

Inequivalent Hadamard Matrices of Order 40 using Block Designs

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Abstract:

This article presents the classification of Hadamard matrices of order 40 with automorphisms of order 19. We determine the 2-(19,9,8) and 2-(20,10,9) designs with automorphisms of order 19. These designs are embedded into 2-(39,19,9) designs which are then extended to 3-(40,20,9) designs. We get 33 non-isomorphic 2-(39,19,9) designs out of which 18 designs gives non-isomorphic 3-(40,20,9) designs and hence 18 inequivalent Hadamard matrices of order 40 with automorphisms of order 19.

Keywords: Hadamard; automorphism; incidence matrix; block designs.

1. Introduction

A Hadamard matrix is an n -square matrix with entries from $\{1, -1\}$, for which the inner product of any pair of distinct rows is 0. The first example of Hadamard matrix was published by James Joseph Sylvester in [1]. Jacques Hadamard further studied this matrix and gave the famous conjecture on existence of Hadamard matrix, which still remains an open problem and has excited interest among researchers. The conjecture states – *A Hadamard matrix of order n (for $n > 2$) exists if and only if $n \equiv 0 \pmod{4}$.* Various methods of constructing Hadamard matrices were developed and various results equivalent to the existence of Hadamard matrix were proposed. Till date the smallest order unknown Hadamard matrix is for $n = 668$.

Two Hadamard matrices H_1 and H_2 of order n are said to be equivalent if there exist monomial matrices P_1 and P_2 such that $P_1 H_1 P_2 = H_2$. Automorphism of a Hadamard matrix is the equivalence of the matrix with itself, that is existence of monomial matrices P_1 and P_2 such that $P_1 H P_2 = H$. The set of all automorphisms of a Hadamard matrix forms a group under composition called the automorphism group of the matrix H and is denoted as $\text{Aut } H$.

Classification of Hadamard matrices is one of the widely studied area in the study of existence of Hadamard matrices. Inequivalent Hadamard matrices of orders upto 20 has been determined by M. Hall in [2] and [3]. In [4], equivalence classes of Hadamard matrices of order 24 is discussed. For order 28, the classification upto automorphism of order 13 and 7 can be found in [5, 6]. [7] gives the equivalence classes of Hadamard matrices of order 36 with automorphisms of order 17.

In this article we study the classification of Hadamard matrices of order 40 with automorphisms of order 19. We use the relation between Hadamard matrices of order $n = 4k + 4$ and the symmetric 2-($4k + 3, 2k + 1, k$) designs. Every 2-($4k + 3, 2k + 1, k$) design which can be extended to non-isomorphic 3-($4k + 4, 2k + 2, k$) designs gives inequivalent Hadamard matrices of order $4k + 4$. We determine the 2-(19,9,8) and 2-(20,10,9) designs with automorphisms of order 19. These designs are embedded into 2-(39,19,9) designs, which are then extended to 3-(40,20,9) designs. We observe that there are 33 non-isomorphic 2-

(39,19,9) designs, of which 18 designs gives non-isomorphic 3-(40,20,9) designs. The incidence matrix of each of these 18 2-(39,19,9) designs gives inequivalent Hadamard matrices of order 40 with automorphisms of order 19.

1.1. Preliminaries

Definition 1.1. A design is a pair (X, B) where X is a set of elements called *points* and B is a collection of nonempty subsets of X called *blocks*.

Definition 1.2. Let v , k and λ be positive integers such that $v > k \geq 2$. A t -(v , k , λ)-balanced incomplete block design is a design (X, B) such that the following properties holds:

- (1) $|X| = v$
- (2) Each block contains exactly k points.
- (3) Every t -subset of distinct points is contained in exactly λ blocks

Definition 1.3. A 2-(v , k , λ)-design is said to be symmetric if $v = b$, where b is the number of blocks and v is the number of objects.

Definition 1.4. Let (X, A) be a design with $X = \{x_1, x_2, \dots, x_v\}$ and $B = \{B_1, B_2, \dots, B_b\}$. The incidence matrix of (X, B) is a $v \times b$ matrix $M = (m_{ij})$ with 0's and 1's defined as

$$(m_{ij}) = \begin{cases} 1, & \text{if } x_i \in B_j \\ 0, & \text{if } x_i \notin B_j \end{cases}$$

Definition 1.5. Let (X, B) be a symmetric 2-(v , k , λ)-design. Then derived design is the design formed by taking one block fixed (say B_0) and constructing blocks $B'_1, B'_2, \dots, B'_{b-1}$ where each B'_i contains the λ objects common to B_0 and B_i for $i = 1, 2, \dots, b-1$. A derived design has parameters: $v' = k$, $b' = b-1 = v-1$, $r' = r-1 = k-1$, $k' = \lambda$, $\lambda' = \lambda-1$.

Definition 1.6. Let (X, B) be a symmetric 2-(v , k , λ)-design. Then residual design is the design formed by deleting B_0 and its objects from the blocks B_1, B_2, \dots, B_b which gives the blocks $B_1^*, B_2^*, \dots, B_b^*$. A residual design has parameters: $v^* = v-k$, $b^* = b-1 = v-1$, $r^* = r = k$, $k^* = k-\lambda$, $\lambda^* = \lambda$.

2. 2-(19,9,8) Designs with Automorphisms of order 19

Let $S = \{1, 2, \dots, 19\}$. Under the action of the cyclic group of order 19, the 9-subsets of the set S are partitioned into 4862 orbits. It is observed that exactly 225 pairs of these orbits form a 2-(19, 9, 8) design given in Table IA. We give the representatives of

the orbits which form the 2-(19, 9, 8) design in Table I. Under the action of the group of affine transformations of $GF(19)$, these 225 designs are partitioned into 17 classes as follows:

- {1, 2, 16, 46, 47, 49, 69, 106, 115, 139, 140, 143, 173, 119, 193, 200, 201, 211};
- {3, 5, 12, 23, 28, 32, 42, 54, 58, 60, 80, 82, 103, 114, 141, 159, 197, 220};
- {4, 6, 11, 22, 29, 33, 43, 53, 57, 59, 81, 83, 88, 102, 111, 127, 196, 221};
- {7, 14, 21, 24, 30, 52, 76, 110, 122, 134, 149, 150, 161, 171, 176, 188, 213, 217};
- {8, 13, 20, 25, 31, 51, 77, 109, 123, 135, 148, 151, 160, 170, 175, 189, 210, 224};

{9, 26, 56, 62, 67, 84, 100, 130, 136, 147, 152, 158, 163, 165, 182, 199, 209, 214};
 {10, 27, 55, 61, 68, 85, 89, 101, 131, 137, 146, 157, 162, 164, 181, 198, 215, 216};
 {15, 50, 71, 90, 94, 96, 105, 116, 120, 124, 126, 129, 133, 154, 174, 190, 195, 205};
 {17, 48, 70, 91, 95, 97, 104, 117, 121, 125, 128, 132, 142, 153, 172, 191, 194, 225};
 {18, 45, 65, 73, 74, 86, 98, 113, 119, 145, 155, 167, 183, 186, 203, 212, 218, 223};
 {19, 44, 66, 72, 75, 87, 99, 112, 118, 144, 156, 166, 184, 185, 202, 204, 219, 222};
 {34, 41, 64, 168, 177, 208};
 {35, 40, 78, 108, 178, 207};
 {36, 39, 79, 107, 179, 206};
 {37, 38, 63, 169, 180, 187};
 {92, 138};
 {93}.

Table I: Orbit Representatives that form 2-(19,9,8) designs

No.	Orbit Representatives	No.	Orbit Representatives
1.	[1,2,3,4,5,6,10,13,16]	58.	[1,2,3,4,6,13,14,16,17]
2.	[1,2,3,4,5,7,9,12,16]	59.	[1,2,3,4,7,8,9,11,13]
3.	[1,2,3,4,5,7,9,13,14]	60.	[1,2,3,4,7,8,10,11,16]
4.	[1,2,3,4,5,7,9,13,16]	61.	[1,2,3,4,7,8,10,11,18]
5.	[1,2,3,4,5,7,10,14,15]	62.	[1,2,3,4,7,8,10,14,18]
6.	[1,2,3,4,5,7,11,12,17]	63.	[1,2,3,4,7,8,10,16,18]
7.	[1,2,3,4,5,7,11,12,18]	64.	[1,2,3,4,7,8,11,13,15]
8.	[1,2,3,4,5,7,11,14,16]	65.	[1,2,3,4,7,8,11,16,18]
9.	[1,2,3,4,5,7,11,15,16]	66.	[1,2,3,4,7,9,11,12,16]
10.	[1,2,3,4,5,7,12,13,17]	67.	[1,2,3,4,7,9,12,13,16]
11.	[1,2,3,4,5,7,12,15,16]	68.	[1,2,3,4,7,10,11,16,18]
12.	[1,2,3,4,5,7,13,14,18]	69.	[1,2,3,4,7,10,12,14,17]
13.	[1,2,3,4,5,8,9,12,14]	70.	[1,2,3,4,7,10,13,14,18]
14.	[1,2,3,4,5,8,10,11,15]	71.	[1,2,3,4,7,11,13,15,18]
15.	[1,2,3,4,5,8,12,13,18]	72.	[1,2,3,4,7,11,15,17,18]
16.	[1,2,3,4,5,8,13,14,18]	73.	[1,2,3,4,7,13,14,16,18]
17.	[1,2,3,4,5,8,13,15,16]	74.	[1,2,3,4,8,9,11,17,18]
18.	[1,2,3,4,5,9,10,12,17]	75.	[1,2,3,4,8,9,12,15,18]
19.	[1,2,3,4,5,9,10,13,18]	76.	[1,2,3,4,8,9,12,16,18]
20.	[1,2,3,4,5,9,10,14,18]	77.	[1,2,3,4,8,10,11,16,18]
21.	[1,2,3,4,5,9,11,14,16]	78.	[1,2,3,4,8,10,12,17,18]
22.	[1,2,3,4,5,9,11,14,18]	79.	[1,2,3,4,8,11,12,15,17]
23.	[1,2,3,4,5,9,12,16,18]	80.	[1,2,3,4,8,11,12,16,18]
24.	[1,2,3,4,5,9,13,16,18]	81.	[1,2,3,4,8,12,13,15,17]
25.	[1,2,3,4,5,10,11,15,18]	82.	[1,2,3,4,8,12,13,15,18]
26.	[1,2,3,4,5,10,14,15,17]	83.	[1,2,3,4,8,12,14,15,18]
27.	[1,2,3,4,5,11,12,16,18]	84.	[1,2,3,4,8,13,14,16,17]
28.	[1,2,3,4,5,11,13,16,17]	85.	[1,2,3,4,8,13,14,16,18]

29.	[1,2,3,4,6,7,9,11,14]	86.	[1,2,3,4,9,11,13,16,17]
30.	[1,2,3,4,6,7,9,13,17]	87.	[1,2,3,4,9,11,14,17,18]
31.	[1,2,3,4,6,7,10,13,15]	88.	[1,2,3,4,9,12,15,16,18]
32.	[1,2,3,4,6,7,12,13,18]	89.	[1,2,3,4,9,13,14,15,18]
33.	[1,2,3,4,6,7,12,14,16]	90.	[1,2,3,4,10,13,15,17,18]
34.	[1,2,3,4,6,7,12,14,18]	91.	[1,2,3,4,11,13,15,16,17]
35.	[1,2,3,4,6,7,13,15,16]	92.	[1,2,3,5,6,7,9,12,14]
36.	[1,2,3,4,6,7,13,15,18]	93.	[1,2,3,5,6,7,10,12,14]
37.	[1,2,3,4,6,8,9,12,15]	94.	[1,2,3,5,6,7,10,13,15]
38.	[1,2,3,4,6,8,10,11,16]	95.	[1,2,3,5,6,7,10,13,16]
39.	[1,2,3,4,6,8,10,11,17]	96.	[1,2,3,5,6,7,11,14,17]
40.	[1,2,3,4,6,8,11,12,18]	97.	[1,2,3,5,6,7,12,14,17]
41.	[1,2,3,4,6,8,12,13,16]	98.	[1,2,3,5,6,7,13,15,17]
42.	[1,2,3,4,6,8,12,15,16]	99.	[1,2,3,5,6,7,13,15,18]
43.	[1,2,3,4,6,8,13,14,16]	100.	[1,2,3,5,6,8,9,12,14]
44.	[1,2,3,4,6,8,13,14,17]	101.	[1,2,3,5,6,8,9,15,17]
45.	[1,2,3,4,6,8,13,16,17]	102.	[1,2,3,5,6,8,10,11,14]
46.	[1,2,3,4,6,8,14,16,17]	103.	[1,2,3,5,6,8,10,14,15]
47.	[1,2,3,4,6,9,10,11,15]	104.	[1,2,3,5,6,8,11,16,18]
48.	[1,2,3,4,6,9,10,12,16]	105.	[1,2,3,5,6,8,13,14,18]
49.	[1,2,3,4,6,9,11,12,16]	106.	[1,2,3,5,6,8,15,16,17]
50.	[1,2,3,4,6,9,11,13,17]	107.	[1,2,3,5,6,9,10,11,18]
51.	[1,2,3,4,6,9,11,17,18]	108.	[1,2,3,5,6,9,11,14,18]
52.	[1,2,3,4,6,9,12,15,16]	109.	[1,2,3,5,6,9,13,14,15]
53.	[1,2,3,4,6,10,11,14,17]	110.	[1,2,3,5,6,11,12,17,18]
54.	[1,2,3,4,6,10,12,17,18]	111.	[1,2,3,5,6,12,14,16,17]
55.	[1,2,3,4,6,10,14,16,17]	112.	[1,2,3,5,7,8,9,12,13]
56.	[1,2,3,4,6,11,12,17,18]	113.	[1,2,3,5,7,8,9,12,17]
57.	[1,2,3,4,6,12,13,16,18]	114.	[1,2,3,5,7,8,9,16,17]
115.	[1,2,3,5,7,8,10,11,18]	166.	[1,2,3,6,7,10,12,13,15]
116.	[1,2,3,5,7,8,11,13,17]	167.	[1,2,3,6,7,10,12,13,18]
117.	[1,2,3,5,7,8,14,16,17]	168.	[1,2,3,6,7,12,14,15,18]
118.	[1,2,3,5,7,8,15,16,18]	169.	[1,2,3,6,8,9,11,14,18]
119.	[1,2,3,5,7,9,10,12,13]	170.	[1,2,3,6,8,9,11,15,17]
120.	[1,2,3,5,7,10,11,15,18]	171.	[1,2,3,6,8,9,12,13,18]
121.	[1,2,3,5,7,10,13,14,16]	172.	[1,2,3,6,8,10,11,14,15]
122.	[1,2,3,5,7,10,14,15,18]	173.	[1,2,3,6,8,10,11,14,17]
123.	[1,2,3,5,7,11,14,16,17]	174.	[1,2,3,6,8,11,12,14,15]
124.	[1,2,3,5,7,12,13,15,16]	175.	[1,2,3,6,8,12,14,15,17]
125.	[1,2,3,5,7,12,15,17,18]	176.	[1,2,3,6,8,13,14,16,17]
126.	[1,2,3,5,8,9,10,13,18]	177.	[1,2,3,6,8,14,15,17,18]
127.	[1,2,3,5,8,9,10,15,18]	178.	[1,2,3,6,9,10,11,15,18]
128.	[1,2,3,5,8,9,11,13,14]	179.	[1,2,3,6,9,12,13,15,17]
129.	[1,2,3,5,8,9,11,16,17]	180.	[1,2,3,6,10,12,15,16,18]
130.	[1,2,3,5,8,9,12,13,14]	181.	[1,2,3,7,8,10,11,16,18]
131.	[1,2,3,5,8,9,13,15,18]	182.	[1,2,3,7,8,11,12,14,18]
132.	[1,2,3,5,8,9,13,16,18]	183.	[1,2,3,7,9,10,13,16,18]

