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Application of the Generalized Intuitionistic Fuzzy Rayleigh (GIFRD) Lifetime Distribution to Agricultural Data

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Abstract:

A number of writers have conducted research on reliability analysis, which is an important topic in engineering. Classical distributions rely on precise parameters to ensure their reliability. The parameters of distributions are typically believed to be accurate real values of number system. But in the actual world, it's not always possible to measure and record data precisely. Generalised intuitionistic fuzzy (GIF) reliability is included in the current study's expansion of the fuzzy reliability concept. Under the assumption that this lifespan parameter is a generalised intuitionistic fuzzy number, we investigate the dependability characteristic of systems using the Rayleigh lifetime distribution. The cut sets for all the survival functions of the generalised intuitionistic fuzzy hazard function and fuzzy mean time to failure are analysed when these systems obey the generalised intuitionistic fuzzy Rayleigh lifetime distribution (GIFRD). This method uses a band with upper and lower boundaries to depict the danger and dependability curves for each unique cut set. Additionally, reliability evaluations of parallel and series systems are carried out. The current work investigated the PDF, CDF, Hazard Rate Function, and Survival Function of the GIFRD using intuitionistic fuzzy values for real-time data. Kerala's agricultural production in 2018 was the main topic of this study. Therefore, the comparison showed that it is possible to estimate fuzziness values more precisely than real-time data.

Keywords: Intuitionistic fuzzy set, hazard curves, PDF, CDF, Rayleigh lifespan distribution.

Mathematical Subject Classification (2020): 03E72, 03F55.

1. Introduction

In engineering, agriculture, and medicine, reliability analysis is crucial. Reliability analysis in statistical distributions used in engineering is one of the main problems that those interested in lifetime data analysis have to deal with. There are several uses for the Rayleigh distribution in reliability theory and survival analysis.

Zadeh coined the term "fuzzy variable" in 1965 [1] to describe improper linguistic idioms and vernacular. This was the origin of fuzzy concepts. A fuzzy set consists of components with varying membership levels.

The dependability of fuzzy systems is evaluated using fuzzy set ideas. The reliability of fuzzy systems was assessed using the Intuitionistic Fuzzy Set theory (IFS). Each component's dependability is represented as a Trapezoidal Intuitionistic Fuzzy Number in order to analyse the dependability of fuzzy systems [2]. First, the arithmetic operations of (TrIFNs) are explained. TrIFNs have been used to generate expressions for assessing the fuzzy reliability of a series and their parallel system. Here, an electric network operating in a dark environment is modelled using an imprecise reliability model. TrIFNs are used to indicate the reliability of each system component in order to calculate the imprecise reliability of the system in question. A corresponding numerical example is given.

Fuzzy reliability of series and their corresponding parallel systems is also implemented. A malfunctioning system of printed circuit board assembly is used as a numerical example to validate the target approach [3]. The proposed method's results are compared to the ways listed in the reliability analysis methodologies.

In order to take into consideration the fuzziness of the lifespan data, the study aimed to generalise the parameters and reliability function estimate for 3-parameter lifetime distributions [4]. While conventional techniques only take stochastic innovation into account, the resulting estimators take into account both stochastic and fuzzy variation in lifetime data. The proposed estimators are more realistic and suitable for lifespan analysis due to the reproduction of fuzziness. Therefore, compared to alternative approaches for modelling lifespan data, the provided methodologies are significantly better.

The Weibull lifespan distribution and intuitionistic fuzzy (IF) notions were used to estimate the survival of a linear consecutive k-out-of-n, F system with non-identical components. We compute intuitionistic triangular fuzzy values for the parameters of the Weibull distribution. The dependability of the system's evolution states is computed using the Markov chain approach. The dependability of the system is illustrated using a numerical example [5].

To compare the investigated methods using simulation with different parameter values and different sample sizes, the mean-square error was employed as a metric [6]. According to the simulation results, the Mixed Thompson Method outperforms the other methods in terms of MSE, and the fuzzy reliability and fuzziness values are superior to the real numbers for all sample sizes. For these reasons, we advise using this type of estimation.

The ambiguity of the collected failure rate and reconstruction time data was taken into consideration by using the triangular membership function [7]. The intuitionistic fuzzy Lambda-Tau approach was used to estimate reliability parameters at different spreads, and the results were compared to the traditional fuzzy Lambda-Tau method. After receiving the data, the reliability engineer for the facility created a maintenance schedule based on time intervals to increase the unit's availability.

In an intuitionistic fuzzy environment, the best estimator is found via Monte Carlo simulations [8]. The mean biassed and MSE criteria are used to compare the performance of the Bayesian and machine learning estimations of the parameters and reliability function.

In addition to the specific instance of the two-parameter Pareto generalised intuitionistic fuzzy reliability analysis [9], the generalised intuitionistic fuzzy reliability attribute of reliability, conditional reliability, hazard rate, and mean time to failure are described in detail. Moreover, the dependability of parallel and series systems is assessed using generalised intuitionistic fuzzy sets. Lastly, the generalised intuitionistic fuzzy reliability features are displayed and associated using several illustrated charts for certain scenarios of fuzzy shape and scale parameters, as well as cut set values.

