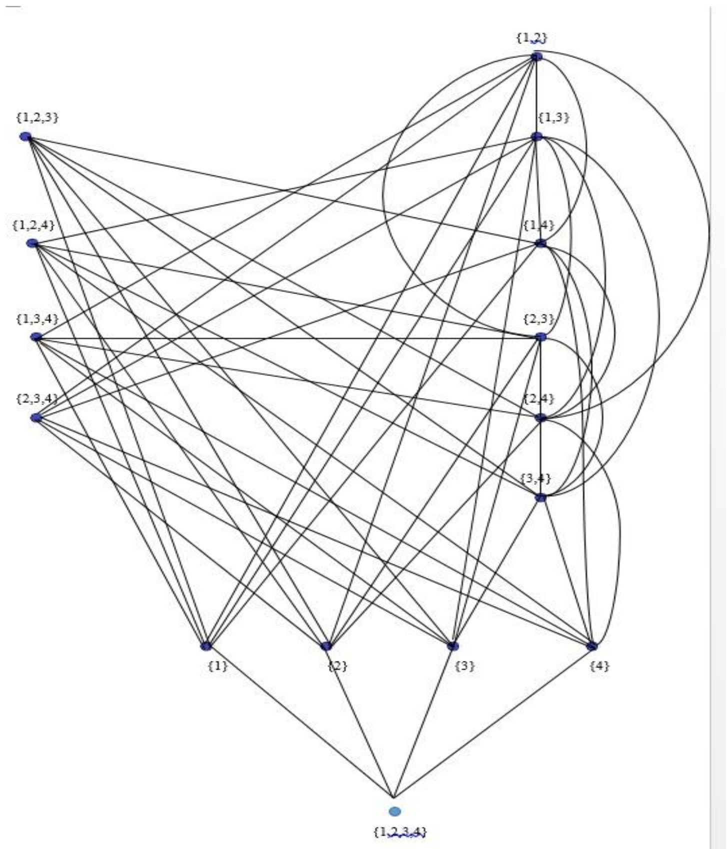

Figure 3

Proposition 3.6: Consider the topological graph Gτ derived from a non-empty set X. It can be asserted that φ(τ) is a simple graph.

Proof: Let {a} ,{b}, represent any two distinct vertices in φ(τ). By definition of φ(τ) {a} ≠ {b} therefore no multiple loops in φ(τ). By definition 3.1 there exists a single edge between any two vertices. Therefore, φ(τ) does not contain multiple edge between vertices.

Proposition 3.7: Let |X| = n, (n > 2) the number of cut vertex of the topological graph φ(τ) is equal to n

Proof : from the examples the set of n element adjacent only with singleton set. So if we remove singleton set from each graph the graph is disconnected. By the definition of the cut vertex the number of cut vertex of the topological graph equals to n.

Proposition 3.8: Suppose the cardinality of set X is denoted by |X| = n. In this case, the order of the topological graph φ(τ) is determined by 2^n – 1.

Proof: Let τ represent a collection comprising all subsets of X with cardinality 2^n. Consequently, the discrete topological graph φ(τ) includes all elements from τ, excluding only the empty set ϕ , as per (Definition 3.1)

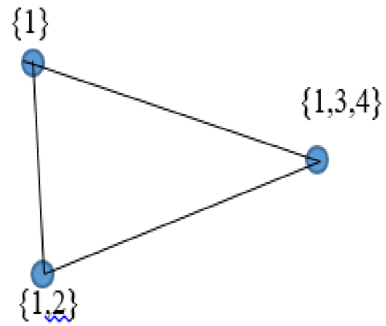

Figure 4

Proposition 3.9; Let |X| = n, (n ≥ 3) and consider the topological graph φ(τ) derived from a non-empty set X, then the girth number of φ(τ) equal 3

Proof: from figure 3 the possible cycle was shown in figure 4. Then by figure 4 the least cycle is 3.

Example 3.10 X = $\{\alpha_1, \alpha_2, \alpha_3\}$, then τ = { X, ϕ, $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$, $\{\alpha_2\alpha_3\}$}, and V(φ) = { $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$,$\{\alpha_2\alpha_3\}$,$\{\alpha_1, \alpha_2, \alpha_3\}$}