133.	[1,2,3,5,8,10,11,12,16]	184.	[1,2,3,7,9,11,14,16,17]
134.	[1,2,3,5,8,10,14,15,18]	185.	[1,2,3,7,10,11,13,14,15]
135.	[1,2,3,5,8,10,14,16,17]	186.	[1,2,3,8,9,11,12,15,17]
136.	[1,2,3,5,8,11,15,16,17]	187.	[1,2,3,8,9,11,13,14,18]
137.	[1,2,3,5,8,12,13,14,17]	188.	[1,2,3,8,9,12,13,15,17]
138.	[1,2,3,5,8,12,13,16,18]	189.	[1,2,3,8,9,13,15,17,18]
139.	[1,2,3,5,8,12,14,16,17]	190.	[1,2,3,8,10,11,13,16,17]
140.	[1,2,3,5,8,13,14,15,18]	191.	[1,2,3,9,10,12,14,15,18]
141.	[1,2,3,5,9,10,12,14,15]	192.	[1,2,3,9,11,14,15,17,18]
142.	[1,2,3,5,9,10,15,17,18]	193.	[1,2,3,9,12,13,15,17,18]
143.	[1,2,3,5,9,11,12,15,16]	194.	[1,2,3,10,11,13,14,16,18]
144.	[1,2,3,5,9,12,14,15,17]	195.	[1,2,4,5,7,8,12,14,16]
145.	[1,2,3,5,9,12,14,17,18]	196.	[1,2,4,5,7,9,10,16,17]
146.	[1,2,3,5,10,11,13,16,17]	197.	[1,2,4,5,7,9,13,14,18]
147.	[1,2,3,5,10,11,14,15,17]	198.	[1,2,4,5,7,10,12,16,17]
148.	[1,2,3,5,10,13,14,15,18]	199.	[1,2,4,5,7,11,12,16,18]
149.	[1,2,3,5,11,12,14,16,17]	200.	[1,2,4,5,8,9,11,13,15]
150.	[1,2,3,5,11,13,14,16,17]	201.	[1,2,4,5,8,9,13,15,18]
151.	[1,2,3,5,12,13,14,17,18]	202.	[1,2,4,5,8,9,15,16,18]
152.	[1,2,3,5,12,13,15,16,18]	203.	[1,2,4,5,8,10,11,13,15]
153.	[1,2,3,6,7,8,12,15,18]	204.	[1,2,4,5,10,12,14,15,17]
154.	[1,2,3,6,7,8,15,17,18]	205.	[1,2,4,5,10,12,14,16,17]
155.	[1,2,3,6,7,9,10,12,18]	206.	[1,2,4,6,8,9,12,13,15]
156.	[1,2,3,6,7,9,10,15,17]	207.	[1,2,4,6,8,9,14,16,17]
157.	[1,2,3,6,7,9,11,12,18]	208.	[1,2,4,6,9,10,13,15,16]
158.	[1,2,3,6,7,9,11,15,18]	209.	[1,2,4,7,8,11,13,15,16]
159.	[1,2,3,6,7,9,11,17,18]		
160.	[1,2,3,6,7,9,12,14,16]		
161.	[1,2,3,6,7,9,12,16,18]		
162.	[1,2,3,6,7,9,13,15,18]		
163.	[1,2,3,6,7,9,14,16,17]		
164.	[1,2,3,6,7,9,15,16,18]		
165.	[1,2,3,6,7,10,12,13,14]		

Table I(A): Pairs of orbits that form 2-(19,9,8) designs

1.	1,160	59.	27,198	117.	56,205	175.	103,150
2.	1,184	60.	27,201	118.	57,129	176.	103,155
3.	2,174	61.	28,197	119.	57,168	177.	104,109
4.	2,186	62.	28,199	120.	58,160	178.	104,130
5.	3,198	63.	29,143	121.	58,184	179.	104,165
6.	3,201	64.	29,182	122.	59,173	180.	104,185
7.	4,166	65.	30,141	123.	59,179	181.	105,149
8.	4,190	66.	30,187	124.	60,71	182.	105,157
9.	5,139	67.	31,42	125.	61,160	183.	106,121
10.	5,158	68.	31,76	126.	61,184	184.	106,183
11.	6,206	69.	32,195	127.	62,82	185.	107,170
12.	6,207	70.	32,200	128.	63,137	186.	107,175

13.	7,208	71.	32,205	129.	63,178	187.	109,125
14.	7,209	72.	33,196	130.	64,102	188.	111,127
15.	8,156	73.	33,202	131.	64,193	189.	111,140
16.	8,163	74.	34,147	132.	65,128	190.	112,203
17.	8,176	75.	34,171	133.	65,191	191.	112,204
18.	9,144	76.	35,122	134.	66,100	192.	113,128
19.	9,169	77.	35,132	135.	66,192	193.	113,191
20.	10,135	78.	36,143	136.	67,92	194.	114,203
21.	10,162	79.	36,182	137.	67,99	195.	114,204
22.	11,131	80.	37,41	138.	68,68	196.	115,172
23.	11,134	81.	37,80	139.	69,112	197.	115,188
24.	12,208	82.	38,53	140.	69,114	198.	117,126
25.	12,209	83.	38,70	141.	70,85	199.	117,148
26.	13,197	84.	39,146	142.	71,84	200.	118,137
27.	13,199	85.	39,167	143.	71,110	201.	118,178
28.	14,120	86.	40,129	144.	72,141	202.	119,136
29.	14,138	87.	40,168	145.	72,187	203.	119,153
30.	15,135	88.	41,88	146.	73,146	204.	121,154
31.	15,162	89.	42,87	147.	73,167	205.	121,177
32.	16,206	90.	43,95	148.	74,122	206.	124,130
33.	16,207	91.	43,96	149.	74,132	207.	125,165
34.	17,116	92.	44,44	150.	75,93	208.	125,185
35.	17,123	93.	44,68	151.	75,98	209.	125,164
36.	17,161	94.	45,128	152.	76,87	210.	126,159
37.	17,180	95.	45,191	153.	77,95	211.	127,177
38.	18,116	96.	46,137	154.	77,96	212.	133,194
39.	18,123	97.	46,178	155.	78,196	213.	136,159
40.	18,161	98.	47,108	156.	78,202	214.	140,149
41.	18,123	99.	47,145	157.	79,92	215.	142,157
42.	19,131	100.	48,94	158.	79,99	216.	142,164
43.	19,134	101.	48,97	159.	80,88	217.	148,189
44.	20,144	102.	49,55	160.	81,100	218.	151,170
45.	20,169	103.	49,62	161.	81,192	219.	151,175
46.	21,95	104.	50,60	162.	83,94	220.	152,172
47.	21,96	105.	50,84	163.	83,97	221.	152,188
48.	22,156	106.	50,110	164.	86,102	222.	153,194
49.	22,163	107.	51,143	165.	86,193	223.	154,183
50.	22,176	108.	51,182	166.	89,108	224.	155,189
51.	23,166	109.	52,93	167.	89,145	225.	177,181
52.	23,190	110.	52,98	168.	90,143		
53.	24,174	111.	53,85	169.	90,182		
54.	24,186	112.	54,147	170.	91,173		
55.	25,139	113.	54,171	171.	91,179		
56.	25,158	114.	55,82	172.	101,124		
57.	26,120	115.	56,195	173.	101,133		
58.	26,138	116.	56,200	174.	101,181		

Using GAP software, we observe that no design in a class is isomorphic to a design from any

other class, thus giving exactly 17 isomorphism classes of 2-(19, 9, 8) designs. We take the first design as representative from each of these classes and denote it as D_i . Thus we have seventeen non-isomorphic designs as $D_1, D_3, D_5, D_7, D_8, D_9, D_{10}, D_{15}, D_{17}, D_{18}, D_{19}, D_{34}, D_{35}, D_{36}, D_{37}, D_{92}$ and D_{93} .

3. 2-(20,10,9) Designs with Automorphisms of order 19

In a similar way we get 2-(20, 10, 9) designs with automorphisms of order 19. We consider the set $S^* = \{1, 2, \dots, 20\}$ and an automorphism $(1, 2, \dots, 19)(20)$. Under this automorphism, all 10-subset of the set S^* are partitioned into 9724 orbits. Of these, exactly 448 pairs of orbits form a 2-(20, 10, 9) design as in Table II(A). The orbits of these designs are of two types, one not involving the fixed point 20 and the second involving the point 20. The representatives of the orbits involving the point 20 are same as representatives in Table I with an inclusion of point 20. Table II gives the representatives of the orbits not involving the point 20.

Under the action of the group of affine transformations of $GF(19)$, fixing the point 20, these 448 designs are partitioned into 32 classes as follows:

{1, 34, 47, 53, 55, 60, 65, 70, 77, 81, 88, 93, 123, 125, 127, 160, 162, 163};
{2, 33, 48, 52, 54, 61, 64, 71, 76, 80, 87, 94, 124, 126, 128, 159, 161, 164};
{3, 7, 23, 25, 86, 145, 201, 229, 236, 245, 268, 301, 341, 343, 344, 378, 426, 444};
{4, 8, 22, 24, 85, 146, 202, 228, 237, 246, 269, 274, 335, 342, 345, 377, 425, 448};
{5, 21, 56, 58, 79, 129, 168, 178, 183, 190, 203, 234, 276, 286, 298, 364, 376, 380};
{6, 20, 57, 59, 78, 130, 167, 177, 184, 189, 204, 235, 277, 297, 334, 363, 375, 379};
{9, 17, 62, 69, 84, 154, 187, 206, 214, 244, 247, 250, 257, 263, 332, 336, 339, 340};
{10, 16, 63, 68, 82, 156, 188, 205, 213, 248, 249, 258, 264, 279, 292, 331, 337, 338};
{11, 28, 31, 32, 50, 113, 121, 151, 152, 153, 294, 305, 428, 430, 435, 436, 440, 443};
{12, 30, 49, 111, 122, 150, 261, 266, 295, 302, 307, 321, 324, 328, 356, 366, 429, 439};
{13, 29, 51, 112, 120, 149, 262, 267, 296, 303, 306, 322, 323, 327, 355, 365, 427, 441};
{14, 15, 83, 155, 211, 212, 215, 278, 293, 304, 406, 407, 408, 409, 423, 424, 437, 438};
{18, 27, 89, 102, 106, 116, 186, 217, 284, 371, 389, 396, 398, 399, 432, 434, 442, 445};
{19, 26, 90, 101, 105, 117, 185, 216, 289, 372, 390, 395, 397, 400, 431, 433, 446, 447};
{37, 40, 103, 109, 158, 182, 208, 210, 283, 287, 308, 316, 373, 382, 391, 393, 412, 418};
{38, 39, 104, 110, 157, 181, 207, 209, 282, 288, 309, 317, 374, 381, 392, 394, 405, 417};
{41, 45, 66, 72, 75, 114, 119, 166, 170, 176, 230, 232, 238, 242, 333, 354, 362, 370};
{42, 46, 67, 73, 74, 115, 118, 165, 169, 175, 231, 233, 239, 243, 252, 275, 285, 369};
{91, 97, 108, 144, 173, 180, 191, 222, 240, 291, 319, 325, 330, 348, 359, 368, 387, 414};
{92,,98, 107, 143, 174, 179, 192, 223, 241, 251, 290, 318, 326, 329, 349, 367, 388, 413};
{95, 100, 200, 224, 300, 310, 313, 314, 350, 353, 360, 386, 401, 404, 411, 415, 419, 422};

{96, 99, 199, 225, 299, 311, 312, 315, 351, 352, 361, 385, 402, 403, 410, 416, 420, 421};
{35, 134, 139, 172, 347, 357};
{36, 135, 138, 171, 346, 358};
{43, 132, 133, 140, 142, 147};
{44, 131, 136, 137, 141, 148};
{193, 195, 218, 256, 273, 384};
{194, 198, 221, 253, 270, 383};
{196, 219, 227, 255, 272, 280};
{197, 220, 226, 254, 271, 281};
{259, 320};
{260, 265}.

Using GAP software, we observe that no design in a class is isomorphic to a design from any other class, thus giving exactly 32 isomorphism classes of 2-(20, 10, 9) designs. We take the first design as representative from each class and denote it as E_i as done for 2-(19, 9, 8) design. Hence we have 32 non-isomorphic 2-(20, 10, 9) design with an automorphism of order 19.

