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An Investigation on the Stationary Shop Queuing Model; an Overview of Mathematical Modeling

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Abstract:

Queuing model theory is a branch of mathematics that studies the behaviour of waiting queues. It is used to analyse and model the overall production of the systems where all the resources and equipment are shared among competing demands. These systems can include anything from computer networks and telecommunications to manufacturing processes and customer service operations.

The purpose of this research paper is to analyse the customer flow and service efficiency of a stationary shop using queuing theory. The study seeks to understand the patterns of customer arrivals, service times, and overall system performance to provide recommendations to improving the shop's operational efficiency.

Data Collection: The data for this study was taken over a period of one month from a well-known and busy stationary shop (Aditya Stationers and Sports) in Jodhpur. We interacted with the owner of the shop Mr. Jitendra Kumar and collected the data. The data includes: Arrival times of customers, Total number of service counters, Queue lengths at different times of the day..

Key Components Analysed: Arrival Process, Service Process, Queue Discipline, Queue Capacity and Total number of Servers.

Metrics Evaluated: Mean Waiting Time, Mean Length of the Queue, Server Utilization.

Introduction

Waiting in line is something we all deal with, and queuing theory looks at the math behind how lines get backed up and how long people have to wait. A queuing system isn't just about people standing in a physical line - it could be planes trying to land at a busy airport, ships waiting to unload cargo, or even cars stuck in traffic [18]. Basically, it's any situation where someone or something has to wait their turn to get something done.

When we talk about waiting times, we usually think about two extremes - really slow service that takes forever, or super quick service that gets you in and out. Queuing theory tries to obtain the information about how these systems work by looking at the numbers and patterns behind them [32] .

The goal is to understand how lines behave and maybe find ways to make them move faster. It's not only about counting the number of people that are in the line, but also figuring out why lines form and how to manage them better. Whether it's at the grocery store or a factory assembly line, queuing theory can help make things run smoother [17].

Queuing theory has two main key elements to be accessed – Customer arrival at facility and Service.

In this research we have studies the queuing patterns of one of the busiest stationery and sports shop (Aditya Stationers and Sports) in Jodhpur. We have observed that average *200* customers arrive on week days and average 300 customers arrive on weekend days to the store. The store has only one server. On average 200 customers arrive in between time of *6* PM to *8* PM on the weekend days (Friday to Sunday). Mean of waiting time of a customer is *5* Minutes and average length of queue is of 3 customers.

Literature review

Managing waiting lines is key to boosting the customer satisfaction and efficiency in service industries. Several studies have explored queuing models to handle these challenges in restaurants and banks.

Queuing theory has proven invaluable across many sectors, enhancing service efficiency and overall customer satisfaction. For example, studies like those conducted by [1], [7], and [17] show how effectively applying these models can help reduce waiting times and improve the service quality. A notable model, the M/M/1, which analyses single-server systems, is particularly relevant to the situation at Aditya Stationers and Sports. In this setting, the insights from [4], [5], and [22] become very important since they focus on metrics like waiting times (W_q) and queue lengths (L_q), essential factors in managing the high customer flow during peak hours. In the realm of production and operations management, [3] presents foundational insights emphasizing strategies, processes, and systems crucial for efficient production across various industries.

Research has also highlighted the potential of Monte Carlo simulations, as seen in [2]. These simulations can provide a deeper understanding of service dynamics, similar to the approaches described by [13] that aim to enhance business processes through modelling. Moreover, optimization techniques discussed in [10], [11], [12], and [16] can be applied to retail contexts like Aditya Stationers, offering methods to streamline operations and enhance customer experiences.

For instance, studies in the fast-food industry, such as those by [6], [20], and [21], show how queuing theory can effectively manage high volumes of customers and reduce bottlenecks. These findings are particularly relevant given that Aditya Stationers experiences a surge of about 200 customers in the evenings over the weekends, especially between 6 PM and 8 PM. Understanding and applying these queuing strategies could lead to smoother operations.

The queuing dynamics of a busy restaurant, Sushi Tei in Jakarta, are analysed in [8], revealing a high utilization rate of 99.1% during peak periods, which contributes to overcrowding and potential customer loss. [23] delves into the application of queuing theory within call centres, emphasizing its critical role in understanding performance metrics and enhancing operational efficiency.

