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Intuitionistic Fuzzy Threshold Hypergraphs and Their Role in Chasing Fugitives with Multi-Bots

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Abstract

In this paper, Intuitionistic Fuzzy Threshold Hypergraph (IFTHG) is described with some definitions, such as adjacency level, strength, walk, hyperpath, score values, connected and disconnected IFTHGs. IFTHGs are essential for modeling complex relationships and uncertainties in emergency response scenarios within crowed areas. Furthermore, a novel method for capturing fugitives using IFTHG model is demonstrated. The proposed system initializes robots and implements a step-by-step algorithm upon detecting any intrusion, ultimately determining the nearest robot to capture the fugitives.

Keywords: Intuitionistic fuzzy threshold hypergraph, multi robots, algorithm, fugitive chase.

2000 Mathematics Subject Classification: 05C65

1. INTRODUCTION

To deal with the complexities of application base, the graph concept was expanded to provide a hypergraph, which is a set of all vertices including a collection of V subsets. The notions of hypergraph was introduced by Berge [5]. Chvatal and Hammer were the first to introduce threshold graphs(1973) in [6].

In set theory, Zadeh[13] created fuzzy sets as a technique of conveying ambiguity and vagueness. Fuzzy set theory has sparked attention to variety of fields. Atanassov[1, 3] came up with of Intuitionistic Fuzzy Sets(IFS) concept as a generalization of fuzzy sets & Atanassov added an additional module to fuzzy set(which specifies the degree of non-membership). The concept behind intuitionistic fuzzy relations and graphs were discussed in [2, 4].

Intuitionistic Fuzzy Graphs(IFGs), Intuitionistic Fuzzy Hypergraphs(IFHGs) and Intuitionistic Fuzzy Directed Hypergraphs(IFDHG) have been introduced in [9, 10, 11]. Some types of IFDHGs are discussed in [8]. A novel decision-making approach based on hypergraphs in IF environment has been discussed in [7]. Lanzhen Yang and Hua Mao [12] introduced intuitionistic fuzzy threshold graph and explained its applications.

In this research paper, Section 2 elaborates on fundamental definitions, while Section 3 includes mathematical definitions of IFTHG and its related extensions. Furthermore, Section 4 outlines the implementation of IFTHG for fugitive determination, offering a discussion on simulation results and examples.

2. PRELIMINARIES

In this section, we go through certain important concepts that related to our main concept.

Definition 2.1. [3] Let a set E be fixed. An *Intuitionistic Fuzzy Set (IFS)* U in E is an object of the form $U = \{< u_i, \mu_i(u_i), \nu_i(u_i) > | u_i \in E\}$, where the function $\mu_i: E \rightarrow [0,1]$ and $\nu_i: E \rightarrow [0,1]$ determine the degree of membership & the degree of non-membership of the element $u_i \in E$, respectively and for every $u_i \in E$, $0 \leq \mu_i(u_i) + \nu_i(u_i) \leq 1$.

Definition 2.2. [3] Let E be the fixed set and $U = \{< u_i, \mu_i(u_i), \nu_i(u_i) > | u_i \in E\}$, be an IFS. Six types of Cartesian products of n subsets (crisp sets) $U_1, U_2, \ldots, U_n$ of U over E are defined as follows

$$U_{i1} \times_1 U_{i2} \times_1 U_{i3} \times_1 \ldots \times_1 U_{in} = \left\{ \langle (u_1, u_2, \ldots, u_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle \,|\, u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \right\}$$

$$U_{i1} \times_2 U_{i2} \times_2 U_{i3} \times_2 \ldots \times_2 U_{in} = \left\{ \langle (u_1, u_2, \ldots, u_n), \sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \ldots \neq n} \mu_i \mu_j \mu_k \ldots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle \,|\, u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \right\}$$

$$U_{i1} \times_3 U_{i2} \times_3 U_{i3} \times_3 \ldots \times_3 U_{in} = \left\{ \langle (u_1, u_2, \ldots, u_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i \neq j} \nu_i \nu_j + \sum_{i \neq j \neq k} \nu_i \nu_j \nu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \ldots \neq n} \nu_i \nu_j \nu_k \ldots \nu_n + (-1)^{n-1} \prod_{i=1}^{n} \nu_i \rangle \,|\, u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \right\}$$

$$U_{i1} \times_4 U_{i2} \times_4 U_{i3} \times_4 \ldots \times_4 U_{in} = \{ \langle (u_1, u_2, \ldots, u_n), min(\mu_1, \mu_2, \ldots, \mu_n), max(\nu_1, \nu_2, \ldots, \nu_n) \rangle | u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \}$$

$$U_{i1} \times_5 U_{i2} \times_5 U_{i3} \times_5 \ldots \times_5 U_{in} = \{ \langle (u_1, u_2, \ldots, u_n), max(\mu_1, \mu_2, \ldots, \mu_n), \min(\nu_1, \nu_2, \ldots, \nu_n) \rangle | u_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n \}$$

$$U_{i1} \times_6 U_{i2} \times_6 U_{i3} \times_6 \dots \times_6 U_{in} = \left\{ \langle (u_1, u_2, \dots, u_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n} \rangle \,|u_1 \in U_1, u_2 \in U_2, \dots, u_n \in U_n \right\}$$

It must be noted that $u_i \times_s u_j$ is an IFS, where $s = 1,2,3,4,5,6$.

