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Generalized New Quadratic-Exponential Distribution

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Abstract:

This proposed distribution has been created by just adding an additional parameter to the New Quadratic-exponential distribution (NQED) which we have named Generalised New Quadratic-exponential Distribution (GNQED). We are almost sure that the contribution of this proposed distribution to probability mixture theory, size-biased probability mixture theory and in many areas can be seen in very near future. Some important characteristics of this distribution have been derived and defined very well manner. It is good for statistical modelling of over-dispersed data related to survival time.

Keywords: Probability distribution, Quadratic-Exponential distribution (QED), New Quadratic-exponential distribution (NQED), Distribution, Probability density function, Moments.

1.0 Introduction:

The research field is very unique. It doesn't matter if you don't do it, but if you do it, you don't want to give up. This distribution is a generalised case of NQED [1]. The probability density function of NQED was given by the expression (1).

$$f_1(v)=\frac{\theta^3}{(\pi\theta^2+2)}(\pi+v^2)e^{-\theta v} \tag{1}$$

$v>0$ and $\theta>0$

The proposed distribution is made up of the mixture of the two functions $(\pi\tau+v^2)$ and $e^{-\theta v}$, which we have named Generalized New Quadratic-exponential Distribution (GNQED). We are very fortunate to take the help of the references [2] to [6] to prepare this paper which boost up the quality of this paper. Work of the paper have been placed in the section (2.0), (3.0) and (4.0).

2.0 Results:

2.1 Probability Density Function ($f(v;\tau,\theta)$):

It is given by the expression (2) which is a generalized case of NQED.

$$f(v;\tau,\theta)=\frac{\theta^3}{(\pi\tau\theta^2+2)}(\pi\tau+v^2)e^{-\theta v} \tag{2}$$

$v>0$, $\theta>0$ and $(\pi\tau\theta^2+2)>0$

Probability Distribution Function ($F(V)$):

It has been derived as follows and given by the expression (3).

$$F(V=v)=\frac{\theta^2}{(\pi\tau\theta^2+2)}\int_0^v(\pi\tau+v^2)e^{-\theta v}dv$$

$$F(V=v)=\frac{\theta^2}{(\pi\tau\theta^2+2)}\left[\left\{-\left(\frac{v^2}{\theta}+\frac{2v}{\theta^2}+\frac{2}{\theta^3}\right)e^{-\theta v}-\left(\frac{\pi\tau}{\theta}\right)e^{-\theta v}\right\}+\left(\frac{\pi\tau}{\theta}+\frac{2}{\theta^3}\right)\right]$$

$$F(V=v)=\left\{1-e^{-\theta v}-\frac{v\theta(2+v\theta)}{(\pi\tau\theta^2+2)}e^{-\theta v}\right\}$$