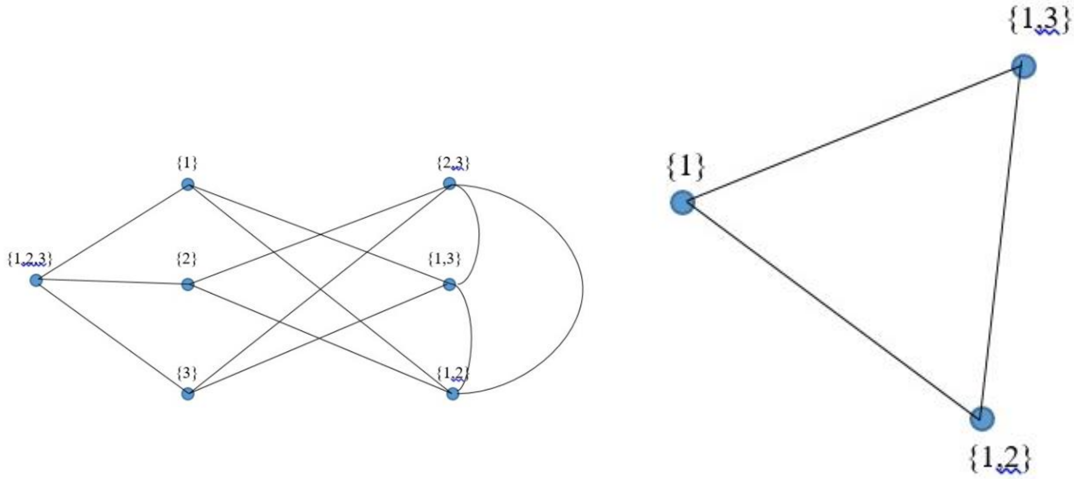

Figure 2.

Example 3.11: : If |X| = 4, then $\tau = \{X, \phi, \{\alpha_1\}, \{\alpha_2\}, \{\alpha_3\}, \{\alpha_4\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}, \{\alpha_1, \alpha_4\}, \{\alpha_2, \alpha_3\}, \{\alpha_2, \alpha_4\}, \{\alpha_3, \alpha_4\}, \{\alpha_1, \alpha_2, \alpha_3\}, \{\alpha_1, \alpha_2, \alpha_4\}, \{\alpha_1, \alpha_3, \alpha_4\}, \{\alpha_2, \alpha_3, \alpha_4\}, \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}\}$

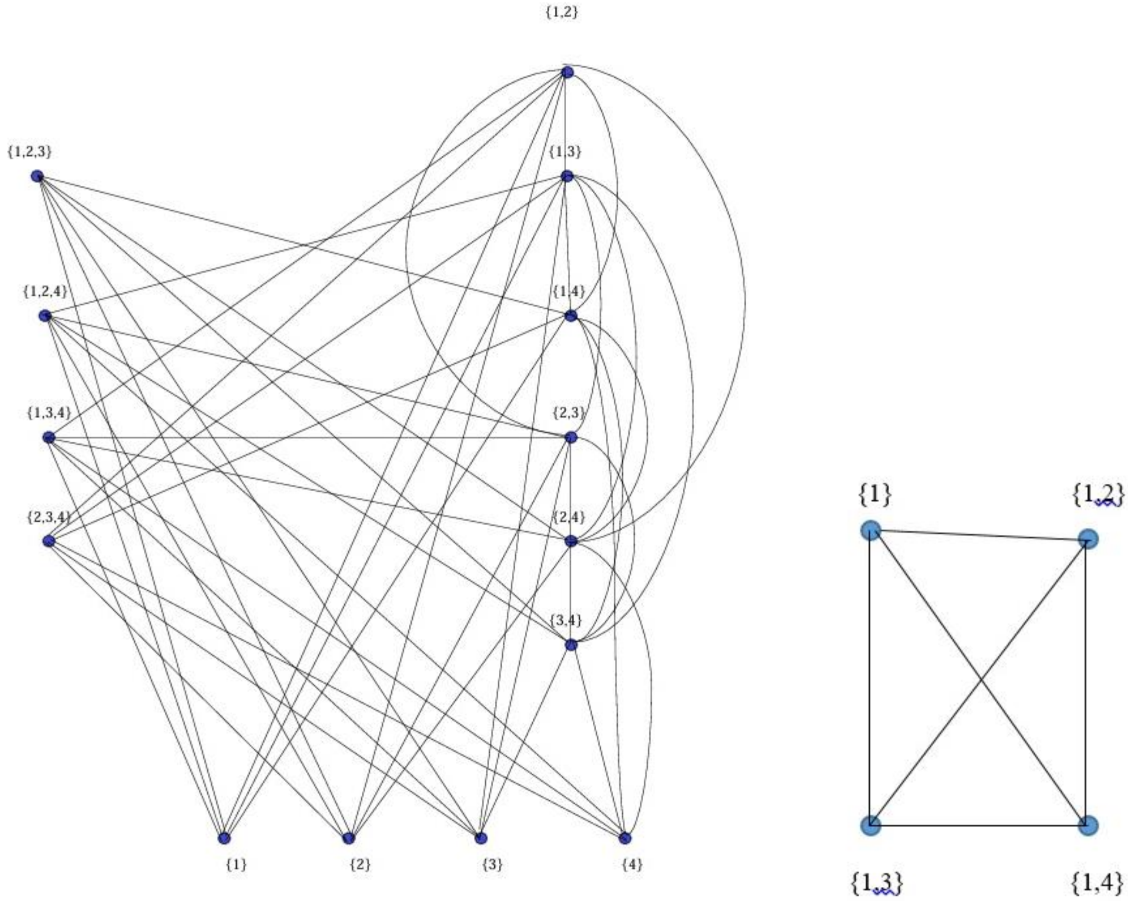

Figure 3.1

Proposition 3.12: Let |X| = n, (n ≥ 3) and φ(τ) be a topological graph on X, then the clique number of φ(τ) is n