Definition 2.3. [9] An *Intuitionistic Fuzzy Graph (IFG)* is of the form $G = (U, E)$, where

(i) $U = \{u_1, u_2, \dots, u_n\}$ such that $\mu_i: U \to [0,1]$ and $\nu_i: U \to [0,1]$ denote the degrees of membership & non-membership of the element $u_i \in U$ respectively and $0 \le \mu_i(u_i) + \nu_i(u_i) \le 1$ for every $u_i \in U$, $i = 1,2, \dots, n$.

(ii) $E \subseteq U \times U$ where $\mu_{ij}: U \times U \to [0,1]$ and $\nu_{ij}: U \times U \to [0,1]$ are such that $\mu_{ij} \le \mu_i \wedge \mu_j$, $\nu_{ij} \le \nu_i \vee \nu_j$ and $0 \le \mu_i(u_i) + \nu_i(u_i) \le 1$, where μ_{ij} and ν_{ij} are the membership & non-membership values of the edge (ν_i, ν_j); the values of $\mu_i \wedge \mu_j$ and $\nu_i \vee \nu_j$ can be determined by any one of the cartesian products $\times_s$ where $s = 1,2,3,4,5,6$, $\forall\, i \,\&\, j$ given in Definition 2.2.

Note: Throughout this paper, it is assumed that the fourth Cartesian product $U_{i1} \times_4 U_{i2} \times_4 U_{i3} \times_4 \dots \times_4 U_{in} = \{\langle (u_1, u_2, \dots, u_n),\ min(\mu_1, \mu_2, \dots, \mu_n),$ $max\,(\nu_1, \nu_2, \dots, \nu_n) \rangle |\, u_1 \in U_1, u_2 \in U_2, \dots, u_n \in U_n\}$, is used to determine the edge membership μ_{ij} and the edge non-membership ν_{ij}.

Definition 2.4. [10] An *Intuitionistic Fuzzy Hypergraph(IFHG)* is an ordered pair $H = (U, E)$ where

(i) $U = \{u_1, u_2, \dots, u_{n,}\}$, is a finite set of IF vertices,

(ii) $E = \{E_1, E_2, \dots, E_m\}$ is a family of crisp subsets of U,

(iii) $E_j = \{u_i, \mu_j(u_i), \nu_j(u_i) | 0 \le \mu_j(u_i) + \nu_j(u_i) \le 1\}$, $j = 1,2, \dots, m$

(iv) $E_j \ne \emptyset$, $j = 1,2, \dots, m$

(v) $\cup_j \operatorname{supp}(E_j) = U$, $j = 1,2, \dots, m$

Here, the hyperedges E_j are crisp sets of IF vertices, $\mu_j(u_i) \,\&\, \nu_j(u_i)$ denotes the degrees of membership & non-membership of vertex u_i to hyperedge E_j.

Notations - list[8]

- $< \mu(u_i), \nu(u_i) >$ or simply $< \mu_i, \nu_i >$ denote the degrees of membership & non-membership of the vertex $u_i \in U$ such that $0 \le \mu_i + \nu_i \le 1$.
- $< \mu(u_j), \nu(u_j) >$ or simply $< \mu_j, \nu_j >$ denote the degrees of membership & non-membership of the hyperedge $(u_i, u_j) \in U \times U$, such that $0 \le \mu_j + \nu_j \le 1$.
- μ_{ij} and ν_{ij} are the membership & non-membership value of i^{th} vertex in j^{th} hyperedge.
- Support of an IFS U in E is denoted by $supp(E_j) = \{u_i \,|\, \mu_j(u_i) > 0 \,\&\, \nu_j(u_i) > 0\}$.

3. Intuitionistic Fuzzy Threshold Hypergraph

Definition 3.1. The *Intuitionistic Fuzzy Threshold Hypergraph*(IFTHG) is defined as $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ where,

(i) $U = \{u_1, u_2, \dots, u_{n,}\}$ is a finite set of IF vertices
(ii) $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_m\}$ is a family of crisp subsets of U
(iii) $\mathcal{E}_j = \{u_i, \mu_j(u_i), \nu_j(u_i) | 0 \le \mu_j(u_i) + \nu_j(u_i) \le 1\},\ j = 1,2, \dots, m$
(iv) $\mathcal{E}_j \neq \emptyset,\ j = 1,2, \dots, m$
(v) $\cup_j \operatorname{supp}(\mathcal{E}_j) = U,\ j = 1,2, \dots, m$
(vi) an independent set $V \subseteq U$ has a set of all distinct combinations of a non-adjacent vertices in $\mathbb{H}_{\mathbb{G}}$ iff there exists a threshold values s_1 & $s_2 > 0$ such that $\sum_{u_i \in V} \mu_j(u_i) \le s_1$ & $\sum_{u_i \in V} (1 - \nu_j(u_i)) \le s_2$.