Table II: Orbit Representatives that form 2-(20,10,9) designs

No.	Orbit Representatives	No.	Orbit Representatives
1.	[1,2,3,4,5,6,8,10,14,15]	58.	[1,2,3,4,6,7,8,11,13,14]
2.	[1,2,3,4,5,6,8,11,15,16]	59.	[1,2,3,4,6,7,8,12,13,15]
3.	[1,2,3,4,5,6,8,12,14,17]	60.	[1,2,3,4,6,7,8,13,14,17]
4.	[1,2,3,4,5,6,8,12,15,16]	61.	[1,2,3,4,6,7,8,13,15,17]
5.	[1,2,3,4,5,6,8,13,14,17]	62.	[1,2,3,4,6,7,9,10,13,15]
6.	[1,2,3,4,5,6,9,10,14,16]	63.	[1,2,3,4,6,7,9,12,16,18]
7.	[1,2,3,4,5,6,9,11,15,17]	64.	[1,2,3,4,6,7,9,13,15,16]
8.	[1,2,3,4,5,6,9,12,13,18]	65.	[1,2,3,4,6,7,10,11,12,18]
9.	[1,2,3,4,5,6,9,12,14,16]	66.	[1,2,3,4,6,7,10,11,16,18]
10.	[1,2,3,4,5,6,9,12,14,18]	67.	[1,2,3,4,6,7,11,12,14,18]
11.	[1,2,3,4,5,6,10,11,14,18]	68.	[1,2,3,4,6,7,11,13,17,18]
12.	[1,2,3,4,5,6,10,11,15,18]	69.	[1,2,3,4,6,7,13,14,15,18]
13.	[1,2,3,4,5,6,10,12,14,17]	70.	[1,2,3,4,6,7,13,15,16,17]
14.	[1,2,3,4,5,6,10,12,16,17]	71.	[1,2,3,4,6,8,9,10,12,17]
15.	[1,2,3,4,5,6,11,12,16,18]	72.	[1,2,3,4,6,8,9,11,12,15]
16.	[1,2,3,4,5,7,8,10,12,15]	73.	[1,2,3,4,6,8,9,11,12,16]
17.	[1,2,3,4,5,7,8,11,13,16]	74.	[1,2,3,4,6,8,9,13,14,17]
18.	[1,2,3,4,5,7,8,13,15,17]	75.	[1,2,3,4,6,8,10,11,12,17]
19.	[1,2,3,4,5,7,9,10,12,16]	76.	[1,2,3,4,6,8,10,11,14,15]
20.	[1,2,3,4,5,7,9,10,12,17]	77.	[1,2,3,4,6,8,10,13,14,17]
21.	[1,2,3,4,5,7,9,10,13,14]	78.	[1,2,3,4,6,8,11,13,14,17]
22.	[1,2,3,4,5,7,9,10,13,16]	79.	[1,2,3,4,6,8,12,13,14,17]
23.	[1,2,3,4,5,7,9,10,14,17]	80.	[1,2,3,4,6,8,12,15,17,18]
24.	[1,2,3,4,5,7,9,11,12,16]	81.	[1,2,3,4,6,8,13,14,15,17]
25.	[1,2,3,4,5,7,9,12,13,17]	82.	[1,2,3,4,6,8,13,14,16,17]
26.	[1,2,3,4,5,7,9,14,15,18]	83.	[1,2,3,4,6,8,13,14,17,18]

27.	[1,2,3,4,5,7,10,11,12,17]	84.	[1,2,3,4,6,9,10,11,17,18]
28.	[1,2,3,4,5,7,10,11,15,17]	85.	[1,2,3,4,6,9,10,12,16,17]
29.	[1,2,3,4,5,7,10,11,16,18]	86.	[1,2,3,4,6,9,10,13,15,17]
30.	[1,2,3,4,5,7,10,13,14,18]	87.	[1,2,3,4,6,9,10,14,15,18]
31.	[1,2,3,4,5,7,11,12,15,17]	88.	[1,2,3,4,6,9,11,12,15,16]
32.	[1,2,3,4,5,7,11,12,15,18]	89.	[1,2,3,4,6,9,13,15,16,17]
33.	[1,2,3,4,5,7,12,14,16,17]	90.	[1,2,3,4,6,10,11,13,16,17]
34.	[1,2,3,4,5,7,13,15,16,17]	91.	[1,2,3,4,6,10,11,14,15,17]
35.	[1,2,3,4,5,8,9,10,12,14]	92.	[1,2,3,4,6,10,12,13,17,18]
36.	[1,2,3,4,5,8,9,10,12,18]	93.	[1,2,3,4,6,11,14,15,16,17]
37.	[1,2,3,4,5,8,9,11,13,18]	94.	[1,2,3,4,6,12,13,14,17,18]
38.	[1,2,3,4,5,8,9,12,15,17]	95.	[1,2,3,4,6,12,13,15,16,17]
39.	[1,2,3,4,5,8,10,12,17,18]	96.	[1,2,3,4,7,8,9,10,13,17]
40.	[1,2,3,4,5,8,10,13,14,16]	97.	[1,2,3,4,7,8,9,10,13,18]
41.	[1,2,3,4,5,8,10,13,14,18]	98.	[1,2,3,4,7,8,9,11,12,14]
42.	[1,2,3,4,5,8,10,13,16,17]	99.	[1,2,3,4,7,8,9,11,12,18]
43.	[1,2,3,4,5,8,10,14,15,18]	100.	[1,2,3,4,7,8,9,11,13,16]
44.	[1,2,3,4,5,8,11,15,16,18]	101.	[1,2,3,4,7,8,9,11,15,18]
45.	[1,2,3,4,5,8,12,13,16,18]	102.	[1,2,3,4,7,8,9,11,17,18]
46.	[1,2,3,4,5,8,13,14,15,18]	103.	[1,2,3,4,7,8,10,11,16,18]
47.	[1,2,3,4,5,8,13,15,16,18]	104.	[1,2,3,4,7,8,11,13,14,18]
48.	[1,2,3,4,5,9,10,13,14,16]	105.	[1,2,3,4,7,8,11,13,15,16]
49.	[1,2,3,4,5,9,11,12,15,16]	106.	[1,2,3,4,7,8,12,14,15,18]
50.	[1,2,3,4,5,9,11,12,15,17]	107.	[1,2,3,4,7,9,10,11,16,18]
51.	[1,2,3,4,5,9,12,14,17,18]	108.	[1,2,3,4,7,9,10,13,14,18]
52.	[1,2,3,4,5,9,12,15,16,18]	109.	[1,2,3,4,7,9,11,14,15,18]
53.	[1,2,3,4,5,9,13,14,16,18]	110.	[1,2,3,4,7,9,11,16,17,18]
54.	[1,2,3,4,5,9,13,15,16,18]	111.	[1,2,3,4,7,9,12,13,14,16]
55.	[1,2,3,4,5,10,13,15,17,18]	112.	[1,2,3,4,7,10,11,12,16,18]
56.	[1,2,3,4,5,11,12,15,16,18]	113.	[1,2,3,4,7,10,11,13,15,16]
57.	[1,2,3,4,5,11,13,15,16,17]	114.	[1,2,3,4,7,10,11,13,16,18]
115.	[1,2,3,4,7,10,11,14,16,18]	172.	[1,2,3,5,7,8,10,14,16,17]
116.	[1,2,3,4,7,10,11,15,16,18]	173.	[1,2,3,5,7,8,11,13,17,18]
117.	[1,2,3,4,7,10,11,16,17,18]	174.	[1,2,3,5,7,8,12,13,15,18]
118.	[1,2,3,4,7,12,13,14,16,18]	175.	[1,2,3,5,7,9,10,11,14,17]
119.	[1,2,3,4,7,12,14,15,16,18]	176.	[1,2,3,5,7,9,10,12,15,16]
120.	[1,2,3,4,8,9,10,12,13,16]	177.	[1,2,3,5,7,9,10,12,17,18]
121.	[1,2,3,4,8,9,11,13,14,17]	178.	[1,2,3,5,7,10,11,15,17,18]
122.	[1,2,3,4,8,9,11,13,16,17]	179.	[1,2,3,5,7,10,13,15,16,17]
123.	[1,2,3,4,8,9,11,15,17,18]	180.	[1,2,3,5,7,11,12,15,17,18]
124.	[1,2,3,4,8,9,12,13,15,18]	181.	[1,2,3,5,7,12,13,14,17,18]
125.	[1,2,3,4,8,10,11,12,15,17]	182.	[1,2,3,5,8,9,11,12,14,16]
126.	[1,2,3,4,8,11,12,14,15,16]	183.	[1,2,3,5,8,9,12,13,14,18]
127.	[1,2,3,4,8,11,13,15,16,17]	184.	[1,2,3,5,8,9,13,15,16,18]
128.	[1,2,3,4,8,12,13,15,16,18]	185.	[1,2,3,5,8,10,11,15,16,18]
129.	[1,2,3,4,9,10,13,14,16,18]	186.	[1,2,3,5,8,11,13,15,16,17]
130.	[1,2,3,4,9,11,12,16,17,18]	187.	[1,2,3,5,9,10,11,14,15,18]

131.	[1,2,3,4,9,11,14,15,17,18]	188.	[1,2,3,5,9,10,12,15,17,18]
132.	[1,2,3,4,9,12,13,15,16,18]	189.	[1,2,3,5,9,12,13,15,17,18]
133.	[1,2,3,4,10,11,13,16,17,18]	190.	[1,2,3,5,10,11,13,14,16,17]
134.	[1,2,3,4,10,12,13,15,16,17]	191.	[1,2,3,6,7,9,10,11,16,18]
135.	[1,2,3,5,6,7,9,10,12,14]	192.	[1,2,3,6,7,9,10,12,13,15]
136.	[1,2,3,5,6,7,9,10,15,18]	193.	[1,2,3,6,7,9,10,12,13,18]
137.	[1,2,3,5,6,7,9,12,13,15]	194.	[1,2,3,6,7,9,11,15,17,18]
138.	[1,2,3,5,6,7,9,12,13,18]	195.	[1,2,3,6,7,9,13,15,16,18]
139.	[1,2,3,5,6,7,9,12,17,18]	196.	[1,2,3,6,7,9,13,15,17,18]
140.	[1,2,3,5,6,7,9,14,15,18]	197.	[1,2,3,6,7,10,13,15,16,18]
141.	[1,2,3,5,6,7,9,15,16,17]	198.	[1,2,3,6,8,9,12,13,15,17]
142.	[1,2,3,5,6,7,10,11,12,18]	199.	[1,2,3,6,8,10,11,14,15,17]
143.	[1,2,3,5,6,7,10,12,13,15]	200.	[1,2,3,7,8,10,11,15,17,18]
144.	[1,2,3,5,6,7,10,12,14,15]	201.	[1,2,3,7,8,11,13,14,16,18]
145.	[1,2,3,5,6,7,11,13,14,16]	202.	[1,2,3,7,8,12,14,15,17,18]
146.	[1,2,3,5,6,7,12,13,15,17]	203.	[1,2,3,7,9,11,12,14,15,18]
147.	[1,2,3,5,6,7,12,14,15,17]	204.	[1,2,3,7,11,12,14,15,17,18]
148.	[1,2,3,5,6,7,12,14,15,18]	205.	[1,2,3,8,9,11,13,14,17,18]
149.	[1,2,3,5,6,7,13,15,17,18]	206.	[1,2,3,8,10,11,13,14,16,17]
150.	[1,2,3,5,6,8,9,10,12,14]	207.	[1,2,4,5,7,8,11,13,15,16]
151.	[1,2,3,5,6,8,9,10,16,18]	208.	[1,2,4,5,7,8,12,13,15,17]
152.	[1,2,3,5,6,8,9,11,12,13]	209.	[1,2,4,5,7,9,11,12,14,15]
153.	[1,2,3,5,6,8,9,11,12,16]		
154.	[1,2,3,5,6,8,9,11,15,16]		
155.	[1,2,3,5,6,8,10,11,12,18]		
156.	[1,2,3,5,6,8,10,11,14,18]		
157.	[1,2,3,5,6,8,10,14,16,17]		
158.	[1,2,3,5,6,8,11,12,16,18]		
159.	[1,2,3,5,6,8,11,13,14,18]		
160.	[1,2,3,5,6,8,12,13,15,16]		
161.	[1,2,3,5,6,8,12,13,16,18]		
162.	[1,2,3,5,6,8,12,14,16,17]		
163.	[1,2,3,5,6,9,10,11,13,15]		
164.	[1,2,3,5,6,9,10,11,16,18]		
165.	[1,2,3,5,6,9,10,12,14,15]		
166.	[1,2,3,5,6,10,12,15,16,18]		
167.	[1,2,3,5,6,11,13,14,16,18]		
168.	[1,2,3,5,7,8,10,11,12,14]		
169.	[1,2,3,5,7,8,10,12,13,14]		
170.	[1,2,3,5,7,8,10,13,16,17]		
171.	[1,2,3,5,7,8,10,14,15,18]		

Table II(A): Pairs of orbits that form 2-(20,10,9) designs

1.	1,173	59.	3,197	117.	42,78	175.	56,197
2.	1,179	60.	4,88	118.	43,126	176.	56,199
3.	2,121	61.	4,124	119.	43,148	177.	27,170
4.	2,183	62.	26,137	120.	44,124	178.	27,197
5.	3,172	63.	26,178	121.	44,133	179.	57,198

6.	3,188	64.	27,122	122.	44,181	180.	57,201
7.	4,170	65.	27,132	123.	45,127	181.	28,179
8.	4,175	66.	28,126	124.	45,140	182.	28,186
9.	5,160	67.	28,148	125.	15,171	183.	58,120
10.	5,184	68.	29,137	126.	15,184	184.	58,138
11.	6,69	69.	29,178	127.	46,122	185.	59,108
12.	6,203	70.	30,103	128.	46,132	186.	59,145
13.	6,204	71.	30,189	129.	16,86	187.	60,50
14.	7,137	72.	5,161	130.	16,109	188.	60,71
15.	7,178	73.	5,178	131.	47,143	189.	61,53
16.	8,160	74.	31,105	132.	47,182	190.	61,70
17.	8,184	75.	31,142	133.	17,158	191.	62,41
18.	9,136	76.	32,103	134.	17,166	192.	62,80
19.	9,153	77.	32,189	135.	17,173	193.	29,183
20.	10,172	78.	6,86	136.	17,180	194.	29,187
21.	10,188	79.	6,109	137.	18,158	195.	63,109
22.	11,170	80.	7,198	138.	18,166	196.	63,130
23.	11,175	81.	7,199	139.	18,173	197.	63,165
24.	12,121	82.	8,74	140.	18,180	198.	63,185
25.	12,183	83.	8,87	141.	48,104	199.	64,111
26.	13,172	84.	8,116	142.	48,125	200.	64,159
27.	13,188	85.	9,159	143.	19,172	201.	30,111
28.	14,69	86.	9,188	144.	19,195	202.	30,125
29.	14,203	87.	10,171	145.	20,159	203.	65,131
30.	14,204	88.	10,184	146.	20,188	204.	65,134
31.	1,176	89.	33,129	147.	49,104	205.	66,128
32.	1,201	90.	33,168	148.	49,125	206.	66,191
33.	15,173	91.	11,172	149.	50,45	207.	31,164
34.	15,179	92.	11,195	150.	50,65	208.	31,181
35.	16,143	93.	12,198	151.	50,113	209.	67,117
36.	16,182	94.	12,199	152.	21,153	210.	67,164
37.	17,64	95.	34,208	153.	21,204	211.	68,128
38.	17,86	96.	34,209	154.	22,74	212.	68,191
39.	18,146	97.	35,19	155.	22,87	213.	32,77
40.	18,167	98.	35,201	156.	22,116	214.	32,115
41.	19,67	99.	36,208	157.	51,64	215.	32,175
42.	19,79	100.	36,209	158.	51,86	216.	34,90
43.	20,143	101.	37,129	159.	52,66	217.	34,104
44.	20,182	102.	37,168	160.	52,81	218.	69,116
45.	21,197	103.	13,179	161.	23,88	219.	69,123
46.	21,199	104.	13,186	162.	23,124	220.	69,161
47.	22,66	105.	38,33	163.	53,52	221.	69,180
48.	22,81	106.	38,78	164.	53,75	222.	70,206
49.	23,124	107.	14,162	165.	54,67	223.	70,207
50.	23,133	108.	14,194	166.	54,79	224.	35,157
51.	23,181	109.	39,173	167.	24,113		
52.	24,52	110.	39,179	168.	24,121		

