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Application of the G'/G Expansion Method for Solving New Form of Nonlinear Schrödinger Equation in Bi-Isotropic Fiber

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Abstract:

Bi-isotropic media (chiral and non-reciprocal) present an outstanding challenge for the scientific community. Their characteristics have facilitated the emergence of new and remarkable applications. In this paper, we focus on the novel effect of chirality, characterized through a newly proposed formalism, to highlight the nonlinear effect induced by the magnetization vector under the influence of a strong electric field. This research work is concerned with a new formulation of constitutive relations. We delve into the analysis and discussion of the family of solutions of the nonlinear Schrödinger equation, describing the pulse propagation in nonlinear bi-isotropic media, with a novel approach to constitutive equations. We apply the extended G'/G-expansion method with varying dispersion and nonlinearity to define certain families of solutions of the nonlinear Schrödinger equation in bi-isotropic (chiral and non-reciprocal) optical fibers. This clarification aids in understanding the propagation of light with two modes of propagation: a right circular polarized wave (RCP) and a left circular polarized wave (LCP), each having two different wave vectors in nonlinear bi-isotropic media. Various novel exact solutions of bi-isotropic optical solitons are reported in this study.

Introduction: The investigation of exact solutions for nonlinear partial differential equations (PDEs) holds significant importance in understanding nonlinear physical phenomena. Nonlinear waves manifest across various scientific domains, notably in optical fibers and solid-state physics. In recent years, several potent methodologies have emerged for identifying solitons and periodic wave solutions of nonlinear PDEs. These include the G'/G -expansion method [1-6], the new mapping method [9-10], the method of generalized projective Riccati equations [11-16], and the G'/G expansion method [17].

Consequently, an original mathematical approach is proposed to evaluate nonlinear effects in bi-isotropic optical fibers, stemming from magnetization under the influence of a strong electric field [19-20]. The extended G'/G-expansion method emerges as a potent technique for deriving solution families of the nonlinear Schrödinger equation in bi-isotropic optical fibers. This method employs a perturbation expansion in powers of the dimensionless parameter and is applicable for both weak and strong nonlinearities. It accommodates varying dispersion and nonlinearity, rendering it suitable for modeling a wide array of optical fibers.

Results and Conclusion: This investigate is concerned with a new formulation of constitutive relation linking to the magnetic effect, to understand rigorously the physical nature of biisotropic effects and to generalize the main macroscopic models. We inferred the nonlinear Schrodinger equation for a bi-isotropic medium term with a nonlinear term of magnetizing. In this article, the extended G'/G-expansion method is a

powerful technique for determining a family of solutions of the nonlinear Schrödinger equation in bi-isotropic optical fibers.

This method is based on the use of a perturbation expansion in powers of the dimensionless parameter, and it is valid for both weak and strong nonlinearities. The method allows for the inclusion of varying dispersion and nonlinearity, making it well-suited for modeling a wide range of optical fibres. Overall, the extended G'/G -expansion method is a valuable tool for understanding the dynamics of nonlinear optical systems, and it is expected to have a wide range of applications in the field of nonlinear optics.

Keywords: Nonlinear Bi-isotropic media , Schrödinger equation , Optical fiber ,The G'/G expansion method, chiratity, non-reciprocity.

1. Introduction

The investigation of exact solutions for nonlinear partial differential equations (PDEs) holds significant importance in understanding nonlinear physical phenomena. Nonlinear waves manifest across various scientific domains, notably in optical fibers and solid-state physics. In recent years, several potent methodologies have emerged for identifying solitons and periodic wave solutions of nonlinear PDEs. These include the G'/G -expansion method [1-6], the new mapping method [9-10], the method of generalized projective Riccati equations [11-16], and the G'/G -expansion method [17].

Conte and Musette [11] introduced an indirect approach to uncover solitary wave solutions of specific nonlinear PDEs expressible as polynomials in two elementary functions satisfying a projective Riccati equation [18]. This method has been successfully applied to numerous nonlinear PDEs, with resulting solitary wave solutions documented in [12-16]. The G'/G -expansion method exhibits broad applicability for tackling various other nonlinear evolution equations in mathematical physics.

In recent years, research endeavors have actively explored novel types of heterogeneous absorbent materials, notably bi-isotropic materials. Bi-isotropic materials comprise a random dispersion of inclusions within a polymeric or ceramic matrix. Lindman in 1920 and Pickering in 1945 specifically investigated the interaction of electromagnetic waves with a collection of randomly distributed metal helices of the same enantiomorph form [19-23]. They observed polarization plane rotation of electromagnetic waves post-interaction with the helices. In 1979, Jaggard, Mickelson, and Papas proposed a macroscopic model detailing the interaction of electromagnetic waves with chiral structures (bi-isotropic materials) [19-20].

The general bi-isotropic medium is chiral and non-reciprocal, involving a complex parameter in the general case. The effects of two parameters, nonlinear chirality, and non-reciprocity, are crucial for estimating non-linearity in bi-isotropic fibers. This paper examines a direct method to determine numerous solution families for the nonlinear Schrödinger equation in bi-isotropic optical fiber, alongside G'/G-expansion. This contributes to a deeper understanding of the interaction between electromagnetic waves and nonlinear bi-isotropic media, facilitating the design of potential applications in microwave and optical domains.

Consequently, an original mathematical approach is proposed to evaluate nonlinear effects in bi-isotropic optical fibers, stemming from magnetization under the influence of a strong electric field

[19-20]. The extended G'/G -expansion method emerges as a potent technique for deriving solution families of the nonlinear Schrödinger equation in bi-isotropic optical fibers. This method employs a perturbation expansion in powers of the dimensionless parameter and is applicable for both weak and strong nonlinearities. It accommodates varying dispersion and nonlinearity, rendering it suitable for modeling a wide array of optical fibers. The method effectively describes the propagation of optical pulses in bi-isotropic fibers and predicts their behavior under diverse conditions. Furthermore, it facilitates the study of soliton dynamics, including stability and interactions. Overall, the extended G'/G -expansion method stands as a valuable tool for comprehending the dynamics of nonlinear optical systems, with anticipated applications across the field of nonlinear optics [24].