Figure-1

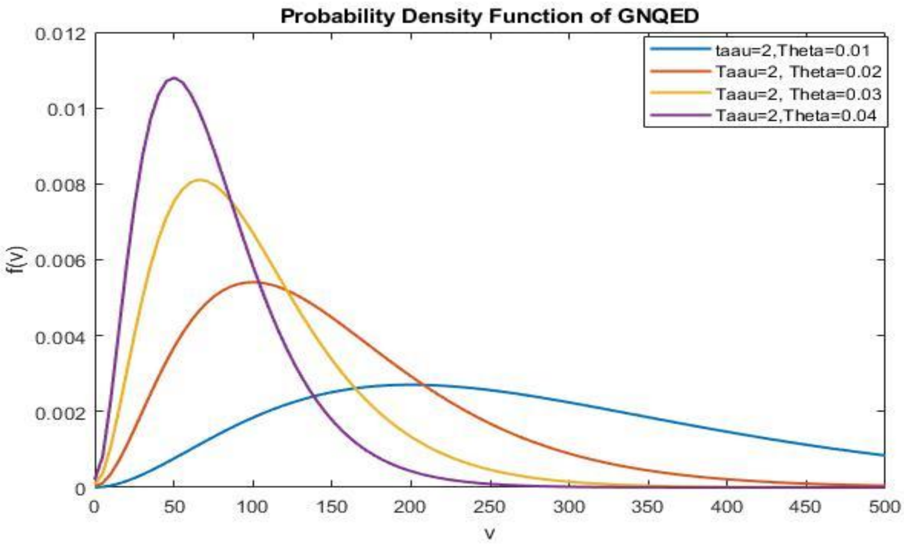


Figure-2

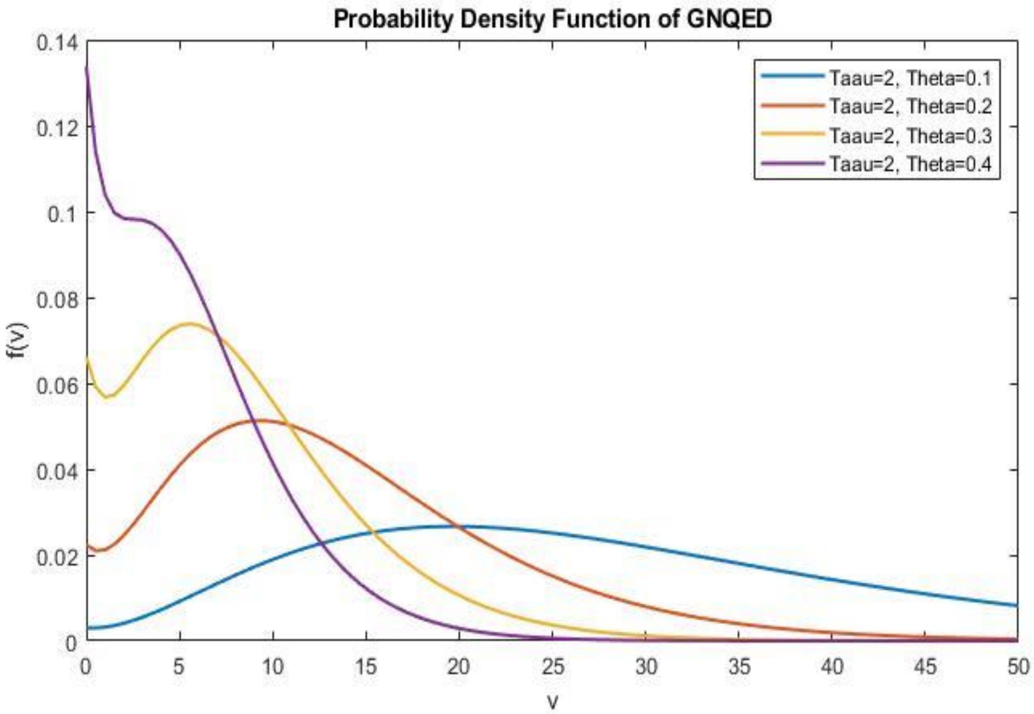


Figure-3

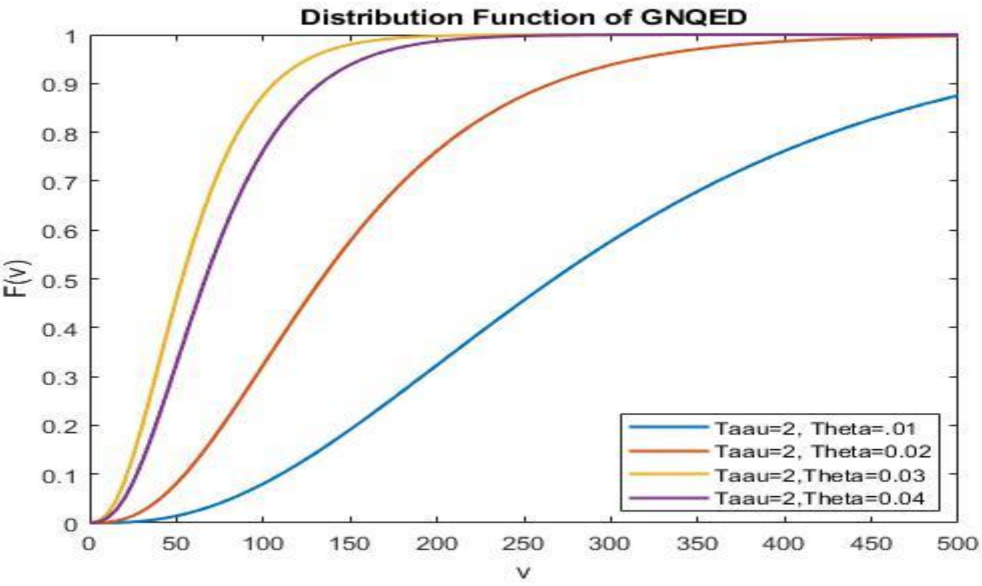
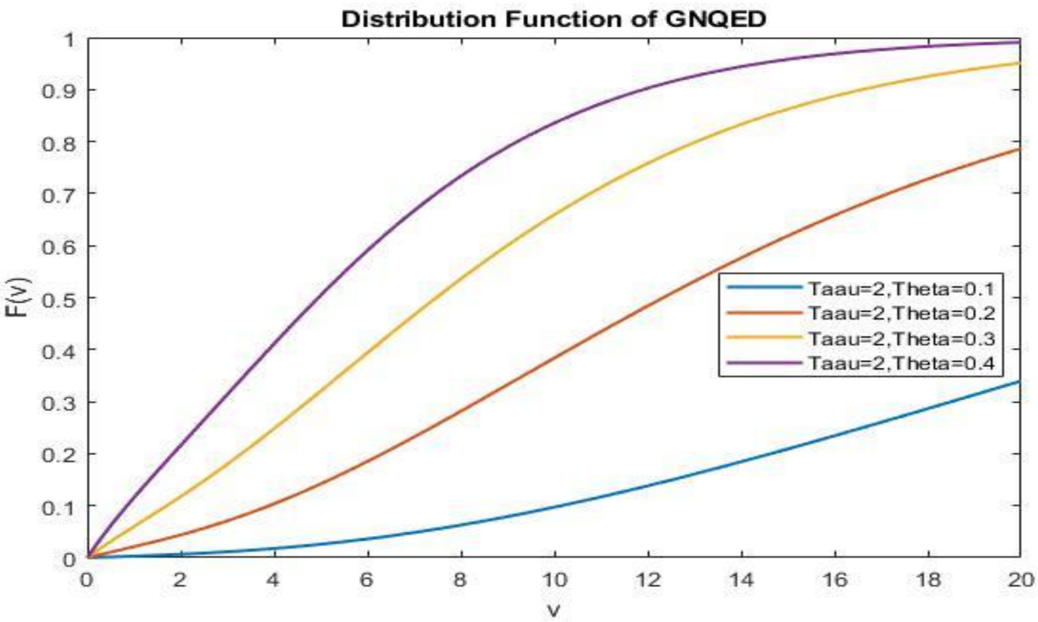


Figure-4



2.2 Moments about the Origin of GNQED:

The moments about the origin (μ_r'), the first four moments about the origin and the first four central moments of GNQED can be derived as follows and given by the expression (4) to (12) respectively.

$$\mu_r' = \frac{\theta^2}{(\pi\tau\theta^2+2)}\int_0^\infty v^r(\pi\tau+v^2)e^{-\theta v}dv$$

Or, $\mu_r' = \frac{\theta^2}{(\pi\tau\theta^2+2)}\left[\pi\tau\int_0^\infty v^r e^{-\theta v}dv + \int_0^\infty v^{r+2}e^{-\theta v}dv\right]$

Or, $\mu_r' = \frac{\theta^2}{(\pi\tau\theta^2+2)}\left[\pi\theta\frac{r!}{\theta^{r+1}} + \frac{(r+2)(r+1)r!}{\theta^{r+3}}\right]$