53.	24,75	111.	40,45	169.	25,161		
54.	25,127	112.	40,65	170.	25,178		
55.	25,140	113.	40,113	171.	55,143		
56.	2,113	114.	41,105	172.	55,182		
57.	2,121	115.	41,142	173.	26,162		
58.	3,170	116.	42,33	174.	26,194		
225.	35,196	283.	89,157	341.	72,125	399.	142,144
226.	36,183	284.	90,54	342.	118,196	400.	142,169
227.	36,187	285.	91,73	343.	118,202	401.	144,100
228.	71,147	286.	53,110	344.	119,147	402.	144,192
229.	71,171	287.	92,117	345.	119,171	403.	93,192
230.	72,42	288.	92,164	346.	120,104	404.	93,206
231.	72,76	289.	54,104	347.	120,125	405.	94,148
232.	73,48	290.	55,143	348.	122,115	406.	95,169
233.	73,83	291.	55,147	349.	122,152	407.	145,101
234.	37,163	292.	56,115	350.	123,111	408.	145,177
235.	37,191	293.	56,175	351.	123,151	409.	96,169
236.	75,196	294.	93,195	352.	75,157	410.	146,100
237.	75,202	295.	93,200	353.	75,196	411.	146,192
238.	76,102	296.	93,205	354.	76,132	412.	148,97
239.	76,193	297.	94,131	355.	77,153	413.	149,174
240.	38,190	298.	94,134	356.	77,204	414.	149,186
241.	38,193	299.	95,135	357.	126,104	415.	98,192
242.	39,91	300.	95,162	358.	126,125	416.	98,206
243.	39,108	301.	57,106	359.	80,131	417.	99,165
244.	77,56	302.	58,176	360.	127,150	418.	99,205
245.	40,85	303.	58,201	361.	127,155	419.	150,166
246.	40,106	304.	96,71	362.	128,83	420.	150,190
247.	78,112	305.	97,195	363.	82,138	421.	151,166
248.	78,114	306.	97,200	364.	82,140	422.	151,190
249.	79,101	307.	97,205	365.	84,167	423.	152,160
250.	79,177	308.	98,139	366.	84,177	424.	152,184
251.	41,131	309.	98,158	367.	85,190	425.	154,119
252.	42,132	310.	99,135	368.	85,193	426.	154,194
253.	80,109	311.	99,162	369.	129,102	427.	155,156
254.	80,130	312.	100,150	370.	129,193	428.	155,163
255.	80,165	313.	100,155	371.	130,108	429.	155,176
256.	80,185	314.	59,207	372.	130,145	430.	160,118
257.	81,95	315.	59,208	373.	87,164	431.	106,182
258.	81,96	316.	101,149	374.	87,181	432.	106,403
259.	82,44	317.	101,157	375.	88,163	433.	107,174
260.	82,68	318.	102,206	376.	88,191	434.	107,185
261.	43,153	319.	102,207	377.	89,156	435.	110,167
262.	43,204	320.	103,68	378.	89,189	436.	110,177
263.	83,128	321.	60,167	379.	133,120	437.	112,209
264.	83,191	322.	60,177	380.	133,138	438.	114,209
265.	44,103	323.	61,176	381.	134,139	439.	168,156

266.	46,160	324.	61,201	382.	134,158	440.	168,163
267.	46,200	325.	62,143	383.	90,183	441.	168,176
268.	47,156	326.	62,147	384.	90,187	442.	174,151
269.	47,189	327.	63,160	385.	91,207	443.	118,200
270.	84,116	328.	63,200	386.	91,208	444.	119,202
271.	84,123	329.	105,115	387.	135,174	445.	182,154
272.	84,161	330.	105,152	388.	135,186	446.	185,151
273.	84,180	331.	107,95	389.	136,141	447.	154,203
274.	85,57	332.	107,96	390.	136,187	448.	202,119
275.	48,128	333.	108,73	391.	137,94		
276.	49,138	334.	110,70	392.	137,97		
277.	49,140	335.	111,72	393.	92,165		
278.	50,96	336.	112,101	394.	92,205		
279.	50,117	337.	112,177	395.	139,141		
280.	51,183	338.	114,112	396.	139,187		
281.	51,187	339.	114,114	397.	141,144		
282.	89,149	340.	117,71	398.	141,169		

4. 2-(39,19,9) and 3-(40,20,9) Designs with Automorphisms of order 19

Consider the design D_1 which is formed by the orbit number 1 and 160. We take the transpose of the orbit 160 which gives orbit 184. From table IA, orbit 184 form design with orbit number 1, 58 and 61. We consider the orbits of 2-(20, 10, 9) corresponding to orbit number 1, 58 and 61 which form a 2-(20, 10, 9) designs E_{31} , E_{32} , E_{302} , E_{303} , E_{323} and E_{324} . It is observed that D_1 and E_{31} form a 2-(39, 19, 9) design. Also the complement of E_{31} that is E_{424} form a 2-(39, 19, 9) design with D_1 .

Similarly we see that D_3 is embedded into a 2-(39, 19, 9) design by E_{167} and E_{388} , D_4 by E_{168} and E_{386} , D_7 by E_{161} and E_{419} , D_8 by E_{162} and E_{418} , D_9 by E_{169} and E_{309} , D_{10} by E_{170} and E_{308} , D_{15} by E_{156} and E_{440} , D_{17} by E_{154} and E_{438} , D_{18} by E_{146} and E_{397} , D_{19} by E_{145} and E_{396} , D_{34} by E_{139} and E_{221} , D_{35} by E_{140} and E_{220} , D_{36} by E_{137} and E_{219} , D_{37} by E_{138} and E_{218} , D_{92} by E_{260} and E_{265} and D_{93} by E_{320} and E_{259} .

Using GAP, we observe that only the designs (D_{93}, E_{320}) and (D_{93}, E_{259}) are isomorphic and hence we get 33 non-isomorphic 2-(39, 19, 9) designs with automorphisms of order 19.

We extend each of these 33 non-isomorphic 2-(39, 19, 9) designs to 3-(40, 20, 9) design by enlarging all blocks with the point 40 and adding 39 new blocks as complements of the blocks of the initial design. It is observed that only 18 of these 2-(39, 19, 9) designs gives non-isomorphic 3-(40, 20, 9) with automorphisms of order 19. The incidence matrix of these 18 2-(39, 19, 9) gives rise to 18 inequivalent Hadamard matrices of order 40 with automorphisms of order 19 as

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & & & \\ \vdots & & M & \\ 1 & & & \\ 1 & & & \end{bmatrix} \quad (1)$$

where M is the incidence matrix of the 2-(39, 19, 9) design with 0 replaced by -1 . We give

□

the list of the 18 2-(39, 19, 9) in the appendix, which can be used to explicitly construct the inequivalent Hadamard matrices of order 40 with automorphisms of order 19 using (1).

5. Declarations

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Conflicts of interest/Competing interests: Briji J. Chathely and Rajendra

P. Deore declare that they have no conflict of interest.

Availability of data and material: Data from previous studies were used to support this study. They are cited at relevant places within the text as references.

Code availability: Not Applicable.

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Appendix

List of 2-(39,19,9) designs which gives 18 inequivalent Hadamard Matrices of order 40 with with automorphism of order 19:

H(1&31):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,6,10,13,16,20,21,22,24,26,28,29,31,34,35],[2,3,4,5,6,7,11,14,17,21,22,23,25,27,29,30,32,35,36],[3,4,5,6,7,8,12,15,18,22,23,24,26,28,30,31,33,36,37],[4,5,6,7,8,9,13,16,19,23,24,25,27,29,31,32,34,37,38],[1,5,6,7,8,9,10,14,17,20,24,25,26,28,30,32,33,35,38],[2,6,7,8,9,10,11,15,18,20,21,25,26,27,29,31,33,34,36],[3,7,8,9,10,11,12,16,19,21,22,26,27,28,30,32,34,35,37],[1,4,8,9,10,11,12,13,17,22,23,27,28,29,31,33,35,36,38],[2,5,9,10,11,12,13,14,18,20,23,24,28,29,30,32,34,36,37],[3,6,10,11,12,13,14,15,19,21,24,25,29,30,31,33,35,37,38],[1,4,7,11,12,13,14,15,16,20,22,25,26,30,31,32,34,36,38],[2,5,8,12,13,14,15,16,17,20,21,23,26,27,31,32,33,35,37],[3,6,9,13,14,15,16,17,18,21,22,24,27,28,32,33,34,36,38],[4,7,10,14,15,16,17,18,19,20,22,23,25,28,29,33,34,35,37],[1,5,8,11,15,16,17,18,19,21,23,24,26,29,30,34,35,36,38],[1,2,6,9,12,16,17,18,19,20,22,24,25,27,30,31,35,36,37],[1,2,3,7,10,1

3,17,18,19,21,23,25,26,28,31,32,36,37,38],[1,2,3,4,8,11,14,18,19,20,22,24,26,27,29,32,33,37,38],[1,2,3,4,5,9,12,15,19,20,21,23,25,27,28,30,33,34,38],[1,2,3,6,7,9,12,14,16,23,26,29,33,34,35,36,37,38,39],[2,3,4,7,8,10,13,15,17,20,24,27,30,34,35,36,37,38,39],[3,4,5,8,9,11,14,16,18,20,21,25,28,31,35,36,37,38,39],[4,5,6,9,10,12,15,17,19,20,21,22,26,29,32,36,37,38,39],[1,5,6,7,10,11,13,16,18,20,21,22,23,27,30,33,37,38,39],[2,6,7,8,11,12,14,17,19,20,21,22,23,24,28,31,34,38,39],[1,3,7,8,9,12,13,15,18,20,21,22,23,24,25,29,32,35,39],[2,4,8,9,10,13,14,16,19,21,22,23,24,25,26,30,33,36,39],[1,3,5,9,10,11,14,15,17,22,23,24,25,26,27,31,34,37,39],[2,4,6,10,11,12,15,16,18,23,24,25,26,27,28,32,35,38,39],[3,5,7,11,12,13,16,17,19,20,24,25,26,27,28,29,33,36,39],[1,4,6,8,12,13,14,17,18,21,25,26,27,28,29,30,34,37,39],[2,5,7,9,13,14,15,18,19,22,26,27,28,29,30,31,35,38,39],[1,3,6,8,10,14,15,16,19,20,23,27,28,29,30,31,32,36,39],[1,2,4,7,9,11,15,16,17,21,24,28,29,30,31,32,33,37,39],[2,3,5,8,10,12,16,17,18,22,25,29,30,31,32,33,34,38,39],[3,4,6,9,11,13,17,18,19,20,23,26,30,31,32,33,34,35,39],[1,4,5,7,10,12,14,18,19,21,24,27,31,32,33,34,35,36,39],[1,2,5,6,8,11,13,15,19,22,25,28,32,33,34,35,36,37,39]]));

H(3&167):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,9,12,16,20,21,22,23,26,29,30,32,34,35],[2,3,4,5,6,8,10,13,17,21,22,23,24,27,30,31,33,35,36],[3,4,5,6,7,9,11,14,18,22,23,24,25,28,31,32,34,36,37],[4,5,6,7,8,10,12,15,19,23,24,25,26,29,32,33,35,37,38],[1,5,6,7,8,9,11,13,16,20,24,25,26,27,30,33,34,36,38],[2,6,7,8,9,10,12,14,17,20,21,25,26,27,28,31,34,35,37],[3,7,8,9,10,11,13,15,18,21,22,26,27,28,29,32,35,36,38],[4,8,9,10,11,12,14,16,19,20,22,23,27,28,29,30,33,36,37],[1,5,9,10,11,12,13,15,17,21,23,24,28,29,30,31,34,37,38],[2,6,10,11,12,13,14,16,18,20,22,24,25,29,30,31,32,35,38],[3,7,11,12,13,14,15,17,19,20,21,23,25,26,30,31,32,33,36],[1,4,8,12,13,14,15,16,18,21,22,24,26,27,31,32,33,34,37],[2,5,9,13,14,15,16,17,19,22,23,25,27,28,32,33,34,35,38],[1,3,6,10,14,15,16,17,18,20,23,24,26,28,29,33,34,35,36],[2,4,7,11,15,16,17,18,19,21,24,25,27,29,30,34,35,36,37],[1,3,5,8,12,16,17,18,19,22,25,26,28,30,31,35,36,37,38],[1,2,4,6,9,13,17,18,19,20,23,26,27,29,31,32,36,37,38],[1,2,3,5,7,10,14,18,19,20,21,24,27,28,30,32,33,37,38],[1,2,3,4,6,8,11,15,19,20,21,22,25,28,29,31,33,34,38],[1,2,3,6,8,11,12,14,15,23,27,30,32,34,35,36,37,38,39],[2,3,4,7,9,12,13,15,16,20,24,28,31,33,35,36,37,38,39],[3,4,5,8,10,13,14,16,17,20,21,25,29,32,34,36,37,38,39],[4,5,6,9,11,14,15,17,18,20,21,22,26,30,33,35,37,38,39],[5,6,7,10,12,15,16,18,19,20,21,22,23,27,31,34,36,38,39],[1,6,7,8,11,13,16,17,19,20,21,22,23,24,28,32,35,37,39],[1,2,7,8,9,12,14,17,18,21,22,23,24,25,29,33,36,38,39],[2,3,8,9,10,13,15,18,19,20,22,23,24,25,26,30,34,37,39],[1,3,4,9,10,11,14,16,1