The study uses a Pythagorean fuzzy exponential distribution to examine the dependability indices of many systems in a fuzzy environment. For systems with the above distribution, expressions for fuzzy hazard function, reliability, mean time to failure, η-cut set, and sensitivity have been developed [10]. Lastly, numerical examples were hypothesised to evaluate the reliability features of the systems functioning in the designated setting. This approach is very helpful in a number of application areas, such as gathering operators, decision-making, and information measures.

Bayesian estimation techniques for one-parameter WXLD and intuitionistic fuzzy lifetime data for reliability analysis. To do this, a gamma prior is used, and the square error loss function is utilised to estimate parameters and dependability. The Lindley approximation and Tierney and Kadane (T-K) approximation are used to approximate Bayesian estimates because of the intricacy of integrals. To show the practical relevance and applicability of the suggested estimate methods, they are evaluated on a simulated dataset [11]. Lastly, real-world data is used to validate the suggested approaches, proving their effectiveness in real-world situations.

The probability density function, cumulative density function, hazard rate function, and survival function of the generalised intuitionistic fuzzy Rayleigh lifetime distribution were investigated in the present work utilising real-time data and a fuzzy intuitionistic fuzzy set. In 2018, Kerala agriculture was the focus of this study [12].

2. “Fuzzy Mathematical tools

2.1 Generalized Intuitionistic Fuzzy Number (GIFN)

Definition 2.1.1.

Assume$X \neq \emptyset$. A generalized intuitionistic fuzzy set (GIFS_B (X))A in X is rigorous as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$, denote the degree of membership and degree of non membership functions of x in A, respectively, and $0 \leq \mu_A(x)^\delta + \nu_A(x)^\delta \leq 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}, n = 1,2, \dots, N$.

Definition 2.1.2.

A generalized $(L - R)$ type intuitionistic fuzzy number A is defined as any $\mathrm{GIFS}_B(X)$ on the real line $\mathbb{R}$ with the following membership function μ_A(x) and non-membership function ν_A(x),

$$\mu_A(x)=\begin{cases}\left(\frac{x-q}{r-q}\right)^{\frac{1}{\delta}} & ,q\le x\le r\\ 1 & ,b\le x\le c\\ \left(\frac{d-x}{d-c}\right)^{\frac{1}{\delta}} & ,c\le x\le d\\ 0, & otherwise\end{cases},\quad v_A(x)=\begin{cases}\left(\frac{r-x}{r-q_1}\right)^{\frac{1}{\delta}}, & q_1\le x\le r\\ 0 & r\le x\le c\\ \left(\frac{x-c}{d_1-c}\right)^{\frac{1}{\delta}}, & c\le x\le d_1\\ 1, & otherwise\end{cases}$$

where $q_1 \le q \le r \le c \le d \le d_1$ and $0 \le \mu_A(x)^\delta + v_A(x)^\delta \le 1, \forall x \in X$. The $GIFN_B A$ is denoted as $A = (q_1, q, r, c, d, d_1, \delta)$.

Definition 2.1.3.

A $GIFN_B$ is said to be symmetric $GIFN_B$ if $r - q = d - c$ and $r - q_1 = d_1 - c$.

2.2. Cut sets on GIFN$_B$

This subsection delves into the concept of cut set on GIFN_B (Baloui Jamkhaneh 2016).

Let β_1 and β_2 be fixed values in [0,1] such that $0 \le {\beta_1}^\delta + {\beta_2}^\delta \le 1$. The set of (β_1, β_2)-cuts produced by a $GIFN_B A$ is defined as

$$A[\beta_1,\beta_2,\delta] = \{\langle x, \mu_A(x) \ge \beta_1, v_A(x) \le \beta_2\rangle : x \in X\}$$

The β_1 - cut set of a GIFN$_B A$ is a crisp subset of R. It is defined as

$$A[\beta_1,\delta] = \{\langle x, \mu_A(x) \ge \beta_1, \rangle : x \in X\} 0 \le \beta_1 \le 1,$$

The definition of $GIFN_B$, clearly states that

$$A[\beta_1,\delta] = [L_1(\beta_1), U_1(\beta_1)] 0 \le \beta_1 \le 1,,$$
$$L_1(\beta_1) = a + (b-a){\beta_1}^\delta, U_1(\beta) = d - (d-c){\beta_1}^\delta.$$

Similarity The β_2 - cut set of a GIF$N_B A$ is a crisp subset of R, defined as

$$A[\beta_2,\delta] = \{\langle x, v_A(x) \le \beta_2\rangle : x \in X\}, 0 \le \beta_2 \le 1$$

According to the definition of $GIFN_B$, it is simply proven that

$$A[\beta_2,\delta] = [L_2(\beta_2), U_2(\beta_2)], 0 \le \beta_2 \le 1$$
$$\mathrm{L}_2(\beta_2) = \mathrm{b}\left(1 - {\beta_2}^\delta\right) + \mathrm{a}_1{\beta_2}^\delta, \mathrm{U}_2(\beta) = \mathrm{c}\left(1 - {\beta_2}^\delta\right) + \mathrm{d}_1{\beta_2}^\delta$$

The (β_1,β_2)-cut set of a $GIFN_B$ can be expressed as

$$A[\beta_1,\beta_2,\delta] = \{x, x \in [L_1(\beta_1), U_1(\beta_1)] \cap [L_2(\beta_2), U_2(\beta_2)]\}$$