Example

Consider an IFTHG $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; 0.4, 0.6)$ with $U = \{u_1, u_2, \dots, u_{7,}\}$, $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$.

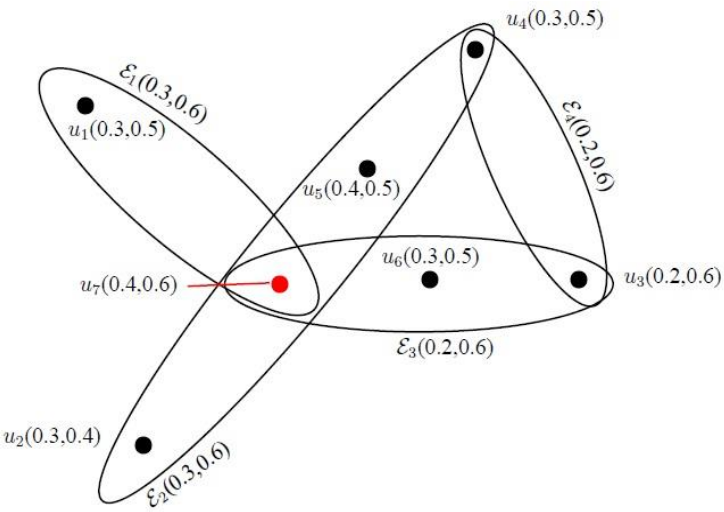

Fig 3.1: Intuitionistic Fuzzy Threshold Hypergraph HG

Adjacency matrix of the above IFTHG is represented as follows:

	u_1	u_2	u_3	u_4	u_5	u_6	u_7
u_1	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$
u_2	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$
u_3	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.2,0.6\rangle$
u_4	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$
u_5	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.6\rangle$
u_6	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.6\rangle$
u_7	$\langle 0.3,0.6\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0,1\rangle$

Note: Intuitionistic fuzzy hypergraphs is a special case of the intuitionistic fuzzy threshold hypergraphs.

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Definition 3.2. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. The *adjacency level between two vertices* u_i and u_{i+1}, denoted by $\gamma(u_i, u_{i+1})$, is defined by

$$\gamma(u_i, u_{i+1}) = \max_j(\min\left(\mu_j(u_i)\right), (\mu_j(u_{i+1}))), \min_j(\max\left(\nu_j(u_i)\right), (\nu_j(u_{i+1}))),$$

where $i = 1,2, \dots, n$ & $j = 1,2, \dots, m$ for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Definition 3.3. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. The *adjacency level between the hyperedges* $\mathcal{E}_j$ and $\mathcal{E}_k$, denoted by $\sigma(\mathcal{E}_j, \mathcal{E}_k)$, is defined by

$$\sigma(\mathcal{E}_j, \mathcal{E}_k) = \max_j(\min\left(\mu_j(u_i)\right), \mu_k(u_i)), \min_j(\max\left(\nu_j(u_i)\right), \nu_k(u_i)),$$

where $i = 1,2, \dots, n$ & $j, k = 1,2, \dots, m$ for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Definition 3.4. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. The *Strength* δ of a hyperedge $\mathcal{E}_j$ is the minimum membership $\mu_j(u_i)$ and maximum non- membership $\nu_j(u_i)$ of vertices in the hyperedge $\mathcal{E}_j$. Then,

$$\delta(\mathcal{E}_j) = (\min_{u_i}(\mu_j(u_i)), \max_{u_i}(\nu_j(u_i))$$

for every $\mu_j(u_i) > 0, \nu_j(u_i) > 0$ for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Definition 3.5. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. Then the *hyper walk* is a sequence of vertices $u_1, u_2, \dots, u_n$, not necessarily distinct, if at least one of the $\mu_j(u_i, u_{i+1})$ & $\nu_j(u_i, u_{i+1})$ are different from zero, for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Definition 3.6. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. Then the *intuitionistic fuzzy threshold hyperpath* $\mathcal{P}$ of length k in an IFTHG is defined as a sequence say, $u_1, \mathcal{E}_1, u_2, \mathcal{E}_2, \dots, u_k, \mathcal{E}_k, u_{k+1}$ of distinct vertices u_i's and hyperedges $\mathcal{E}_j$'s such that

(i) $\mu_j(\mathcal{E}_j) > 0$, for all $1 \leq j \leq k$
(ii) $u_i, u_{i+1} \in \mathcal{E}_j$, for all $1 \leq j \leq k$

for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Definition 3.7. Consider $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG on a non-empty set U is *connected* if every two distinct vertices in $\mathbb{H}_{\mathbb{G}}$ are linked by an intuitionistic fuzzy threshold hyperpath for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$. Otherwise, $\mathbb{H}_{\mathbb{G}}$ is *disconnected.*

