Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 4s (2024)

Applying a Fuzzy Ordering Approach in Transportation Problems with Decagonal Intuitionistic Fuzzy Numbers

[a] Nathiya K, [b] Balasubramanian KR, [c] Gunasekar T, [d] Ramesh R, [e] Seenivasan M

[a] Research Scholar, Department of Mathematics, H.H. The Rajah's College, (Affiliated to Bharathidasan University), Pudukkottai, Tamil Nadu, India. Email: nathiyagk30@gmail.com

[b] Department of Mathematics, H.H. The Rajah's College, (Affiliated to Bharathidasan University), Pudukkottai, Tamil Nadu, India. Email: balamohitha@gmail.com

[C] Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, Tamil Nadu, India, Email: tguna84@gmail.com

[d] Department of Mathematics, Arignar Anna Government Arts College, Musiri, Tamil Nadu, India, Email: rameshsanju123@gmail.com

[e] Mathematics Wings - CDOE, Annamalai University, Annamalai Nagar, Tamil Nadu, India, Email: emseeni@yahoo.com

Article History:

Received: *20-04-2024*

Revised: *10-06-2024*

Accepted: *24-06-2024*

Abstract:

In this study, the paper delves into precision challenges within traditional transportation problem solutions, which rigidly define cost, supply, and demand. Acknowledging the inherent vagueness in real world contexts, the research explores the efficacy of intuitive fuzzy sets as a potent tool. Organized into four distinct sections, this work utilizes decagonal intuitionistic fuzzy numbers for managing supply and demand, while upholding conventional approaches for cost considerations.

Employing a fuzzy ordering method, optimal solutions are derived by adjusting the configuration of decagonal intuitive fuzzy numbers across each segment. Through a comparative analysis, the

Study identifies the most effective solution, with initial sections addressing balanced geometric intuitionistic fuzzy transportation problems and the final part focusing on unbalanced scenarios, specifically emphasizing supply and demand complexities.

Keywords: Fuzzy Set; Intuitionistic; Ranking; Transportation Problem; Numerical Analysis.

2020AMS subject classifications: 94D05; 03F55; 62F07; 90C08; 62L86.

1. Introduction

Transportation problems, a foundational aspect of operations research, are pivotal in optimizing the distribution of goods among diverse sources and destinations. Traditional solutions to transportation problems often assume deterministic parameters, overlooking the inherent uncertainties in real-world scenarios.Sharma and Taha gave the basic concepts of Operations research ((12; 16)). In recent years, the integration of fuzzy set theory has provided a robust frameworkfor capturing and modeling this uncertainty ((19)). This introduction aims to explore fundamental concepts in transportation problems, fuzzy sets, and their multifaceted applications, drawing insights from various relevant research papers.

The seminal work of Zadeh in 1965 introduced fuzzy set theory as a means tohandle imprecise and uncertain information ((19)). Fuzzy sets generalize classical set theory, allowing elements to belong to a set to varying degrees, mirroring the inherent ambiguity in human reasoning. Transportation problems involvedecision-making in uncertain environments, where the utilization of fuzzy sets has significantly enhanced traditional optimization techniques. Atanassov. K.T introduced intuitionistic Fuzzy Sets, a substantial extension within Fuzzy sets and systems ((2)). burillo et. al discussed with intuitionistic Fuzzy Sets ((4)).

Annie M. S Christi has made substantial contributions by applying fuzzy sets to transportation problems. Her research explores the use of pentagonal intuitionistic fuzzy numbers to solve transportation problems, employing ranking techniquesand Russell's method ((5; 6)). Syamala et. al discussed some Fuzzy and Intuitionistic Fuzzy concepts ((1; 13; 14; 15)). By incorporating intuitionistic fuzzy numbers, which extend traditional fuzzy sets by considering both the degree of membership and non-membership, the model becomes more adept at handling imprecise data.

Building upon Christi's work, Mohideen et al. expanded the scope to octagonfuzzy numbers and introduced α-cut and ranking techniques to solve fuzzy transportation problems ((11)). This research further exemplifies the adaptability of fuzzy set theory to different geometric configurations, demonstrating its applicability in diverse scenarios.

Felix et. al used octagonal fuzzy numbers ((7)). The exploration of generalized hexagonal and octagonal fuzzy numbers in transportation problems is presentedby Ghadle and Pathade ((8; 9)). Beaula et. al discussed also fuzzy transportation problems ((3)).Their work introduces a ranking method as a means to find optimal solutions, showcasing the versatility of fuzzy sets in addressing various problem formulations. The incorporation of generalized fuzzy numbers allows fora broader representation of uncertainty, contributing to a more realistic modelingof transportation systems.

Thamaraiselvi and Santhi investigate the application of hexagonal intuitionistic fuzzy numbers in solving transportation problems ((17; 18)). Their work emphasizes the importance of intuitionistic fuzzy sets in capturing uncertainty and vagueness, providing a more nuanced approach to modeling real-world decision-making processes. Li used ratio ranking method ((10)).