2. Description of the (G'/G)-expansion method

Study a nonlinear PDE in the following form

$$P(u, u_z, u_t, u_{zz}, u_{tt}) = 0 \tag{1}$$

where $u = u(t, z)$ is an new function, P is a polynomial in $u = u(t, z)$, and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Let us now provide the main steps of the extended (G'/G)- expansion method .

Step 1. We usage the following conversion

$$u(z, t) = u(\xi); \xi = z - vt \tag{2}$$

to reduce Eq.(3) to the next nonlinear ODE:

$$K(u, u', u'', \dots) = 0 \tag{3}$$

where v is velocity of the propagation, K is a polynomial of $u(\xi)$ and its derivatives $u'(\xi)$, $u''(\xi)$, . . . where $u'(\xi) = \frac{du}{\partial \xi}$, $u''(\xi) = \frac{d^2u(\xi)}{d\xi^2}$,...

Step 2. We adopt that the solution of Eq.(1) has the form:

$$u(\xi) = a_0(z) + \sum_{i=1}^{N} a_i \left(\frac{G'}{G}\right)^i + b_i(z)\left(\frac{G'}{G}\right)^{i-1} \sqrt{1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2} \tag{4}$$

where $a_i = a_i(z), b_i = b_i(z)$ $(i = 1,2,..N)$ are functions of z to be determined. $G = G(\xi)$ satisfies the resulting second order ODE equation:

$$G''(\xi) + \mu G(\xi) = 0 \tag{5}$$

Where $G' = \frac{dG}{d\xi}, G'' = \frac{d^2G}{d\xi^2}$. We describe the degree of $u(\xi)$ as $D[u(\xi)] = M$, then

$$D[\frac{d^p u(\xi)}{d\xi^p}] = M + p \;;\; D[u^p \left(\frac{d^q u(\xi)}{d\xi^q}\right)^s] = Mp + (M + q)s \tag{6}$$

The parameter N can be start by balancing the highest order derivative term and nonlinear terms in Eq.(3). N is typically a positive integer. Substituting Eq.(1) along with Eq.(2) into Eq.(4), collecting all terms with the same power of $(\frac{G'(\xi)}{G(\xi)})^i (i = 0; 1; 2;...N)$, and setting each coefficients to zero, we get a system of algebraic equations for a_i and b_i.The solution of Eq.(5) are given as

3. $$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \sqrt{-\mu}\left(\frac{A_1 \sinh\sqrt{-\mu}\xi + A_2 \cosh\sqrt{-\mu}\xi}{A_1 \cosh\sqrt{-\mu}\xi + A_2 \sinh\sqrt{-\mu}\xi}\right); \mu < 0 \\ \sqrt{\mu}\left(\frac{A_1 \sin\sqrt{\mu}\xi + A_2 \cos\sqrt{\mu}\xi}{A_2 \sin\sqrt{\mu}\xi - A_1 \cos\sqrt{\mu}\xi}\right); \mu > 0 \\ \frac{A_2}{A_1 + A_2\xi}; \mu = 0 \end{cases} \tag{7}$$

which can be written in the following simplified form

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \sqrt{-\mu}\tanh(\sqrt{-\mu}\xi + \xi_0); \mu < 0, \tanh\xi_0 = \frac{A_1}{A_2} \ ; \left|\frac{A_2}{A_1}\right| > 1 \\ \sqrt{-\mu}\coth(\sqrt{-\mu}\xi + \xi_0); \mu < 0, \coth\xi_0 = \frac{A_1}{A_2} \ ; \left|\frac{A_2}{A_1}\right| < 1 \\ \sqrt{\mu}\cot(\sqrt{\mu}\xi + \xi_0); \mu > 0, \cos\xi_0 = -\frac{A_2}{A_1} \\ \frac{A_2}{A_1 + A_2\xi}; \mu = 0 \end{cases} \tag{8}$$

4. By solving the over-determined algebraic system with the help of Mathematica, we obtain the values of a_0; a_i and b_i. Substituting these results into Eq.(1), and combining with the solution of Eq.(5), we obtain some exact solutions of Eq.(1).

3. Application of the (G'/G)-expansion method in bi-isotropic fiber

The nonlinear Schrödinger equation (ESNL) is an equation that occurs in several fields of wave physics [1-24]. Among other things, it can be obtained from an asymptotic expansion of the Korteweg-de Vries (KdV) equation for weakly nonlinear wave packets [1-5]. The Schrödinger equation then follows from a first order approximation and gives the evolution of the wave packet envelope [1-5].

The characterization of the bi-isotropic nonlinear medium is achieved through an electromagnetic approach utilizing a novel formulation of constitutive relations [22]. This formulation enables the derivation of the nonlinear Schrödinger equation for a chiroptic fiber, obtained through a first-order approximation, providing insight into the evolution of the wave packet envelope.

This endeavor promises a deeper understanding of the interplay between electromagnetic waves and bi-isotropic nonlinear mediums, paving the way for potential applications in optics and microwaves, exemplified by our case study on chiroptic fibers.

Within this section, our focus lies on the analysis and modeling of light pulse propagation within a bi-isotropic fiber, leveraging our newly formulated constitutive equations [22]. Hence, we have employed the extended G'/G -expansion method as our chosen approach for solving the nonlinear Schrödinger equation, facilitating the acquisition of precise solutions for bi-isotropic fiber nonlinearities. As delineated in our formalism expounded in [19-23], the constitutive equations governing the bi-anisotropic nonlinear effects are delineated as follows:

$$\vec{D} = \bar{\varepsilon}\,\vec{E} + \bar{\zeta}_{EH}\,\vec{H} \tag{9}$$

$$\vec{B} = \bar{\mu}\vec{H} + \bar{\zeta}^{g}_{HE}\vec{E} \tag{10}$$

The medium effects are contained in the dyadic: $\bar{\varepsilon}$, $\bar{\mu}$, $\bar{\zeta}_{EH}$ and $\bar{\zeta}^{g}_{HE}$ due to anisotropy.