Moreover, the work of [9], [14], and [24] offers a broader perspective on operations research and its role in improving decision-making within queuing systems. These insights can inform strategies to optimize customer queue management at Aditya Stationers. Similar findings from studies on banks ([17], [20], [21]) and restaurants ([6], [18], [32]) provide valuable benchmarks for reducing waiting times and enhancing service quality, which can be directly applied to the store's busy weekend periods. [26] proposes a solution through caching web objects at proxy servers, enhancing web access performance by reducing latency and improving response times. The study identifies a gap in existing research, noting that while there has been significant focus on web-replacement algorithms and cache management, limited work has been done on estimating the optimal cache size for proxy servers under high-traffic conditions. [28] serves as a foundational resource, emphasizing the application of quantitative methods in decision-making processes. The authors provide a thorough exploration of key topics, including linear programming, network flows, simulation, and queuing theory, which together equip readers with a robust understanding of the methodologies used to solve complex problems.

In addition, [19] sheds light on the issue of reneging, where customers leave the queue due to excessive wait times. This phenomenon is a critical consideration for Aditya Stationers, as it highlights the importance of keeping wait times manageable to prevent customer loss. Innovative solutions, such as the IoT-based printing system proposed in [25], and simulation tools like ACE, as described in [29], illustrate modern approaches to optimizing customer interactions and minimizing operational inefficiencies.

Furthermore, the mathematical models examined in [27] and [31] offer various optimization techniques that could significantly enhance service system efficiency. Similarly, [32] emphasizes the importance of effective queue management in delivering quality service.

By drawing on this rich body of research ([1]-[32]), this study aims to analyse and improve the queuing patterns at Aditya Stationers. The goal is to reduce waiting times, effectively manage customer flow, and ultimately enhance overall service quality. Given the store's customer dynamics, which mirror the scenarios highlighted in previous studies, applying these queuing models can lead to meaningful improvements in service delivery.

Basic properties

A queue system is defined by five main features: the input process (how things arrive), queue discipline (rules for waiting), service mechanism (how things are processed), system capacity (how much it can hold), and service channels (where processing happens). These elements together describe how a queue operates [18].

At any service centre, there may be one or more than one service channels or the service stations.

- With infinite servers, no queues form. Every customer gets instant service upon arrival, so there's no waiting time. Everyone's served right away.
- If the total number of servers is finite, then the customers are being served according to a specific order.
- If all the provided service stations are empty, then the arriving customers will be served immediately. If service is not available then the arriving customer will not have to wait anymore. And the Customer will leave the system after taking service (see [33- 35]).

Characteristics of The Queuing model system

Queuing models basically have two main parts: customers and servers [21]. Now, when we say "customers," we're not just talking about people. It could be machines, cars, or whatever fits the system we're looking at. The servers are what provide the service, and you can have one or multiple of them.

Here's how it usually goes down: The Customers are coming from one source. If there are enough servers, everyone gets helped right away. But if there are a greater number of customers than servers, some folks have to wait. That's when you get a line forming.

When a server finishes up with one customer, it grabs the next person in line. If there's nobody waiting, the server just chills until someone new shows up.

To really understand a queuing model, you need to look at *6* key things. These factors paint the whole picture of how the system works:

- Arrival time distribution: This is how customers show up. We use a fancy math thing called a Poisson distribution to guess how many folks will arrive in the given time.
- Service time distribution: This is about how long it takes to help each customer. It's not linked to when people arrive and usually follows something called an exponential distribution.
- Service channel [22]: You've got two main types:
  - ✓ Single Service Channel: Just one server doing all the work.
  - ✓ Multi Service Channel: Multiple servers working at once, either side by side or one after another.
- Queue discipline: This is just a fancy way of saying "how we decide who's next." You've got:
  - ✓ FCFO (first come, first out): Like waiting in line at the bank.
  - ✓ LIFO (last in, first out): Think of a stack of plates - you grab from the top.
  - ✓ SIROC (service in random order): Basically, picking names out of a hat.
- Queue lengths: It can be infinite (theoretically never-ending) or finite (with a set limit).
- System capacity: This is how many customers the whole system can handle, including those waiting. Some places have a limit and won't let new folks join if it's full.