9,21,23,24,25,26,27,31,35,38,39],[1,2,4,5,10,11,12,15,17,20,22,24,25,26,27,28,32,36,39],
 ,[2,3,5,6,11,12,13,16,18,21,23,25,26,27,28,29,33,37,39],[3,4,6,7,12,13,14,17,19,22,24,2
 6,27,28,29,30,34,38,39],[1,4,5,7,8,13,14,15,18,20,23,25,27,28,29,30,31,35,39],[2,5,6,8,
 9,14,15,16,19,21,24,26,28,29,30,31,32,36,39],[1,3,6,7,9,10,15,16,17,22,25,27,29,30,31,3
 2,33,37,39],[2,4,7,8,10,11,16,17,18,23,26,28,30,31,32,33,34,38,39],[3,5,8,9,11,12,17,18
 ,19,20,24,27,29,31,32,33,34,35,39],[1,4,6,9,10,12,13,18,19,21,25,28,30,32,33,34,35,36,3
 9],[1,2,5,7,10,11,13,14,19,22,26,29,31,33,34,35,36,37,39]]));

H(4&168):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,9,12,16,
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 ,30,31,33,35,36],[1,5,6,7,8,9,11,13,16,21,24,25,26,27,31,32,34,36,37],[2,6,7,8,9,10,12,
 14,17,22,25,26,27,28,32,33,35,37,38],[3,7,8,9,10,11,13,15,18,20,23,26,27,28,29,33,34,36
 ,38],[4,8,9,10,11,12,14,16,19,20,21,24,27,28,29,30,34,35,37],[1,5,9,10,11,12,13,15,17,2
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 [3,7,11,12,13,14,15,17,19,21,23,24,27,30,31,32,33,37,38],[1,4,8,12,13,14,15,16,18,20,22
 ,24,25,28,31,32,33,34,38],[2,5,9,13,14,15,16,17,19,20,21,23,25,26,29,32,33,34,35],[1,3,
 6,10,14,15,16,17,18,21,22,24,26,27,30,33,34,35,36],[2,4,7,11,15,16,17,18,19,22,23,25,27
 ,28,31,34,35,36,37],[1,3,5,8,12,16,17,18,19,23,24,26,28,29,32,35,36,37,38],[1,2,4,6,9,1
 3,17,18,19,20,24,25,27,29,30,33,36,37,38],[1,2,3,5,7,10,14,18,19,20,21,25,26,28,30,31,3
 4,37,38],[1,2,3,4,6,8,11,15,19,20,21,22,26,27,29,31,32,35,38],[1,2,3,8,9,11,12,15,17,20
 ,22,23,24,25,26,30,34,37,39],[2,3,4,9,10,12,13,16,18,21,23,24,25,26,27,31,35,38,39],[3,
 4,5,10,11,13,14,17,19,20,22,24,25,26,27,28,32,36,39],[1,4,5,6,11,12,14,15,18,21,23,25,2
 6,27,28,29,33,37,39],[2,5,6,7,12,13,15,16,19,22,24,26,27,28,29,30,34,38,39],[1,3,6,7,8,
 13,14,16,17,20,23,25,27,28,29,30,31,35,39],[2,4,7,8,9,14,15,17,18,21,24,26,28,29,30,31,
 32,36,39],[3,5,8,9,10,15,16,18,19,22,25,27,29,30,31,32,33,37,39],[1,4,6,9,10,11,16,17,1
 9,23,26,28,30,31,32,33,34,38,39],[1,2,5,7,10,11,12,17,18,20,24,27,29,31,32,33,34,35,39]
 ,[2,3,6,8,11,12,13,18,19,21,25,28,30,32,33,34,35,36,39],[1,3,4,7,9,12,13,14,19,22,26,29
 ,31,33,34,35,36,37,39],[1,2,4,5,8,10,13,14,15,23,27,30,32,34,35,36,37,38,39],[2,3,5,6,9
 ,11,14,15,16,20,24,28,31,33,35,36,37,38,39],[3,4,6,7,10,12,15,16,17,20,21,25,29,32,34,3
 6,37,38,39],[4,5,7,8,11,13,16,17,18,20,21,22,26,30,33,35,37,38,39],[5,6,8,9,12,14,17,18
 ,19,20,21,22,23,27,31,34,36,38,39],[1,6,7,9,10,13,15,18,19,20,21,22,23,24,28,32,35,37,3
 9],[1,2,7,8,10,11,14,16,19,21,22,23,24,25,29,33,36,38,39]]));

H(7&161):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,9,13,16,20,21,22,23,25,28,30,31,34,35],[2,3,4,5,6,8,10,14,17,21,22,23,24,26,29,31,32,35,36],[3,4,5,6,7,9,11,15,18,22,23,24,25,27,30,32,33,36,37],[4,5,6,7,8,10,12,16,19,23,24,25,26,28,31,33,34,37,38],[1,5,6,7,8,9,11,13,17,20,24,25,26,27,29,32,34,35,38],[2,6,7,8,9,10,12,14,18,20,21,25,26,27,28,30,33,35,36],[3,7,8,9,10,11,13,15,19,21,22,26,27,28,29,31,34,36,37],[1,4,8,9,10,11,12,14,16,22,23,27,28,29,30,32,35,37,38],[2,5,9,10,11,12,13,15,17,20,23,24,28,29,30,31,33,36,38],[3,6,10,11,12,13,14,16,18,20,21,24,25,29,30,31,32,34,37],[4,7,11,12,13,14,15,17,19,21,22,25,26,30,31,32,33,35,38],[1,5,8,12,13,14,15,16,18,20,22,23,26,27,31,32,33,34,36],[2,6,9,13,14,15,16,17,19,21,23,24,27,28,32,33,34,35,37],[1,3,7,10,14,15,16,17,18,22,24,25,28,29,33,34,35,36,38],[2,4,8,11,15,16,17,18,19,20,23,25,26,29,30,34,35,36,37],[1,3,5,9,12,16,17,18,19,21,24,26,27,30,31,35,36,37,38],[1,2,4,6,10,13,17,18,19,20,22,25,27,28,31,32,36,37,38],[1,2,3,5,7,11,14,18,19,20,21,23,26,28,29,32,33,37,38],[1,2,3,4,6,8,12,15,19,20,21,22,24,27,29,30,33,34,38],[1,2,3,6,7,10,12,13,15,23,26,30,32,34,35,36,37,38,39],[2,3,4,7,8,11,13,14,16,20,24,27,31,33,35,36,37,38,39],[3,4,5,8,9,12,14,15,17,20,21,25,28,32,34,36,37,38,39],[4,5,6,9,10,13,15,16,18,20,21,22,26,29,33,35,37,38,39],[5,6,7,10,11,14,16,17,19,20,21,22,23,27,30,34,36,38,39],[1,6,7,8,11,12,15,17,18,20,21,22,23,24,28,31,35,37,39],[2,7,8,9,12,13,16,18,19,21,22,23,24,25,29,32,36,38,39],[1,3,8,9,10,13,14,17,19,20,22,23,24,25,26,30,33,37,39],[1,2,4,9,10,11,14,15,18,21,23,24,25,26,27,31,34,38,39],[2,3,5,10,11,12,15,16,19,20,22,24,25,26,27,28,32,35,39],[1,3,4,6,11,12,13,16,17,21,23,25,26,27,28,29,33,36,39],[2,4,5,7,12,13,14,17,18,22,24,26,27,28,29,30,34,37,39],[3,5,6,8,13,14,15,18,19,23,25,27,28,29,30,31,35,38,39],[1,4,6,7,9,14,15,16,19,20,24,26,28,29,30,31,32,36,39],[1,2,5,7,8,10,15,16,17,21,25,27,29,30,31,32,33,37,39],[2,3,6,8,9,11,16,17,18,22,26,28,30,31,32,33,34,38,39],[3,4,7,9,10,12,17,18,19,20,23,27,29,31,32,33,34,35,39],[1,4,5,8,10,11,13,18,19,21,24,28,30,32,33,34,35,36,39],[1,2,5,6,9,11,12,14,19,22,25,29,31,33,34,35,36,37,39]]));

H(8&162):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,9,13,16,20,21,22,23,27,28,31,32,34,37],[2,3,4,5,6,8,10,14,17,21,22,23,24,28,29,32,33,35,38],[3,4,5,6,7,9,11,15,18,20,22,23,24,25,29,30,33,34,36],[4,5,6,7,8,10,12,16,19,21,23,24,25,26,30,31,34,35,37],[1,5,6,7,8,9,11,13,17,22,24,25,26,27,31,32,35,36,38],[2,6,7,8,9,10,12,14,18,20,23,25,26,27,28,32,33,36,37],[3,7,8,9,10,11,13,15,19,21,24,26,27,28,29,33,34,37

,38],[1,4,8,9,10,11,12,14,16,20,22,25,27,28,29,30,34,35,38],[2,5,9,10,11,12,13,15,17,20,21,23,26,28,29,30,31,35,36],[3,6,10,11,12,13,14,16,18,21,22,24,27,29,30,31,32,36,37],[4,7,11,12,13,14,15,17,19,22,23,25,28,30,31,32,33,37,38],[1,5,8,12,13,14,15,16,18,20,23,24,26,29,31,32,33,34,38],[2,6,9,13,14,15,16,17,19,20,21,24,25,27,30,32,33,34,35],[1,3,7,10,14,15,16,17,18,21,22,25,26,28,31,33,34,35,36],[2,4,8,11,15,16,17,18,19,22,23,26,27,29,32,34,35,36,37],[1,3,5,9,12,16,17,18,19,23,24,27,28,30,33,35,36,37,38],[1,2,4,6,10,13,17,18,19,20,24,25,28,29,31,34,36,37,38],[1,2,3,5,7,11,14,18,19,20,21,25,26,29,30,32,35,37,38],[1,2,3,4,6,8,12,15,19,20,21,22,26,27,30,31,33,36,38],[1,2,3,8,10,11,13,16,17,20,22,23,24,25,26,30,33,37,39],[2,3,4,9,11,12,14,17,18,21,23,24,25,26,27,31,34,38,39],[3,4,5,10,12,13,15,18,19,20,22,24,25,26,27,28,32,35,39],[1,4,5,6,11,13,14,16,19,21,23,25,26,27,28,29,33,36,39],[1,2,5,6,7,12,14,15,17,22,24,26,27,28,29,30,34,37,39],[2,3,6,7,8,13,15,16,18,23,25,27,28,29,30,31,35,38,39],[3,4,7,8,9,14,16,17,19,20,24,26,28,29,30,31,32,36,39],[1,4,5,8,9,10,15,17,18,21,25,27,29,30,31,32,33,37,39],[2,5,6,9,10,11,16,18,19,22,26,28,30,31,32,33,34,38,39],[1,3,6,7,10,11,12,17,19,20,23,27,29,31,32,33,34,35,39],[1,2,4,7,8,11,12,13,18,21,24,28,30,32,33,34,35,36,39],[2,3,5,8,9,12,13,14,19,22,25,29,31,33,34,35,36,37,39],[1,3,4,6,9,10,13,14,15,23,26,30,32,34,35,36,37,38,39],[2,4,5,7,10,11,14,15,16,20,24,27,31,33,35,36,37,38,39],[3,5,6,8,11,12,15,16,17,20,21,25,28,32,34,36,37,38,39],[4,6,7,9,12,13,16,17,18,20,21,22,26,29,33,35,37,38,39],[5,7,8,10,13,14,17,18,19,20,21,22,23,27,30,34,36,38,39],[1,6,8,9,11,14,15,18,19,20,21,22,23,24,28,31,35,37,39],[1,2,7,9,10,12,15,16,19,21,22,23,24,25,29,32,36,38,39]]);

H(9&169):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,10,14,15,20,21,22,24,25,27,31,32,35,37],[2,3,4,5,6,8,11,15,16,21,22,23,25,26,28,32,33,36,38],[3,4,5,6,7,9,12,16,17,20,22,23,24,26,27,29,33,34,37],[4,5,6,7,8,10,13,17,18,21,23,24,25,27,28,30,34,35,38],[5,6,7,8,9,11,14,18,19,20,22,24,25,26,28,29,31,35,36],[1,6,7,8,9,10,12,15,19,21,23,25,26,27,29,30,32,36,37],[1,2,7,8,9,10,11,13,16,22,24,26,27,28,30,31,33,37,38],[2,3,8,9,10,11,12,14,17,20,23,25,27,28,29,31,32,34,38],[3,4,9,10,11,12,13,15,18,20,21,24,26,28,29,30,32,33,35],[4,5,10,11,12,13,14,16,19,21,22,25,27,29,30,31,33,34,36],[1,5,6,11,12,13,14,15,17,22,23,26,28,30,31,32,34,35,37],[2,6,7,12,13,14,15,16,18,23,24,27,29,31,32,33,35,36,38],[3,7,8,13,14,15,16,17,19,20,24,25,28,30,32,33,34,36,37],[1,4,8,9,14,15,16,17,18,21,25,26,29,31,33,34,35,37,38],[2,5,9,10,15,16,17,18,19,20,22,26,27,30,32,34,35,36,38],[1,3,6,10,11,16,17,18,19,20,21,23,27,28,31,33,35,36,37],[1,2,4,7,11,12,17,18,19,21,22,24,28,29,32,34,36,37,38],[1,2,3,5,8,12,13,18,19,20,22,23,25,29,30,33,3

5,37,38],[1,2,3,4,6,9,13,14,19,20,21,23,24,26,30,31,34,36,38],[1,2,3,5,8,12,14,16,17,21,24,26,27,28,29,30,35,36,39],[2,3,4,6,9,13,15,17,18,22,25,27,28,29,30,31,36,37,39],[3,4,5,7,10,14,16,18,19,23,26,28,29,30,31,32,37,38,39],[1,4,5,6,8,11,15,17,19,20,24,27,29,30,31,32,33,38,39],[1,2,5,6,7,9,12,16,18,20,21,25,28,30,31,32,33,34,39],[2,3,6,7,8,10,13,17,19,21,22,26,29,31,32,33,34,35,39],[1,3,4,7,8,9,11,14,18,22,23,27,30,32,33,34,35,36,39],[2,4,5,8,9,10,12,15,19,23,24,28,31,33,34,35,36,37,39],[1,3,5,6,9,10,11,13,16,24,25,29,32,34,35,36,37,38,39],[2,4,6,7,10,11,12,14,17,20,25,26,30,33,35,36,37,38,39],[3,5,7,8,11,12,13,15,18,20,21,26,27,31,34,36,37,38,39],[4,6,8,9,12,13,14,16,19,20,21,22,27,28,32,35,37,38,39],[1,5,7,9,10,13,14,15,17,20,21,22,23,28,29,33,36,38,39],[2,6,8,10,11,14,15,16,18,20,21,22,23,24,29,30,34,37,39],[3,7,9,11,12,15,16,17,19,21,22,23,24,25,30,31,35,38,39],[1,4,8,10,12,13,16,17,18,20,22,23,24,25,26,31,32,36,39],[2,5,9,11,13,14,17,18,19,21,23,24,25,26,27,32,33,37,39],[1,3,6,10,12,14,15,18,19,22,24,25,26,27,28,33,34,38,39],[1,2,4,7,11,13,15,16,19,20,23,25,26,27,28,29,34,35,39]]));