2.3. Generalized Intuitionistic Fuzzy Distribution

Model a component's lifetime random variable (X) as $f(x,\tilde{\lambda})$, where $\tilde{\lambda}$ represents a $GIFN_B$. In this situation, the generalised intuitionitic fuzzy probability of finding a value in B is $\tilde{P}(B)$,The cut sets are computed as follows.

$$P(B)[\beta_i] = \left\{\int_B f(x,\lambda)dx \mid \lambda \in \lambda[\beta_i,\delta] = \left[P^L[\beta_i],P^U[\beta_i]\right], i = 1,2\right.$$

for all $0 \leq \beta_1 \leq 1, 0 \leq \beta_2 \leq 1, 0 \leq {\beta_1}^{\delta} + {\beta_2}^{\delta} \leq 1$, where

$$P^L[\beta_i] = \min\left\{\int_B f(x,\lambda)dx \mid \lambda \in \lambda[\beta_i,\delta], i = 1,2.\right.$$

$$P^U[\beta_i] = \max\left\{\int_B f(x,\lambda)dx \mid \lambda \in \lambda[\beta_i,\delta], i = 1,2.\right.$$

According to this definition, $\tilde{P}(B)$ is a GIFN_B, where$[P^L[\beta_1],P^U[\beta_1]]$ and $\left[P^L[\beta_2],P^U[\beta_2]\right]$ are the cut sets of membership and non-membership functions, respectively. Also, the cut set

(β_1,β_2)- is as follows.

$$P(B)[\beta_1,\beta_2] = \left[P^L[\beta_1],P^U[\beta_1]\right] \cap \left[P^L[\beta_2],P^U[\beta_2]\right]$$

Let lifetime random variable of a component (X) be modeled by a Rayleigh distribution. Then

$$f(x,\tilde{\lambda}) = \frac{2x}{\tilde{\lambda}} e^{-\frac{x^2}{\tilde{\lambda}}}, x > 0$$

where $\tilde{\lambda}$ is a $GIFN_B$. In this case we have

$$\tilde{P}(n \leq X \leq m)[\beta_i] = \left\{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \middle| \lambda \in \lambda[\beta_i,\delta]\right\} = \left[P^L[\beta_i],P^U[\beta_i]\right], i = 1,2$$

for all $0 \leq \beta_1 \leq 1, 0 \leq \beta_2 \leq 1, 0 \leq {\beta_1}^{\delta} + {\beta_2}^{\delta} \leq 1$, where

$$P^L[\beta_i] = \min\left\{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \middle| \lambda \in \lambda[\beta_i,\delta]\right\} = \min\left\{e^{-\frac{n^2}{\lambda}} - e^{-\frac{m^2}{\lambda}} \middle| \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1,2 \text{ ,,}$$

$$P^U[\beta_i] = \max\left\{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \middle| \lambda \in \lambda[\beta_i,\delta]\right\} = \max\left\{e^{-\frac{n^2}{\lambda}} - e^{-\frac{m^2}{\lambda}} \middle| \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1,2$$

2.4. Generalized Intuitionistic Fuzzy Reliability Band

$GIFN_B$, which serves as the foundation for generalized intuitionistic fuzzy reliability, The generalized intuitionistic fuzzy probability unit (GIFR) endures beyond time t. Assume X follows a Rayleigh distribution with a generalized intuitionistic fuzzy lifespan parameter $\tilde{\lambda} = (a_1, a, b, c, d, d_1, \delta)$. In this situation, the GIFR of the component is $\tilde{S}(t)$, and the cut sets are computed as follows:

$$S(t)[\beta_i] = P(X > t)[\beta_i] = \left\{\int_t^\infty \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \middle| \lambda \in \lambda[\beta_i,\delta]\right\} = \left\{e^{-\frac{t^2}{\lambda}} \middle| \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1,2$$

Given that $e^{-\frac{t^2}{\lambda}}$ is a monotonically growing function with regard to λ, the α_1-cut set of membership function and α_2-cut set of non-membership function are as follows:

$$S(t)[\beta_1] = [S^L[\beta_1], S^U[\beta_1]] = \left[e^{-\frac{t^2}{(a+(b-a)\alpha_1^\delta)}}, e^{-\frac{t^2}{(d-(d-c))_1^\delta)}}\right]$$

$$S(t)[\beta_2] = [S^L[\beta_2], S^U[\beta_2]] = \left[e^{-\frac{t^2}{(b(1-\alpha_2^\delta)+a_1\alpha_2^\delta)}}, e^{-\frac{t^2}{(c(1-\alpha_2^\delta)+\alpha_1\delta_2^\delta)}}\right]$$

