

Analysis of the Effect of Noise on the Rabies Epidemic Model

Mohamed Rinas P R¹, M N Srinivas^{2*}, D V Saradhi³, B S N Murthy⁴, Lalitha Chada⁵, M Naga Raju⁶

^{1,2}Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632014, Tamil Nadu, India.

E-mail: ¹rinasrafipr@gmail.com, ²mnsrinivaselr@gmail.com

^{3,4,5,6} Department of Mathematics, Aditya University, Surampalem 533437, Andhra Pradesh, India.

Email: ³saradhid9@gmail.com, ⁴bsn3213@gmail.com, ⁵ch.lalitha16@gmail.com, ⁶nagarajuacet@gmail.com

Article History:

Received: 27-09-2024

Revised: 28-11-2024

Accepted: 12-12-2024

Abstract:

The aim of this research is to explore some novel dynamics of a special model that describes the dynamics of the rabies virus and takes into consideration the impacts of appropriate vaccination. Three compartments are used to categorise the population: susceptible, infected, and recovered. Without taking noise into account, the model's basic reproduction number, equilibrium point, and local and global stabilities are examined. The Gaussian three-dimensional white noise process is being used in this study to determine the stochasticity of the noisy system. Additionally, we used Matlab to verify the stochastic results through numerical simulations.

Keywords: Rabies virus, Stochasticity, Basic reproduction number, Equilibrium point.

1. Introduction

Emerging illnesses have been proliferating over time, leading to numerous infections and preventable deaths in numerous nations. One of those illnesses with a lengthy human history is rabies, which was initially identified in China in 566 BC [1]. The saliva of infected animals can spread the deadly rabies virus to humans. The rabies virus is typically disseminated by bites. The animals most prone to transmit the disease were skunks, foxes, coyotes, bats, and raccoons. In developing countries, stray dogs are the most prevalent way for humans to contract rabies [2]. After a person develops the symptoms of rabies, they almost always die. Because of this, anyone who may be at danger of contracting rabies is advised to get vaccinated. A few significant human rabies epidemics occurred in China within the past 60 years. About 2000 cases were reported in 1956 and 1957, marking the first peak. This was followed by significant declines in the early 1960s. In 1969, the number of cases hit 2000 once more, and in 1981, it hit a record-breaking 7037 cases[1].

More than 59,000 people worldwide lose their lives to rabies each year [3-4]. People can contract this virus by being bitten by a dog, fox, or bat. Fever, pain where exposure occurs, trouble moving certain body parts, disorientation and unstable consciousness, dread of water, and uncontrollable movements are some of its early symptoms. If rabies is not treated, the infected person may suffer from severe side effects, such as brain inflammation in humans and infected mammals, which can quickly result in death. The onset of these symptoms may occur one, three, or even a year later. The length of time is primarily dictated by how far the virus travels throughout the body before arriving to the central nervous system, which could ultimately be lethal [5]. In figure 1 shows the Rabies virus.

According to reports, rabies is most common in Asia and Africa, particularly among the impoverished and, most importantly, children. Because they are more susceptible to bites and scratches when playing with dogs, children are the ones most afflicted by this illness, and reports show that between 30 percentage and 60 percentage of those who have died from it are under the age of 15. For these reasons, more stringent precautions are required to reduce the virus's risk and its impact on various types of individuals. Vaccination and animal control are two popular strategies to prevent severe viral symptoms in the event of an infection. It is possible to vaccinate people with underlying medical issues, particularly those who reside in regions where rabies is prevalent. Another way to lessen the virus's ability to spread throughout the body is to wash the bitten or affected region with soap and water or iodine within 15 minutes of the bite. It is imperative to take further steps to stop the virus from spreading, which is why mathematical modeling is useful when working with these epidemiological models. Additional research on the rabies virus's dynamics can be found in [6-7].



Fig. 1 Rabies virus [12]

The following variables may raise your chance of contracting rabies: When visiting or residing in poor nations where rabies is more prevalent, Camping without taking precautions to keep wild animals away from your campsite or visiting tunnels where bats live are two activities that could expose you to wild creatures that transmit rabies. Working in a lab as a veterinarian and handling

the rabies virus, Head or neck injuries could hasten the rabies virus's progression to the brain [2]. There aren't any WHO-approved diagnostic methods available right now to identify rabies infection before symptoms appear. It is challenging to make a clinical diagnosis of rabies in the absence of distinct hydrophobia or aerophobia symptoms or a trustworthy history of exposure with a rabid animal [8-11]. The most effective way to prevent rabies is to avoid contact with wildlife. Never approach an injured animal; instead, call animal control for assistance. Pet owners can lower their risk by ensuring their pets are up to date on their rabies vaccines. Additionally, make an effort to keep your pets away from wild creatures and other pets they are unfamiliar with. [14-15].