Kendall's Notation

Queuing model can be expanding in the following symbolic form- $(a\ /\ b\ /\ c)$: $(d\ /\ e\ /\ f)$

Where

- a = Arrival time
- b = Service time distribution
- c = Number of channels $(1, 2, 3, 4\ \ldots)$
- d = Capacity of system
- e = Queue discipline
- f = Size of calling source

It's called Kendall's notation [32]. A guy named D. Kendall came up with the first three parts $(a/b/c)$ back in*1953*. Then, in*1983*, A. Lee added the last three bits $(d/e/f)$ to make this more complete.

Now, this system will work even better for simple queues, but it's not so hot for the complicated stuff. If you're dealing with queues that are linked up in a series or spread out in a network, Kendall's notation starts to fall short. It just can't capture all the complexity of those setups.

Little's Theorem

Little's theorem [15] is proposed by John Little. This theorem describes the relationship between arrival and service rate. The average or the mean number of customers in the whole system is equal to the observed arrival rate λ multiplied by the mean waiting time in the queue which can also be written as:

$$L_s = \lambda W_s \qquad (1)$$

Here λ = mean arrival rate for the arriving customers that are coming to the given system.

W_s = The expected waiting time in the given system for a customer.

The result applies to any system and particularly. It applies to system within system.

There are three fundamental relations which can be stated using the little's theorem:

- If λ or W_s increase then L_s is also increase.
- If L_s increase or W_s decrease then W_s increase.
- If L_s increase or λ decrease then W_s increase.

Aditya Stationers Store Model

We observed the one-month daily data with the help of Store owner. We know all about Store servers, Average waiting times of the customers, Length of the queue and the Rate of arrival of the customers. The store has only one server. Based on this information we can say that in this scenario, we can use Queuing model [8] $(M/M/1: FCFS/\infty)$.

In this model, customers arrive in the Poisson distribution. The mean rate of arrival of customers is λ. Here the total number of servers is one which is fixed. Now the customers arrive in the System and takes service if the server is free otherwise it starts to wait for the server to get free. As soon as the server gets free, the next customer arrives to get the service and so on.

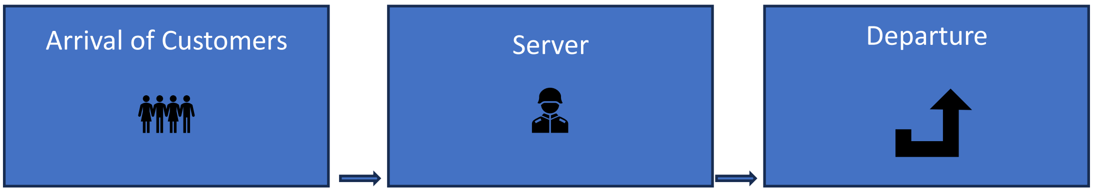


Figure 1: M/M/1 Queuing model

Assumptions

- Customers come from an endless pool of people. There's no limit to how many might show up.
- They arrive following something called the Poisson distribution. It's a math thing that helps predict random events.
- The line works on a "first come, first served" basis. Just like waiting in the line at a coffee shop.
- How long it takes to serve each of the customer follows an exponential distribution.

In this paper, our aim is that all customers should get service and the waiting time of customer is minimum in the queue.

The utilization factor $= \rho < 1$ *or* $\frac{\lambda}{\mu} < 1$ i.e. the average service is faster than the overall mean of the observed arrival rate.

Or analysing the Aditya store's $M/M/1$ queuing model [8] for the following variables. There we will be investigation variables.

λ = Mean of the arrival rate of customers.

$$\rho = \text{Utilization factor} = \frac{\lambda}{\mu}. \qquad (2)$$

Probability of being no (zero) customer in the store where $\rho < 1$.

$$P_0 = 1 - \rho \qquad (3)$$

The probability or the chances of being n customers in the Aditya store –

$$P_n = (1-\rho)\,\rho^n \qquad (4)$$

L_q =The mean number of the customers in the waiting queue.

$$L_q = \frac{\rho^2}{1-\rho} \tag{5}$$

L_s = The mean of the number of the customer in the Store.

