Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 4s (2024)

On Generalized Nano $\widetilde{\mathbb{N}\alpha}$- Closed and Nano $\widehat{\mathbb{N}\alpha}$- Open Sets in Nano-Topological Spaces

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Article History:

***Received:** 25-04-2024*

***Revised:** 10-06-2024*

***Accepted:** 21-06-2024*

Abstract:

This work aims to define a new class of sets in nano topological spaces called Nano (Nα)ˇ-closed and (Nα)ˆ-open sets, and to prove its verifiable properties and theorems.

Subject Classification: 54A05, 54A 10

Keywords: nano topology, (Nα)ˇ- closed set and Nano (Nα)ˆ- open Set.

1. Introduction

In 2021, Ą regular closed sets in nano topological spaces were presented by Narmatha S., Harshitha S., and others [3]. Within micro topological spaces, πgβ-closed sets are studied by Rajasekaran I. and others [4]. In nano topological spaces, ng∗α− closed sets were first presented by Rajendran V. and colleagues [5]..Crossley and Hildebrand [7] conducted research on semi-closure in 1971. Dunham [14] provided a definition of the closure operator C* notion along with various attributes. Operator of regular closed sets was first defined by S. Bhattacharya [6] in 2011. Soft W -int. and soft W -Cl. in Soft topological spaces are studied by Savita R. [8]. Soft g* closed in Soft topological spaces are studied by Kalavathi, A. and Krishnan, G.[2]. Regular generalized* in topological spaces, closure regular generalized* Closure Regular Generalized*, and regular generalized closed sets are new classes of operators introduced by Siham I. Aziz and Nabila I. Aziz [9, 10, 11] and, respectively. 2014 saw the introduction of Nano closure and Nano Interior operator in Nano topological spaces by Thivagar M. Lellis and Carmel Rechard [12], a novel class of operators Open and Closed Nano operators Ɲˆ(Ā) and Ɲˇ(Ā) introduced by ABDULAZIZ .S.[1]. The aim of this work is to investigate and characterize a new class of operators in nano topological spaces called Nano $\widetilde{\mathbb{N}\alpha}$- closed and $\widehat{\mathbb{N}\alpha}$-open sets , and to establish their verifiable characteristics and theorems.

2. Preliminaries

Definition 2.1 [2] : Suppose ϑ be the world, ψ ⊆ ϑ, and Π be an equivalence relation on ϑ. With regard to Ψ, τ ϕ(Ψ) ={ ϑ, ∅, LR(Ψ), ϑ Φ(Ψ), BΦ(Ψ)} and (ϑ, τ Φ(Ψ)) define the Nano topology on U.

Definition 2.2 [1]: Assume A ⊆ ϑ and (U, τ R(Ψ)) be a Nano topological space. Next, we established 1- ℵ(A) = ∩ {G : A⊆ G, G ∈ N O(ϑ, Ψ)} and 2-Nˇ(A) = ∪ {G : G⊆ A, G ∈ NC (ϑ, Ψ)}.

Definition 2.3 [2]: Assuming Ψ, Ψ ⊆ ϑ, let (U, τ R(Ψ)) be a Nano topological space. If A is not equal to ϑ, then: The union of all of A's open subsets is A's nanointerior, and it is represented by Nint(A). Ncl(A) represents the Nano closure of A, which is the intersection of all Nano closed subsets containing A.

Definition 2.4 [13] : When M ⊆ Nint (NCl (Nint M)), a subset M of (ϑ, τ R(Ψ)) is referred to as a nano α open set (briefly, Nα-o-s.). In (U, τ R(Ψ)), the complement of a Nα-o-s. is referred to as a nano α-closed set (briefly, Nα-c-s.). Nα-o-(ϑ, Ψ) (resp. Nα-c-(ϑ, Ψ)) represents the family of all Nα-o-s. (resp. Nα-c-s.) of U.

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Definition 3.1 Assume that A ⊆ U and that (U, τ R(Ψ)) is a Nano topological space. If A ⊆ ℵ ((N)ˇ (ℵ(A))), then a subset A is referred to as an open set (Nα)̂.The closed set Nano (ℵδ) - is the complement of the open set Nano (Nα)ˆand is defined as [A ⊇ (N)ˇ(ℵ(((N)ˇ(A)))]

Example 3.2: Assuming U/R = {{r, p}, { q}} and Ψ = {r, q}, let U = {r, μ, q}.
Assuming τR(ψ)={∅,U, { q},{r, p}}, we get τcR(Ψ) = {∅,U, { q},{r, p}}.
(Nα) -o(x)={U, ϕ,{r},{μ },{q},{r, },{r,q},{μ,q}}.