The bi-isotropic medium has a Kerr type nonlinearity characterized by [19- 23]:

$$\varepsilon_g = \varepsilon + \varepsilon_{Kerr}|E^2| \quad (11)$$

$$\zeta_{HE}^{g} = \zeta_{EH}^{*} + \zeta_{HE}^{Kerr}|E^2| \quad (12)$$

ζ_{EH}^{*} is the linear bi-isotropy coefficient , and the term $\zeta_{HE}^{Kerr}|E^2|$corrects the bi-isotropic coefficient with a quantity proportional to the field intensity [19-23].

The linear bi-isotropy factors are written as follows [19-23]:

$$\zeta_{EH} = \gamma - j\kappa \quad (13)$$

γ is the non-reciprocity parameter, and κ is the linear chirality parameter.

γ^{Kerr} is the nonlinear non-reciprocity parameter, and κ^{Kerr} is the nonlinear chirality parameter.

There exist three distinct cases for the biisotropic medium:

1. The chiral medium, which is reciprocal (purely imaginary), denoted by $\kappa \neq 0$ and $\gamma = 0$.
2. The Tellegen medium, which is non-reciprocal (purely real), indicated by $\kappa = 0$ and $\gamma \neq 0$.
3. The biisotropic medium, characterized by both chirality and non-reciprocity (complex numbers), with $\kappa \neq 0$ and $\gamma \neq 0$.

In this study, our focus centers on the third case. A biisotropic fiber refers to an optical fiber featuring a chiral core enveloped by an optical cladding. The core of the biisotropic fiber possesses a slightly higher refractive index compared to the sheath. This variation in refractive index induces total internal reflection of light within the chiral core, enabling the propagation of light with two distinct modes: a right circular polarized wave (RCP) and a left circular polarized wave (LCP), each exhibiting different wave vectors.

From Maxwell's equations, which serve as the cornerstone of electromagnetism and locally describe the evolution and properties of electric and magnetic fields, we specifically consider Maxwell's first equation, known as the Maxwell-Faraday equation. This equation elucidates the phenomenon of electromagnetic induction first discovered by Faraday:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (14)$$

As for the second equation, which is the Maxwell-Ampere equation, and which stems from Ampère's theorem, it links the evolution of the electric field as a function of the magnetic field. It is given by:

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (15)$$

What allowed us to deduce the equation of propagation in a Kerr-biisotropic medium our result is also a generalization:

$$\vec{\nabla}^2.\vec{E} - (\mu\varepsilon - \mu_0\varepsilon_0|\zeta_{EH}^2|)\frac{\partial^2\vec{E}}{\partial t^2} - \sqrt{\mu_0\varepsilon_0}(\zeta_{EH}^{*} - \zeta_{EH})\frac{\partial \vec{\nabla}\times\vec{E}}{\partial t} = \left(\mu\varepsilon_{Kerr} - \mu_0\varepsilon_0\zeta_{EH}\zeta_{HE}^{Kerr}\right)\left|\vec{E}\right|^2\frac{\partial^2\vec{E}}{\partial t^2} + \sqrt{\mu_0\varepsilon_0}\zeta_{HE}^{Kerr}\left|\vec{E}\right|^2\frac{\partial\vec{\nabla}\times\vec{E}}{\partial t} + \mu\sigma\frac{\partial\vec{E}}{\partial t} \quad (16)$$

σ is the absorption coefficient.

The electric field in the bi-isotropic fiber can be represented by wave propagating in the z direction:

$$\overrightarrow{E_{\pm}} = \left(\overrightarrow{e_x} \pm i\overrightarrow{e_y}\right)\Psi_{\pm}(r,t)e^{-i(k_{\pm}z-\omega_0 t)} \qquad (17)$$

$$\overrightarrow{E_{\pm}} = \overrightarrow{\Psi_{\pm}}(r,t)e^{-i(k_{\pm}z-\omega_0 t)} \qquad (18)$$

where the wave numbers k_+ (RCP) and k_- (LCP) can be written as:

$$k_+ = k\sqrt{\mu_0\varepsilon_0} + \sqrt{\mu\varepsilon - \gamma^2\mu_0\varepsilon_0} \qquad (19)$$

$$k_- = -k\sqrt{\mu_0\varepsilon_0} + \sqrt{\mu\varepsilon - \gamma^2\mu_0\varepsilon_0} \qquad (20)$$

The conditions of slowly variant envelope are given by [12, 13]:

$$\left|\frac{\partial^2}{\partial z^2}\Psi_{\pm}\right| \ll \left|2ik_{\pm}\right| \;;\; \left|\frac{\partial}{\partial z}\Psi_{\pm}\right| \ll \left|i\omega_0\Psi_{\pm}\right| \qquad (21)$$

$$\left|\frac{\partial^2}{\partial z^2}\left|\Psi_{\pm}\right|^2\Psi_{\pm}\right| \ll \left|i\omega_0\frac{\partial}{\partial t}\left|\Psi_{\pm}\right|^2\Psi_{\pm}\right| \ll \left|i\omega_0\left|\Psi_{\pm}\right|^2\Psi_{\pm}\right| \qquad (22)$$

$$\frac{A_{\pm}(z,t)}{2k_{\pm}}\cdot\overrightarrow{e_z} = \overrightarrow{\Psi_{\pm}} \qquad (23)$$

The phenomenon of dispersion is included in heuristic form through the relation

$$\Delta k = \frac{1}{v} \qquad (24)$$

After algebraic manipulations, within the slowly varying amplitude approximation of Maxwell’s equations [18-23], signals’ propagating through bi-isotropic fiber is described using the following equation:

$$\left(\frac{\partial}{\partial z}A_{\pm}(z,t) + \left(\beta_1\frac{\partial}{\partial t} + i\frac{1}{2}\beta_2\frac{\partial^2}{\partial t^2} - \frac{1}{6}\beta_3\frac{\partial^3}{\partial t^3}\right)A_{\pm}(z,t)\right) = i\delta A_{\pm}(z,t)i\rho\left|A_{\pm}(z,t)\right|^2A_{\pm}(z,t) \qquad (25)$$