Or, $\mu_r' = \frac{r!}{\theta^r}\frac{\left\{\pi\tau\theta^2 + (r+2)(r+1)\right\}}{(\pi\tau\theta^2 + 2)}$ (4)

$$\mu_1' = \frac{1!}{\theta^1}\frac{\left\{\pi\tau\theta^2 + (1+2)(1+1)\right\}}{(\pi\tau\theta^2 + 2)} = \frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)} \quad (5)$$

$$\mu_2' = \frac{2!}{\theta^2}\frac{\left\{\pi\tau\theta^2 + (2+2)(2+1)\right\}}{(\pi\tau\theta^2 + 2)} = \frac{2(\pi\tau\theta^2 + 12)}{\theta^2(\pi\tau\theta^2 + 2)} \quad (6)$$

$$\mu_3' = \frac{3!}{\theta^3}\frac{\left\{\pi\tau\theta^2 + (3+2)(3+1)\right\}}{(\pi\tau\theta^2 + 2)} = \frac{6(\pi\tau\theta^2 + 24)}{\theta^3(\pi\tau\theta^2 + 2)} \quad (7)$$

$$\mu_4' = \frac{4!}{\theta^4}\frac{\left\{\pi\tau\theta^2 + (4+2)(4+1)\right\}}{(\pi\tau\theta^2 + 2)} = \frac{24(\pi\tau\theta^2 + 30)}{\theta^4(\pi\tau\theta^2 + 2)} \quad (8)$$

$$\mu_1 = 0 \quad (9)$$

$$\mu_2 = \frac{2(\pi\tau\theta^2 + 12)}{\theta^2(\pi\tau\theta^2 + 2)} - \left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\}^2$$

Or, $\mu_2 = \frac{\left\{(\pi\tau\theta^2)^2 + 16(\pi\tau\theta^2) + 12\right\}}{\theta^2(\pi\tau\theta^2 + 2)^2}$ (10)

$$\mu_3 = \frac{6(\pi\tau\theta^2 + 24)}{\theta^3(\pi\tau\theta^2 + 2)} - 3\left\{\frac{2(\pi\tau\theta^2 + 12)}{\theta^2(\pi\tau\theta^2 + 2)}\right\}\left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\} + 2\left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\}^3$$

Or, $\mu_3 = \frac{\left\{2(\pi\tau\theta^2)^3 + 60(\pi\tau\theta^2)^2 + 720(\pi\tau\theta^2 + 48\right\}}{\theta^3(\pi\tau\theta^2 + 2)^3}$ (9)

Or, $\mu_4 = \frac{(24\pi\tau\theta^2 + 720)}{\theta^4(\pi\tau\theta^2 + 2)} - 4\frac{6(\pi\tau\theta^2 + 20)}{\theta^3(\pi\tau\theta^2 + 2)}\left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\} + 6\left\{\frac{2(\pi\tau\theta^2 + 12)}{\theta^2(\pi\tau\theta^2 + 2)}\right\}\left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\}^2 - 3\left\{\frac{(\pi\tau\theta^2 + 6)}{\theta(\pi\tau\theta^2 + 2)}\right\}^4$

Or, $\mu_4 = \frac{\left\{9(\pi\tau\theta^2)^4 + 384(\pi\tau\theta^2)^3 + 1224(\pi\tau\theta^2)^2 + 1728(\pi\tau\theta^2) + 720\right\}}{[\theta(\pi\tau\theta^2 + 2)]^4}$ (10)

2.3 Nature of GNQED:

(a) Index of Dispersion (I):

$$I = \frac{Variance}{Mean} = \frac{\left\{(\pi\tau\theta^2) + 16(\pi\tau\theta^2) + 12\right\}}{\left\{\theta(\pi\tau\theta^2 + 2)(\pi\tau\theta^2 + 6)\right\}} \quad (11)$$

GNQED will be under-dispersed, Equi-dispersed, and over-dispersed if I<, (=),>1 in order respectively.

(b) Shape of GNQED:

$$\gamma_1 = \frac{\left\{2(\pi\tau\theta^2)^3 + 60(\pi\tau\theta^2)^2 + 720(\pi\tau\theta^2 + 48\right\}}{\left\{(\pi\tau\theta^2)^2 + 16(\pi\tau\theta^2) + 12\right\}^{3/2}} \quad (12)$$

Range of γ_1 : $[2/(\sqrt{3})] < \gamma_1 < \infty$.