Fig. 1(a) Rabies virus infected person [13]

Rabies can be prevented by getting vaccinated before coming into touch with potentially rabid animals. This is recommended for populations at higher risk of rabies exposure. As mandated by law, vaccinate cats and dogs against rabies. Rabies vaccination is required for all dogs and cats older than four months. Vaccinations ought to be current at all times[16]. Control your cats and dogs. Free roaming of animals is forbidden by animal control legislation. Rabies can be prevented by getting vaccinated before coming into touch with potentially rabid animals. This is recommended for populations at higher risk of rabies exposure compared to pets kept in their owners' care, roaming pets are more likely to have been exposed to rabies. It is not appropriate to keep wild animals as pets. It is possible for a captive raccoon or skunk to carry rabies. Such animals cannot be purchased or kept as pets due to local regulations. As of right now, For wild animals, there are no authorized vaccinations or quarantines. [16–20]. Make it difficult for wild animals to find your property. Seal off any openings under porches, attics, and basements, and cover chimneys. Keep trash cans properly closed and feed your dogs indoors.

2. Computational Model

In this section, we shall describe the basic principles of the proposed model. The model includes sections for Total human population $N(t)$ susceptible individuals $S(t)$, infected individuals $I(t)$, and

recovered individuals $R(t)$. Each compartment's population may fluctuate over time since it is a function of time. Where $N(t) = S(t) + I(t) + R(t)$

In 2024 Wafa F. Alfwzan et.al [21] The dynamics and solution of a modified model for modelling the SIR rabies virus model are created by the authors. The system considers the effects of several parameters, including the vaccination rate. Studying the long-term effects of disease transmission is the goal of looking at the stability analysis and bifurcation of the illness and epidemic models, specifically the rabies epidemic. Rabies, which is characterized by dynamic transmission among different animals, is still a problem in many regions. In general, these conventional models follow accepted procedures. They might overlook some nonlinearities or unconventional characteristics that are present in real life, like seasonality, population dynamics, and the geographic or regional clustering of the epidemic. After the system is developed, its equilibrium points were identified by the authors. Where stability prerequisites are met in terms of important system parameters, the local and global stabilities of equilibrium points of 2 the system were also examined. The potential presence of codimension-one bi-furcations in the system were examined using bifurcation analysis. Bifurcation surfaces are also shown in the system's parameter space.

$$\left. \begin{aligned} S'(t) &= \gamma - \lambda(1 - \phi)S_I - \mu_1 S - mS + \alpha_1 \xi_1(t) \\ I'(t) &= \lambda(1 - \phi)S_I - m_1 I - (\mu_1 + \Lambda)I + \alpha_2 \xi_2(t) \\ R'(t) &= mS + m_1 I - \mu_1 R + \alpha_3 \xi_3(t) \end{aligned} \right\} \quad (1)$$

Table 1 Shows the Conditional variables of Equation (1)

Conditional variables	Description of variables
N	Total population of humans
S	Susceptible Individuals
I	Infected Individuals
R	Recovered Individuals

Table 2 Shows the Conditional Parameters of Equation (1)

Parameters Description	Description of the parameters
γ	The fertility rate
λ	The rate of disease-induced mortality
Λ	The rate of infection
ϕ	The awareness rate of rabies disease
μ_1	The rate of natural mortality
m	The introduction rate of immunization
m_1	The effectiveness rate of treatment for rabies patient

3. Stability Without Noise

In 2024 Wafa F. Alfwzan et.al [21] determined the endemic equilibrium point and the disease-free equilibrium point of the system after studying the SIR Rabies virus model without noise. Theoretically, if the fundamental reproduction number is smaller than 1, the disease-free equilibrium is asymptotically stable both locally and globally. It is shown theatrically that if the fundamental reproduction number is bigger than one, the endemic equilibrium point is locally asymptotically stable.