Where the chromatic dispersion coefficients associated with β_1, β_2 and β_3, and α is the attenuation coefficient and fiber nonlinearity related to coefficient ρ .When setting the variable $\tilde{t} = t - \beta_1 z$, we obtain the nonlinear Schrödinger equation as follows:

$$\frac{\partial}{\partial z}A_{\pm}(z,\tilde{t}) + \left(i\frac{1}{2}\beta_2\frac{\partial^2}{\partial\tilde{t}^2} - \frac{1}{6}\beta_3\frac{\partial^3}{\partial\tilde{t}^3}\right)A_{\pm}(z,\tilde{t}) = \left(\frac{-\alpha}{2} + i\delta\right)A_{\pm}(z,\tilde{t}) - i\rho\left|A_{\pm}(z,\tilde{t})\right|^2A_{\pm}(z,\tilde{t}) \qquad (26)$$

In this step the bi-isotropic fiber is operattin in the third optical windows, where $\beta_3 = 0$ and neglecting absorption $\alpha = 0$ the Eq.(26) becomes

$$\frac{\partial}{\partial z}A_{\pm}(z,\tilde{t}) + \left(i\frac{1}{2}\beta_2\frac{\partial^2}{\partial\tilde{t}^2}\right)A_{\pm}(z,\tilde{t}) = i\delta A_{\pm}(z,\tilde{t}) - i\rho\left|A_{\pm}(z,\tilde{t})\right|^2A_{\pm}(z,\tilde{t}) \qquad (27)$$

Since $A_{\pm} = A_{\pm}(z,\tilde{t})$ is a complex function , we assume that travelling wave transformation is in the form

$$A_{\pm}(z,\tilde{t}) = v_{\pm}(z,\tilde{t})\exp(i\theta_{\pm}(z,\tilde{t})) \qquad (28)$$

where $v_{\pm}(z,\tilde{t})$ and $\theta_{\pm}(z,\tilde{t})$ are amplitude and phase functions respectively.Substituting the wave transformation Eq.(17) into Eq.(18) and separating the real and imaginary parts , we have

$$-v_{\pm}\theta_{\pm_z} - \frac{1}{2}\beta_2(z)(v_{\pm_{\tilde{t}\tilde{t}}} - v_{\pm}{\theta_{\pm_z}}^2) - \rho_{\pm}{v_{\pm}}^3 = 0 \quad (29)$$

$$v_{\pm_z} - \beta_2(z)(2\theta_{\pm_z}v_{\pm_z} + v_{\pm}\theta_{\pm_{zz}}) - v_{\pm} = 0 \quad (30)$$

where $\theta_{\pm_z} = \frac{d\theta_{\pm}}{dz}, \theta_{\pm_{zz}} = \frac{d^2\theta_{\pm}}{dz^2}$ and $v_{\pm_z} = \frac{dv_{\pm}}{dz}, v_{\pm_{\tilde{t}}} = \frac{dv_{\pm}}{d\tilde{t}}$

Considering the homogenous balance in Eq.(29) and Eq.(30), we assume that Eq.(29) and Eq.(30) have the following solutions form

$$\begin{cases} v_{\pm}(z,\tilde{t}) = v_{\pm}(\xi) = a_0 + a_1\left(\frac{{G_{\pm}}'}{G_{\pm}}\right) + b_1\sqrt{1 + \frac{1}{\mu}\left(\frac{{G_{\pm}}'}{G_{\pm}}\right)^2} \\ \xi_{\pm} = p_{\pm}(z)\tilde{t} + q_{\pm}(z) \end{cases} \quad (31)$$

$$\theta_{\pm}(z,\tilde{t}) = a_{\pm}(z)\tilde{t}^2 + b_{\pm}(z)\tilde{t} + c_{\pm}(z) \quad (32)$$

where $G_{\pm} = G_{\pm}(\xi)$ satifies

$$\begin{cases} -a_0{a_{\pm}}'(z) + 2a_0{a_{\pm}}^2(z)\beta_{\pm 2}(z) = 0 \\ -a_1{a_{\pm}}'(z) + 2a_1{a_{\pm}}^2(z)\beta_{\pm 2}(z) = 0 \\ -b_1{a_{\pm}}'(z) + 2b_1{a_{\pm}}^2(z)\beta_{\pm 2}(z) = 0 \\ b_1{b_{\pm}}'(z) + 2b_1{a_{\pm}}^2(z)b_{\pm}(z)\beta_{\pm 2}(z) = 0 \\ -a_1\beta_{\pm 2}(z) - a_1^3\rho_{\pm} - \frac{3a_1b_1^2\rho_{\pm}}{\mu} = 0 \\ 3a_0a_1^2 + \frac{3a_0b_1^2}{\mu} = 0 \\ -\frac{1}{2}\mu b_1\beta_{\pm 2}(z)p^2(z) - \rho_{\pm}b_1^3 - b_1{c_{\pm}}'(z) - 3a_0b_1\rho_{\pm} + \frac{1}{2}b_1\beta_{\pm 2}(z)b^2 = 0 \\ -a_1{c_{\pm}}'(z) - a_1\mu\beta_{\pm 2}(z){p_{\pm}}^2(z) - 3\rho_{\pm}a_0^2a_1 - 3a_1b_1^2\rho_{\pm} + \frac{1}{2}a_1b^2\beta_{\pm 2}(z) = 0 \\ {p_{\pm}}'(z) - 2p_{\pm}(z)\rho_{\pm}\beta_{\pm 2}(z) \\ {q_{\pm}}'(z) + p_{\pm}(z)\beta_{\pm 2}(z)b = 0 \\ -\rho_{\pm}\beta_{\pm 2}(z) = 1 \end{cases} \quad (33)$$

Solving the algebric system with the help of Mathematica , we get the following cases.