(c) Size of GNQED:

$$\beta_2 = \frac{\left\{9(\pi\tau\theta^2)^4 + 384(\pi\tau\theta^2)^3 + 1224(\pi\tau\theta^2)^2 + 1728(\pi\tau\theta^2) + 720\right\}}{\left\{(\pi\tau\theta^2)^2 + 16(\pi\tau\theta^2) + 12\right\}^2} \qquad (14)$$

Range of γ_1 : $5 < \beta_2 < \infty$. GNQED is positively skewed in shape and kurtosis by size.

2.4 Some more Characteristics of GNQED:

(a) Reliability Function [R(t)] of GNQED:

R (t) = 1-F (V)

$$R(t) = \frac{\left\{(2+\pi\tau\theta^2) + \theta t(2+\theta v)\right\}}{(2+\pi\tau\theta^2)} e^{-\theta t} \qquad (15)$$

Figure-5

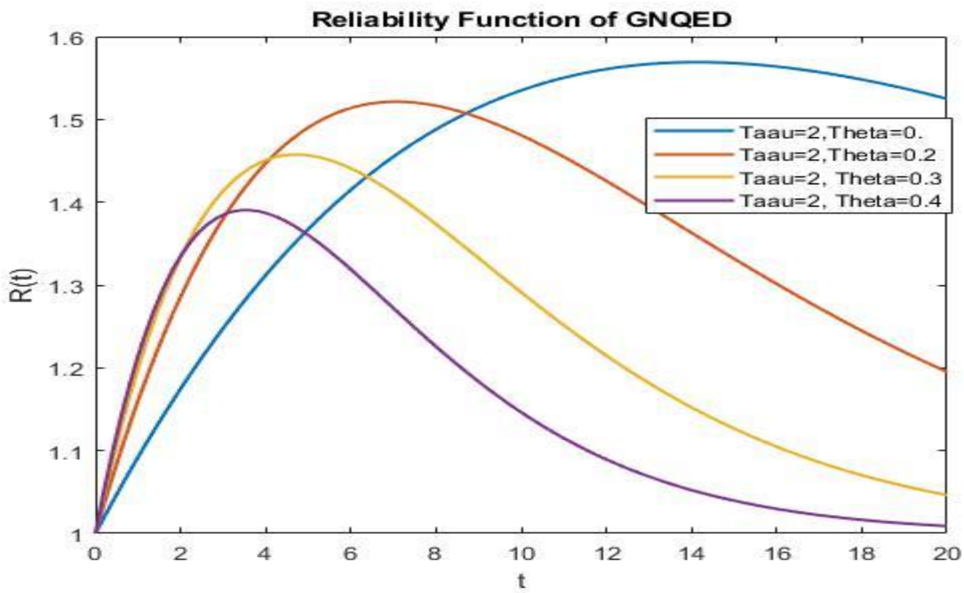
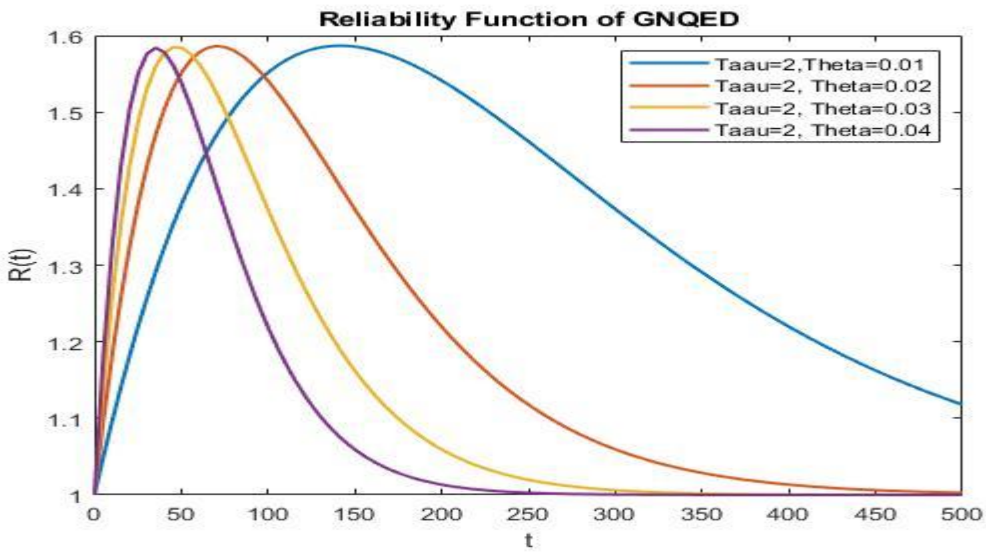


