On Generalized Nano $\mathbb{N}_{\alpha}$-Closed and Nano $\mathbb{N}_{\alpha}$-Open Sets in Nano-Topological Spaces

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Abstract:

This work aims to define a new class of sets in nano topological spaces called Nano ($\mathbb{N}_{\alpha}^\cdot$)-closed and (N$\alpha$)-open sets, and to prove its verifiable properties and theorems.

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1. Introduction

In 2021, $\mathbb{N}_{\alpha}$ regular closed sets in nano topological spaces were presented by Narmatha S., Harshitha S., and others [3]. Within micro topological spaces, $\pi g\beta$-closed sets are studied by Rajasekaran I. and others [4]. In nano topological spaces, $ng^*\alpha$-closed sets were first presented by Rajendran V. and colleagues [5]. Crossley and Hildebrand [7] conducted research on semi-closure in 1971. Dunham [14] provided a definition of the closure operator $C^*$ notion along with various attributes. Operator of regular closed sets was first defined by S. Bhattacharya [6] in 2011. Soft W-int. and soft W-CI. in Soft topological spaces are studied by Savita R. [8]. Soft $g^*$ closed in Soft topological spaces are studied by Kalavathi, A. and Krishnan, G.[2]. Regular generalized* in topological spaces, closure regular generalized* Closure Regular Generalized*, and regular generalized closed sets are new classes of operators introduced by Siham I. Aziz and Nabila I. Aziz [9, 10, 11] and, respectively. 2014 saw the introduction of Nano closure and Nano Interior operator in Nano topological spaces by Thivagar M. Lellis and Carmel Rechard [12], a novel class of operators Open and Closed Nano operators $N^\cdot(\bar{A})$ and $N^\cdot(\bar{A})$ introduced by ABDULAZIZ S.[1]. The aim of this work is to investigate and characterize a new class of operators in nano topological spaces called Nano $\mathbb{N}_{\alpha}$-closed and $\mathbb{N}_{\alpha}$-open sets, and to establish their verifiable characteristics and theorems.

2. Preliminaries

Definition 2.1 [2]: Suppose $\Omega$ be the world, $\psi \subseteq \Omega$, and $\Pi$ be an equivalence relation on $\Omega$. With regard to $\Psi$, $\tau \phi(\Psi) = \{ \psi, \emptyset, LR(\psi), \emptyset \Phi(\psi), B\Phi(\psi) \}$ and $(\emptyset, \tau \Phi(\Psi))$ define the Nano topology on $U$. 

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Definition 2.2 [1]: Assume \( A \subseteq \emptyset \) and \((U, \tau R(\Psi))\) be a Nano topological space. Next, we established 
1. \( \emptyset(A) = \cap \{G : A \subseteq G, G \in \mathbb{N}(\emptyset, \Psi)\} \)
2. \( 2^{-N}(A) = U \{G : G \subseteq A, G \in \mathbb{N}(\emptyset, \Psi)\} \).

Definition 2.3 [2]: Assuming \( \Psi, \Psi \subseteq \emptyset \), let \((U, \tau R(\Psi))\) be a Nano topological space. If \( A \) is not equal to \( \emptyset \), then: The union of all of \( A \)'s open subsets is \( A \)'s nanointerior, and it is represented by \( \text{Nint}(A) \). \text{Ncl}(A) \) represents the Nano closure of \( A \), which is the intersection of all Nano closed subsets containing \( A \).

Definition 2.4 [13]: When \( M \subseteq \text{Nint}(\mathbb{N}(\text{Nint}(M))) \), a subset \( M \) of \((\emptyset, \tau R(\Psi))\) is referred to as a nano \( \alpha \)-open set (briefly, Nano \( \alpha \)-o.s.). In \((U, \tau R(\Psi))\), the complement of a Nano \( \alpha \)-closed set is referred to as a nano \( \alpha \)-closed set (briefly, Nano \( \alpha \)-c.s.). Nano \( \alpha \)-(\emptyset, \Psi) \) (resp. Nano \( \alpha \)-(\emptyset, \Psi \)) represents the family of all Nano \( \alpha \)-o.s. (resp. Nano \( \alpha \)-c.s.) of \( U \).