Case 1

$$a_0 = 0, b_1 = 0, a_1 = a_1, p_{\pm}(z) = c_2a_{\pm}(z), q_{\pm}(z) = \frac{1}{2}c_1c_2a_{\pm}(z) + c_3 \quad (34)$$

$$b_{\pm}(z) = c_1a_{\pm}(z), c_{\pm}(z) = \frac{1}{4}(c_1^2 - 2\mu c_2^2)a_{\pm}(z) + c_4 \quad (35)$$

$$\rho_{\pm}(z) = \frac{c_2^2\beta_{\pm}(z){a_{\pm}}^2(z)}{a_1^2} \quad (36)$$

where $c_i(i = 1,2, \ldots ,5)$ are arbitry constants and $a_{\pm}(z)$ is given by

$$a_{\pm}(z)[-2\int \beta_{\pm 2}(z)dz + c_5] = 1 \quad (37)$$

The exact solution of Eq.(26) is given by,

$$A_{\pm}(z,\tilde{t}) = \begin{cases} \sqrt{-\mu}a_1\left(\frac{A_1\sinh\sqrt{-\mu}\xi+A_2\cosh\sqrt{-\mu}\xi}{A_1\cosh\sqrt{-\mu}\xi+A_2\sinh\sqrt{-\mu}\xi}\right).\exp(i\theta_{\pm}(z,\tilde{t}));\mu<0 \\ \sqrt{\mu}a_1\left(\frac{A_1\sin\sqrt{\mu}\xi+A_2\cos\sqrt{\mu}\xi}{A_2\sin\sqrt{\mu}\xi-A_1\cos\sqrt{\mu}\xi}\right).\exp(i\theta_{\pm}(z,\tilde{t}));\mu>0 \\ \frac{A_2}{A_1+A_2}\ ;\ \mu=0 \end{cases} \quad (38)$$

The dark solitary of Eq.(27) are given in the form

$$A_{\pm_1}(z,\tilde{t}) = \sqrt{-\mu}a_1\tanh(\sqrt{-\mu}\xi+\xi_0)\cdot\exp\{ia_{\pm}(z,t)[\tilde{t}^2+c_1\tilde{t}+(c_1^2-2\mu c_2^2]\} \quad (39)$$

The hyperbolic function solution of Eq.(27) are written as

$$A_{\pm_2}(z,\tilde{t}) = \sqrt{-\mu}a_1\coth(\sqrt{-\mu}\xi+\xi_0)\cdot\exp\{ia_{\pm}(z,t)[\tilde{t}^2+c_1\tilde{t}+(c_1^2-2\mu c_2^2]\} \quad (40)$$

And the triangular periodic wave solution of Eq.(27) are written as

$$A_{\pm_3}(z,\tilde{t}) = \sqrt{-\mu}a_1\cot(\sqrt{-\mu}\xi+\xi_0)\cdot\exp\{ia_{\pm}(z,t)[\tilde{t}^2+c_1\tilde{t}+(c_1^2-2\mu c_2^2]\} \quad (41)$$

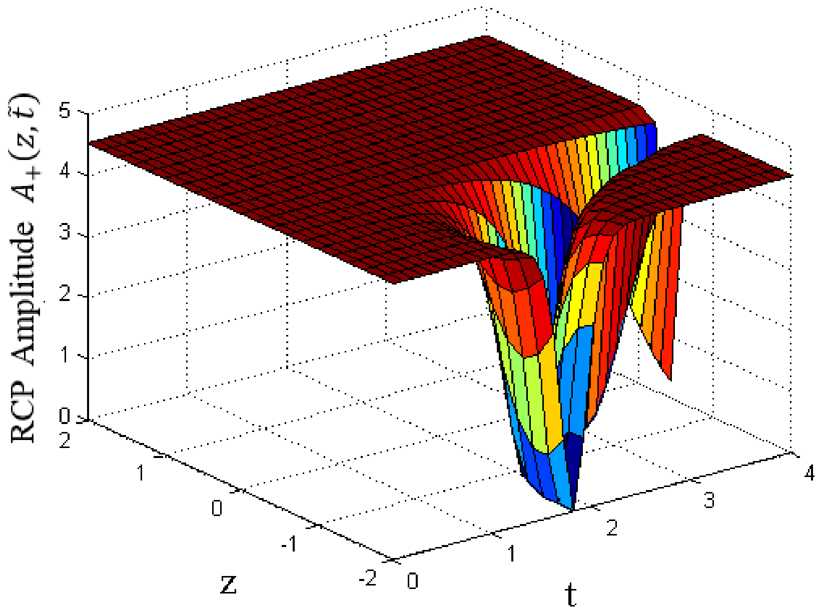

Fig.1. Numerical simulation of RCP Amplitude $A_{+1}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4$, $\varepsilon_r = 5.2$, $\gamma = 0.5$, $\kappa = 0.5$,$\gamma^{Kerr} = 2.58\times10^{-17}$ $\kappa^{Kerr} = 3.58\times10^{-17}$.

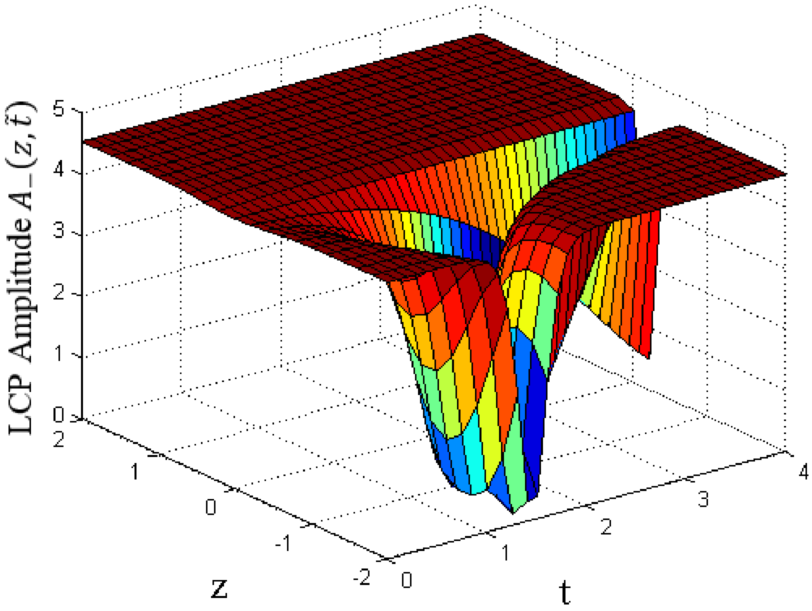

Fig.2. Numerical simulation of LCP Amplitude $A_{-1}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4, \varepsilon_r = 5.2, \gamma = 0.5, \kappa = 0.5, \gamma^{Kerr} = 2.58 \times 10^{-17}$ $\kappa^{Kerr} = 3.58 \times 10^{-17}$.