Beyond transportation-specific applications, fuzzy set theory, as introduced by Zadeh and further elaborated by researchers like Zimmermann ((20)), has foundapplications in various disciplines. Fuzzy set theory proves beneficial in situations where available information is inherently imprecise or vague. In addressingtransportation problems, effective solutions often hinge on precise specificationsof costs, supply, and demand. Intuitionistic fuzzy sets emerge as a potent tool for handling the inherent vagueness in such scenarios.

The subsequent sections of this paper delve into exploring the application of decagonal intuitive fuzzy numbers in transportation problem formulations. Thesesections employ a proposed ranking method to determine optimal solutions across various methodological approaches, conducting comparative analyses to identify favorable solutions. While the initial sections focus on balanced decagonal intuitionistic Fuzzy Transportation Problems, the concluding section addresses unbalanced transportation problems, particularly involving supply and demand, culminating the paper's investigation.

2. Preliminaries

Definition 2.1. Let X be a nonempty set. A fuzzy set $\tilde{A}$ of X is defined as

$\tilde{A} = \{< x, \mu_{\tilde{A}}(x) > / x \in \boldsymbol{X}\}$. Where $\mu_{\tilde{A}}(x)$ is called membership function, which maps each element of x to [0,1].

Definition 2.2. Given a non-empty set $\boldsymbol{X}$, an Intuitionistic Fuzzy Set $\tilde{A}^{\boldsymbol{I}}$ defined over $\boldsymbol{X}$ is described by its elements in the form $< x, \mu_{\tilde{A}}I(x), \theta_{\tilde{A}}I(x) >$, where x belongs to $\boldsymbol{X}$. Here, $\mu_{\tilde{A}}I(x)$ and $\theta_{\tilde{A}}I(x)$ denote the membership and non- membership functions, respectively. Both functions, $\mu_{\tilde{A}}I$ and $\theta_{\tilde{A}}I$, are mappings from $\boldsymbol{X}$ to the interval [0, 1] and must adhere to the constraint $0 \leq \mu_{\tilde{A}}I(x) + \theta_{\tilde{A}}I(x) \leq 1$ for every $x \in \boldsymbol{X}$.

Definition 2.3. A fuzzy number, represents a range of possible values rather thana single precise value. It is characterized by a connected set of values on thereal line R, where each value within this set is assigned a weight, known as its membership function, ranging from 0 to 1.

For a fuzzy number to be valid:

1. There must be at least one value $x \in$ R for which the membership function

$\mu_{\tilde{A}}(x)$ equals

2. The membership function $\mu_{\tilde{A}}(x)$ must exhibit piecewise continuity, ensuring the smooth transition between different values within the set.

Definition 2.4. An Intuitionistic Fuzzy Subset $\tilde{A}^{\boldsymbol{I}}$ defined as $< x, \mu_{\tilde{A}}I(x), \theta_{\tilde{A}}I(x) > x \quad |\boldsymbol{X} \in$

on the real line R is termed an Intuitionistic Fuzzy Number if it satisfies the following conditions, there exists a value m in the real line $\mathbf{R}$ such that $\mu_{\tilde{A}}I(\mathrm{m}) = 1$, and simultaneously, $\theta_{\tilde{A}}I(\mathrm{m}) = 0$.

1. $\mu_{\tilde{A}}I$ is a continuous function from $\mathbf{R} \to [0,1]$

2. $0 \leq \mu_{\tilde{A}}I(x) + \theta_{\tilde{A}}I(x) \leq 1$ for all $x \in \boldsymbol{X}$.

The membership and non-membership functions of $\tilde{A}^{\boldsymbol{I}}$ are in the following form

$$\mu_{\tilde{A}}I(x) = \begin{cases} 0 & for \ -\infty < x < \vartheta 1 \\ f(x) & for \ \vartheta 1 \leq x < \vartheta 2 \\ 1 & for \ x = \vartheta 2 \\ g(x) & for \vartheta 2 \leq x < \vartheta 3 \\ 0 & for \vartheta 3 \leq x < \infty \end{cases}$$

$$\theta_{A^{\sim}}I(x) = \begin{cases} 1 & for \ -\infty < x < \vartheta 1 \\ f'(x) & for \vartheta 1\, 0 \leq x < \vartheta 2 \\ 0 & for x = \vartheta 2 \\ g'(x) & for \vartheta 2 \leq x < \vartheta 3\, 0 \\ 1 & for \vartheta 3\, 0 \leq x < \infty \end{cases}$$

Where, ϑ1, ϑ2, ϑ3, ϑ1', ϑ2', and ϑ3' are real numbers. f, f', g, g' are functions from R → [0, 1]. f and g 0 are strictly increasing functions and g and f 0 are strictly decreasing functions with the conditions 0 ≤ f(x) + f '(x) ≤ 1 and 0 ≤ g(x) + g'(x) ≤ 1.