Steady state : The authors [21] computed the disease free equilibrium points as :

$$D_1 = (S^1, I^1, R^1) = \left(\left(\frac{\gamma}{\mu_1} + m \right), 0, \left(\frac{m_1}{\mu_1(\mu_1 + m)} \right) \right)$$

they also computed the endemic equilibrium point as:

$$E_1 = (S^*, I^*, R^*) = \left(\frac{m_1 + \mu_1 + \Lambda}{\lambda(1 - \phi)}, \frac{\gamma - (\mu_1 + m)s^*}{\lambda(1 - \mu)s^*}, \frac{ms^* + m_1 I^*}{\mu_1} \right)$$

they also computed the basic reproduction number R_0 as :

$$R_0 = \frac{\lambda\gamma(1 - \phi)}{(\mu_1 + m)(\mu_1 + m_1 + \Lambda)}$$

The authors [19] also proved the disease-free equilibrium point D_1 is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$

The authors [19] proved the theorem, If $R_0 > 1$ then the endemic equilibrium of the system (1) E_1 is locally asymptotically stable.

The authors[19] proved the theorem, For any time $t > 0$ the system (1) is globally asymptotically stable for the disease-free equilibrium point E_1 which is contained in the region κ if $R_0 < 1$

We presented a novel stochastic model using seven-dimensional Gaussian white noise after being motivated by several earlier studies [21-25].

4. Stochasticity

Climatic fluctuations and natural disruptions characterize ecological systems. Researchers are motivated to examine how a stochastic environment may impact and define the dynamics of natural systems since random drivers recur in bio-geophysical processes. The deterministic system (1) is now extended to examine how stability is affected by arbitrary environmental changes. The system's parameters fluctuate about their mean values due to random fluctuations. By adding additive white noises to the system, we take this unpredictability into the system(1). Now, the equilibrium of the deterministic and stochastic models will fluctuate around their mean states. However, the white noise perturbation included will change any parameter D of the model to $D + \alpha\xi(t)$, where α is the noise's amplitude and $\xi(t)$ is a Gaussian white noise process at time t .

Considering the randomly fluctuating driving forces in the system (1) as additive noise results in the stochastic model that follows. The stochastic model that we obtain is as follows:

The system (1) With three dimensional Gaussian white noise satisfying,

$$E[\xi_i(t)] = 0; i = 1, 2, 3$$

$$E[\xi_i(t), \xi_j(t')] = \delta_{ij} \delta(t - t'); i = j = 1, 2, 3$$

In this case, δ is the delta -dirac function, and δ_{ij} is the Kronecker symbol.

Let,

$$S(t) = u_1(t) + S^*, I(t) = u_2(t) + I^*, R(t) = u_3(t) + R^* \quad (2)$$

We can now state that,

$$\frac{dS}{dt} = \frac{du_1}{dt}, \frac{dI}{dt} = \frac{du_2}{dt}, \frac{dR}{dt} = \frac{du_3}{dt} \quad (3)$$

and only considering the impact of linear stochastic disturbances of system (1), by replacing (2) and (3) we obtain ,

$$u_1'(t) = -\lambda u_2(t) S^* + \lambda \phi u_2(t) + S^* + \alpha_1 \xi_1(t) \quad (4)$$

$$u_2'(t) = \lambda u_1(t) I^* - \lambda \phi u_1(t) + I^* + \alpha_2 \xi_2(t) \quad (5)$$

$$u_3'(t) = 0 + \alpha_3 \xi_3(t) \quad (6)$$

Applying the Fourier transform to (4)– (6) results in

$$i\omega \bar{u}_1(\omega) + (\lambda - \lambda\phi) \bar{u}_2(\omega) = \alpha_1 \bar{\xi}_1(\omega) \quad (7)$$

$$-(\lambda - \lambda\phi) \bar{u}_1(\omega) + i\omega \bar{u}_2(\omega) = \alpha_2 \bar{\xi}_2(\omega) \quad (8)$$

$$i\omega \bar{u}_3(\omega) = \alpha_3 \bar{\xi}_3(\omega) \quad (9)$$

Equations (11) – (18) are in the matrix form of

$$M(\omega) \bar{u}(\omega) = \bar{\xi}(\omega) \quad (10)$$

Where

$$M(\omega) = \begin{bmatrix} i\omega & \lambda - \lambda\phi & 0 \\ -(\lambda - \lambda\phi) & i\omega & 0 \\ 0 & 0 & i\omega \end{bmatrix} \quad (11)$$

$$\bar{u}(\omega) = \begin{bmatrix} \bar{u}_1(\omega) \\ \bar{u}_2(\omega) \\ \bar{u}_3(\omega) \end{bmatrix} \quad (12)$$

$$\bar{\xi}(\omega) = \begin{bmatrix} \bar{\xi}_1(\omega) \\ \bar{\xi}_2(\omega) \\ \bar{\xi}_3(\omega) \end{bmatrix} \quad (13)$$