Case 2

$$a_0 = 0; a_1 = 0, b_1 = b_1, p_\pm(z) = c_2 a(z) \; , b_\pm(z) = c_1 a_\pm(z) \tag{42}$$

$$q_\pm(z) = \frac{1}{2} c_1 c_2 a_\pm(z) + c_3 \; ; -a_\pm(z)\beta_\pm(z) = 1 \tag{43}$$

$$c_\pm(z) = \frac{1}{4}(c_1^2 + \mu c_2^2) a_\pm(z) + c_4 \tag{44}$$

$$a_\pm(z)[-2\int \beta_{\pm 2}(z)dz + c_5] = 1 \tag{45}$$

In this case , we have the exact solution of Eq.(27) as follows

$$A_\pm(z,\tilde{t}) = \begin{cases} b_1\sqrt{1 - \left(\frac{A_1 \sinh\sqrt{-\mu}\xi + A_2 \cosh\sqrt{-\mu}\xi}{A_1 \cosh\sqrt{-\mu}\xi + A_2 \sinh\sqrt{-\mu}\xi}\right)^2} . \exp(i\theta_\pm(z,\tilde{t})); \mu < 0 \\ b_1\sqrt{1 + \left(\frac{a_1 \sin\sqrt{\mu}\xi + a_2 \cos\sqrt{\mu}\xi}{a_2 \sin\sqrt{\mu}\xi - a_1 \cos\sqrt{\mu}\xi}\right)^2} . \exp(i\theta_\pm(z,\tilde{t})); \;\; \mu > 0 \end{cases} \tag{46}$$

The bright solitary wave solution of Eq.(27) are given by

$$A_{\pm_4}(z,\tilde{t}) = b_1 \sec(\sqrt{-\mu}\xi + \xi_0). \exp\{ia_\pm(z,\tilde{t})[\tilde{t}^2 + c_1\tilde{t} + (c_1^2 - 2\mu c_2^2]\} \tag{47}$$

The singular solution of Eq.(27) are given by

$$A_{\pm_5}(z,\tilde{t}) = b_1 \operatorname{csch}(\sqrt{-\mu}\xi + \xi_0). \exp\{ia_\pm(z,\tilde{t})[\tilde{t}^2 + c_1\tilde{t} + (c_1^2 - 2\mu c_2^2]\} \tag{48}$$

The triangular periodic solution of Eq.(27) are given by

$$A_{\pm_6}(z,\tilde{t}) = b_1 \operatorname{csch}(\sqrt{-\mu}\xi + \xi_0) \cdot \exp\{ia_{\pm}(z,\tilde{t})[\tilde{t}^2 + c_1\tilde{t} + (c_1^2 - 2\mu c_2^2]\} \qquad (49)$$

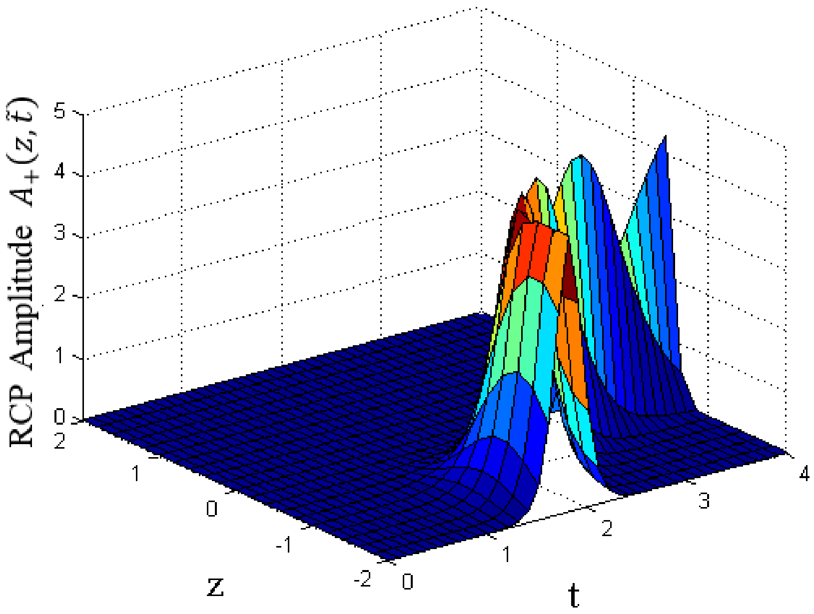

Fig.3. Numerical simulation of RCP Amplitude $A_{+4}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4$, $\varepsilon_r = 5.2$, $\gamma = 0.5$, $\kappa = 0.5$,$\gamma^{Kerr} = 2.58 \times 10^{-17}$ $\kappa^{Kerr} = 3.58 \times 10^{-17}$.