Definition 2.5. A Decagonal Fuzzy Number, is represented by D˜ = (ϑ1, ϑ2, ϑ3, ϑ4, ϑ5, ϑ6, ϑ7, ϑ8, ϑ9, ϑ10). Here, ϑ1, ϑ2, ϑ3, ϑ4, ϑ5, ϑ6, ϑ7, ϑ8, ϑ9, and ϑ10 are real numbers. The membership function μD˜ (x) characterizing this Generalized Decagonal Fuzzy Number is defined as follows,

$$\mu_{\tilde{D}}\mathrm{I}(x) = \begin{cases} \mathrm{p}\left(\frac{\mathrm{x}-\vartheta1}{\vartheta2-\vartheta1}\right) for\vartheta1 < x < \vartheta2 \\ p+(q-p)\left(\frac{\mathrm{x}-\vartheta2}{\vartheta3-\vartheta2}\right) for\vartheta2 \le x < \vartheta3 \\ q+(r-q)\left(\frac{\mathrm{x}-\vartheta3}{\vartheta4-\vartheta3}\right) for\vartheta3 \le x < \vartheta4 \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta4}{\vartheta5-\vartheta4}\right) for\vartheta4 \le x < \vartheta5 \\ 1 \quad for\vartheta5 \le \mathrm{x} < \vartheta6 \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta6}{\vartheta7-\vartheta6}\right) for\vartheta6 \le x < \vartheta7 \\ n+(1-n)\left(\frac{\vartheta7-\mathrm{x}}{\vartheta8-\vartheta7}\right) for\vartheta7 < x \le \vartheta8 \\ m+(n-m)\left(\frac{\vartheta8-\mathrm{x}}{\vartheta9-\vartheta8}\right) for\vartheta8 < x \le \vartheta9 \\ m\left(\frac{\vartheta9-\mathrm{x}}{\vartheta10-\vartheta9}\right) for\vartheta9 < x < \vartheta10 \\ 0 \quad for\, \mathrm{x} \le \vartheta1, \mathrm{x} \le \vartheta10 \end{cases}$$

where $0 < p < q < r < n < m1$.

Definition 2.6. A Decagonal Intuitionistic Fuzzy Number is specified by

$\tilde{A}_D^I$ = (ϑ1, ϑ2, ϑ3, ϑ4, ϑ5, ϑ6, ϑ7, ϑ8, ϑ9, ϑ10),

$(\vartheta1', \vartheta2', \vartheta3', \vartheta4', \vartheta5', \vartheta6', \vartheta7', \vartheta8', \vartheta9', \vartheta10')$ Where $\vartheta1, \vartheta2, \vartheta3, \vartheta4, \vartheta5, \vartheta6, \vartheta7, \vartheta8, \vartheta9, \vartheta10$,

$\vartheta1', \vartheta2', \vartheta3', \vartheta4', \vartheta5', \vartheta6', \vartheta7', \vartheta8', \vartheta9'\ and\ \vartheta10'$ are real numbers such that$\vartheta1' \le \vartheta1 \le \vartheta2' \le \vartheta2 \le \vartheta3' \le \vartheta3 \le \vartheta4' \le \vartheta4 \le \vartheta5 \le \vartheta6 \le \vartheta7' \le \vartheta7 \le \vartheta8' \le \vartheta8 \le \vartheta9' \le \vartheta9 \le \vartheta10' \le \vartheta10$ and its membership and non membership functions are given below $\mu_{\tilde{D}^I}(x)$μ is given below,

$$\mu_{\tilde{D}}\mathrm{I}(x)=\begin{cases} \mathrm{p}\left(\frac{\mathrm{x}-\vartheta 1}{\vartheta 2-\vartheta 1}\right) for \vartheta 1 < x < \vartheta 2 \\ p+(q-p)\left(\frac{\mathrm{x}-\vartheta 2}{\vartheta 3-\vartheta 2}\right) for \vartheta 2 \le x < \vartheta 3 \\ q+(r-q)\left(\frac{\mathrm{x}-\vartheta 3}{\vartheta 4-\vartheta 3}\right) for \vartheta 3 \le x < \vartheta 4 \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta 4}{\vartheta 5-\vartheta 4}\right) for \vartheta 4 \le x < \vartheta 5 \\ 1 \quad for \vartheta 5 \le \mathrm{x} < \vartheta 6 \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta 6}{\vartheta 7-\vartheta 6}\right) for \vartheta 6 \le x < \vartheta 7 \\ n+(1-n)\left(\frac{\vartheta 7-\mathrm{x}}{\vartheta 8-\vartheta 7}\right) for \vartheta 7 < x \le \vartheta 8 \\ m+(n-m)\left(\frac{\vartheta 8-\mathrm{x}}{\vartheta 9-\vartheta 8}\right) for \vartheta 8 < x \le \vartheta 9 \\ m\left(\frac{\vartheta 9-\mathrm{x}}{\vartheta 10-\vartheta 9}\right) for \vartheta 9 < x < \vartheta 10 \\ 0 \quad for\, \mathrm{x} \le \vartheta 1, \mathrm{x} \le \vartheta 10 \end{cases}$$