Equation (10) can be written as $\bar{u}(\omega) = [M(\omega)]^{-1} \bar{\xi}(\omega)$

Let us suppose that $[M(\omega)]^{-1} = K(\omega)$

Therefore,

$$\bar{u}(\omega) = \bar{u}(\omega) \bar{\xi}(\omega) \quad (14)$$

here

$$K(\omega) = \begin{bmatrix} \frac{i\omega}{(\lambda-\lambda\phi)^2-\omega^2} & \frac{-(\lambda-\lambda\phi)}{(\lambda-\lambda\phi)^2-\omega^2} & 0 \\ \frac{\lambda-\lambda\phi}{(\lambda-\lambda\phi)^2-\omega^2} & \frac{i\omega}{(\lambda-\lambda\phi)^2-\omega^2} & 0 \\ 0 & 0 & \frac{1}{\omega} \end{bmatrix} \quad (15)$$

the components of the (14) solution are provided by

$$\bar{u}_i(\omega) = \sum_{j=1}^3 K_{ij}(\omega) \bar{\xi}_j(\omega); i = 1,2,3 \quad (16)$$

the $u_i, i = 1,2,3$ spectrum is provided by

$$S_{u_i}(\omega) = \sum_{j=1}^3 \alpha_j |K_{ij}(\omega)|^2; i = 1,2,3 \quad (17)$$

The magnitudes of the variations in the variable $u_i, i = 1,2,3$ are provided by

$$\delta_{u_i}^2 = \frac{1}{2\pi} \sum_{j=1}^3 \int_{-\infty}^{\infty} \alpha_j |K_{ij}(\omega)|^2 d(\omega), i = 1,2,3 \quad (18)$$

So we are substituting (15) in (18) we have,

$$\delta_{u_1}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(\lambda-\lambda\phi)^2-\omega^2} [\alpha_1 i\omega - \alpha_2 (\lambda - \lambda\phi)]^2 d\omega \quad (19)$$

$$\delta_{u_2}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(\lambda-\lambda\phi)^2-\omega^2} [\alpha_1 (\lambda - \lambda\phi) + \alpha_2 i\omega]^2 d\omega \quad (20)$$

$$\delta_{u_3}^2 = \frac{-\alpha_3}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2} d\omega, \quad \text{where } \omega \neq 0 \quad (21)$$

If we consider the noise effect on any one of the classes and want to ascertain the characteristics of the system (1) with either $\alpha_1 = 0$ or $\alpha_2 = 0$ or $\alpha_3 = 0$, the population variances are as follows:

If $\alpha_2 = 0, \alpha_3 = 0$, then

$$\delta_{u_1}^2 = \frac{-\alpha_1^2}{2\pi} \int_{-\infty}^{\infty} \frac{\omega^2}{(\lambda - \lambda\phi)^2 - \omega^2} d\omega$$

$$\delta_{u_2}^2 = \frac{\alpha_1^2}{2\pi} \int_{-\infty}^{\infty} \frac{(\lambda - \lambda\phi)^2}{(\lambda - \lambda\phi)^2 - \omega^2} d\omega$$

$$\delta_{u_3}^2 = 0$$

If $\alpha_1 = 0, \alpha_2 = 0$, then

$$\delta_{u_1}^2 = 0, \delta_{u_2}^2 = 0$$

$$\delta_{u_3}^2 = \frac{-\alpha_3}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2} d\omega, \quad \text{where } \omega \neq 0$$

5.Numerical Simulations

In the corresponding sections, computer simulations for the Rabbits disease model with 3 classes which are $S(t)$, $I(t)$, and $R(t)$ are provided using the most relevant, well-fitting and appropriate parameter values. Every visual projection has been positioned, styled, and presented in the most informative manner.