$S(t)[\beta_i], i = 1,2$ are two-dimensional functions in terms of β_i and t($0 \le \beta_1 \le 1, 0 \le \beta_2 \le 1, 0 \le {\beta_1}^\delta + {\beta_2}^\delta \le 1$ and t>0). For $t_0, \tilde{S}(t_0)$ is a GIFN_B with the following membership and non-membership functions:

$$\mu_{S(t_0)}(x) = \begin{cases} \left(\frac{-\frac{{t_0}^2}{\ln x} - a}{b - a}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{a}} \le x \le e^{-\frac{{t_0}^2}{b}} \\ 1, & e^{-\frac{{t_0}^2}{b}} \le x \le e^{-\frac{{t_0}^2}{c}}. \\ \left(\frac{\frac{{t_0}^2}{\ln x} + d}{d - c}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{c}} \le x \le e^{-\frac{{t_0}^2}{d}} \\ 0, & o.w. \end{cases}$$

$$v_{S(t_0)}(x) = \begin{cases} \left(\frac{t_0 + b}{b - a_1}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{a_1}} \le x \le e^{-\frac{{t_0}^2}{b}} \\ 0, & e^{-\frac{{t_0}^2}{b}} \le x \le e^{-\frac{{t_0}^2}{c}}. \\ \left(\frac{-\frac{{t_0}^2}{\ln x} - c}{d_1 - c}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{c}} \le x \le e^{-\frac{{t_0}^2}{d_1}} \\ 1, & o.w. \end{cases}$$

A (β_1, β_2) - cut set of $\tilde{S}(t)$ is given by

$$S(t)[\beta_1, \beta_2] = S(t)[\beta_1] \cap S(t)[\beta_2]$$

“For each β_{10} and β_{20}, the dependability curve is represented by a band with upper and lower boundaries. In this case, it's known as the GIFR band. The reliability band includes the following properties:

(i) $S(0)[\beta_{10}, \beta_{20}] = [1,1]$, i.e. no one starts off dead,

(ii) $S(\infty)[\beta_{10}, \beta_{20}] = [0,0]$, i.e. everyone dies eventually,

(iii) $S(t_1)[\beta_{10}, \beta_{20}] \ge S(t_2)[\beta_{10}, \beta_{20}] \Leftrightarrow t_1 \le t_2$, i.e. band of $S(t)[\beta_{10}, \beta_{20}]$ declines monotonically.

Corollary

Let $\eta = \frac{\beta_2^\delta}{1-\beta_1^\delta}, k_1 = \frac{b-a}{b-a_1}$ and $k_2 = \frac{d-c}{d_1-c}$, then we have if $k_1 < k_2$

$$S(t_0)[\beta_1,\beta_2] = \begin{cases} [S^L[\beta_2],S^U[\beta_2]], & \eta < k_1 \\ [S^L[\beta_1],S^U[\beta_2]], & k_1 \le \eta \le k_2 \\ [S^L[\beta_1],S^U[\beta_1]], & k_2 < \eta \end{cases}$$

if $k_1 > k_2$

$$S(t_0)[\beta_1,\beta_2] = \begin{cases} [S^L[\beta_2],S^U[\beta]], & \eta < k_2 \\ [S^L[\beta_2],S^U[\beta_1]], & k_2 \le \eta \le k_1 \\ [S^L[\beta_1],S^U[\beta_1]], & k_1 < \eta \end{cases}$$

if $k_1 = k_2 = k$ (i.e. $\tilde{\lambda}$ is symmetric GIFN_B)

$$S(t_0)[\beta_1,\beta_2] = \begin{cases} [S^L[\beta_2],S^U[\beta_2]], & \eta < k \\ [S^L[\beta_1],S^U[\beta_1]], & \eta \ge k \end{cases}$$

if $\eta = 1$ (i.e. $\beta_2^\delta = 1 - \beta_1^\delta$), then $S(t_0)[\beta_1,\beta_2] = [S^L[\beta_1],S^U[\beta_1]]$.

2.5. Generalized Intuitionistic Fuzzy Hazard Band

Assume X follows a Rayleigh distribution with a generalized intuitionistic fuzzy lifespan parameter $\tilde{\lambda} = (q_1, q, r, c, d, d_1, \delta)$. The GIFHF for component ($\tilde{h}(t)$) is as follows:

$$h(t)[\beta_i] = \left\{\frac{f(t)}{s(t)}\middle| \lambda \in \lambda[\beta_i,\delta]\right\} = \left\{\frac{2t}{\lambda}\middle| \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1{,}2$$

$$h(t)[\beta_i] = [h(t)^L[\beta_i], h(t)^U[\beta_i]], i = 1{,}2$$

for all $0 \le \beta_1 \le 1{,}0 \le \beta_2 \le 1{,}0 \le {\beta_1}^\delta + {\beta_2}^\delta \le 1$, where

$$\mathrm{h(t)^L}[\beta\alpha_\mathrm{i}] = \min\left\{\frac{2\mathrm{t}}{\lambda}\middle| \lambda \in \lambda[\beta_\mathrm{i},\delta]\right\}, \mathrm{h(t)^U}[\beta_\mathrm{i}] = \max\left\{\frac{2\mathrm{t}}{\lambda}\middle| \lambda \in \lambda[\beta_\mathrm{i},\delta]\right\}, i = 1{,}2.$$

Therefore,

$$h(t)[\alpha_1] = \left[\frac{2t}{d-(d-c)\beta_1^\delta}, \frac{2t}{q+(r-q)\beta_1\delta'}\right], h(t)[\alpha_2]$$
$$= \left[\frac{2t}{c(1-\beta_2^\delta)+d_1\beta_2\delta}, \frac{2t}{r(1-\beta_2\delta)+q_1\beta_2^\delta}\right].$$