$$\theta_{\tilde{D}}\mathrm{I}(x)=\begin{cases} \mathrm{p}\left(\frac{\mathrm{x}-\vartheta 1'}{\vartheta 2'-\vartheta 1'}\right) for\, \vartheta 1' < x < \vartheta 2' \\ p+(q-p)\left(\frac{\mathrm{x}-\vartheta 2'}{\vartheta 3'-\vartheta 2'}\right) for \vartheta 2' \le x < \vartheta 3' \\ q+(r-q)\left(\frac{\mathrm{x}-\vartheta 3'}{\vartheta 4'-\vartheta 3'}\right) for \vartheta 3' \le x < \vartheta 4' \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta 4'}{\vartheta 5'-\vartheta 4'}\right) for \vartheta 4' \le x < \vartheta 5' \\ 0 \quad for\, \vartheta 5 \le \mathrm{x} < \vartheta 6 \\ r+(1-r)\left(\frac{\mathrm{x}-\vartheta 6}{\vartheta 7'-\vartheta 6}\right) for \vartheta 6 \le x < \vartheta 7' \\ n+(1-n)\left(\frac{\vartheta 7'-\mathrm{x}}{\vartheta 8'-\vartheta 7'}\right) \quad for\, \vartheta 7' < x \le \vartheta 8' \\ m+(n-m)\left(\frac{\vartheta 8'-\mathrm{x}}{\vartheta 9'-\vartheta 8'}\right) \quad for\, \vartheta 8' < x \le \vartheta 9' \\ m\left(\frac{\vartheta 9'-\mathrm{x}}{\vartheta 10'-\vartheta 9'}\right) \quad for\, \vartheta 9' < x < \vartheta 10' \\ 1 \quad for\, \mathrm{x} \le \vartheta 1', \mathrm{x} \le \vartheta 10' \end{cases}$$

3. Proposed New Ordering of Decagonal Intuitionistic Fuzzy Numbers

The ordering function of Decagonal Intuitionistic Fuzzy Number (OIFN)

A'_{OC}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}) (a_1^r, a_2^r, a_3^r, a_4^r, a_5^r, a_6^r, a_7^r, A_8^r, a_9^r, a_{10}^r, a_{11}^r, a_{12}^r)

defined as maps the set of all Fuzzy numbers to a set of real numbers defined as

$$R[\bar{A}_{OC}]=\text{Max}[Mag_{\mu}(A_{OC}),Mag_{\theta}(A_{OC})]\ Where$$

$$Mag_{\mu}(\bar{A}_{OC})=\frac{1a_1+2a_2+3a_3+4a_4+5a_5+5a_6+4a_7+3a_8+2a_9+1a_{10}}{30}$$

$$Mag_{\theta}(\bar{A}_{OC})=\frac{1a'^{1}+2a'^{2}+3a'^{3}+4a'^{4}+5a'^{5}+5a'^{6}+4a'^{7}+3a'^{8}+2a'^{9}+1a'^{10}}{30}$$

Solving Transportation Problem with Supply and Demand Values are Decagonal Intuitionistic Fuzzy Numbers using New Proposed Ordering Method

3.1 Numerical Example

ConsiderSuppliesandDemandsaredecagonalIntuitionisticFuzzyNumber.

	B_1	B_2	B_3	*Supply*
A_1	12.5	11.5	9.5	(2,4,5,6,7,8,9,1 (6,7,8,9,10,11,12,13,14,15)
A_2	10.5	6.5	13.5	(8,9,10,11,12,13,14,15,16,17) (4,6,7,8,9,10,12,13,15,16)
A_3	11.5	9.5	8.5	(4,6,7,8,9,10,11,12,13,14) (1,2,3,4,5,6,7,8,9,10)
Demand	(4,5,6,7,8,9,10,11,12,13) (6,7,8,9,10,11,12,13,14,15)	(1,2,3,5,6,7,8,12,14,15) (0,1,2,3,4,7,9,10,12,13)	(3,4,5,6,7,8,9,10,11,12) (8,9,10,11,12,13,14,15,16,17)	

$\Sigma Demand=\Sigma Supply$

The problem is a balanced transportation problem. Using the proposed algorithm, the solution of the problem is as follows.

Applying accuracy function on decagonal Intuitionistic Fuzzy Number

[(1,2,3,5,6,7,8,10)(3,6,7,8,9,10,12,13)], *we have*

$R(A_{OC})=10.5$

Similarly applying for all the values, we have the following table after ordering

3.2 Applying VAM Method

	B_1	B_2	B_3	*Supply*
A_1	12.5	11.5	9.5 [10.5]	10.5
A_2	10.5 [7.25]	6.5 [7.25]	13.5	12.5
A_3	11.5 [6.25]	9.5	8.5 [5]	6.25
Demand	11.5	7.25	13.5	

Since the number of occupied cell m+n−1 = 5 and are also independent. There exist non-negative basic feasible solutions.

The initial transportation cost is

[(10.5×9.5)+(7.25×10.5)+(7.25×6.5)+(6.25×11.5)+(5×8.5)]=337.38.