Proof: consider the i th vertices, let A be the set of all sets which contains two elements and the one element as i th element. Therefore the maximum number of sets in A is (n-1). Therefore the maximum clique number = number of elements in A + 1(i th element)

= n-1+1= n

Proposition 3.13: Let |X| = n, (n ≤ 3) and consider the topological graph φτ derived from a non-empty set X then the (φτ) is a planer graph.

Proof; is clear by (definition 2.5), so there are no vertices intersecting since the graph (φτ) is planer.

Example 3.14: X = $\{\alpha_1, \alpha_2, \alpha_3\}$, then τ = { X, ϕ, $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$, $\{\alpha_2\alpha_3\}$}, and V(φ) = { $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_3\}$, $\{\alpha_1\alpha_2\}$, $\{\alpha_1\alpha_3\}$,$\{\alpha_2\alpha_3\}$,$\{\alpha_1, \alpha_2, \alpha_3\}$}

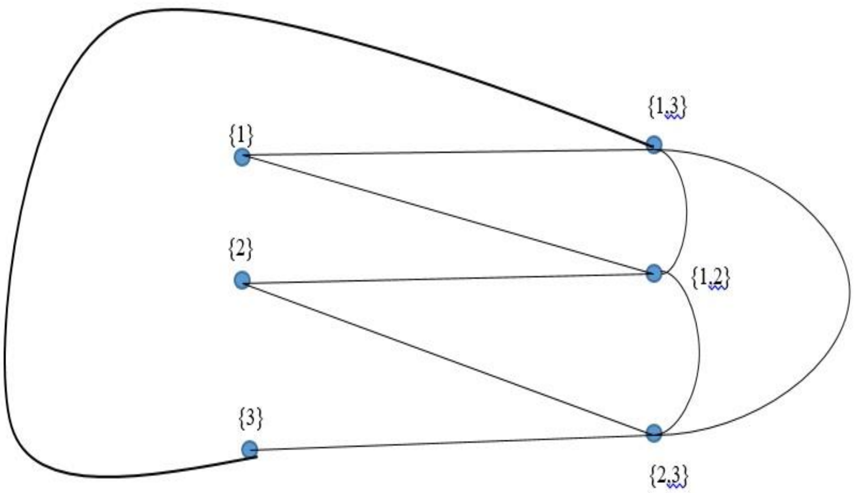

Figure 5

Proposition 3.15: Let |X| = n, (n > 3) and consider the topological graph φτ derived from a non-empty set X then the (φτ) is not a planer graph.

Proof: we can prove that by two cases as follow

Case 1: if |X| = 4, we can see that the subgraph containing the vertices of the set of two elements is {{1,2},{1,3},{1,4},{2,3},{2,4}} has the crossing number is shown in figure, so this subgraph is non-planar and thus deduce that the graph is non-planar.

Case 2: if |X| = 5, in this case we have a subgraph containing the vertices {{1},{1,2},{1,3},{1,4},{1,5}} is isomorphic to the complete graph K_5 which thus the graph is non planar(see figure 7)

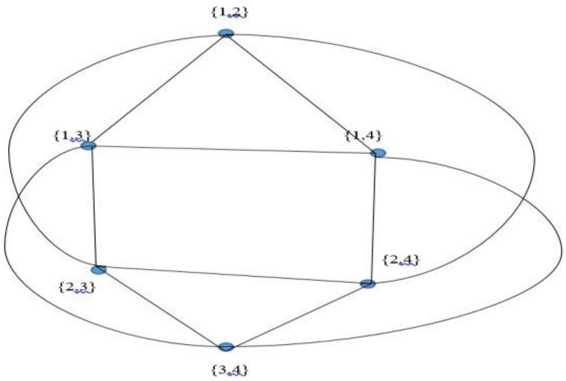

Figure 6

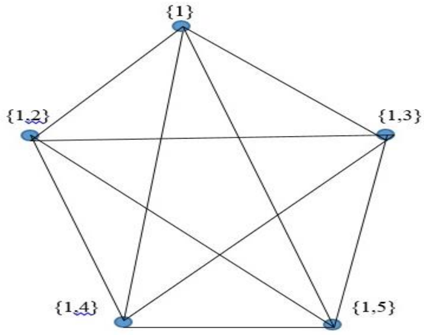

figure 7

Conclusion:

This study presented a new contruction of a discrete Topological graph. Also, many properties of the graph have been studied in this space. Our chart is a simple, connected graph. Moreover, the girth number, Clique number, and planer graph, were calculated by studying many other properties.

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