Theorem 3.3 All subsets of U R(Ψ) are not (Nα)ˆopen sets if τ R(Ψ) is not highly disconnected.

Proof

Case 1 in the event that τ R(Ψ) ={ U, Ø, U R(Ψ) }.

To begin with, assume A = U R(Ψ) ⇒ℵ(A) = U R(Ψ) ⇒ (N)ˇ(ℵ(U R(Ψ))=∅⇒ℵ(N)ˇ(ℵ(∅))=∅.
If A⊉ ∅ ∅ ⇒ A ⊈ ℵ((N)ˇ(ℵ(A)), then A∉ (Nα)̂.

2-Take A⊆ Uc R(Ψ)⇒ N̂(A)=U· (N)· (N̂(U)=U· N̂((N)Œ(N̂(U))=U ⇒ A ⊆N̂((N)ˇ(N̂(A)).
3. If [U R(Ψ) and Uc R(Ψ)] intersect at A, then ℵ(A)= U ⇒ (N)ˇ(ℵ(U)=U ⇒ ℵ((N)ˇ(ℵ(U))=U ⇒ A ⊆ℵ((N)ˇ(ℵ(A)).

Case 2: If $\tau_{R(\Psi)}$ ={ U, Ø, $L_{R(\Psi)}$, $B_{R(\Psi)}$,$U_{R(\Psi)}$ }.

First, suppose that A⊆ L R(Ψ) ⇒ℵ(A)= L R(Ψ)⇒ (N)ˇ(ℵ(L R(Ψ))=∅ ⇒ℵ(N)ˇ(ℵ(∅))=∉.
A⊈ ∅ ∀̸ ⇒ A⊈ ℵ((N)ˇ(ℵ(A)). Afterwards, A∉ (Nα)ˆ
2. In the event when A ⊆ B R(Ψ) ⇒ ℵ(A)= B R(Ψ) ⇒ (N)ˇ(ℵ(B R(Ψ))=∅ ⇒ℵ(N)ˇ(ℵ(∅))=∪.
A⊈_∅_⇒ A⊈_ℵ((N)ˇ(ℵ(A)). Afterwards, A∉ (Nα)ˆ
3. At the intersection of A with [L R(Ψ) and B R(Ψ)], ℵ(A)= U R(Ψ) and (N)ˇ(ℵ(U R(Ψ)) = ∅ ⇒ℵ(N)ˇ(ℵ(∅)) =∅. A⊈ ∅ ∀̸ ⇒ A⊈ ℵ((N)ˇ(ℵ(A)). Afterwards, A∉ (Nα)

4-If A⊆ Uc R(Ψ) ⇒ N̂(A)= U ⇒ (N)ˇ(N̂(U)=U ⇒ N̂((N)ˇ(N̂(U))=U ⇒ A ⊆N̂((N)ˇ(N̂(A)).
5. In the event when A intersects [L R(Ψ) and Uc R(Ψ)] ⇒ ℵ(A)= U ⇒ (N)ˇ(ℵ(U)=U ⇒ ℵ((N)ˇ(ℵ(U))=U ⇒ A ⊆ℵ((N)ˇ(ℵ(A)).

6-If [B R(Ψ) and Uc R(Ψ)] intersect A, then N̂(A)= U ⇒ (ℵ)ˇ(N̂(U)=U ⇒ ℵ((ℵ)ˇ(N̂(U))=U ⇒ A ⊆ℵ((N)ˇ(ℵ(A)).

Theorem 3.4: All subset of $U^c{}_{R(\Psi)}$ is $\widehat{ℕα}$ open set.

Proof: Case 1- If $\tau_{R(\Psi)}$ ={ U, Ø, $U_{R(\Psi)}$ }.

If A⊆ $U^c{}_{R(\Psi)}$ ⇒ $\widehat{ℕ}$(A)=U ⇒ $\breve{ℕ}$ ($\widehat{ℕ}$(U) = U ⇒ $\widehat{ℕ}$ ($\breve{ℕ}$($\widehat{ℕ}$(U))=U ⇒ A ⊆ $\widehat{ℕ}$ ($\breve{ℕ}$($\widehat{ℕ}$(A)). Then A∈ $\widehat{ℕα}$.

Case 2 : when $\tau_{R(\Psi)}$ ={ U, Ø, $L_{R(\Psi)}$, $B_{R(\Psi)}$,$U_{R(\Psi)}$ }.