3.3 Applying MODI Method

Table corresponding to optimal solution is

	B_1	B_2	B_3	*Supply*
A_1	12.5	11.5	9.5 [10.5]	10.5
A_2	10.5 [7.25]	6.5 [7.25]	13.5	12.5
A_3	11.5 [6.25]	9.5	8.5 [5]	9.25
Demand	11.5	7.25	13.5	

Since all $d_{ij} \geq 0$ the solution in optimum and unique. The solution is given by

$x_{13}=10.5, x_{21}=7.25, x_{22}=7.25, x_{31}=6.25, x_{33}=5$. *The optimal solution is*

=[(10.5×9.5)+(7.25×10.5)+(7.25×6.5)+(6.25 ×11.5)+(5×8.5)]

=337.38

4. Solving Transportation Problem with Cost Values are Decagonal Intuitionistic Fuzzy Numbers using New Proposed Ordering Method

4.1 Numerical Example

Consider Costs are Decagonal Intuitionistic Fuzzy Number.

	B_1	B_2	B_3	*Supply*
A_1	(8,9,10,11,12,13,14,15,16,17) (6,7,8,9,10,11,12,13,14,15)	(6,7,8,9,10,11,12,13,14,15) (1,2,3,4,5,6,7,8,9,10)	(5,6,7,8,9,10,11,12,13,14,) (1,2,3,4,5,6,7,10,11,12)	13.5
A_2	(5,6,7,8,9,10,11,12,13,14) (1,2,3,4,5,6,7,8,9,10)	(1,2,3,4,5,6,7,8,9,10) (0,1,2,3,4,5,6,7,8,9)	*(9,10,11,12,13,14,15,16,17,18)* (3,4,5,6,7,8,9,10,11,12)	11.5
A_3	*(6,7,8,9,10,11,12,13,14,15)* (3,4,5,6,7,8,9,10,11,12)	(4,5,6,7,8,9,10,11,13,15) (1,2,3,5,6,7,8,10,11,13)	(3,4,5,6,7,8,9,10,11,12) (1,2,3,4,5,6,7,8,9,10)	7.25
Demand	12.5	9.25	10.5	

$\Sigma\ Demand = \Sigma Supply$

The problem is a balanced transportation problem. Using the proposed algorithm, the solution of the problem is as follows.

Applying accuracy function on decagonal Intuitionistic Fuzzy Number

(8,9,10,11,12,13,14,15,16,17)(6,7,8,9,10,12,13,14,15),*we have*

$R(A_{OC})=12.5$

Similarly applying for all the values, we have the following table after ranking Reduced Table

	B_1	B_2	B_3	*Supply*
A_1	12.5	10.5	9.5	13.5
A_2	10.5	6.5	13.5	11.5
A_3	11.5	9.5	8.5	7.25
Demand	12.5	9.25	10.5	

4.2 Applying VAM Method

Table corresponding to initial basic feasible solution is

	B_1	B_2	B_3	*Supply*
A_1	12.5 [5]	10.5	9.5 [10.5]	13.5
A_2	10.5 [4.25]	6.5 [9.25]	13.5	11.5
A_3	11.5 [7.25]	9.5	8.5	7.25
Demand	12.5	9.25	10.5	

Since the number of occupied cell m +n-1 = 5 and are also independent. There exist non-negative basic feasible solutions.

The initial transportation cost is

[(5×12.5)+(9.5×10.5)+(4.25×10.5)+(9.25×6.5)+(7.25×11.5)]=350.38

4.3 Applying MODI Method

Table corresponding to optimal solution is

	B_1	B_2	B_3	*Supply*
A_1	12.5 [5]	10.5	9.5 [10.5]	13.5
A_2	10.5 [4.25]	6.5 [9.25]	13.5	11.5
A_3	11.5 [7.25]	9.5	8.5	7.25
Demand	12.5	9.25	10.5	

Since all $d_{ij} \geq 0$ *the solution in optimum and unique. The solution is given* by $x_{11}=5$, $x_{13}=10.5$, $x_{21}=4.25$, $x_{22}=9.25$, $x_{31}=7.25$. *The optimal solution is*

[(5×12.5)+(9.5×10.5)+(4.25×10.5)+(9.25×6.5)+(7.25×11.5)]=350.38.

5. Solving Transportation Problem with Cost, Sup- ply and Demand Values are Decagonal Intuitionistic Fuzzy Numbers using New Proposed Ordering Method

5.1 Numerical Example

Consider Costs, Supplies and Demands are Decagonal Intuitionistic Fuzzy Number.