The (β_1,β_2) - cut set of $\tilde{h}(t)$ is as follows

$$h(t)[\beta_1,\beta_2] = h(t)[\beta_1] \cap h(t)[\beta_2]$$

Thus, with a generalized intuitionistic fuzzy Rayleigh distribution, the GIFH function increases over time. For $t_0, \tilde{h}(t_0)$ is a generalized intuitionistic fuzzy number. Its membership and non-membership functions are as follows:

$$\mu_{h(t_0)}(x)=\begin{cases}\left(\frac{d-\frac{2t_0}{x}}{d-c}\right)^{\frac{1}{\delta}}, & \frac{2t_0}{d}\le x\le\frac{2t_0}{c}\\ 1, & \frac{2t_0}{c}\le x\le\frac{2t_0}{b}\\ \left(\frac{\frac{2t_0}{x}-a}{b-a}\right)^{\frac{1}{\delta}}, & \frac{2t_0}{b}\le x\le\frac{2t_0}{a}\\ 0, & o.w\end{cases},\ \nu_{h(t_0)}(x)=\begin{cases}\left(\frac{\frac{2t_0}{x}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \frac{2t_0}{d_1}\le x\le\frac{2t_0}{c}\\ 0, & \frac{2t_0}{c}\le x\le\frac{2t_0}{b}\\ \left(\frac{b-\frac{2t_0}{x}}{b-a_1}\right)^{\frac{1}{\delta}}, & \frac{2t_0}{b}\le x\le\frac{2t_0}{a_1}\\ 1, & o.w\end{cases}$$

Corollary

For every $\delta, S(t)[1,0]=\left[e^{-\frac{t^2}{r}}, e^{-\frac{t^2}{c}}\right], h(t)[1,0]=\left[\frac{2t}{c},\frac{2t}{r}\right], S(t)[0,1]=\left[e^{-\frac{t^2}{q}}, e^{-\frac{t^2}{d}}\right]$, $h(t)[1,0]=\left[\frac{2t}{d},\frac{2t}{q}\right]$.

Corollary

Let $\eta=\frac{\beta_2^{\delta}}{1-\alpha_1^{\delta}}, k_1=\frac{r-q}{r-q_1}$ and $k_2=\frac{d-c}{d_1-c}$, then we have if $k_1<k_2$

$$h(t_0)[\beta_1,\alpha_2]=\begin{cases}[h^L[\beta_2],h^U[\beta_2]], & \eta<k_1\\ [h^L[\beta_2],h^U[\beta_1]], & k_1\le\eta\le k_2\\ [h^L[\beta_1],h^U[\beta_1]], & k_2<\eta\end{cases}$$

if $k_1>k_2$

$$h(t_0)[\beta_1,\beta_2]=\begin{cases}[h^L[\beta_2],h^U[\beta_2]], & \eta<k_2\\ [h^L[\beta_1],h^U[\beta_2]], & k_2\le\eta\le k_1\\ [h^L[\beta_1],h^U[\beta_1]], & k_1<\eta\end{cases}$$

if $k_1=k_2=k$

$$h(t_0)[\beta_1,\beta_2]=\begin{cases}[h^L[\beta_2],h^U[\beta_2]], & \eta<k\\ [h^L[\beta_1],h^U[\beta_1]], & \eta\ge k\end{cases}$$

if $\eta=1$ i.e. $(\beta_2^{\delta}=1-\beta_1^{\delta})$, then $h(t_0)[\beta_1,\beta_2]=[h^L[\beta_1],h^U[\beta_1]]$.

2.6 Generalized Intuitionistic Fuzzy Mean Time to Failure

The anticipated mean time to failure (MTTF) is the generalised intuitionistic fuzzy mean time to failure (GIFMTTF). The MTTF of any component with a generalized intuitionistic fuzzy Rayleigh distribution is a $GIFN_B$, as defined below:

$$\begin{aligned}\mathrm{GIFMTTF}[\alpha_i] &= \{\textstyle\int_0^\infty s(x)dx \mid \lambda \in \lambda[\beta_i,\delta]\}\\ &= \left\{\int_0^\infty e^{-\frac{x^2}{\lambda}}dx \,\middle|\, \lambda \in \lambda[\beta_i,\delta]\right\} = \left\{\frac{\sqrt{\pi\lambda}}{2}\,\middle|\, \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1,2,\\ \mathrm{GIFMTTF}[\alpha_1] &= \left[\frac{\sqrt{\pi\left(q+(r-q)\beta_1^\delta\right)}}{2}, \frac{\sqrt{\pi\left(d-(d-c)\beta_1^\delta\right)}}{2}\right]\\ \mathrm{GIFMTTF}[\alpha_2] &= \left[\frac{\sqrt{\pi\left(r\left(1-\beta_2^\delta\right)+q_1\beta_2^\delta\right)}}{2}, \frac{\sqrt{\pi\left(c\left(1-\beta_2^\delta\right)+d_1\beta_2^\delta\right)}}{2}\right]\end{aligned}$$