Definition 3.8. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG. Then the *score* value of a vertex u_i and hyperedge $\mathcal{E}_j$ is denoted as $\mathcal{S}(u_i) = \frac{1 - \nu_i(u_i)}{2 - \mu_i(u_i) - \nu_i(u_i)}$ & $\mathcal{S}(\mathcal{E}_j) = \frac{1 - \nu_j(\mathcal{E}_j)}{2 - \mu_j(\mathcal{E}_j) - \nu_j(\mathcal{E}_j)}$ respectively, for which $\sum_{u_i \in V} \mu_j(u_i) \leq s_1$ & $\sum_{u_i \in V}(1 - \nu_j(u_i)) \leq s_2$.

Theorem 3.1. *If $d_{uv} \leq s_1$ and $(1 - d_{uv}) \leq s_2$, $\forall u, v \in \mathcal{E}$ in the IFTHG, then the IFTHG is connected.*

Proof. Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG, where U represents the set of all vertices $u_1, u_2, \ldots, u_n$ and $\mathcal{E}$ represents the set of all hyperedges $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n$. Let d_{uv} denote the distance between any two connected vertices *u* and *v* and let s_1, s_2 denotes the threshold values.

Assume two arbitrary vertices *u* and *v* which are connected in the IFTHG. $\exists$ a path $P: u = u_1\ \mathcal{E}_1\ u_2\ \mathcal{E}_2 \ldots u_m\ \mathcal{E}_m\ u_{m+1}$ between any 2 vertices u & v in the IFTHG with the condition that the distance d_{uv} between u & v must satisfy the condition $d_{uv} \leq s_1$ and $(1 - d_{uv}) \leq s_2$. Since all the hyperedges are connected with each other, it is proved that the IFTHG is connected.

Theorem 3.2. *Let $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be the IFTHG. Then $\mathbb{H}_{\mathbb{G}}$ is disconnected iff every non-empty subsets U_1, U_2 of U such that $U_1 \cup U_2 = U, U_1 \cap U_2 = \emptyset$ and there is no hyperedge $\mathcal{E}_j \in \mathcal{E}$ which has one vertex in U_1 and another vertex in U_2.*

Proof. Suppose that $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ is a disconnected IFTHG. Then $\exists$ a vertices $u, v \in U$ such that there is no path between *u* and *v*.

Let $U_u = \{u \in U | \exists \text{ a path between } u \,\&\, v\}$.

Then, clearly $U_u \neq \emptyset$.

Let $U_v = U - U_u$. Since $v \notin U_u \Longrightarrow v \in U_v$.

Now, $U_u \neq \emptyset, U_v \neq \emptyset$ and $U_u \cup U_v = U$.(also $U_u \cap U_v = \emptyset$).

Suppose there exists a hyperedge $\mathcal{E}_j = \{u_1, u_2\} \in \mathcal{E}$ such that $u_1 \in U_u, u_2 \in U_v$.
Now u_1 and u are connected as $\mathcal{E}_j = \{u_1, u_2\} \Longrightarrow u$ and u_2 are also disconnected.
$\Rightarrow u_2 \in U_u$
which is a contradiction. Hence the result holds good.
Conversely, assume that $\exists$ a partition of U such that $U_1 \cup U_2 = U, U_1 \cap U_2 = \emptyset,\ U_1 \neq \emptyset, U_2 \neq \emptyset$ and there exists no hyperedge $\mathcal{E}_j \in \mathcal{E}$ having one vertex in U_1 and another vertex in U_2.
To prove: $\mathbb{H}_{\mathbb{G}}$ is disconnected.
Suppose $\mathbb{H}_{\mathbb{G}}$ is connected.
Take $u \in U_1 \subseteq U, v \in U_2 \subseteq U$.

$\Longrightarrow u, v \in U_1 \cup U_2 = U$ [Since $\mathbb{H}_{\mathbb{G}}$ is connected].

Therefore, $\exists$ a path $P: u = u_1\mathcal{E}_1 u_2 \mathcal{E}_2 \ldots u_m \mathcal{E}_m u_{m+1} = v$ connecting u and *v*.

Since $u \in U_1, v \in U_2$ and $U_1 \cap U_2 = \emptyset$.

$\Longrightarrow$ there exists an i such that $u_i \in U_1, u_{i+1} \in U_2$. Now $\mathcal{E} = \{u_i, u_{i+1}\} \in \mathcal{E}$ such that one vertex is in U_1 and another vertex is in U_2. which is contradiction.
$\Longrightarrow$ $\mathbb{H}_{\mathbb{G}}$ is disconnected IFTHG.