	$B1$	$B2$	$B3$	*Supply*
$A1$	(2,3,4,5,6,7,8,9,10,11) (1,2,3,4,5,6,7,8,9,10)	(4,5,6,7,8,9,10,11,12,12) (1,2,3,4,5,6,7,8,9,10)	*(6,7,8,9,10,11,12,13,14,15)* (4,5,6,7,8,9,10,11,12,13)	(3,4,5,6,7,8,9,10,11,12) *(9,10,11,12,13,14,15,16,17,18)*
$A2$	(5, 6, 7, 8, 9, 10, 11, 12, 13, 14) (1,2,3,4,5,6,7,10,11,12)	(9,10, 11, 12, 13, 14,15, 16, 17, 18) (4,5,6,7,8,9,10,11,12,13)	(6,7, 8, 9, 10, 12, 13, 14, 15, 16) (2,3,4,5,6,7,S,9)	(3,4,5,6,7,8,9,10,11,12) (6,7, 8, 9, 10, 11, 12, 13, 14, 15)
$A3$	(5, 6, 7, 8, 9, 10, 11, 12, 13, 14) (1,2,3,4,5,6,7,8,9,10)	(7, 8, 9, 10,11, 12, 13,14) (6,7, 8, 9, 10, 12, 13, 14, 15, 16)	(5, 6, 7, 8, 9, 10, 11, 12, 13, 14) *(1,2,3,5,6,7,8,10,11,12)*	*(1,2,3,5,6,7,8,13,14,15)* (1,2,3,4,7,9,10,11,12,13)
Demand	(7,8, 9, 10, 11, 12, 13, 14, 15, 16) (3, 6, 7, 8, 9, 10, 12, 13, 14, 15)	(2,4,6,7,8,9,10,11,13,15) (1,2,3,4,5,6,7,10,11,13)	*(1,2,3,5,6,7,8,10,11,12)* (4,5,6,7,8,9,10,11,12,13)	

5.2 Applying VAM Method

Table corresponding to initial basic feasible solution is

	B_1	B_2	B_3	*Supply*
A_1	[6.25]6.5	[9.25]8.5	11.5	13.5
A_2	[8.25]9.5	13.5	[5.25]11.5	11.5
A_3	10.5	12.5	[7.25]9.5	7.25
Demand	12.5	9.25	10.5	28.25

Since the number of occupied cell m+n-1=5 and are also independent. There exist non-negative basic feasible solutions.

The initial transportation cost is

$[(6.25 \times 6.5)+(9.25\times 8.5)+(8.25\times 9.5)+(5.25\times 10.5)+(7.25\times 9.5)]=$

321.63.

5.3 Applying MODI Method

Table corresponding to optimal solution is

	B_1	B_2	B_3	*Supply*
A_1	[6.25]6.5	[9.25]8.5	(6)11.5	13.5
A_2	[8.25]9.5	(4)13.5	[5.25]10.5	11.5
A_3	(4)10.5	(2.5)12.5	[7.25]9.5	7.25
Demand	12.5	9.25	10.5	28.25

Since all $d_{ij}\geq 0$ the solution in optimum and unique.

The solution is given by x_{11}=4.25,

$x_{12}=9.25, x_{21}=8.25, x_{23}=5.25, x_{33}=7.25$

The optimal solution is

=[(6.25×6.5)+(9.25×8.5)+(8.25×9.5)+(5.25×10.5)+(7.25×9.5)]

=321.63.

6. Solving Unbalanced Transportation Problem using Decagonal Intuitionistic Fuzzy Numbers

6.1 Numerical Example

	Dl	*D2*	*D3*	*Supply*
Sl	(2,3,4,5,6,7,8,9,10,11) (4,5,6,7,8,9,10,11,12,23)	(3,4,5,6,7,8,9,10,11,12) (1,2,3,4,5,6,7,8,9,10)	(6,7, 8, 9, 10, 11, 12, 13, 14, 15) (3,4,5,6,7,8,9,10,11,12)	(3,4,5,6,7,8,9,10,11,12) (8,9, 10, 11, 12, 13, 14, 15, 16,17)
S2	(4,5,6,7,8,9,10,11,12,13) (1,2,3,4,5,6,7,10,11,12)	(8,9, 10, 11, 12, 13, 14, 15, 16,17)1 (3,4,5,6,7,8,9,10,11,12)	(3, 6, 7, 8, 9, 10, 12, 13, 14, 15) (2,3,4,5,6,7,8,9,10,11)	(3,4,5,6,7,8,9,10,11,12) (6,7, 8, 9, 10, 11, 12, 13, 14, 15
S3	(5, 6, 7, 8, 9, 10, 11, 12, 13, 14) (1,2,3,4,5,6,7,8,9,10)	(7,8, 9, 10, 11, 12, 13, 14, 15, 16) (3, 6, 7, 8, 9, 10, 12, 13, 14, 15)	(4,5,6,7,8,9,10,11,12,13) (1,2,3,5,6,7,8,10,11,12)	(1,2,3,5,6,7,8,10,11,12) (1,2,3,4,5,6,7,8,9,10)

Demand	(1,2,3,4,5,6,7,9,10,11) (3,4,5,6,7,8,10,12,13,14)	(4,5,6,7,8,9,10,11,12,13) (1,2,3,4,5,6,7,8,9,10)	(1,2,3,4,5,6,7,8,9,10) (1,2,3,4,5,6,7,8,9,10)	