The membership and non-membership functions of GIFMTTF are defined as follows:"

$$\mu_G(x) = \begin{cases}\left(\frac{\frac{4x^2}{\pi}-q}{r-q}\right)^{\frac{1}{\delta}}, & \frac{\sqrt{\pi q}}{2} \le x \le \frac{\sqrt{\pi r}}{2}\\ 1, & \frac{\sqrt{\pi r}}{2} \le x \le \frac{\sqrt{\pi c}}{2}\\ \left(\frac{d-\frac{4x^2}{\pi}}{d-c}\right)^{\frac{1}{\delta}}, & \frac{\sqrt{\pi c}}{2} \le x \le \frac{\sqrt{\pi d}}{2}\\ 0, & o.w\end{cases}, \nu_G(x) = \begin{cases}\left(\frac{-\frac{4x^2}{\pi}-r}{r-q_1}\right)^{\frac{1}{\delta}}, & \frac{\sqrt{\pi q_1}}{2} \le x \le \frac{\sqrt{\pi r}}{2}\\ 0, & \frac{\sqrt{\pi r}}{2} \le x \le \frac{\sqrt{\pi c}}{2}\\ \left(\frac{\frac{4x^2}{\pi}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \frac{\sqrt{\pi c}}{2} \le x \le \frac{\sqrt{\pi d_1}}{2}\\ 1, & o.w\end{cases}$$

2.7. GIFR of Series and Parallel Systems

In this portion, we present a generalized intuitionistic fuzzy reliability evaluation approach for serial and parallel systems.

"Series System

When n components are connected in series, the β_i-cut $(i = 1,2)$ of GIFR with generalised intuitionistic fuzzy Rayleigh distribution is as follows:

$$S(t)[\beta_i] = \{P(Y_1 > t) \mid \lambda \in \lambda[\beta_i,\delta]\} = \left\{e^{-\frac{nt^2}{\lambda}}\,\middle|\, \lambda \in \lambda[\beta_i,\delta]\right\}, i = 1,2.$$

$$S(t)[\beta_1] = \left[e^{-\frac{nt^2}{q+(r-q)\beta_1^\delta}}, e^{-\frac{nt^2}{d-(d-c)\beta_1^\delta}}\right], S(t)[\beta_2] = \left[e^{-\frac{nt^2}{r\left(1-\beta_2^\delta\right)+q_1\beta_2^\delta}}, e^{-\frac{nt^2}{c\left(1-\beta_2^\delta\right)+d_1\beta_2^\delta}}\right],$$

For t_0, GIFR is a $GIFN_B$ and membership function and non-membership function of $\tilde{S}(t)$ are as follows:

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\dfrac{-\frac{nt_0^2}{\ln x}-q}{r-q}\right)^{1/\delta}, & e^{-\frac{nt_0^2}{q}}\le x\le e^{-\frac{nt_0^2}{r}}\\ 1, & e^{-\frac{nt_0^2}{r}}\le x\le e^{-\frac{nt_0^2}{c}}\\ \left(\dfrac{\frac{nt_0^2}{\ln x}+d}{d-c}\right)^{\frac{1}{\delta}}, & e^{-\frac{nt_0^2}{c}}\le x\le e^{-\frac{nt_0^2}{d}}\\ 0, & o.w\end{cases}$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\dfrac{\frac{nt_0^2}{\ln x}+b}{b-a_1}\right)^{1/\delta}, & e^{-\frac{nt_0^2}{a_1}}\le x\le e^{-\frac{nt_0^2}{b}}\\ 0, & e^{-\frac{nt_0^2}{b}}\le x\le e^{-\frac{nt_0^2}{c}}\\ \left(\dfrac{-\frac{nt_0^2}{\ln x}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & e^{-\frac{nt_0^2}{c}}\le x\le e^{-\frac{nt_0^2}{d_1}}\\ 0, & o.w\end{cases}$$

2.8. Parallel System

When n components are connected in parallel, the β_i-cut $(i = 1,2)$ of GIFR with generalized intuitionistic fuzzy Rayleigh distribution is given by.

$$S(t)[\beta_i]=\{P(Y_n>t)\mid \lambda\in\lambda[\beta_i,\delta]\}=\left\{1-\left(1-e^{-\frac{t^2}{\lambda}}\right)^n \middle| \lambda\in\lambda[\beta_i,\delta]\right\}, i=1,2$$

$$S(t)[\beta_1]=\left[1-\left(1-e^{-\frac{t^2}{q+(r-q)\beta_1^{\delta}}}\right)^n, 1-\left(1-e^{-\frac{t^2}{d-(d-c)\beta_1^{\delta}}}\right)^n\right]$$

$$S(\mathrm{t})[\beta_2]=\left[1-\left(1-\mathrm{e}^{-\frac{\mathrm{t}^2}{r(1-\beta_2^{\delta})+\beta_1\beta_2^{\delta}}}\right)^{\mathrm{n}}, 1-\left(1-\mathrm{e}^{-\frac{\mathrm{t}^2}{c(1-\beta_2^{\delta})+\beta_1\beta_2^{\delta}}}\right)^{\mathrm{n}}\right]$$