Figure-6



(b) Hazard Rate Function [$h(v=t)$] of GNQED:

$$h(v)=\frac{\left\{\theta^3(\pi\tau+v^2)\right\}}{[(2+\pi\tau\theta^2)+\theta v(2+\theta v)]} \tag{16}$$

At $v=t=0$

$$h(v=t=0)=\frac{\left\{\pi\tau\theta^3\right\}}{[(2+\pi\tau\theta^2)]}>0 \tag{17}$$

$h(t)$ is an increasing function of t and θ.

(c) Mean Residual Life Function [$m(v)$]:

$$m(v)=\frac{\int_v^\infty \left\{1-F(t)\right\}dt}{\left\{1-F(v)\right\}}$$

$$m(v)=\frac{\left\{(2+\pi\tau\theta^2)+(4+4\theta v+\theta^2v^2\right\}}{\theta\left\{(2+\pi\tau\theta^2)+\theta v(2+\theta v)\right\}} \tag{18}$$

At $v=0$

$$m(v)=\frac{\left\{(6+\pi\tau\theta^2)\right\}}{\theta\left\{(2+\pi\tau\theta^2)\right\}} \tag{19}$$

Which is the mean of GNQED.

(d) Moment Generating Function of GNQED:

$$M_v^t=\int_0^\infty e^{vt}f(v)dt=\frac{\theta^3}{(2+\pi\tau\theta^2)}\frac{\left\{2+\pi\tau(\theta-t)^2\right\}}{(\theta-t)^3} \tag{20}$$

2.5 Estimation of the Parameters of GNQED:

This distribution has two parameters, namely, τ and θ which can be estimated by the help of first and second moment about the origin as follows

Dividing the expression (6) by (5), we get

$$K=\frac{(24+2\pi\tau\theta^2)}{(6\theta+\pi\tau\theta^2)}, where\ K=\mu_2'/\mu_1'$$

Finally, we get

$$\tau=\frac{(24-6k\theta)}{(K\pi\theta^3-2\pi\theta^2)} \tag{21}$$

Substituting the value of τ in equation (5), we get the following quadratic equation

$$\mu_2'\theta^2-5\mu_1'\theta+3=0 \tag{22}$$

3.0 Application of GNQED:

This distribution can be useful for statistical modelling for survival time data related to Biology, ecology, Production management and Quality control.

Table-1

Survival time (in days) of guinea pigs infected with virulent tubercle bacilli reported by Bzerkedal [7].

Class	0-80	80-160	160=240	240-320	320-400	400-480	480-560
O	8	30	18	8	4	3	1

Chi-square goodness of fit test is applied to the table-1 and the obtained results have been tabulated in the Table-2.

Table-2

Survival Time (in days)	Observed Frequency	Expected Frequency
		GQEED
0-80	8	10.9
80-160	30	24.7
160-240	18	18.9
240-320	8	10.1
320-400	4	4.5
400-480	3	1.2
480-560	1	1.7
Total	72.0	72.0
$\overline{u}$ =181.11111	$\mu_2' = 43911.11111$	-
$\hat{\tau}$	-	18.9980746
$\hat{\theta}$	-	0.0164758
$\mathrm{d}.f.$	-	2
$\chi^2_{d.f.}$	-	2.08
$P-Value$	-	0.3535

4.0 Conclusion:

This distribution is suitable for statistical modelling of survival time over-dispersed data.

Conflict of Interest:

We, the authors, have no any conflict of interest.

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