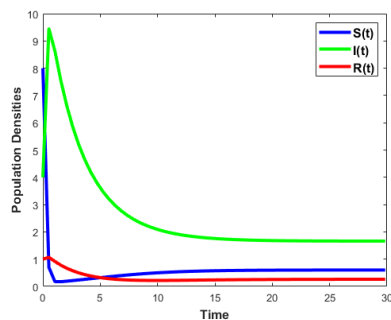


Fig.2

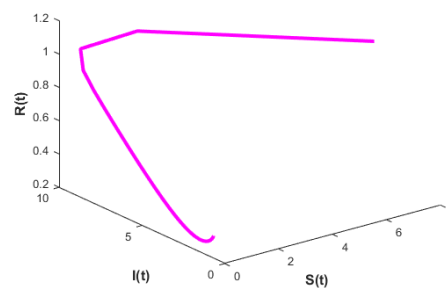


Fig 2(a)

Fig.2 and Fig.2(a) shows the projections of population classes ($S(t)$, $I(t)$ and $R(t)$) w.r.t the parameter values $\gamma = 0.5$; $\lambda = 0.5$; $\phi = 0.0001$; $\mu_1 = 0.25$; $m = 0.25$; $m_1 = 0.001$; $\Lambda = 0.01$;

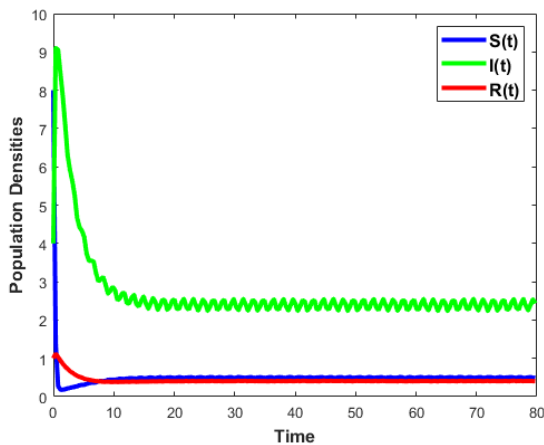


Fig.3(a)

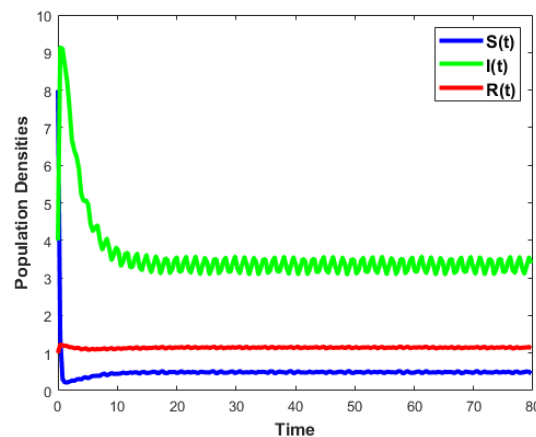


Fig.3(b)

Fig.3(a) and Fig. 3(b) shows the projections of population classes ($S(t)$, $I(t)$ and $R(t)$), w.r.t the parameter values $\gamma = 0.5$; $\lambda = 0.5$; $\phi = 0.0001$; $\mu_1 = 0.25$; $m = 0.25$; $m_1 = 0.001$; $\Lambda = 0.01$; and at various noise intensities For Fig.3(a) noise intensities are $\alpha_1 = 0.1$; $\alpha_2 = 0.3$; $\alpha_3 = 0.1$; and For Fig.3(b) noise intensities are $\alpha_1 = 0.5$; $\alpha_2 = 0.9$; $\alpha_3 = 0.5$; .

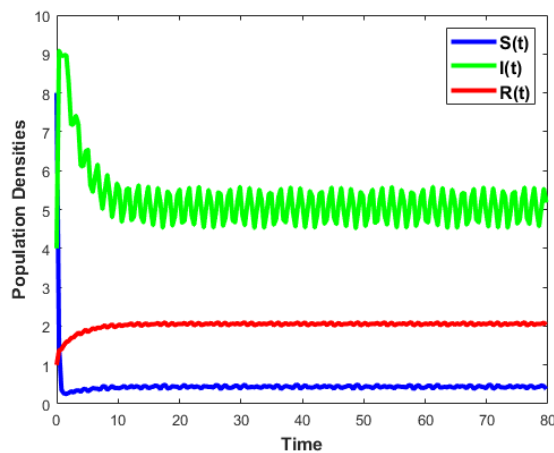


Fig.3(c)