H(10&170):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,10,14,15,20,21,22,24,26,29,30,34,36,37],[2,3,4,5,6,8,11,15,16,21,22,23,25,27,30,31,35,37,38],[3,4,5,6,7,9,12,16,17,20,22,23,24,26,28,31,32,36,38],[4,5,6,7,8,10,13,17,18,20,21,23,24,25,27,29,32,33,37],[5,6,7,8,9,11,14,18,19,21,22,24,25,26,28,30,33,34,38],[1,6,7,8,9,10,12,15,19,20,22,23,25,26,27,29,31,34,35],[1,2,7,8,9,10,11,13,16,21,23,24,26,27,28,30,32,35,36],[2,3,8,9,10,11,12,14,17,22,24,25,27,28,29,31,33,36,37],[3,4,9,10,11,12,13,15,18,23,25,26,28,29,30,32,34,37,38],[4,5,10,11,12,13,14,16,19,20,24,26,27,29,30,31,33,35,38],[1,5,6,11,12,13,14,15,17,20,21,25,27,28,30,31,32,34,36],[2,6,7,12,13,14,15,16,18,21,22,26,28,29,31,32,33,35,37],[3,7,8,13,14,15,16,17,19,22,23,27,29,30,32,33,34,36,38],[1,4,8,9,14,15,16,17,18,20,23,24,28,30,31,33,34,35,37],[2,5,9,10,15,16,17,18,19,21,24,25,29,31,32,34,35,36,38],[1,3,6,10,11,16,17,18,19,20,22,25,26,30,32,33,35,36,37],[1,2,4,7,11,12,17,18,19,21,23,26,27,31,33,34,36,37,38],[1,2,3,5,8,12,13,18,19,20,22,24,27,28,32,34,35,37,38],[1,2,3,4,6,9,13,14,19,20,21,23,25,28,29,33,35,36,38],[1,2,3,6,7,9,11,15,18,20,24,27,29,30,31,32,33,38,39],[2,3,4,7,8,10,12,16,19,20,21,25,28,30,31,32,33,34,39],[1,3,4,5,8,9,11,13,17,21,22,26,29,31,32,33,34,35,39],[2,4,5,6,9,10,12,14,18,22,23,27,30,32,33,34,35,36,39],[3,5,6,7,10,11,13,15,19,23,24,28,31,33,34,35,36,37,39],[1,4,6,7,8,11,12,14,16,24,25,29,32,34,35,36,37,38,39],[2,5,7,8,9,12,13,15,17,20,25,26,30,33,35,36,37,38,39],[3,6,8,9,10,13,14,16,18,20,21,26,27,31,34,36,37,38,39],[4,7,9,10,11,14,15,17,19,20,21,22,27,28,32,35,37,38,39],[1,5,8,10,11,12,15,16,18,20,21,22,23,28,29,33,36,38,39],[2,

6,9,11,12,13,16,17,19,20,21,22,23,24,29,30,34,37,39],[1,3,7,10,12,13,14,17,18,21,22,23,24,25,30,31,35,38,39],[2,4,8,11,13,14,15,18,19,20,22,23,24,25,26,31,32,36,39],[1,3,5,9,12,14,15,16,19,21,23,24,25,26,27,32,33,37,39],[1,2,4,6,10,13,15,16,17,22,24,25,26,27,28,33,34,38,39],[2,3,5,7,11,14,16,17,18,20,23,25,26,27,28,29,34,35,39],[3,4,6,8,12,15,17,18,19,21,24,26,27,28,29,30,35,36,39],[1,4,5,7,9,13,16,18,19,22,25,27,28,29,30,31,36,37,39],[1,2,5,6,8,10,14,17,19,23,26,28,29,30,31,32,37,38,39]]));

H(15&156):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,11,14,16,20,21,22,23,26,29,30,34,35,37],[2,3,4,5,6,8,12,15,17,21,22,23,24,27,30,31,35,36,38],[3,4,5,6,7,9,13,16,18,20,22,23,24,25,28,31,32,36,37],[4,5,6,7,8,10,14,17,19,21,23,24,25,26,29,32,33,37,38],[1,5,6,7,8,9,11,15,18,20,22,24,25,26,27,30,33,34,38],[2,6,7,8,9,10,12,16,19,20,21,23,25,26,27,28,31,34,35],[1,3,7,8,9,10,11,13,17,21,22,24,26,27,28,29,32,35,36],[2,4,8,9,10,11,12,14,18,22,23,25,27,28,29,30,33,36,37],[3,5,9,10,11,12,13,15,19,23,24,26,28,29,30,31,34,37,38],[1,4,6,10,11,12,13,14,16,20,24,25,27,29,30,31,32,35,38],[2,5,7,11,12,13,14,15,17,20,21,25,26,28,30,31,32,33,36],[3,6,8,12,13,14,15,16,18,21,22,26,27,29,31,32,33,34,37],[4,7,9,13,14,15,16,17,19,22,23,27,28,30,32,33,34,35,38],[1,5,8,10,14,15,16,17,18,20,23,24,28,29,31,33,34,35,36],[2,6,9,11,15,16,17,18,19,21,24,25,29,30,32,34,35,36,37],[1,3,7,10,12,16,17,18,19,22,25,26,30,31,33,35,36,37,38],[1,2,4,8,11,13,17,18,19,20,23,26,27,31,32,34,36,37,38],[1,2,3,5,9,12,14,18,19,20,21,24,27,28,32,33,35,37,38],[1,2,3,4,6,10,13,15,19,20,21,22,25,28,29,33,34,36,38],[1,2,3,6,7,9,10,15,17,20,23,27,29,30,31,32,33,37,39],[2,3,4,7,8,10,11,16,18,21,24,28,30,31,32,33,34,38,39],[3,4,5,8,9,11,12,17,19,20,22,25,29,31,32,33,34,35,39],[1,4,5,6,9,10,12,13,18,21,23,26,30,32,33,34,35,36,39],[2,5,6,7,10,11,13,14,19,22,24,27,31,33,34,35,36,37,39],[1,3,6,7,8,11,12,14,15,23,25,28,32,34,35,36,37,38,39],[2,4,7,8,9,12,13,15,16,20,24,26,29,33,35,36,37,38,39],[3,5,8,9,10,13,14,16,17,20,21,25,27,30,34,36,37,38,39],[4,6,9,10,11,14,15,17,18,20,21,22,26,28,31,35,37,38,39],[5,7,10,11,12,15,16,18,19,20,21,22,23,27,29,32,36,38,39],[1,6,8,11,12,13,16,17,19,20,21,22,23,24,28,30,33,37,39],[1,2,7,9,12,13,14,17,18,21,22,23,24,25,29,31,34,38,39],[2,3,8,10,13,14,15,18,19,20,22,23,24,25,26,30,32,35,39],[1,3,4,9,11,14,15,16,19,21,23,24,25,26,27,31,33,36,39],[1,2,4,5,10,12,15,16,17,22,24,25,26,27,28,32,34,37,39],[2,3,5,6,11,13,16,17,18,23,25,26,27,28,29,33,35,38,39],[3,4,6,7,12,14,17,18,19,20,24,26,27,28,29,30,34,36,39],[1,4,5,7,8,13,15,18,19,21,25,27,28,29,30,31,35,37,39],[1,2,5,6,8,9,14,16,19,22,26,28,29,30,31,32,36,38,39]]));

H(17&154):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,11,14,16,20,21,22,23,25,27,28,32,33,36],[2,3,4,5,6,8,12,15,17,21,22,23,24,26,28,29,33,34,37],[3,4,5,6,7,9,13,16,18,22,23,24,25,27,29,30,34,35,38],[4,5,6,7,8,10,14,17,19,20,23,24,25,26,28,30,31,35,36],[1,5,6,7,8,9,11,15,18,21,24,25,26,27,29,31,32,36,37],[2,6,7,8,9,10,12,16,19,22,25,26,27,28,30,32,33,37,38],[1,3,7,8,9,10,11,13,17,20,23,26,27,28,29,31,33,34,38],[2,4,8,9,10,11,12,14,18,20,21,24,27,28,29,30,32,34,35],[3,5,9,10,11,12,13,15,19,21,22,25,28,29,30,31,33,35,36],[1,4,6,10,11,12,13,14,16,22,23,26,29,30,31,32,34,36,37],[2,5,7,11,12,13,14,15,17,23,24,27,30,31,32,33,35,37,38],[3,6,8,12,13,14,15,16,18,20,24,25,28,31,32,33,34,36,38],[4,7,9,13,14,15,16,17,19,20,21,25,26,29,32,33,34,35,37],[1,5,8,10,14,15,16,17,18,21,22,26,27,30,33,34,35,36,38],[2,6,9,11,15,16,17,18,19,20,22,23,27,28,31,34,35,36,37],[1,3,7,10,12,16,17,18,19,21,23,24,28,29,32,35,36,37,38],[1,2,4,8,11,13,17,18,19,20,22,24,25,29,30,33,36,37,38],[1,2,3,5,9,12,14,18,19,20,21,23,25,26,30,31,34,37,38],[1,2,3,4,6,10,13,15,19,20,21,22,24,26,27,31,32,35,38],[1,2,3,6,8,13,14,16,17,21,25,27,28,29,30,31,35,37,39],[2,3,4,7,9,14,15,17,18,22,26,28,29,30,31,32,36,38,39],[3,4,5,8,10,15,16,18,19,20,23,27,29,30,31,32,33,37,39],[1,4,5,6,9,11,16,17,19,21,24,28,30,31,32,33,34,38,39],[1,2,5,6,7,10,12,17,18,20,22,25,29,31,32,33,34,35,39],[2,3,6,7,8,11,13,18,19,21,23,26,30,32,33,34,35,36,39],[1,3,4,7,8,9,12,14,19,22,24,27,31,33,34,35,36,37,39],[1,2,4,5,8,9,10,13,15,23,25,28,32,34,35,36,37,38,39],[2,3,5,6,9,10,11,14,16,20,24,26,29,33,35,36,37,38,39],[3,4,6,7,10,11,12,15,17,20,21,25,27,30,34,36,37,38,39],[4,5,7,8,11,12,13,16,18,20,21,22,26,28,31,35,37,38,39],[5,6,8,9,12,13,14,17,19,20,21,22,23,27,29,32,36,38,39],[1,6,7,9,10,13,14,15,18,20,21,22,23,24,28,30,33,37,39],[2,7,8,10,11,14,15,16,19,21,22,23,24,25,29,31,34,38,39],[1,3,8,9,11,12,15,16,17,20,22,23,24,25,26,30,32,35,39],[2,4,9,10,12,13,16,17,18,21,23,24,25,26,27,31,33,36,39],[3,5,10,11,13,14,17,18,19,22,24,25,26,27,28,32,34,37,39],[1,4,6,11,12,14,15,18,19,23,25,26,27,28,29,33,35,38,39],[1,2,5,7,12,13,15,16,19,20,24,26,27,28,29,30,34,36,39]]);

H(18&146):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,11,15,16,20,21,22,24,28,29,31,34,36,37],[2,3,4,5,6,8,12,16,17,21,22,23,25,29,30,32,35,37,38],[3,4,5,6,7,9,13,17,18,20,22,23,24,26,30,31,33,36,38],[4,5,6,7,8,10,14,18,19,20,21,23,24,25,27,31,32,34,37],[1,5,6,7,8,9,11,15,19,21,22,24,25,26,28,32,33,35,38],[1,2,6,7,8,9,10,12,16,20,22,23,25,26,27,29,33,34,36],[2,3,7,8,9,10,11,13,17,21,23,24,26,27,28,30,34,35,37],[3,4,8,9,10,11,12,14,18,22,24,25,27,28,29,31,35,36,38],[4,5,9,10,11,12,13,15,19,20,

23,25,26,28,29,30,32,36,37],[1,5,6,10,11,12,13,14,16,21,24,26,27,29,30,31,33,37,38],[2,6,7,11,12,13,14,15,17,20,22,25,27,28,30,31,32,34,38],[3,7,8,12,13,14,15,16,18,20,21,23,26,28,29,31,32,33,35],[4,8,9,13,14,15,16,17,19,21,22,24,27,29,30,32,33,34,36],[1,5,9,10,14,15,16,17,18,22,23,25,28,30,31,33,34,35,37],[2,6,10,11,15,16,17,18,19,23,24,26,29,31,32,34,35,36,38],[1,3,7,11,12,16,17,18,19,20,24,25,27,30,32,33,35,36,37],[1,2,4,8,12,13,17,18,19,21,25,26,28,31,33,34,36,37,38],[1,2,3,5,9,13,14,18,19,20,22,26,27,29,32,34,35,37,38],[1,2,3,4,6,10,14,15,19,20,21,23,27,28,30,33,35,36,38],[1,2,3,5,9,12,14,15,17,21,23,24,25,26,27,31,32,36,39],[2,3,4,6,10,13,15,16,18,22,24,25,26,27,28,32,33,37,39],[3,4,5,7,11,14,16,17,19,23,25,26,27,28,29,33,34,38,39],[1,4,5,6,8,12,15,17,18,20,24,26,27,28,29,30,34,35,39],[2,5,6,7,9,13,16,18,19,21,25,27,28,29,30,31,35,36,39],[1,3,6,7,8,10,14,17,19,22,26,28,29,30,31,32,36,37,39],[1,2,4,7,8,9,11,15,18,23,27,29,30,31,32,33,37,38,39],[2,3,5,8,9,10,12,16,19,20,24,28,30,31,32,33,34,38,39],[1,3,4,6,9,10,11,13,17,20,21,25,29,31,32,33,34,35,39],[2,4,5,7,10,11,12,14,18,21,22,26,30,32,33,34,35,36,39],[3,5,6,8,11,12,13,15,19,22,23,27,31,33,34,35,36,37,39],[1,4,6,7,9,12,13,14,16,23,24,28,32,34,35,36,37,38,39],[2,5,7,8,10,13,14,15,17,20,24,25,29,33,35,36,37,38,39],[3,6,8,9,11,14,15,16,18,20,21,25,26,30,34,36,37,38,39],[4,7,9,10,12,15,16,17,19,20,21,22,26,27,31,35,37,38,39],[1,5,8,10,11,13,16,17,18,20,21,22,23,27,28,32,36,38,39],[2,6,9,11,12,14,17,18,19,20,21,22,23,24,28,29,33,37,39],[1,3,7,10,12,13,15,18,19,21,22,23,24,25,29,30,34,38,39],[1,2,4,8,11,13,14,16,19,20,22,23,24,25,26,30,31,35,39]]);