Theorem 3.3. *Let $\mathbb{H}_{\mathbb{G}}$ be an IFTHG, with vertices $u, v \in U$. There is a $u-v$ hyperwalk in $\mathbb{H}_{\mathbb{G}}$ iff there exists a $u-v$ hyperpath.*

Proof. Assume $\mathbb{H}_{\mathbb{G}} = (U, \mathcal{E}; s_1, s_2)$ be an IFTHG and W is a hyperwalk in $\mathbb{H}_{\mathbb{G}}$. The theorem is proved by using mathematical induction on length of W. If W be the length 1 or 2, then it is obvious that W is a hyperpath in $\mathbb{H}_{\mathbb{G}}$.
Now, assume the result is possible for every hyperwalks of length less than k, and consider W has length k, which implies W is, $u = u_0, u_1, u_2, \dots, u_{k-1}, u_k = v$ where the vertices are not necessarily distinct. If the vertices are distinct then W itself be a desired $u-v$ hyperpath. If not, then assume j is a smallest integer such that $u_j = u_r$ for some $r > j$. Assume W_1 is a hyperwalk in $\mathbb{H}_{\mathbb{G}}$, is
$u = u_0, u_1, u_2, \dots, u_j, u_{r+1}, \dots, u_k = v$.
This hyperwalk has length strictly less than k, and then the induction hypothesis gives that W_1 has a $u-v$ hyperpath in $\mathbb{H}_{\mathbb{G}}$. This means that W contains a $u-v$ hyperpath and proof is complete. The converse part is obviously holds.

4. Utilizing the IFTHG Model for Robot-Based Security Applications

The IFTHG model proposes employing robots as security guards in order to reduce risks and ensure public safety in situations when the robots are able to catch fugitives and keep control until the police arrives. Security bots would be alerted and transmit a message to the control room in the event of an unusual occurrence such as robbery, heist, gunshot or accident.

Consider a large mall comprising two significant compartments with 11 and 10 robots respectively, each equipped with a primary control room.

- In the IFTHG model, individual robots are represented as vertices, de- noted by $U = \{u_1, u_2, \dots, u_{21}, K, L\}$, while hyperedges could represent complex relationships or interactions between multiple robots in each block of mall which are denoted by $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_{10}\}$.
- Every robot is attached to two main processing systems, K and L, which direct the robots on how to act.
- There are two intuitionistic fuzzy threshold subhypergraphs(IFTsHGs) based on the major control systems K and L. The first IFTsHG has 11 robots $\{u_1, u_2, \dots, u_{11}\}$ linked with main control system K with hyperedges $\{\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_5\}$, and the second IFTsHG has 10 robots$\{u_{12}, \dots, u_{21}\}$ connected to L with hyperedges $\{\mathcal{E}_6, \dots \mathcal{E}_9\}$.
- Additionally, the hyperedge S_{10} is connected to the major control systems $K \& L$.

To regulate the maximum and minimum values for bots, threshold values are essential. Vertices of IFTHGs indicate where the robots are situated and how accessible they are possible to escape routes. Almost every exit gate and crowded area has at least one robot. Robots have been spread out around the area and are ready to go on tasks before any alarm case. This algorithm is used to find in which gate the fugitives are using in the situation that the security department suddenly develops an alarm case concerning any suspicious persons. After this, the security sections goal is to pursue any suspicious fugitives until the police forces get involved in

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this pursuit.

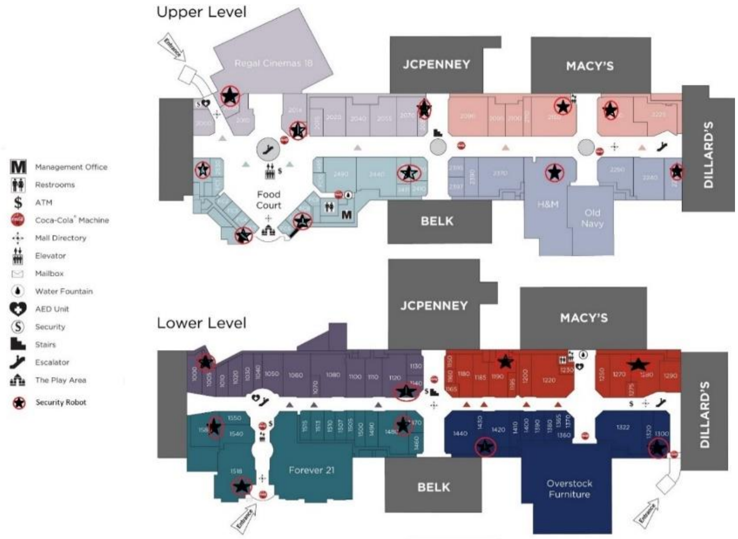

Fig 4.1: Large mall with security robots

Some notions of membership and non-membership in this scenario are denoted as follows:

- $\mu_j(u_n)$ represents where the robots are located and how accessible they are to possible escape routes.
- $v_j(u_n)$ reflect how far a robot is from these critical areas.
- $\mu_{ij}(\mathcal{E}_m)$ represents the overall significance of a hyperedge in enhancing the system's efficiency and effectiveness within addressing block.
- $v_{ij}(\mathcal{E}_m)$ reflects the degree of non-belongingness or lack of significance of a hyperedge within addressing block.