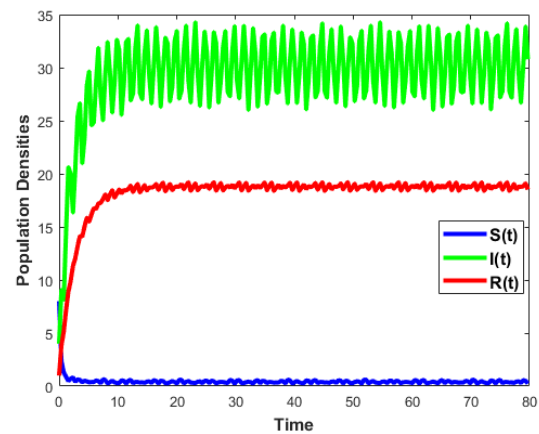


Fig.3(d)

Fig.3(c) and Fig. 3(d) shows the projections of population classes($S(t)$, $I(t)$ and $R(t)$), w.r.t the parameter values $\gamma = 0.5$; $\lambda = 0.5$; $\phi = 0.0001$; $\mu_1 = 0.25$; $m = 0.25$; $m_1 = 0.001$; $\Lambda = 0.01$; and at various intensities For Fig.3(c) noise intensities are $\alpha_1 = 1$; $\alpha_2 = 3$; $\alpha_3 = 1$; and For Fig.3(d) noise intensities are $\alpha_1 = 10$; $\alpha_2 = 15$; $\alpha_3 = 10$;

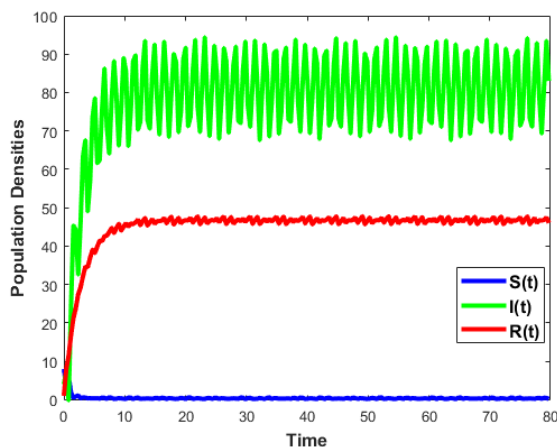


Fig.3(e)

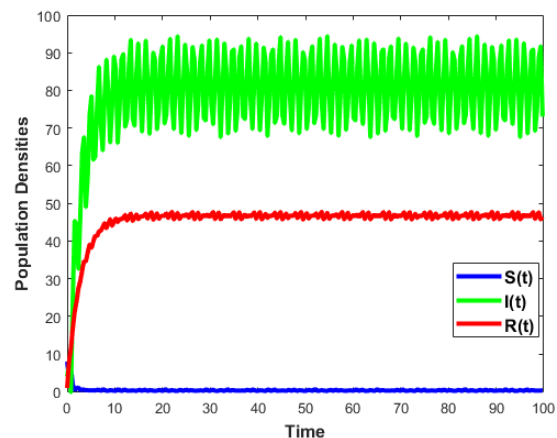


Fig.3(f)

Fig.3(e) and Fig. 3(f) shows the projections of population classes($S(t)$, $I(t)$ and $R(t)$), w.r.t the parameter values $\gamma = 0.5$; $\lambda = 0.5$; $\phi = 0.0001$; $\mu_1 = 0.25$; $m = 0.25$; $m_1 = 0.001$; $\Lambda = 0.01$; and at various intensities. For Fig.3(e) noise intensities are $\alpha_1 = 25$; $\alpha_2 = 50$; $\alpha_3 = 25$; For Fig.3(f) noise intensities are $\alpha_1 = 35$; $\alpha_2 = 60$; $\alpha_3 = 35$;

Numerical Observations:

Fig.2 and Fig.2(a) shows the deterministic projections of all population classes $S(t)$, $I(t)$ and $R(t)$ with the best fitted values, which are $\gamma = 0.5$; $\lambda = 0.5$; $\phi = 0.0001$; $\mu_1 = 0.25$; $m = 0.25$; $m_1 = 0.001$; $\Lambda = 0.01$. Fig.2 exhibits the dynamics of all population classes $S(t)$, $I(t)$ and $R(t)$, which is a time

series evaluation. Fig.2(a) exhibits the dynamics of all population classes $S(t)$, $I(t)$ and $R(t)$, which is a phase portrait projection.