If A⊆ $U^c{}_{R(\Psi)}$ ⇒ $\widehat{ℕ}$(A)=U ⇒ $\breve{ℕ}$ ($\widehat{ℕ}$(U) = U ⇒ $\widehat{ℕ}$ ($\breve{ℕ}$($\widehat{ℕ}$(U))=U ⇒ A ⊆ $\widehat{ℕ}$ ($\breve{ℕ}$($\widehat{ℕ}$(A)). Then A∈ $\widehat{ℕα}$.

Theorem 3. 5: All subset of U which intersect [$U_{R(\Psi)}$ and $U^c{}_{R(\Psi)}$] is $\widehat{ℕα}$ open set.

Proof: Case 1- If $\tau_{R(\Psi)}$ ={ U, Ø, $U_{R(\Psi)}$ }

Let A ⊆ U Such that A intersect [$U_{R(\Psi)}$ and $U^c{}_{R(\Psi)}$]

$\widehat{ℕ}$(A)=U ⇒ $\breve{ℵ}$ ($\widehat{ℕ}$(U) = U ⇒ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(U))=U ⇒ A ⊆ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(A)). Then A∈ $\widehat{ℕα}$ open set.

Case 2: $\tau_{R(\Psi)}$ ={ U, Ø, $L_{R(\Psi)}$, $B_{R(\Psi)}$,$U_{R(\Psi)}$ }.

1- Let **A**⊆ U such that A intersect [$L_{R(\Psi)}$ and $U^c{}_{R(\Psi)}$].

$\widehat{ℕ}$(A)=U ⇒ $\breve{ℕ}$ (ℵ(U) = U ⇒ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(U))=U ⇒ A ⊆ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(A)). Then A∈ $\widehat{ℕα}$ open set.

2- Let **A**⊆ U such that A intersect [$B_{R(\Psi)}$ and $U^c{}_{R(\Psi)}$].

$\widehat{ℕ}$(A)=U ⇒ $\breve{ℕ}$ ($\widehat{ℕ}$(U) = U ⇒ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(U))=U ⇒ A ⊆ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℵ}$(A)). Then A∈ $\widehat{ℕα}$ open set.

3- Let **A**⊆ U such that A intersect [$L_{R(\Psi)}$ and $B_{R(\Psi)}$ and $U^c{}_{R(\Psi)}$].

$\widehat{ℕ}$(A)=U ⇒ $\breve{ℕ}$ ($\widehat{ℕ}$(U) = U ⇒ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℕ}$(U))=U ⇒ A ⊆ $\widehat{ℵ}$ ($\breve{ℕ}$($\widehat{ℕ}$(A)). Then A∈ $\widehat{ℕα}$ open set.

Theorem 3.6: When the Nano space U is extremely disconnected then all subset of U is $\widehat{ℕα}$ −open set.

Proof: $\tau_{R(\Psi)}$ ={ U, Ø, $L_{R(\Psi)}$, $B_{R(\Psi)}$ }.

Assuming that A ⊆ L R(Ψ) ⇒ℵ(A)= L R(Ψ)⇒ (N)ˇ(ℵ(L R(Ψ)) = L R(Ψ)⇒ ℵ (N)ˇ(ℵ(L R(Ψ))= L R(Ψ)A ⊆ ℵ((N)ˇ(ℵ(A)) if A⊆ L R(Ψ) ⇒ A. After that, A∈(Nα)ˆopen set. In the event that A ⊆ B R(Ψ) ⇒ ℵ(A)= B R(Ψ) ⇒ (N)ˇ(ℵ(B R(Ψ))= B R(Ψ) ⇒ ℵ (N)ˇ(ℵ(B R(Ψ))= B R(Ψ) ⇒\ A⊆ B R(Ψ) ⇒ A ⊆ B R(Ψ) ⇒ B R(Ψ) ⇒ A ⊆ ℵ((N)ˇ(ℵ(A)). A∈(Nα)ˆis thus an open set.When A crosses across [L R(Ψ) and B R(Ψ)].N (A)=U ⇒ (N) (N̂(U) = U· N̂((N)(N̂(U))=U· A ⊆N̂((N)(ℵ(A)). A∈(Nα)ˆis thus an open set.

4.Conclusion

The purpose of this study is to define a novel class called ($\breve{ℕα}$- closed and $\widehat{ℕα}$- open) Sets in Nano-Topological Spaces and to demonstrate its verifiable characteristics and theorems. The (β , b, regular, and semi) sets can be included in the future generalization of the new concept.

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