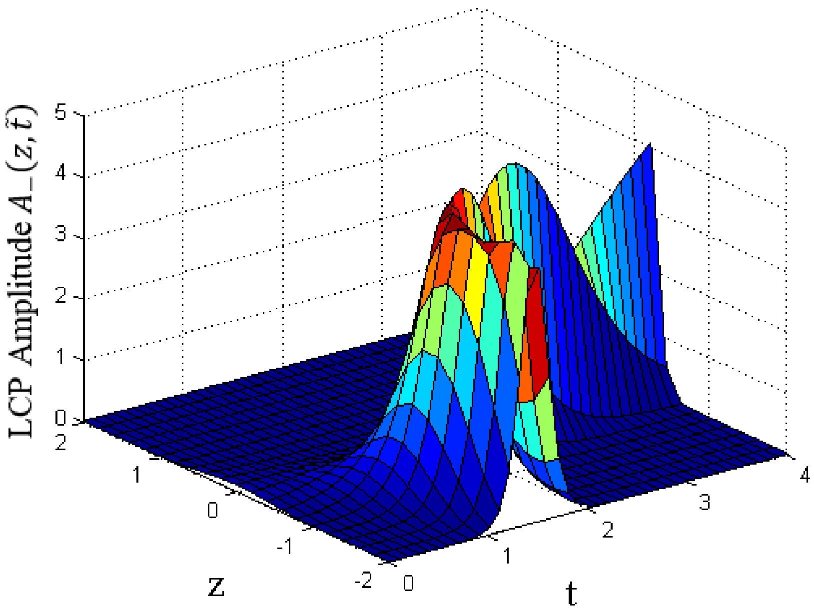

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Fig.4. Numerical simulation of LCP Amplitude $A_{-4}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4$, $\varepsilon_r = 5.2$, $\gamma = 0.5$, $\kappa = 0.5$,$\gamma^{Kerr} = 2.58 \times 10^{-17}$ $\kappa^{Kerr} = 3.58 \times 10^{-17}$.

Case 3

$$a_0 = 0, a_1 = a_1, b_1 = b_1, p_\pm(z) = c_2 a_\pm(z) \tag{50}$$

$$q_\pm(z) = \frac{1}{2} c_1 c_2 a_\pm(z) + c_3; b_\pm(z) = c_1 a_\pm(z) \tag{51}$$

$$c_\pm(z) = \frac{1}{8}(c_1^2 - 2\mu c_2^2) a_\pm(z) + c_1 \tag{52}$$

$$-a_\pm(z)\beta_{\pm 2}(z) = 1 \ ; \rho_\pm(z) = \frac{c_1^2 \beta_{\pm 2}(z) a_\pm{}^2(z)}{4b_1^2} \tag{53}$$

Where $a_\pm(z)$ satisfies the following condition

$$a_\pm(z)[-2\int \beta_{\pm 2}(z)dz + c_5] = 1 \tag{54}$$

In this case we have to type of exact solutions for the Eq.(27)

- $\mu < 0$, the hyperbolic solutions are defined by

$$A_{\pm_7}(z,\tilde{t}) = \sqrt{-\mu} a_1 \left[\frac{A_1 \sinh\sqrt{-\mu}\xi + A_2 \cosh\sqrt{-\mu}\xi}{A_1 \cosh\sqrt{-\mu}\xi + A_2 \sinh\sqrt{-\mu}\xi} \mp i\sqrt{\left(\frac{A_1 \sinh\sqrt{-\mu}\xi + A_2 \cosh\sqrt{-\mu}\xi}{A_1 \cosh\sqrt{-\mu}\xi + A_2 \sinh\sqrt{-\mu}\xi}\right)^2} \right] . \exp[i\theta_\pm(z,\tilde{t})] \tag{55}$$

- If $\mu < 0, \coth\xi_0 = \frac{A_2}{A_1}, \left|\frac{A_1}{A_2}\right| > 1$,then the bright-dark soliton of Eq.(27) are derived as,

$$A_{\pm_{7,1}}(z,\tilde{t}) = \sqrt{-\mu} a_1 [\tanh\sqrt{-\mu}\xi + \xi_0] = \mp i \sec(\sqrt{-\mu}\xi + \xi_0) . \exp[i\theta_\pm(z,\tilde{t})] \tag{56}$$

- If $\mu < 0, \coth\xi_0 = \frac{A_2}{A_1}, \left|\frac{A_1}{A_2}\right| < 1$,then the solution of Eq.(27) becomes

$$A_{\pm_{7,2}}(z,\tilde{t}) = \sqrt{-\mu} a_1 [\coth\sqrt{-\mu}\xi + \xi_0] = \pm \mathrm{csch}(\sqrt{-\mu}\xi + \xi_0) . \exp[i\theta_\pm(z,\tilde{t})] \tag{57}$$

- $\mu > 0$ the triangular periodic sulution are given by

$$A_{\pm_8}(z,\tilde{t}) = \sqrt{-\mu} a_1 \left[\frac{A_1 \cos\sqrt{-\mu}\xi + A_2 \sin\sqrt{-\mu}\xi}{A_1 \cos\sqrt{-\mu}\xi + A_2 \sin\sqrt{-\mu}\xi} \mp i\sqrt{\left(\frac{A_1 \cos\sqrt{-\mu}\xi + A_2 \sin\sqrt{-\mu}\xi}{A_1 \cos\sqrt{-\mu}\xi + A_2 \sin\sqrt{-\mu}\xi}\right)^2} \right] . \exp[i\theta_\pm(z,\tilde{t})] \tag{58}$$

- If $\mu > 0$,$\tan\xi_0 = \frac{A_2}{A_1}$, hen the solution of Eq.(27) becomes as follow

$$A_{\pm_{8,1}}(z,\tilde{t}) = \sqrt{\mu} a_1 [\cot\sqrt{\mu}\xi + \xi_0] = \pm \mathrm{csch}(\sqrt{\mu}\xi + \xi_0) . \exp[i\theta_\pm(z,\tilde{t})] \tag{59}$$

Where

$$\theta_\pm(z,\tilde{t}) = a_\pm(z) \left[\tilde{t}^2 + c_1 \tilde{t} + \frac{c_1^2 - 2\mu c_2^2}{8} a_\pm(z) + c_4 \right] \tag{60}$$

$$\xi_\pm = c_2 a_\pm(z)\tilde{t} + \frac{1}{2} c_1 c_2 a_\pm(z) + c_3 \tag{61}$$