Fig. 3(a) and Fig. 3(b) shows the projections of stochastic system of all population classes $S(t)$, $I(t)$ and $R(t)$ with noise intensities $\alpha_1 = 0.1$; $\alpha_2 = 0.3$; $\alpha_3 = 0.1$; and $\alpha_1 = 0.5$; $\alpha_2 = 0.9$; $\alpha_3 = 0.5$; respectively. Both figures shows clearly the impact of noise intensities α_1 , α_2 and α_3 on the system (1). All classes $S(t)$, $I(t)$ and $R(t)$ are influenced by noise intensities, particularly more fluctuation and influence exhibited by Infected class $I(t)$. As noise intensities varies(raises or increases), system totally went under the influence of gaussian white noise.

Fig. 3(c) and Fig. 3(d) shows the projections of stochastic system of all population classes $S(t)$, $I(t)$ and $R(t)$ with noise intensities $\alpha_1 = 1$; $\alpha_2 = 3$; $\alpha_3 = 1$; and $\alpha_1 = 10$; $\alpha_2 = 15$; $\alpha_3 = 10$; respectively. Both figures shows clearly the impact of noise intensities α_1 , α_2 and α_3 on the system (1). All classes $S(t)$, $I(t)$ and $R(t)$ are influenced by noise intensities, particularly more fluctuation and influence exhibited by Infected class $I(t)$. Fig. 3(c) clearly shows noticeable and little higher fluctuations in the Infected class $I(t)$ and Fig. 3(d) clearly shows a drastic and highly influential fluctuations in the Infected class $I(t)$. As noise intensities varies(raises or increases), system totally went under the influence of gaussian white noise and exhibits high fluctuations.

Fig. 3(e) and Fig. 3(f) shows the projections of stochastic system of all population classes $S(t)$, $I(t)$ and $R(t)$ with noise intensities $\alpha_1 = 25$; $\alpha_2 = 50$; $\alpha_3 = 25$; and $\alpha_1 = 35$; $\alpha_2 = 60$; $\alpha_3 = 35$; respectively. Both figures shows clearly the impact of noise intensities α_1 , α_2 and α_3 on the system (1). All classes $S(t)$, $I(t)$ and $R(t)$ are influenced by noise intensities, particularly more fluctuation and influence exhibited by Infected class $I(t)$. Fig. 3(e) clearly shows higher fluctuations in the Infected class $I(t)$ and Fig. 3(f) clearly shows a drastic and very high influential fluctuations in the Infected class $I(t)$. As noise intensities varies(raises or increases), system totally went under the high influence of gaussian white noise and exhibits very high fluctuations(particularly in Fig. 3(f)).

6. Concluding Remarks

We examined the innovative stochastic structure or analysis for the system (1) to show the numerical representations of the rabies virus using the Fourier transform approach. We discussed the system's disease-free equilibrium point as well as the endemic equilibrium points that exist under particular circumstances (1). We also talked about the SIR Rabies virus model without noise, the basic reproduction number, and the system's global and asymptotic stability. We used three-dimensional Gaussian white noise to study the stochasticity of the system. The parameters of the system fluctuate about their mean values due to random changes. Additionally, we used Matlab to verify the stochastic results through numerical simulations.

Rabies disease develops when people are infected with the Rabies virus. It results in symptoms like paralysis, hallucinations, and convulsions. The most prevalent methods to get rabies are dog bites in Asia and Africa or bat bites in the US. If people receive a vaccination as soon as possible after exposure, rabies spreads, and its development can be reduced. Rabies is deadly as soon as symptoms appear if not protected by vaccination. Raising awareness of the rabies virus encourages individuals to seek the necessary care and involves communities. This involves knowing how to

avoid rabies in animals, how to recognize the symptoms of rabies, and what to do if an animal attacks you. Postexposure prophylaxis (PEP) involves giving a series of rabies vaccines and, in some cases, rabies immunoglobulin (RIG) after a suspected rabies exposure. With proper wound care and prompt access to quality-assured PEP, human rabies deaths can be almost entirely prevented.