$\Sigma Demand \neq \Sigma Supply$

The problem is unbalanced transportation problem, so convert the problem

Into balanced transportation problem by introducing a dummy column

	Dl	*D2*	*D3*	*Dummy*	*Supply*
Sl	(2, 3, 4, 5, 6, 7,8, 9, 10, 11) (4, 5, 6, 7, 8, 9, 10, 11, 12, 23)	(3, 4, 5, 6, 7, 8, 9, 10,11, 12) (1,2,3,4,5,6,7,8,9,10)	(6,7, 8, 9, 10, 11, 12, 13, 14, 15) (3, 4, 5, 6, 7, 8, 9, 10,11, 12)	(0,0,0,0,0,0,0,0,0) (0,0,0,0,0,0,0,0,0)	(3, 4, 5, 6, 7, 8, 9, 10,11, 12) (8,9, 10, 11, 12, 13, 14,15, 16, 17)
S2	(4, 5, 6, 7, 8, 9, 10, 11, 12, 13) (1, 2, 3, 4, 5, 6, 7, 10,11, 12)	(8,9, 10, 11, 12, 13, 14,15, 16, 17)1 (3, 4, 5, 6, 7, 8, 9, 10,11, 12)	(3, 6, 7, 8, 9, 10, 12, 13, 14, 15) (2, 3, 4, 5, 6, 7,8, 9, 10, 11)	(0,0,0,0,0,0,0,0,0) (0,0,0,0,0,0,0,0,0)	(3, 4, 5, 6, 7, 8, 9, 10,11, 12) (6,7, 8, 9, 10, 11, 12, 13, 14, 15
S3	(5, 6, 7, 8, 9, 10, 11, 12, 13, 14) (1,2,3,4,5,6,7,8,9,10)	(7,8, 9, 10, 11, 12, 13, 14, 15, 16) (3, 6, 7, 8, 9, 10, 12, 13, 14, 15)	(4, 5, 6, 7, 8, 9, 10, 11, 12, 13) (1, 2, 3, 5, 6, 7, 8, 10,11, 12)	(0,0,0,0,0,0,0,0,0) (0,0,0,0,0,0,0,0,0)	(1, 2, 3, 5, 6, 7, 8, 10,11, 12) (1,2,3,4,5,6,7,8,9,10)
Demand	(1, 2, 3, 4, 5, 6,7, 9, 10, 11) (3, 4, 5, 6, 7, 8, 10, 12, 13, 14)	(4, 5, 6, 7, 8, 9, 10, 11, 12, 13) (1,2,3,4,5,6,7,8,9,10)	(1,2,3,4,5,6,7,8,9,10) (1,2,3,4,5,6,7,8,9,10)	(1, 2, 3, 5, 6, 7, 8, 10,11, 12) (4, 5, 6, 7, 8, 9, 10, 11, 13, 15)	

$\Sigma Demand = \Sigma Supply$

Using the proposed algorithm, the solution of the problem is as follows. Ap- plying accuracy function on Decagonal Intuitionistic Fuzzy Number

(2,3,4,5,6,7,8,9,10,11)(4,5,6,7,8,9,10,11,12,23),*wehave*

$R(A_{OC})=8.75$

Similarly applying for all the values, we have the following table after ordering

Reduced Table:

	B_1	B_2	B_3	*Dummy*	*Supply*
A_1	6.5	8.5	11.5	0	13.5
A_2	9.5	13.5	10.5	0	11.5
A_3	10.5	12.5	9.5	0	7.25
Demand	8.75	9.5	5.5	10.5	

6.2 Applying VAM Method

Table corresponding to initial basic feasible solution is

	B_1	B_2	B_3	*Dummy*	*Supply*
A_1	[6]6.5	[9.5]8.5	11.5	0	13.5
A_2	[4.75]9.5	13.5	[5.5]10.5	[5.25]0	11.5
A_3	10.5	12.5	9.5	[7.25]0	7.25
Demand	8.75	9.5	5.5	10.5	

Since the number of occupied cell m+n−1 = 6 and are also independent. There exists a non-negative basic feasible solution.

The initial transportation cost is

[(6×6.5)+(9.5×8.5)+(4.75×9.5)+(5.5×10.5)+(5.25×0)+(7.25×0)]= 222.63.

6.3 Applying MODI Method

Table corresponding to optimal solution is	B_1	B_2	B_3	*Dummy*	*Supply*
A_1	[6]6.5	[9.5]8.5	(6)11.5	(5)0	13.5
A_2	[4.75]9.5	(4)13.5	[5.5]10.5↓	[5.25]←0	11.5
A_3	(3)10.5	(3)12.5	(−1)9.5→	[7.25]0↑	7.25
Demand	8.75	9.5	5.5	10.5	

Step−5: Again, find sign of each d_{ij}, the values are all positive (d_{ij}>0) then current basic feasible solution is optimal.

	B_1	B_2	B_3	*Dummy*	*Supply*
A_1	[6]6.5	[9.5]8.5	11.5	0	13.5
A_2	[4.75]9.5	13.5	10.5	[8.75]0	11.5
A_3	10.5	12.5	[5.5]9.5	[10.75]0	7.25
Demand	8.75	9.5	5.5	10.5	

Since all d_{ij}≥0 the solution in optimum and unique.