For t_0, this is a $GIFN_B$ and membership function and non-membership function of $\tilde{S}(t_0)$ are as follows"

$$\mu_{S(t_0)}(x) = \begin{cases} \left(\dfrac{-q - {t_0}^2\left(\ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{r-q}\right)^{\frac{1}{\delta}}, & 1-\left(1-e^{-\frac{{t_0}^2}{q}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{r}}\right)^n \\ 1, & 1-\left(1-e^{-\frac{{t_0}^2}{r}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{c}}\right)^n \\ \left(\dfrac{d + {t_0}^2\left(\ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{d-c}\right)^{\frac{1}{\delta}}, & 1-\left(1-e^{-\frac{{t_0}^2}{c}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{d}}\right)^n \\ 0 & \text{o.w.} \end{cases}$$

$$\nu_{S(t_0)}(x) = \begin{cases} \left(\dfrac{r + {t_0}^2\left(\ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{\mathrm{r}-q_1}\right)^{\frac{1}{\delta}}, & 1-\left(1-e^{-\frac{{t_0}^2}{q_1}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{r}}\right)^n \\ 0, & 1-\left(1-e^{-\frac{{t_0}^2}{r}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{c}}\right)^n \\ \left(\dfrac{-c - {t_0}^2\left(\ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{d_1-c}\right)^{\frac{1}{\delta}}, & 1-\left(1-e^{-\frac{{t_0}^2}{c}}\right)^n \le x \le 1-\left(1-e^{-\frac{{t_0}^2}{d_1}}\right)^n \\ 0 & \text{o.w.} \end{cases}$$

3. Real Data study

In this practical application, we examined the 2018 Kerala agriculture data set by repurposing real-time agricultural data from Kaggle.com [12]. As shown in Tables 1 and 2, a generalised intuitionistic fuzzy Rayleigh lifetime distribution was used to analyse the data. For every crop, the mean square error is: 1277296849.70, 94750756.00, 734485339.93, 1828538185.19, 78889924.00, 125873590.04, 450169608430.24, 397627528.36, 1594436045.55, 4452213614.51, 859025687.54 615670081.29, 1210692025.00, 893751323.91, 24947927.14, and 1479723475.

Crop	Season	Area	Production	Annual Rainfall	Fertilizer	Pesticide	Yield
Arecanut	Whole Year	95739.29	99925	2989.7	15528913	33508.75	0.795714
Arhar/Tur	Whole Year	266	438	2989.7	43145.2	93.1	1.65
Banana	Whole	52898.	429061	2989.7	858015	18514.	7.9307

	Year	61			5	51	14
Black pepper	Whole Year	82761.41	36777	2989.7	13423901	28966.49	0.328571
Cardamom	Whole Year	38882	11535	2989.7	6306660	13608.7	0.068571
Cashewnut	Whole Year	38780.66	15635	2989.7	6290223	13573.23	0.276429
Coconut	Whole Year	760946.8	5.3E+09	2989.7	1.23E+08	266331.4	6426.82
Cotton(lint)	Whole Year	59.4	90	2989.7	9634.68	20.79	1.51
Garlic	Whole Year	69.61	345	2989.7	11290.74	24.3635	4.96
Ginger	Whole Year	3275.09	15124	2989.7	531219.6	1146.282	3.463571
Gram	Whole Year	690.86	546	2989.7	112057.5	241.801	0.70375
Groundnut	Whole Year	187.3	239	2989.7	30380.06	65.555	1.28
Jowar	Whole Year	205	168	2989.7	33251	71.75	0.82
Maize	Whole Year	104.326	145	2989.7	16921.68	36.5141	4.508
Onion	Whole Year	5.21	0	2989.7	845.062	1.8235	0
Other Kharif pulses	Kharif	1532.826	1316	2989.7	248624.4	536.4891	1.076
Potato	Whole Year	536.9	7381	2989.7	87085.18	187.915	13.75

Table.1. Agriculture data of Kerala in the year of 2018.

Crop	Production (tons)	Price (units)	Cost (units)	Value (units)	Yield per unit area (units)	Generalized Membership Values (μ_A(x))	Generalized Non-Membership Values(ν_A(x))	Hesitation (π_A(x)))
Arecanut	95739.29	99925	2989.7	15528913	33508.75	0.60	0.20	0.20
Arhar/Tur	266	438	2989.7	43145.2	93.1	0.10	0.85	0.05
Banana	52898.61	429061	2989	8580155	18514.	0.80	0.10	0.10

			.7		51			
Black pepper	82761.41	36777	2989.7	13423901	28966.49	0.40	0.50	0.10
Cardamom	38882	11535	2989.7	6306660	13608.7	0.30	0.60	0.10
Cashewnut	38780.66	15635	2989.7	6290223	13573.23	0.50	0.30	0.20
Coconut	760946.8	5300000000	2989.7	123000000	266331.4	0.90	0.05	0.05
Cotton (lint)	59.4	90	2989.7	9634.68	20.79	0.20	0.70	0.10
Garlic	69.61	345	2989.7	11290.74	24.3635	0.40	0.40	0.20
Ginger	3275.09	15124	2989.7	531219.6	1146.282	0.70	0.20	0.10
Gram	690.86	546	2989.7	112057.5	241.801	0.30	0.50	0.20
Groundnut	187.3	239	2989.7	30380.06	65.555	0.25	0.55	0.20
Jowar	205	168	2989.7	33251	71.75	0.35	0.45	0.20
Maize	104.326	145	2989.7	16921.68	36.5141	0.30	0.50	0.20
Onion	5.21	0	2989.7	845.062	1.8235	0.05	0.90	0.05
Other Kharif pulses	1532.826	1316	2989.7	248624.4	536.4891	0.40	0.50	0.10

Table.2. intuitionistic fuzzy data values of Kerala in the year of 2018 .