References

- [1] S. Ruan, “Modeling the Transmission Dynamics and Control of Rabies in China,” *Mathematical Biosciences* 286 (2017):65–93, <https://doi.org/10.1016/j.mbs.2017.02.005>.
- [2] <https://www.mayoclinic.org/diseases-conditions/rabies/symptoms-causes/syc-20351821>
- [3] Fisher, C. R., Streicker, D. G., and Schnell, M. J. (2018). The spread and evolution of rabies virus: conquering new frontiers. *Nature Reviews Microbiology*, 16(4), 241-255.
- [4] Davis, P. L., Bourhy, H., and Holmes, E. C. (2006). The evolutionary history and dynamics of bat rabies virus. *Infection, Genetics and Evolution*, 6(6), 464-473.
- [5] V. Kumar, A. K. Abbas, N. Fausto, and J. C. Aster, *Robbins and Cotran Pathologic Basis of Disease* (Amsterdam, Netherlands: Elsevier Health Sciences, 2014).
- [6] K. F. Smith, A. P. Dobson, F. E. McKenzie, L. A. Real, D. L. Smith, and M. L. Wilson, “Ecological Theory to Enhance Infectious Disease Control and Public Health Policy,” *Frontiers in Ecology and the Environment* 3, no. 1 (2005): 29–37, <https://doi.org/10.2307/3868442>.
- [7] R. M. Anderson, H. C. Jackson, R. M. May, and A. M. Smith, “Population Dynamics of Fox Rabies in Europe,” *Nature* 289, no. 5800 (1981): 765–771, <https://doi.org/10.1038/289765a0>
- [8] K. Das, B.S.N. Murthy, S.K. Samad, M. H. A. Biswas, Mathematical transmission analysis of SEIR Tuberculosis disease model. *Sens. Int.* 2, 100120, 2021. *Malaysian Journal of Computing* 4, no. 1, 201–213, 2019.
- [9] <https://www.mayoclinic.org/diseases-conditions/rabies/diagnosis-treatment/drc-20351826>
- [10] K.S.Nisar, B.S.N. Murthy, M.N. Srinivas, V. Madhusudhan, A.Zeb, Exploring the dynamics of white noise and spatial temporal variations on hearing loss due to Mumps virus. *Results in Physics* 51, 106584, 2023.
- [11] Mani RS, Madhusudana SN. Laboratory diagnosis of human rabies: recent advances. *ScientificWorldJournal*. 2013 Nov 14;2013:569712.
- [12] <https://www.rabieswatch.com/documents/4388176/4391777/learn-to-avoid-one-of-the-worldsdeadliestinfectious-diseases-desktop.png/404bc376-3f93-0437-cfabec9d5e3253bb?t=1659941382645>
- [13] <https://www.vaccineinformation.org/wp-content/uploads/photos/rabicdc009a.jpg>
- [14] Wunner, W. H. (2003). Rabies virus. *Rabies*, 23-77.
- [15] Brunker, K., and Mollentze, N. (2018). Rabies virus. *Trends in microbiology*, 26(10), 886-887.
- [16] Jackson, A. C. (2003). Rabies virus infection: an update. *Journal of neurovirology*, 9, 253-258.

[17] Hankins, D. G., and Rosekrans, J. A. (2004, May). Overview, prevention, and treatment of rabies. In Mayo Clinic Proceedings (Vol. 79, No. 5, pp. 671-676). Elsevier.

[18] B.S.N. Murthy, SinaEtemad, M.N. Srinivas, V. Madhusudanan , ShahramRezapour, The impact of Caputo-Fabrizio fractional derivative and the dynamics of noise on worm propagation in wireless IoT networks, Alexandria Engineering Journal 91, 558-579, 2024.

[19] Dean, D. J., Evans, W. M., and McClure, R. C. (1963). Pathogenesis of rabies. Bulletin of the World Health Organization, 29(6), 803.

[20] Bano, I., Sajjad, H., Shah, A. M., Leghari, A., Mirbahar, K. H., Shams, S., and Soomro, M. (2017). A review of rabies disease, its transmission and treatment. J. Anim. Health Prod, 4(4), 140-144.

[21] F. Alfwzan, W., Raza, A., Ahmed, N., Elsonbaty, A., Rafiq, M., and Adel, W. (2024). Nonstandard Computational and Bifurcation Analysis of the Rabies Epidemic Model. Journal of Mathematics, 2024(1), 3909089.

[22] RuanShiGui, R. S. (2017). Modeling the transmission dynamics and control of rabies in China.

[23] Biswas, S. K., Ghosh, U., and Sarkar, S. (2020). Mathematical model of zika virus dynamics with vector control and sensitivity analysis. Infectious Disease Modelling, 5, 23-41.

[24] Das, Prasenjit and Mukherjee, Debasis and Das, Kalyan and Srinivas, M N. (2020). Hepatitis b infection and its control: Deterministic and stochastic analysis. 5. 84-95.

[25] M.A. Akinlar, Mustafa Inc, J.F. G´omez-Aguilar, B. Boutarfa, Solutions of a disease model with fractional white noise, Chaos, Solitons and Fractals, Volume 137, 2020, 109840, ISSN 0960-0779