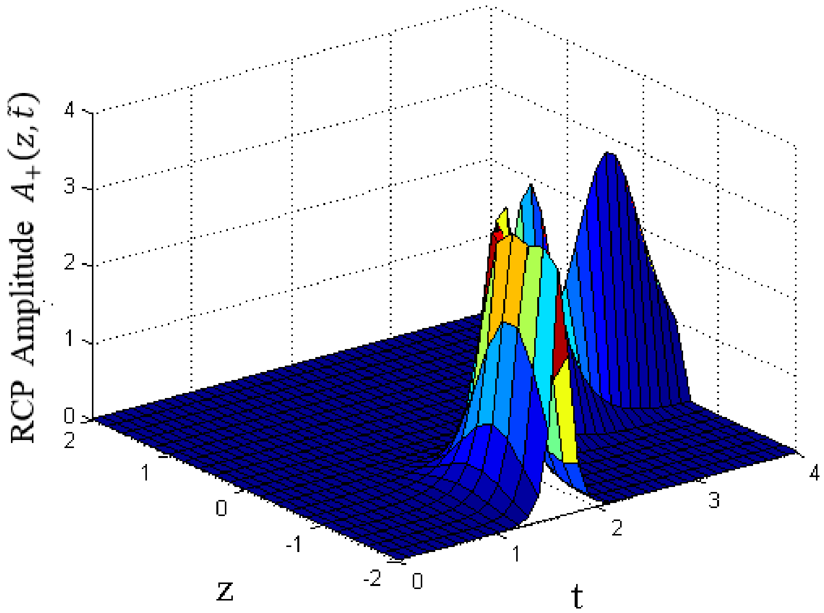

Fig.5. Numerical simulation of RCP Amplitude $A_{+7}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4$, $\varepsilon_r = 5.2$, $\gamma = 0.5$, $\kappa = 0.5$,$\gamma^{Kerr} = 2.58 \times 10^{-17}$ $\kappa^{Kerr} = 3.58 \times 10^{-17}$.

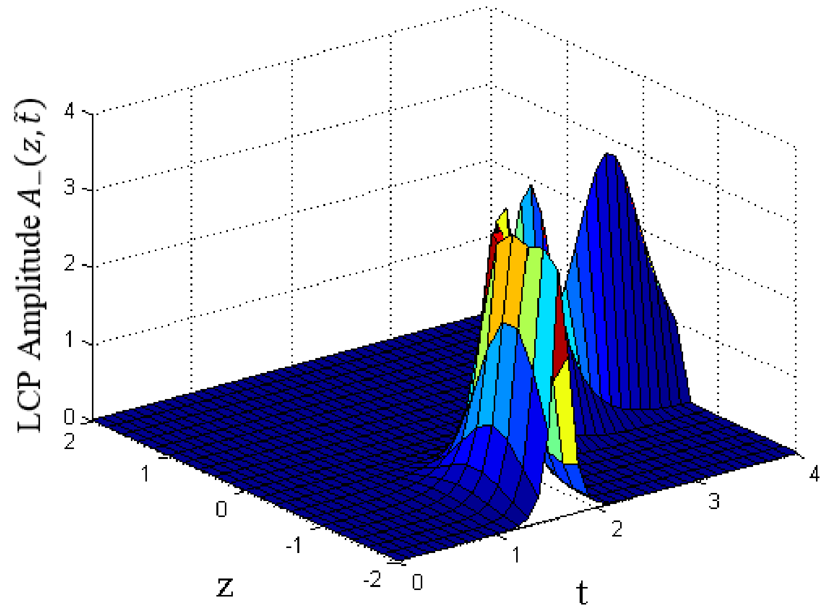

Fig.5. Numerical simulation of LCP Amplitude $A_{-7}(z,\tilde{t})$ for the first case of solutions where: c_1=3 , c_2=4 , c_3=5 ξ_0 =1; μ=-1/2 , a_1 =3 , b_1=3, $\mu_r = 1.4$, $\varepsilon_r = 5.2$, $\gamma = 0.5$, $\kappa = 0.5$,$\gamma^{Kerr} = 2.58 \times 10^{-17}$ $\kappa^{Kerr} = 3.58 \times 10^{-17}$.

In the following figures 1-6, the different exact families of solutions were obtained and we take as an example some graphs for the solutions of case 1, case 2, and case 3 with two modes of propagation (RCP) and (LCP). The extended G'/G-expansion method has been used to present an analytic study of the generalized nonlinear Schrödinger equation in bi-isotropic (chiral and non-reciprocal). optical fibers. The method is constructed on a perturbation expansion in powers of a dimensionless parameter, which allows for the inclusion of varying dispersion and nonlinearity. This types the method well-suited for modeling a wide range of optical fibres, including those with complex dispersion and nonlinearity profiles. The method was used to find exact different families of solutions of the nonlinear Schrödinger equation for a bi-isotropic optical fiber. The method was able to accurately describe the propagation of optical pulses in bi-isotropic (chiral and non-reciprocal). fibers and can be used to predict the behavior of these pulses under different conditions. The method can also be used to study the dynamics of solitons, including the stability and interactions of these localized structures.

5. Results

This investigate is concerned with a new formulation of constitutive relation linking to the magnetic effect, to understand rigorously the physical nature of biisotropic effects and to generalize the main macroscopic models. We inferred the nonlinear Schrodinger equation for a bi-isotropic medium term with a nonlinear term of magnetizing. In this article, the extended (G'/G)-expansion method is a powerful technique for determining a family of solutions of the nonlinear Schrödinger equation in bi-isotropic optical fibers. This method is based on the use of a perturbation expansion in powers of the dimensionless parameter, and it is valid for both weak and strong nonlinearities. The method allows for the inclusion of varying dispersion and nonlinearity, making it well-suited for modeling a wide range of optical fibres. Overall, the extended (G'/G)-expansion method is a valuable tool for understanding the dynamics of nonlinear optical systems, and it is expected to have a wide range of applications in the field of nonlinear optics.

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