The following IFTHG depicts the pictorial representation of fig. 4.1

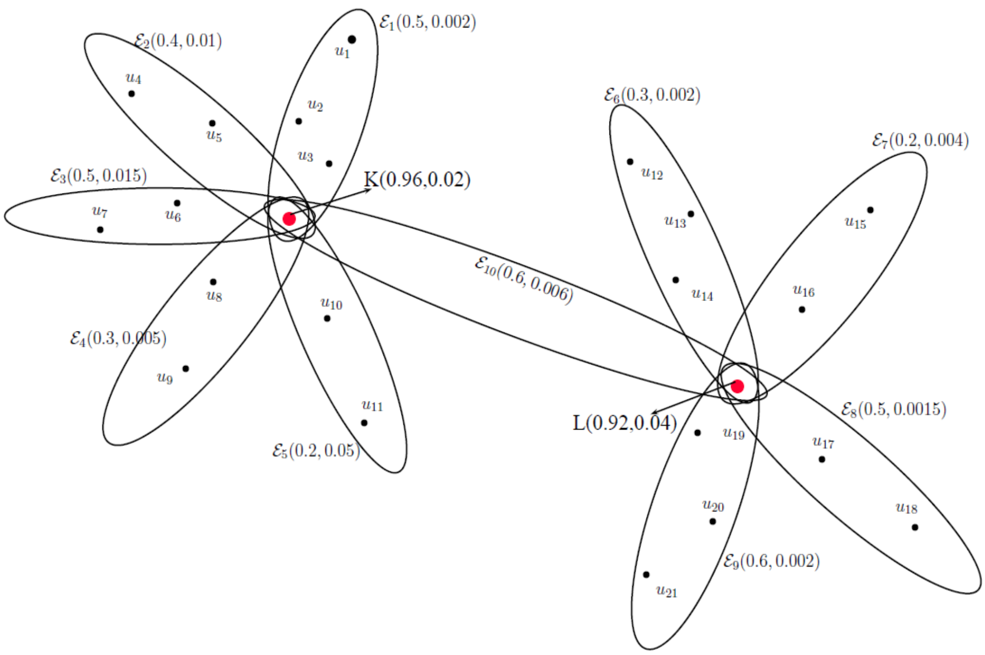

Fig 4.2: IFTHG linked with multi robot system

The values of vertices are tabulated below:

Table 1: Score values

S.No	$u_i/\mathcal{E}_j$	(μ, ν)	Score
1	u_1	(0.6,0.002)	0.7139
2	u_2	(0.5,0.0015)	0.6663
3	u_3	(0.55,0.002)	0.6892
4	u_4	(0.5,0.001)	0.6894
5	u_5	(0.4,0.001)	0.6248
6	u_6	(0.5,0.001)	0.6664
7	u_7	(0.6,0.0015)	0.7140
8	u_8	(0.3,0.004)	0.5875
9	u_9	(0.5,0.005)	0.6656
10	u_{10}	(0.3,0.05)	0.5758
11	u_{11}	(0.2,0.045)	0.5442
12	u_{12}	(0.5,0.001)	0.6664
13	u_{13}	(0.3,0.002)	0.5878
14	u_{14}	(0.4,0.0015)	0.6246
15	u_{15}	(0.6,0.003)	0.7137
16	u_{16}	(0.2,0.004)	0.5547
17	u_{17}	(0.5,0.001)	0.6664
18	u_{18}	(0.7,0.0015)	0.7688
19	u_{19}	(0.6,0.001)	0.7141
20	u_{20}	(0.7,0.002)	0.7689
21	u_{21}	(0.65,0.0015)	0.7404
22	K	(0.96,0.02)	0.9608
23	L	(0.92,0.04)	0.9423
24	$\mathcal{E}_1$	(0.5,0.002)	0.6662
25	$\mathcal{E}_2$	(0.4,0.01)	0.6226
26	$\mathcal{E}_3$	(0.5,0.015)	0.6633
27	$\mathcal{E}_4$	(0.3,0.005)	0.5870
28	$\mathcal{E}_5$	(0.2,0.05)	0.5429
29	$\mathcal{E}_6$	(0.3,0.002)	0.5878
30	$\mathcal{E}_7$	(0.2,0.004)	0.5546
31	$\mathcal{E}_8$	(0.5,0.0015)	0.6663
32	$\mathcal{E}_9$	(0.6,0.002)	0.7139
33	$\mathcal{E}_{10}$	(0.6,0.006)	0.7131

An overview of the proposed simulation is represented as follows:

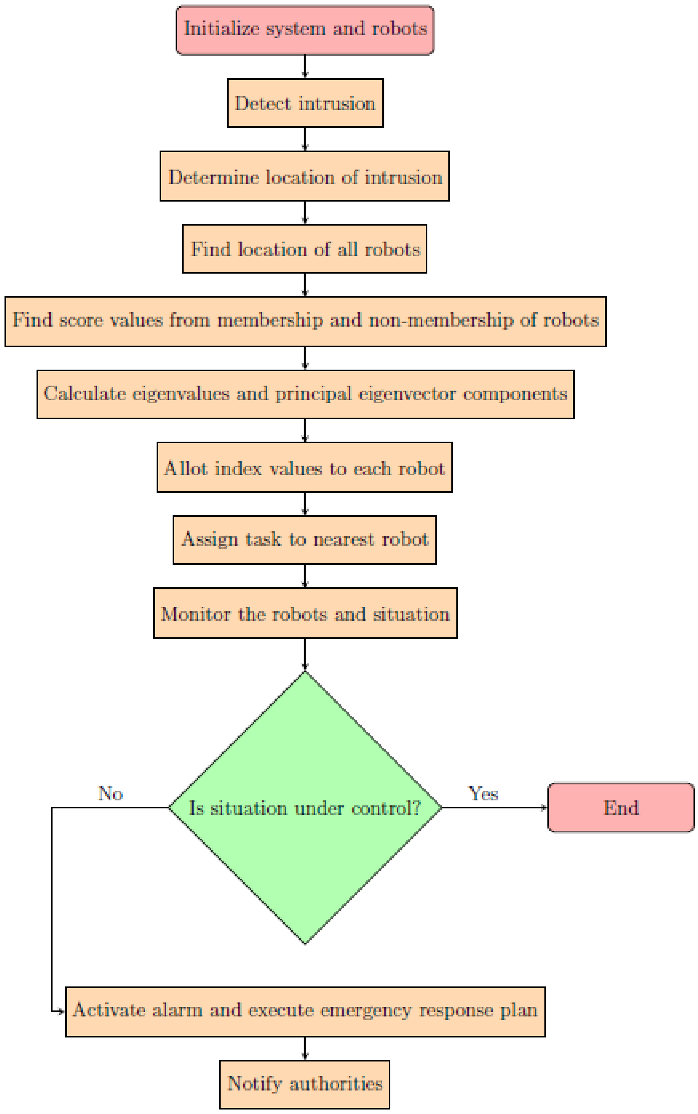

Fig 4.3: Flow chart of simulated IFTHG model

This situation describes a scenario where there's a main control system, referred to as K, which is responsible for overseeing and managing a certain environment or system. When the main control system detects an intrusion or a threat, it triggers an algorithm to initiate a series of calculations.

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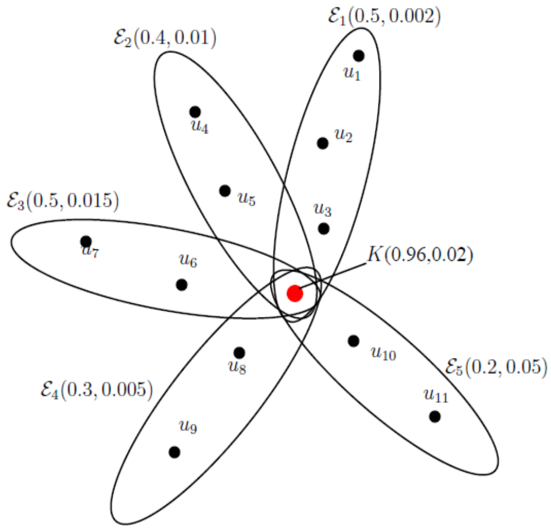

Fig:4.4 IFTsHG: Robots linked to K.

The following tables 2 and 3 represents the adjacency matrix and score values respectively of fig. 4.4

Table 2: Adjacency matrix of robots linked to K

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	K
u_1	$\langle 0,1\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.002\rangle$
u_2	$\langle 0.5,0.002\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.002\rangle$
u_3	$\langle 0.5,0.002\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.002\rangle$
u_4	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.4,0.01\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.4,0.01\rangle$
u_5	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.4,0.01\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.4,0.01\rangle$
u_6	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.015\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.015\rangle$
u_7	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.015\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.5,0.015\rangle$
u_8	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.005\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.005\rangle$
u_9	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.005\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.3,0.005\rangle$
u_{10}	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.05\rangle$	$\langle 0.2,0.05\rangle$
u_{11}	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.05\rangle$	$\langle 0,1\rangle$	$\langle 0.2,0.05\rangle$
K	$\langle 0.5,0.002\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0.5,0.002\rangle$	$\langle 0.4,0.01\rangle$	$\langle 0.4,0.01\rangle$	$\langle 0.5,0.015\rangle$	$\langle 0.5,0.015\rangle$	$\langle 0.3,0.005\rangle$	$\langle 0.3,0.005\rangle$	$\langle 0.2,0.05\rangle$	$\langle 0.2,0.05\rangle$	$\langle 0,1\rangle$