H(19&145):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,7,11,15,16,20,21,22,24,25,27,30,32,33,37],[2,3,4,5,6,8,12,16,17,21,22,23,25,26,28,31,33,34,38],[3,4,5,6,7,9,13,17,18,20,22,23,24,26,27,29,32,34,35],[4,5,6,7,8,10,14,18,19,21,23,24,25,27,28,30,33,35,36],[1,5,6,7,8,9,11,15,19,22,24,25,26,28,29,31,34,36,37],[1,2,6,7,8,9,10,12,16,23,25,26,27,29,30,32,35,37,38],[2,3,7,8,9,10,11,13,17,20,24,26,27,28,30,31,33,36,38],[3,4,8,9,10,11,12,14,18,20,21,25,27,28,29,31,32,34,37],[4,5,9,10,11,12,13,15,19,21,22,26,28,29,30,32,33,35,38],[1,5,6,10,11,12,13,14,16,20,22,23,27,29,30,31,33,34,36],[2,6,7,11,12,13,14,15,17,21,23,24,28,30,31,32,34,35,37],[3,7,8,12,13,14,15,16,18,22,24,25,29,31,32,33,35,36,38],[4,8,9,13,14,15,16,17,19,20,23,25,26,30,32,33,34,36,37],[1,5,9,10,14,15,16,17,18,21,24,26,27,31,33,34,35,37,38],[2,6,10,11,15,16,17,18,19,20,22,25,27,28,32,34,35,36,38],[1,3,7,11,12,16,17,18,19,20,21,23,26,28,29,33,35,36,37],[1,2,4,8,12,13,17,18,19,21,22,24,27,29,30,34,36,37,38],[1,2,3,5,9,13,14,18,19,20,22,23,25,28,30,31,35,37,38],[1,2,3,4,6,10,14,15,19,20,21,23,24,26,29,31,32,36,38],[1,2,3,6,8,9,11,14,18,21,

22,26,30,32,33,34,35,36,39],[2,3,4,7,9,10,12,15,19,22,23,27,31,33,34,35,36,37,39],[1,3,4,5,8,10,11,13,16,23,24,28,32,34,35,36,37,38,39],[2,4,5,6,9,11,12,14,17,20,24,25,29,33,35,36,37,38,39],[3,5,6,7,10,12,13,15,18,20,21,25,26,30,34,36,37,38,39],[4,6,7,8,11,13,14,16,19,20,21,22,26,27,31,35,37,38,39],[1,5,7,8,9,12,14,15,17,20,21,22,23,27,28,32,36,38,39],[2,6,8,9,10,13,15,16,18,20,21,22,23,24,28,29,33,37,39],[3,7,9,10,11,14,16,17,19,21,22,23,24,25,29,30,34,38,39],[1,4,8,10,11,12,15,17,18,20,22,23,24,25,26,30,31,35,39],[2,5,9,11,12,13,16,18,19,21,23,24,25,26,27,31,32,36,39],[1,3,6,10,12,13,14,17,19,22,24,25,26,27,28,32,33,37,39],[1,2,4,7,11,13,14,15,18,23,25,26,27,28,29,33,34,38,39],[2,3,5,8,12,14,15,16,19,20,24,26,27,28,29,30,34,35,39],[1,3,4,6,9,13,15,16,17,21,25,27,28,29,30,31,35,36,39],[2,4,5,7,10,14,16,17,18,22,26,28,29,30,31,32,36,37,39],[3,5,6,8,11,15,17,18,19,23,27,29,30,31,32,33,37,38,39],[1,4,6,7,9,12,16,18,19,20,24,28,30,31,32,33,34,38,39],[1,2,5,7,8,10,13,17,19,20,21,25,29,31,32,33,34,35,39]]);

H(34&139):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,8,13,15,16,20,21,22,24,26,27,30,32,36,37],[2,3,4,5,6,9,14,16,17,21,22,23,25,27,28,31,33,37,38],[3,4,5,6,7,10,15,17,18,20,22,23,24,26,28,29,32,34,38],[4,5,6,7,8,11,16,18,19,20,21,23,24,25,27,29,30,33,35],[1,5,6,7,8,9,12,17,19,21,22,24,25,26,28,30,31,34,36],[1,2,6,7,8,9,10,13,18,22,23,25,26,27,29,31,32,35,37],[2,3,7,8,9,10,11,14,19,23,24,26,27,28,30,32,33,36,38],[1,3,4,8,9,10,11,12,15,20,24,25,27,28,29,31,33,34,37],[2,4,5,9,10,11,12,13,16,21,25,26,28,29,30,32,34,35,38],[3,5,6,10,11,12,13,14,17,20,22,26,27,29,30,31,33,35,36],[4,6,7,11,12,13,14,15,18,21,23,27,28,30,31,32,34,36,37],[5,7,8,12,13,14,15,16,19,22,24,28,29,31,32,33,35,37,38],[1,6,8,9,13,14,15,16,17,20,23,25,29,30,32,33,34,36,38],[2,7,9,10,14,15,16,17,18,20,21,24,26,30,31,33,34,35,37],[3,8,10,11,15,16,17,18,19,21,22,25,27,31,32,34,35,36,38],[1,4,9,11,12,16,17,18,19,20,22,23,26,28,32,33,35,36,37],[1,2,5,10,12,13,17,18,19,21,23,24,27,29,33,34,36,37,38],[1,2,3,6,11,13,14,18,19,20,22,24,25,28,30,34,35,37,38],[1,2,3,4,7,12,14,15,19,20,21,23,25,26,29,31,35,36,38],[1,2,3,5,7,8,11,13,17,20,21,23,28,31,32,33,34,35,39],[2,3,4,6,8,9,12,14,18,21,22,24,29,32,33,34,35,36,39],[3,4,5,7,9,10,13,15,19,22,23,25,30,33,34,35,36,37,39],[1,4,5,6,8,10,11,14,16,23,24,26,31,34,35,36,37,38,39],[2,5,6,7,9,11,12,15,17,20,24,25,27,32,35,36,37,38,39],[3,6,7,8,10,12,13,16,18,20,21,25,26,28,33,36,37,38,39],[4,7,8,9,11,13,14,17,19,20,21,22,26,27,29,34,37,38,39],[1,5,8,9,10,12,14,15,18,20,21,22,23,27,28,30,35,38,39],[2,6,9,10,11,13,15,16,19,20,21,22,23,24,28,29,31,36,39],[1,3,7,10,11,12,14,16,17,21,22,23,24,25,29,30,32,37,39],[2,4,8,11,12,13,15,17,18,22,23,24,25,26,30,31,33,38,39],[3,5,9,12,13,14,16,18,19,20,23,24

,25,26,27,31,32,34,39],[1,4,6,10,13,14,15,17,19,21,24,25,26,27,28,32,33,35,39],[1,2,5,7,11,14,15,16,18,22,25,26,27,28,29,33,34,36,39],[2,3,6,8,12,15,16,17,19,23,26,27,28,29,30,34,35,37,39],[1,3,4,7,9,13,16,17,18,24,27,28,29,30,31,35,36,38,39],[2,4,5,8,10,14,17,18,19,20,25,28,29,30,31,32,36,37,39],[1,3,5,6,9,11,15,18,19,21,26,29,30,31,32,33,37,38,39],[1,2,4,6,7,10,12,16,19,20,22,27,30,31,32,33,34,38,39]]));

H(35&140):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,8,13,15,16,20,21,22,24,26,30,31,34,36,37],[2,3,4,5,6,9,14,16,17,21,22,23,25,27,31,32,35,37,38],[3,4,5,6,7,10,15,17,18,20,22,23,24,26,28,32,33,36,38],[4,5,6,7,8,11,16,18,19,20,21,23,24,25,27,29,33,34,37],[1,5,6,7,8,9,12,17,19,21,22,24,25,26,28,30,34,35,38],[1,2,6,7,8,9,10,13,18,20,22,23,25,26,27,29,31,35,36],[2,3,7,8,9,10,11,14,19,21,23,24,26,27,28,30,32,36,37],[1,3,4,8,9,10,11,12,15,22,24,25,27,28,29,31,33,37,38],[2,4,5,9,10,11,12,13,16,20,23,25,26,28,29,30,32,34,38],[3,5,6,10,11,12,13,14,17,20,21,24,26,27,29,30,31,33,35],[4,6,7,11,12,13,14,15,18,21,22,25,27,28,30,31,32,34,36],[5,7,8,12,13,14,15,16,19,22,23,26,28,29,31,32,33,35,37],[1,6,8,9,13,14,15,16,17,23,24,27,29,30,32,33,34,36,38],[2,7,9,10,14,15,16,17,18,20,24,25,28,30,31,33,34,35,37],[3,8,10,11,15,16,17,18,19,21,25,26,29,31,32,34,35,36,38],[1,4,9,11,12,16,17,18,19,20,22,26,27,30,32,33,35,36,37],[1,2,5,10,12,13,17,18,19,21,23,27,28,31,33,34,36,37,38],[1,2,3,6,11,13,14,18,19,20,22,24,28,29,32,34,35,37,38],[1,2,3,4,7,12,14,15,19,20,21,23,25,29,30,33,35,36,38],[1,2,3,5,7,11,14,16,17,22,25,26,27,28,29,33,34,36,39],[2,3,4,6,8,12,15,17,18,23,26,27,28,29,30,34,35,37,39],[3,4,5,7,9,13,16,18,19,24,27,28,29,30,31,35,36,38,39],[1,4,5,6,8,10,14,17,19,20,25,28,29,30,31,32,36,37,39],[1,2,5,6,7,9,11,15,18,21,26,29,30,31,32,33,37,38,39],[2,3,6,7,8,10,12,16,19,20,22,27,30,31,32,33,34,38,39],[1,3,4,7,8,9,11,13,17,20,21,23,28,31,32,33,34,35,39],[2,4,5,8,9,10,12,14,18,21,22,24,29,32,33,34,35,36,39],[3,5,6,9,10,11,13,15,19,22,23,25,30,33,34,35,36,37,39],[1,4,6,7,10,11,12,14,16,23,24,26,31,34,35,36,37,38,39],[2,5,7,8,11,12,13,15,17,20,24,25,27,32,35,36,37,38,39],[3,6,8,9,12,13,14,16,18,20,21,25,26,28,33,36,37,38,39],[4,7,9,10,13,14,15,17,19,20,21,22,26,27,29,34,37,38,39],[1,5,8,10,11,14,15,16,18,20,21,22,23,27,28,30,35,38,39],[2,6,9,11,12,15,16,17,19,20,21,22,23,24,28,29,31,36,39],[1,3,7,10,12,13,16,17,18,21,22,23,24,25,29,30,32,37,39],[2,4,8,11,13,14,17,18,19,22,23,24,25,26,30,31,33,38,39],[1,3,5,9,12,14,15,18,19,20,23,24,25,26,27,31,32,34,39],[1,2,4,6,10,13,15,16,19,21,24,25,26,27,28,32,33,35,39]]));

H(36&137):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,8,13,15,16,20,21,22,24,25,27,30,31,35,37],[2,3,4,5,6,9,14,16,17,21,22,23,25,26,28,31,32,36,38],[3,4,5,6,7,10,15,17,18,20,22,23,24,26,27,29,32,33,37],[4,5,6,7,8,11,16,18,19,21,23,24,25,27,28,30,33,34,38],[1,5,6,7,8,9,12,17,19,20,22,24,25,26,28,29,31,34,35],[1,2,6,7,8,9,10,13,18,21,23,25,26,27,29,30,32,35,36],[2,3,7,8,9,10,11,14,19,22,24,26,27,28,30,31,33,36,37],[1,3,4,8,9,10,11,12,15,23,25,27,28,29,31,32,34,37,38],[2,4,5,9,10,11,12,13,16,20,24,26,28,29,30,32,33,35,38],[3,5,6,10,11,12,13,14,17,20,21,25,27,29,30,31,33,34,36],[4,6,7,11,12,13,14,15,18,21,22,26,28,30,31,32,34,35,37],[5,7,8,12,13,14,15,16,19,22,23,27,29,31,32,33,35,36,38],[1,6,8,9,13,14,15,16,17,20,23,24,28,30,32,33,34,36,37],[2,7,9,10,14,15,16,17,18,21,24,25,29,31,33,34,35,37,38],[3,8,10,11,15,16,17,18,19,20,22,25,26,30,32,34,35,36,38],[1,4,9,11,12,16,17,18,19,20,21,23,26,27,31,33,35,36,37],[1,2,5,10,12,13,17,18,19,21,22,24,27,28,32,34,36,37,38],[1,2,3,6,11,13,14,18,19,20,22,23,25,28,29,33,35,37,38],[1,2,3,4,7,12,14,15,19,20,21,23,24,26,29,30,34,36,38],[1,2,3,6,7,9,12,16,18,20,22,27,30,31,32,33,34,38,39],[2,3,4,7,8,10,13,17,19,20,21,23,28,31,32,33,34,35,39],[1,3,4,5,8,9,11,14,18,21,22,24,29,32,33,34,35,36,39],[2,4,5,6,9,10,12,15,19,22,23,25,30,33,34,35,36,37,39],[1,3,5,6,7,10,11,13,16,23,24,26,31,34,35,36,37,38,39],[2,4,6,7,8,11,12,14,17,20,24,25,27,32,35,36,37,38,39],[3,5,7,8,9,12,13,15,18,20,21,25,26,28,33,36,37,38,39],[4,6,8,9,10,13,14,16,19,20,21,22,26,27,29,34,37,38,39],[1,5,7,9,10,11,14,15,17,20,21,22,23,27,28,30,35,38,39],[2,6,8,10,11,12,15,16,18,20,21,22,23,24,28,29,31,36,39],[3,7,9,11,12,13,16,17,19,21,22,23,24,25,29,30,32,37,39],[1,4,8,10,12,13,14,17,18,22,23,24,25,26,30,31,33,38,39],[2,5,9,11,13,14,15,18,19,20,23,24,25,26,27,31,32,34,39],[1,3,6,10,12,14,15,16,19,21,24,25,26,27,28,32,33,35,39],[1,2,4,7,11,13,15,16,17,22,25,26,27,28,29,33,34,36,39],[2,3,5,8,12,14,16,17,18,23,26,27,28,29,30,34,35,37,39],[3,4,6,9,13,15,17,18,19,24,27,28,29,30,31,35,36,38,39],[1,4,5,7,10,14,16,18,19,20,25,28,29,30,31,32,36,37,39],[1,2,5,6,8,11,15,17,19,21,26,29,30,31,32,33,37,38,39]]));

H(37&138):