The solution is given by $x_{11} = 6, x_{12} = 9.5, x_{21} = 4.75, x_{24} = 8.75, x_{33} = 5.5, x_{34}$=10.75.

The optimal solution is = [(6×6.5)+(9.5×8.5)+(4.75×9.5)+(8.75×0) +(5.75×9.5)+(10.75×0)] = 219.5.

7. Conclusion

In this article, a fuzzy ordering method has been employed to simplify comprehension and approach a closer-to-optimal solution. The integration of unbalanced Decagonal Intuitionistic Fuzzy numbers for supply and demand values was introduced. Leveraging the Proposed Ordering Method facilitated the reduction of crisp values, effectively transforming the initially balanced transportation problem towards deriving an optimal solution. Ultimately, the findings indicate that Octagonal Intuitionistic Fuzzy numbers tend to minimize costs.

REFERENCES

[1] Amalorpava, J. J., Benedict, M. L., Dhanalakshmi, k., Syamala, P. (2017). Harmonic Index of Graphs with More Than One Cut-vertex. Ars Combina-toria, 135, 283-298.

[2] Atanassov, K. T. (1989). More on Intuitionistic Fuzzy sets. Fuzzy Sets and Systems, 33, 37-46.

[3] Beaula-M. Priyadharshini, T. (2015). A New Algorithm for Finding a FuzzyOptimal Solution for Intuitionistic Fuzzy Transportation Problems. Interna-tional Journal of Applications of Fuzzy Sets and Artificial Intelligence, 5, 183-192.

[4] Burillo, P., Bustince, H., Mohedano, V. (1994). Some definition of intuition- istic fuzzy number. Fuzzy Sets Based.

[5] Christi, A. M. S. (2016). Transportation Problem with Pentagonal Intuition-istic Fuzzy Numbers Solved Using Ranking Technique and Russell's Method. Journal of Engineering Research and Applications, 6(2), 82-86.

[6] Christi, A. M. S. (2017). Solutions of Fuzzy Transportation Problem Using Best Candidates Method and Different Ranking Techniques. World Academyof Science, Engineering and Technology International Journal of Mathemat- ical and Computational Sciences, 11(4).

[7] Felix, A. and Victor Devadoss, A. A New Decagonal Fuzzy Number under Uncertain Linguistic Environment.

International Journal of Mathematics And its Applications, vol.3, 89-97

[8] Ghadle, K. P., Pathade, P. A. (2017). Solving Transportation Problem withGeneralized Hexagonal and Generalized Octagonal Fuzzy Numbers by Ranking Method. Global Journal of Pure and Applied Mathematics, 13(9),6367-6376.

[9] Ghadle, K. P., Ingle, S. M., Hamoud, A. A. (2018). Optimal Solution of Fuzzy Transshipment Problem Using Generalized Hexagonal Fuzzy Numbers. In-ternational Journal of Engineering & Technology, 7(4.10), 558-561.

[10] Li, D. F. (2010). A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems. Computer and Mathemat-ics with Applications, 60, 1557-1570.

[11] Mohideen, I., Devi, K. P., Durga, M. D. (2016). Fuzzy Transportation Prob-lem of Octagon Fuzzy Numbers with α-Cut and Ranking Technique. Journalof Computer-JoC, 1(2), 60-67.

[12] Sharma, J. K. (2005). Operations Research-Theory and Applications. Macmillan India LTD, New Delhi.

[13] Syamala, P., Balasubramaniyan, K. R. (2019). Perfectly Regular and Per- fectly Edge-Regular Intuitionistic Fuzzy Graphs. International Journal of Engineering and Advanced Technology, 9(1), 6348-6352.

[14] Syamala, P., Balasubramaniyan, K. R. (2020). On Lexicographic Productsof Two Intuitionistic Fuzzy Graphs. Journal of Mathematical and Computa-tional Science, 10(1), 136-149.

[15] Syamala, P., Ramesh, R., Seenivasan, M., Singaravel, R. (2023), 3D Based CT Scan Retrial Queueing Models by Fuzzy Ordering Approach. 2023 2nd International Conference on Electrical, Electronics, Informa- tion and Communication Technologies, ICEEICT 2023., IEEE, doi : https://doi.org/101109/iceeict56924.2023.

[16] Taha, H. A. (2004). Operations Research-Introduction. Prentice Hall of In-dia, New Delhi.

[17] Thamaraiselvi, A., Santhi, R. (2015). On Intuitionistic Fuzzy Transporta- tion Problem Using Hexagonal Intuitionistic Fuzzy Numbers. InternationalJournal of Fuzzy Logic Systems (IJFLS), 5(1).

[18] Thamaraiselvi, A., Santhi, R. (2015). Solving fuzzy transportation problem with generalized hexagonal fuzzy numbers. IOSR Journal of Mathematics, 11(5), 8-13.

[19] Zadeh, L. A. (1965). Fuzzy Sets. Information, 8, 338-353.

[20] Zimmermann, H. J. (2001). Fuzzy Set Theory and Its Applications (4th ed.).Kluwer Academic Publishers.