3. On Generalized Nano \( \overline{\mathbb{N}} \alpha \)-closed and Nano \( \overline{\mathbb{N}} \delta \)-open Sets in Nano-Topological Spaces

Definition 3.1: Assume that \( A \subseteq U \) and that \((U, \tau R(\Psi))\) is a Nano topological space. If \( A \subseteq \emptyset \) (\( \mathbb{N}(\emptyset, \Psi)) \), then a subset \( A \) is referred to as an open set (briefly, Nano \( \alpha \)-open set (briefly, \( \mathbb{N}(\mathbb{N}(\emptyset, \Psi)) \)).

Definition 3.2: Assuming \( U/R = \{\{r, p\}, \{q\}\} \) and \( \Psi = \{r, q\} \), let \( U = \{r, \mu, q\} \).
Assuming \( \tau R(\Psi) = \{\emptyset, U, \emptyset, \{r\}, \{q\}\} \), we get \( \tau \in R(\Psi) = \{\emptyset, U, \emptyset, \{r\}, \{q\}\} \).

(1) \( (\mathbb{N}) \alpha-o(x) = \{U, \emptyset, \{r\}, \{q\}\} \).

Theorem 3.3 All subsets of \( U \in R(\Psi) \) are not \( (\mathbb{N}) \alpha \)-open sets if \( \tau R(\Psi) \) is not highly disconnected.

Proof

Case 1 in the event that \( \tau R(\Psi) = \{U, \emptyset, U R(\Psi)\} \).

To begin with, assume \( A = U R(\Psi) \Rightarrow \mathbb{N}(A) = U R(\Psi) \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(U R(\Psi))) = \emptyset \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

If \( A \not\subseteq \emptyset \Rightarrow A \not\subseteq \mathbb{N}(\emptyset) \), then \( A \not\subseteq (\mathbb{N}) \alpha \).

2. Take \( A \subseteq U c R(\Psi) \Rightarrow \mathbb{N}(A) \subseteq U c R(\Psi) \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(U c R(\Psi))) = \emptyset \Rightarrow A \subseteq \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

3. If \( [U R(\Psi) \text{ and } U c R(\Psi)] \) intersect at \( A \), then \( \mathbb{N}(A) = U \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(U)) = \emptyset \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

Case 2: If \( \tau R(\Psi) = \{U, \emptyset, L R(\Psi), B R(\Psi), U R(\Psi)\} \).

First, suppose that \( A \subseteq L R(\Psi) \Rightarrow \mathbb{N}(A) \subseteq L R(\Psi) \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(L R(\Psi))) = \emptyset \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

\( A \not\subseteq \emptyset \Rightarrow A \not\subseteq \mathbb{N}(\emptyset) \). Afterwards, \( A \not\subseteq (\mathbb{N}) \alpha \).

2. In the event when \( A \subseteq B R(\Psi) \Rightarrow \mathbb{N}(A) \supseteq B R(\Psi) \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(B R(\Psi))) = \emptyset \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

\( A \not\subseteq \emptyset \Rightarrow A \not\subseteq \mathbb{N}(\emptyset) \). Afterwards, \( A \not\subseteq (\mathbb{N}) \alpha \).

3. At the intersection of \( A \) with \([L R(\Psi) \text{ and } B R(\Psi)]\), \( \mathbb{N}(A) = U R(\Psi) \), and \((\mathbb{N}) \alpha-(\mathbb{N}(U R(\Psi))) = \emptyset \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

4. If \( A \subseteq U c R(\Psi) \Rightarrow \mathbb{N}(A) \Rightarrow U \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(U)) \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).

5. In the event when \( A \) intersects \([L R(\Psi) \text{ and } U R(\Psi)]\), \( \mathbb{N}(A) = U \Rightarrow (\mathbb{N}) \alpha-(\mathbb{N}(U)) \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) \Rightarrow \mathbb{N}(\emptyset) = \emptyset \).
6-If \([B \cap R(\Psi) \text{ and } C \cap R(\Psi)]\) intersect \(A\), then \(\tilde{N}(A)=U \Rightarrow (N^{*})(\tilde{N}(U))=U \Rightarrow K( (N) \cap (N^{*})(U))=U \Rightarrow A \subseteq N((N) \cap (N^{*})(U))\).