BlockDesign(39,[[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,5,8,13,15,16,20,21,22,24,25,29,31,34,35,37],[2,3,4,5,6,9,14,16,17,21,22,23,25,26,30,32,35,36,38],[3,4,5,6,7,10,15,17,18,20,22,23,24,26,27,31,33,36,37],[4,5,6,7,8,11,16,18,19,21,23,24,25,27,28,32,34,37,38],[1,5,6,7,8,9,12,17,19,20,22,24,25,26,28,29,33,35,38],[1,2,6,7,8,9,10,13,18,20,21,23,25,26,27,29,30,34,36],[2,3,7,8,9,10,11,14,19,21,22,24,26,27,28,30,31,35,37],[1,3,4,8,9,10,11,12,15,22,23,25,27,28,29,31,32,36,38],[2,4,5,9,10,11,12,13,16,20,23,24,26,28,29,30,32,33,37],[3,5,6,10,11,12,13,14,17,21,24,25,27,29,30,31,33,34,38],[4,6

,7,11,12,13,14,15,18,20,22,25,26,28,30,31,32,34,35],[5,7,8,12,13,14,15,16,19,21,23,26,27,29,31,32,33,35,36],[1,6,8,9,13,14,15,16,17,22,24,27,28,30,32,33,34,36,37],[2,7,9,10,14,15,16,17,18,23,25,28,29,31,33,34,35,37,38],[3,8,10,11,15,16,17,18,19,20,24,26,29,30,32,34,35,36,38],[1,4,9,11,12,16,17,18,19,20,21,25,27,30,31,33,35,36,37],[1,2,5,10,12,13,17,18,19,21,22,26,28,31,32,34,36,37,38],[1,2,3,6,11,13,14,18,19,20,22,23,27,29,32,33,35,37,38],[1,2,3,4,7,12,14,15,19,20,21,23,24,28,30,33,34,36,38],[1,2,3,6,10,12,15,16,18,21,24,25,26,27,28,32,33,35,39],[2,3,4,7,11,13,16,17,19,22,25,26,27,28,29,33,34,36,39],[1,3,4,5,8,12,14,17,18,23,26,27,28,29,30,34,35,37,39],[2,4,5,6,9,13,15,18,19,24,27,28,29,30,31,35,36,38,39],[1,3,5,6,7,10,14,16,19,20,25,28,29,30,31,32,36,37,39],[1,2,4,6,7,8,11,15,17,21,26,29,30,31,32,33,37,38,39],[2,3,5,7,8,9,12,16,18,20,22,27,30,31,32,33,34,38,39],[3,4,6,8,9,10,13,17,19,20,21,23,28,31,32,33,34,35,39],[1,4,5,7,9,10,11,14,18,21,22,24,29,32,33,34,35,36,39],[2,5,6,8,10,11,12,15,19,22,23,25,30,33,34,35,36,37,39],[1,3,6,7,9,11,12,13,16,23,24,26,31,34,35,36,37,38,39],[2,4,7,8,10,12,13,14,17,20,24,25,27,32,35,36,37,38,39],[3,5,8,9,11,13,14,15,18,20,21,25,26,28,33,36,37,38,39],[4,6,9,10,12,14,15,16,19,20,21,22,26,27,29,34,37,38,39],[1,5,7,10,11,13,15,16,17,20,21,22,23,27,28,30,35,38,39],[2,6,8,11,12,14,16,17,18,20,21,22,23,24,28,29,31,36,39],[3,7,9,12,13,15,17,18,19,21,22,23,24,25,29,30,32,37,39],[1,4,8,10,13,14,16,18,19,22,23,24,25,26,30,31,33,38,39],[1,2,5,9,11,14,15,17,19,20,23,24,25,26,27,31,32,34,39]]));

H(92&260):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,6,8,13,14,17,20,21,22,23,25,27,32,33,35,36],[2,3,4,5,7,9,14,15,18,21,22,23,24,26,28,33,34,36,37],[3,4,5,6,8,10,15,16,19,22,23,24,25,27,29,34,35,37,38],[1,4,5,6,7,9,11,16,17,20,23,24,25,26,28,30,35,36,38],[2,5,6,7,8,10,12,17,18,20,21,24,25,26,27,29,31,36,37],[3,6,7,8,9,11,13,18,19,21,22,25,26,27,28,30,32,37,38],[1,4,7,8,9,10,12,14,19,20,22,23,26,27,28,29,31,33,38],[1,2,5,8,9,10,11,13,15,20,21,23,24,27,28,29,30,32,34],[2,3,6,9,10,11,12,14,16,21,22,24,25,28,29,30,31,33,35],[3,4,7,10,11,12,13,15,17,22,23,25,26,29,30,31,32,34,36],[4,5,8,11,12,13,14,16,18,23,24,26,27,30,31,32,33,35,37],[5,6,9,12,13,14,15,17,19,24,25,27,28,31,32,33,34,36,38],[1,6,7,10,13,14,15,16,18,20,25,26,28,29,32,33,34,35,37],[2,7,8,11,14,15,16,17,19,21,26,27,29,30,33,34,35,36,38],[1,3,8,9,12,15,16,17,18,20,22,27,28,30,31,34,35,36,37],[2,4,9,10,13,16,17,18,19,21,23,28,29,31,32,35,36,37,38],[1,3,5,10,11,14,17,18,19,20,22,24,29,30,32,33,36,37,38],[1,2,4,6,11,12,15,18,19,20,21,23,25,30,31,33,34,37,38],[1,2,3,5,7,12,13,16,19,20,21,22,24,26,31,32,34,35,38],[1,2,3,4,6,8,13,14,17,24,26,28,29,30,31,34,37,38,39],[2,3,4,5,7,9,14,15,18,20,25,27,29,30,31,32,35,38,39],[3,4,5

,6,8,10,15,16,19,20,21,26,28,30,31,32,33,36,39],[1,4,5,6,7,9,11,16,17,21,22,27,29,31,32,33,34,37,39],[2,5,6,7,8,10,12,17,18,22,23,28,30,32,33,34,35,38,39],[3,6,7,8,9,11,13,18,19,20,23,24,29,31,33,34,35,36,39],[1,4,7,8,9,10,12,14,19,21,24,25,30,32,34,35,36,37,39],[1,2,5,8,9,10,11,13,15,22,25,26,31,33,35,36,37,38,39],[2,3,6,9,10,11,12,14,16,20,23,26,27,32,34,36,37,38,39],[3,4,7,10,11,12,13,15,17,20,21,24,27,28,33,35,37,38,39],[4,5,8,11,12,13,14,16,18,20,21,22,25,28,29,34,36,38,39],[5,6,9,12,13,14,15,17,19,20,21,22,23,26,29,30,35,37,39],[1,6,7,10,13,14,15,16,18,21,22,23,24,27,30,31,36,38,39],[2,7,8,11,14,15,16,17,19,20,22,23,24,25,28,31,32,37,39],[1,3,8,9,12,15,16,17,18,21,23,24,25,26,29,32,33,38,39],[2,4,9,10,13,16,17,18,19,20,22,24,25,26,27,30,33,34,39],[1,3,5,10,11,14,17,18,19,21,23,25,26,27,28,31,34,35,39],[1,2,4,6,11,12,15,18,19,22,24,26,27,28,29,32,35,36,39],[1,2,3,5,7,12,13,16,19,23,25,27,28,29,30,33,36,37,39]]));

H(92&265):

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,6,8,13,14,17,20,21,22,23,26,27,29,30,35,37],[2,3,4,5,7,9,14,15,18,21,22,23,24,27,28,30,31,36,38],[3,4,5,6,8,10,15,16,19,20,22,23,24,25,28,29,31,32,37],[1,4,5,6,7,9,11,16,17,21,23,24,25,26,29,30,32,33,38],[2,5,6,7,8,10,12,17,18,20,22,24,25,26,27,30,31,33,34],[3,6,7,8,9,11,13,18,19,21,23,25,26,27,28,31,32,34,35],[1,4,7,8,9,10,12,14,19,22,24,26,27,28,29,32,33,35,36],[1,2,5,8,9,10,11,13,15,23,25,27,28,29,30,33,34,36,37],[2,3,6,9,10,11,12,14,16,24,26,28,29,30,31,34,35,37,38],[3,4,7,10,11,12,13,15,17,20,25,27,29,30,31,32,35,36,38],[4,5,8,11,12,13,14,16,18,20,21,26,28,30,31,32,33,36,37],[5,6,9,12,13,14,15,17,19,21,22,27,29,31,32,33,34,37,38],[1,6,7,10,13,14,15,16,18,20,22,23,28,30,32,33,34,35,38],[2,7,8,11,14,15,16,17,19,20,21,23,24,29,31,33,34,35,36],[1,3,8,9,12,15,16,17,18,21,22,24,25,30,32,34,35,36,37],[2,4,9,10,13,16,17,18,19,22,23,25,26,31,33,35,36,37,38],[1,3,5,10,11,14,17,18,19,20,23,24,26,27,32,34,36,37,38],[1,2,4,6,11,12,15,18,19,20,21,24,25,27,28,33,35,37,38],[1,2,3,5,7,12,13,16,19,20,21,22,25,26,28,29,34,36,38],[1,2,3,4,6,8,13,14,17,24,25,28,31,32,33,34,36,38,39],[2,3,4,5,7,9,14,15,18,20,25,26,29,32,33,34,35,37,39],[3,4,5,6,8,10,15,16,19,21,26,27,30,33,34,35,36,38,39],[1,4,5,6,7,9,11,16,17,20,22,27,28,31,34,35,36,37,39],[2,5,6,7,8,10,12,17,18,21,23,28,29,32,35,36,37,38,39],[3,6,7,8,9,11,13,18,19,20,22,24,29,30,33,36,37,38,39],[1,4,7,8,9,10,12,14,19,20,21,23,25,30,31,34,37,38,39],[1,2,5,8,9,10,11,13,15,20,21,22,24,26,31,32,35,38,39],[2,3,6,9,10,11,12,14,16,20,21,22,23,25,27,32,33,36,39],[3,4,7,10,11,12,13,15,17,21,22,23,24,26,28,33,34,37,39],[4,5,8,11,12,13,14,16,18,22,23,24,25,27,29,34,35,38,39],[5,6,9,12,13,14,15,17,19,20,23,24,25,26,28,30,35,36,39],[1,6,7,10,13,14,15,16,18,21,24,25,26,27,29,31,36,37,39],[2,7,8,11,14,

15,16,17,19,22,25,26,27,28,30,32,37,38,39],[1,3,8,9,12,15,16,17,18,20,23,26,27,28,29,31,33,38,39],[2,4,9,10,13,16,17,18,19,20,21,24,27,28,29,30,32,34,39],[1,3,5,10,11,14,17,18,19,21,22,25,28,29,30,31,33,35,39],[1,2,4,6,11,12,15,18,19,22,23,26,29,30,31,32,34,36,39],[1,2,3,5,7,12,13,16,19,23,24,27,30,31,32,33,35,37,39]]));

$H(93\&320) \approx H(93\&259)$:

BlockDesign(39,[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19],[1,2,3,4,6,8,13,14,17,20,21,22,23,26,27,29,30,35,37],[2,3,4,5,7,9,14,15,18,21,22,23,24,27,28,30,31,36,38],[3,4,5,6,8,10,15,16,19,20,22,23,24,25,28,29,31,32,37],[1,4,5,6,7,9,11,16,17,21,23,24,25,26,29,30,32,33,38],[2,5,6,7,8,10,12,17,18,20,22,24,25,26,27,30,31,33,34],[3,6,7,8,9,11,13,18,19,21,23,25,26,27,28,31,32,34,35],[1,4,7,8,9,10,12,14,19,22,24,26,27,28,29,32,33,35,36],[1,2,5,8,9,10,11,13,15,23,25,27,28,29,30,33,34,36,37],[2,3,6,9,10,11,12,14,16,24,26,28,29,30,31,34,35,37,38],[3,4,7,10,11,12,13,15,17,20,25,27,29,30,31,32,35,36,38],[4,5,8,11,12,13,14,16,18,20,21,26,28,30,31,32,33,36,37],[5,6,9,12,13,14,15,17,19,21,22,27,29,31,32,33,34,37,38],[1,6,7,10,13,14,15,16,18,20,22,23,28,30,32,33,34,35,38],[2,7,8,11,14,15,16,17,19,20,21,23,24,29,31,33,34,35,36],[1,3,8,9,12,15,16,17,18,21,22,24,25,30,32,34,35,36,37],[2,4,9,10,13,16,17,18,19,22,23,25,26,31,33,35,36,37,38],[1,3,5,10,11,14,17,18,19,20,23,24,26,27,32,34,36,37,38],[1,2,4,6,11,12,15,18,19,20,21,24,25,27,28,33,35,37,38],[1,2,3,5,7,12,13,16,19,20,21,22,25,26,28,29,34,36,38],[1,2,3,4,7,10,11,16,18,21,22,27,29,31,32,33,34,37,39],[2,3,4,5,8,11,12,17,19,22,23,28,30,32,33,34,35,38,39],[1,3,4,5,6,9,12,13,18,20,23,24,29,31,33,34,35,36,39],[2,4,5,6,7,10,13,14,19,21,24,25,30,32,34,35,36,37,39],[1,3,5,6,7,8,11,14,15,22,25,26,31,33,35,36,37,38,39],[2,4,6,7,8,9,12,15,16,20,23,26,27,32,34,36,37,38,39],[3,5,7,8,9,10,13,16,17,20,21,24,27,28,33,35,37,38,39],[4,6,8,9,10,11,14,17,18,20,21,22,25,28,29,34,36,38,39],[5,7,9,10,11,12,15,18,19,20,21,22,23,26,29,30,35,37,39],[1,6,8,10,11,12,13,16,19,21,22,23,24,27,30,31,36,38,39],[1,2,7,9,11,12,13,14,17,20,22,23,24,25,28,31,32,37,39],[2,3,8,10,12,13,14,15,18,21,23,24,25,26,29,32,33,38,39],[3,4,9,11,13,14,15,16,19,20,22,24,25,26,27,30,33,34,39],[1,4,5,10,12,14,15,16,17,21,23,25,26,27,28,31,34,35,39],[2,5,6,11,13,15,16,17,18,22,24,26,27,28,29,32,35,36,39],[3,6,7,12,14,16,17,18,19,23,25,27,28,29,30,33,36,37,39],[1,4,7,8,13,15,17,18,19,24,26,28,29,30,31,34,37,38,39],[1,2,5,8,9,14,16,18,19,20,25,27,29,30,31,32,35,38,39],[1,2,3,6,9,10,15,17,19,20,21,26,28,30,31,32,33,36,39]]));