Mathematical Results of the generalized intuitionistic fuzzy Rayleigh lifespan distribution

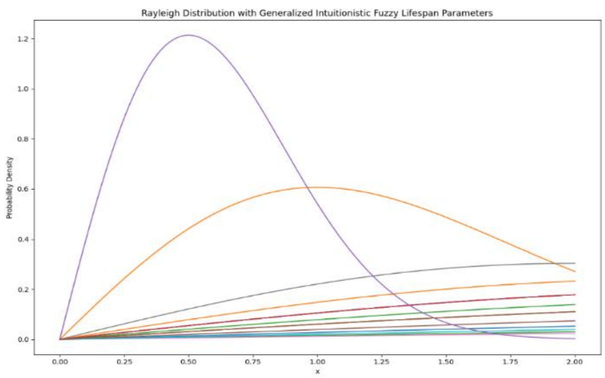

Fig. 1: PDF of generalized intuitionistic fuzzy Rayleigh lifespan distribution.

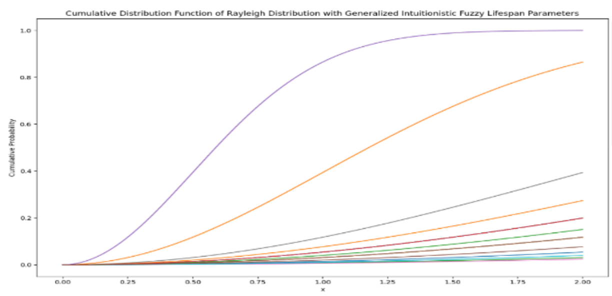

Fig. 2: CDF of generalized intuitionistic fuzzy Rayleigh lifespan distribution.

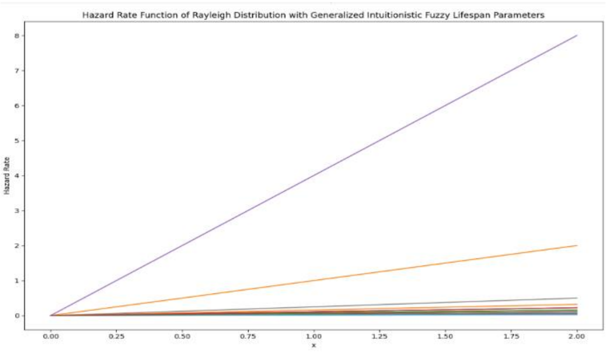

Fig.3 : Hazard rate of generalized intuitionistic fuzzy Rayleigh lifespan distribution.

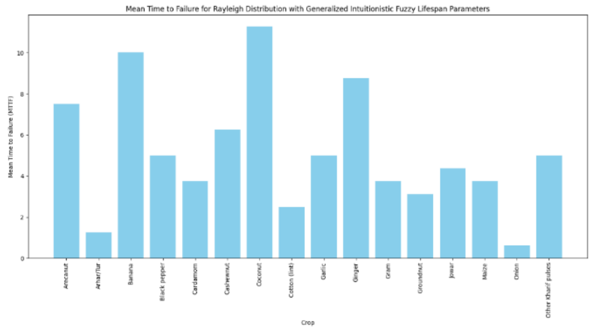

Fig.4 : Mean Time to Failiure of generalized intuitionistic fuzzy Rayleigh lifespan distribution.

4. Results and Discussion

In this research, the scale parameters were obtained in intuitionistic fuzzy form. The employment of intuitionistic fuzzy parameters in the system's reliability study has disguised the crisp parameters' inadequacies. The use of intuitionistic fuzzy sets has reduced randomness or fuzziness to the necessary level of precision, allowing us to make our analysis more exact and authentic. This work presents a method for analyzing system dependability adopting generalized intuitionistic fuzzy sets theory concepts. The lifespan parameter of a component is denoted by $GIFNB$. The generalised intuitionistic fuzzy reliability and hazard functions were examined, and several fuzzy parameters and α-cut values were generated. Using a generalised intuitionistic fuzzy lifespan parameter, we demonstrated reliability analysis for both series and lateral systems. Our approach is more comprehensive than previous approaches. Using the generalised intuitionistic fuzzy lifetime distribution, future studies can investigate the characteristics of conditional reliability, mean residual lifetime function (MRLF), mean past lifetime function, cumulative hazard function, and reversed hazard function.

The present study findings are supported by Kaggle.com [12]. In the generalised intuitionistic fuzzy Rayleigh lifespan distribution to the agriculture problem of Kerala in the year of 2018 with fuzzy parameters and a supporting Python tool, we found results for the Probability density function (Fig. 1), Cumulative density function (Fig. 2), Survival function (Fig. 3), and Mean Time to Failure of each crop (Fig. 4). These results demonstrate how well the proposed technique performs in predicting crop production for all seasons.

5. Conclusion

In this study, applying intuitionistic fuzzy parameters to real life application of kerala agriculture in 2018, we obtained three reliability ranges for each scenario. If the crisp parameters were used, The reliability bands produced in the present study show that the fuzzy reliability of the Probability density function, Cumulative density function, Hazard rate function and Mean Time to Failure of the each crop production for Kerala agriculture system in the year of 2018, which correlates to the results obtained using different fuzzy parameters. All the results of crop that is Areca nut, Archer/Tur, Banana…. Levels are increased in all the seasons.

All functions for the generalised intuitionistic fuzzy lifetime distribution are included in this. According to this study, when the product's quality is uncertain, the suggested method can be applied. According to the study, the approach performs well in unpredictable agricultural situations.

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