**Theorem 3.4:** All subset of \(U_{c_{R(\Psi)}}\) is \(\tilde{N}r\) open set.

**Proof:** Case 1- If \(\tau_{R(\Psi)} = \{U, \emptyset, U_{R(\Psi)}\}\).

If \(A \subseteq U_{c_{R(\Psi)}} \Rightarrow \tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\).

Case 2 : when \(\tau_{R(\Psi)} = \{U, \emptyset, L_{R(\Psi)}, B_{R(\Psi)}, U_{R(\Psi)}\}\).

If \(A \subseteq U_{c_{R(\Psi)}} \Rightarrow \tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\).

**Theorem 3.5:** All subset of \(U\) which intersect \([U_{R(\Psi)} \text{ and } U_{c_{R(\Psi)}}]\) is \(\tilde{N}r\) open set.

**Proof:** Case 1- If \(\tau_{R(\Psi)} = \{U, \emptyset, U_{R(\Psi)}\}\)

Let \(A \subseteq U\) such that \(A\) intersect \([U_{R(\Psi)} \text{ and } U_{c_{R(\Psi)}}]\)

\(\tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\) open set.

Case 2: \(\tau_{R(\Psi)} = \{U, \emptyset, L_{R(\Psi)}, B_{R(\Psi)}, U_{R(\Psi)}\}\).

1- Let \(A \subseteq U\) such that \(A\) intersect \([L_{R(\Psi)} \text{ and } U_{c_{R(\Psi)}}]\).

\(\tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\) open set.

2- Let \(A \subseteq U\) such that \(A\) intersect \([B_{R(\Psi)} \text{ and } U_{c_{R(\Psi)}}]\).

\(\tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\) open set.

3- Let \(A \subseteq U\) such that \(A\) intersect \([L_{R(\Psi)} \text{ and } B_{R(\Psi)} \text{ and } U_{c_{R(\Psi)}}]\).

\(\tilde{N}(A)=U \Rightarrow \tilde{N}(\tilde{N}(U))=U \Rightarrow A \subseteq \tilde{N}(\tilde{N}(A))\). Then \(A \subseteq \tilde{N}r\) open set.

**Theorem 3.6:** When the Nano space \(U\) is extremely disconnected then all subset of \(U\) is \(\tilde{N}r\) — open set.

**Proof:** \(\tau_{R(\Psi)} = \{U, \emptyset, L_{R(\Psi)}, B_{R(\Psi)}\}\).

Assuming that \(A \subseteq L_{R(\Psi)} \Rightarrow (N(A) \subseteq L_{R(\Psi)} \Rightarrow \tilde{N}(N(L_{R(\Psi)})) \Rightarrow \tilde{N}(N(L_{R(\Psi)}))\) \(\tilde{N}(N(L_{R(\Psi)}))\) \(\tilde{N}(N(L_{R(\Psi)}))\)

A \subseteq \(N^{*}(N(A)).\) A \(\subseteq \tilde{N}(N^{*}(N(A)))\) if \(A \subseteq L_{R(\Psi)} \Rightarrow A\). After that, \(A \subseteq \tilde{N}(N^{*}(N(A)))\) is thus an open set. When \(A\) crosses across \([L_{R(\Psi)} \text{ and } B_{R(\Psi)}]\)

\(N(A)=U \Rightarrow (N(\tilde{N}(U))=U \Rightarrow \tilde{N}((N)(\tilde{N}(U)))=U \Rightarrow A \subseteq \tilde{N}(N(A))\).

4.**Conclusion**

The purpose of this study is to define a novel class called \((\tilde{N}r\) - closed and \(\tilde{N}r\) - open) sets in Nano-Topological Spaces and to demonstrate its verifiable characteristics and theorems. The \((\beta', b, \text{ regular, and semi})\) sets can be included in the future generalization of the new concept.
References