$$L_s = L_q + \frac{\lambda}{\mu} \tag{6}$$

W_q = The mean of the waiting time in the queue –

$$W_q = \frac{L_q}{\lambda} \tag{7}$$

W_s = The mean of the waiting time that is spent in the store –

$$W_s = \frac{L_s}{\lambda} \tag{8}$$

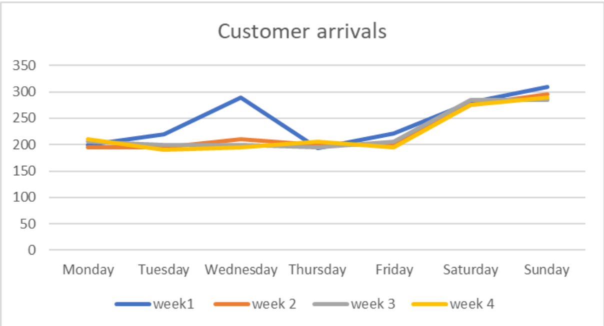
$$W_s = W_q + \frac{1}{\mu} \tag{9}$$

Observation and Discussions

The one-month daily customers' data were shared by the Store owner as shown in following Table 1:

Week	Monday Day1	Tuesday Day 2	Wednesday Day3	Thursday Day4	Friday Day5	Saturday Day6	Sunday Day7
1	200	220	290	194	222	280	310
2	195	195	210	200	200	275	295
3	205	200	200	195	205	285	285
4	210	190	195	205	195	275	290

Table 1: One-Month data



Calculations

Average 100 customers arrive in 3 hours. So, the arrival rate is –

$\lambda = \frac{100}{180} = 0.56$ Customers per minutes (cpm)

By our observations, we have seen that on average a customer spends *7* minutes (W_s) in the system. Queue length L_q is of *3* customers.

On average, the waiting time is of around *5* minutes.

Now using equation number (*7*)

$$W_q = \frac{3}{0.56} = 5.35\ Minutes$$

It can be easily observed that the actual waiting time and calculated waiting do not differ so much.

The mean of the number of customers in the store. Using the Little's theorem by equation (1)

$$L_s = \lambda W_s = 0.56 * 7 = 3.92$$

We can derive the rate of service by the equation (6)

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$\frac{1}{\mu} = \frac{1}{\lambda}[L_s - L_q] = \frac{1}{0.56}[3.92 - 3] = 1.643$$

$$\mu = 0.609$$

Now using equation (2) –

$$\rho = \frac{\lambda}{\mu} = \frac{0.56}{0.609} = 0.9195$$

The probability or chances of being no (zero) customer [17] in the store here, $\rho < 1$, so by equation (3) we have

$$P_0 = 1 - \rho$$

$$P_0 = 1 - 0.9195 = 0.0805$$

Hence probability of having zero customer in the store is 0.0805.

Evaluation

- The expression for the utilization factor [33] is $\rho = \frac{\lambda}{\mu}$ where λ is a mean number of the customers arrival rate and μ is a mean of the service rate of the store.
- As the average or the mean number of the customers grows, the utilization factor (ρ) increases too. They're directly linked - when one goes up, so does the other. In math terms, ρ is proportional to the mean number of customers.
- In this store the utilisation factor is $\rho = 0.9195$ which is high.

Benefits

Aditya stores is the busiest store in Sikar city, and this research article may help them figure out how to improve their overall service quality. When there's a long line of customers, they can use a model to estimate the number of people that will be waiting. It will also help them to stay prepare in advance for busy the times when lots of folks are expected to show up.

The math formulas in this study give Aditya stores a way to set up better queues without having to do major renovations. It's like a blueprint for managing customer flow more smoothly.

By using these ideas, Aditya stores could cut down on wait times and make shopping there a better experience overall.

Conclusions

This research work has discussed the application and benefits of the $M/M/1$ queuing model on Aditya stationery store. We have calculated that the rate of arrival of the customers is $\lambda = 0.56$. The rate of service is $\mu = 0.609$. The probability of having no (zero) customers [32] in the given system is 0.0805. As for the future work, we will develop the simulation model for the store. By creating a simulation model, we can double-check the results from our model in this research paper article. It's like a reality check for our math. Queuing analysis is super useful for understanding how things work and making smart choices about important resources. It's one of the best tools we've got for this kind of stuff - practical and gets the job done.

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