Table 3: Score values of robots linked to K

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	K
u_1	0	0.6662	0.6662	0	0	0	0	0	0	0	0	0.6662
u_2	0.6662	0	0.6662	0	0	0	0	0	0	0	0	0.6662
u_3	0.6662	0.6662	0	0	0	0	0	0	0	0	0	0.6662
u_4	0	0	0	0	0.6226	0	0	0	0	0	0	0.6226
u_5	0	0	0	0.6226	0	0	0	0	0	0	0	0.6226
u_6	0	0	0	0	0	0	0.6633	0	0	0	0	0.6633
u_7	0	0	0	0	0	0.6633	0	0	0	0	0	0.6633
u_8	0	0	0	0	0	0	0	0	0.5870	0	0	0.5870
u_9	0	0	0	0	0	0	0	0.5870	0	0	0	0.5870
u_{10}	0	0	0	0	0	0	0	0	0	0	0.5429	0.5429
u_{11}	0	0	0	0	0	0	0	0	0	0.5429	0	0.5429
K	0.6662	0.6662	0.6662	0.6226	0.6226	0.6633	0.6633	0.5870	0.5870	0.5429	0.5429	0

If a thread is detected under the major control system K, then K will promptly initiate the following algorithm, proceeding step by step.

Algorithm:

Step 1: Initialize system and robots class
Robot:
def__init__(self, id, location):
self.id = id self.location = location
Step 2: Find any intrusion and determine its location intrusion detected
= True
Step 3: Find location of all robots def find
robot locations(robots):
return robot.id: robot.location for robot in robots robot locations =
find robot locations(robots)
Step 4: Find score value of robots
def calculate score(1-non membership value/2-membership value- non
membership value):
score values =
for robot id, location in robot locations.items():
return score values
Step 5: Calculate eigenvalues and eigenvectors A =
np.array([])
eigenvalues, eigenvectors = np.linalg.eig(A)
Step 6: Assign index values to robots based on eigenvalues sorted
indices = np.argsort(eigenvalues)
index values = robot: index for index, robot in enumerate(sorted indices)
Step 7: Assign task to nearest robot
task assigned = f"Task assigned to Robot nearest robot id"
Step 8: Monitor the robots and situation situation
under control = True
Step 9: Check if situation is under control if situation
under control:
print("Situation is under control. Continuing monitoring.") else:
print("Situation is not under control. Activating alarm and executing emergency response plan. Notifying authorities.")

The score values of the robots indicate their accessibility. This plot illustrates the relationship between the score values of robots in two compartments. From this plotted graph, robots connected to the main control system L demonstrate higher accessibility compared to those connected to K.

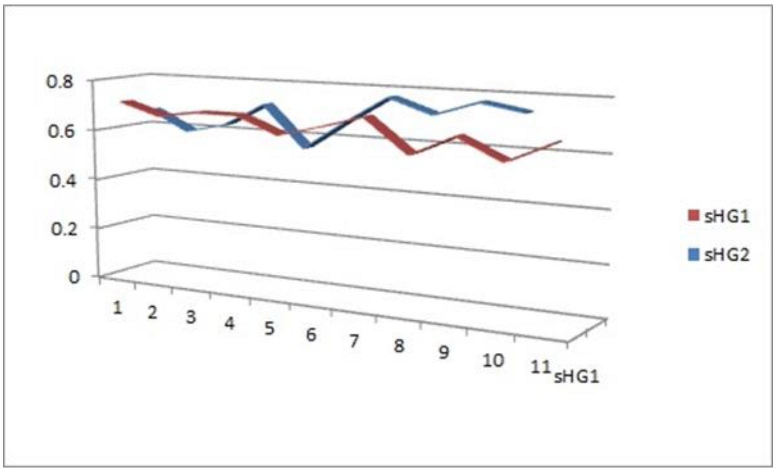

Fig 4.5: Score values of the robots are linked to K & L

4.1 Simulation Results

- Score values are employed to precisely locate security robots, utilizing the IFTHG model to calculate distances between them. These robots establish communication channels using radio frequency technologies, facilitating smooth information exchange and coordination in surveillance operations.
- In order to obtain communication and distance information, the adjacency matrix, its eigenvalues, and related index values have been used.
- The adjacency matrix illustrates the distances between robots that establish communication links with each other.
- The highest eigenvalue represents the principal eigenvector of the system, from which the main control system directs the nearest robot seize the fugitives.

5. CONCLUSION

This paper explains about the basic concepts of IFTHG and some of its properties. IFTHGs provide a more flexible approach to real-time decision making by representing uncertainty and vagueness in robot configuration. Also an algorithm designed to identify the nearest robot index to seize the fugitives when an intrusion is detected in a large area. Future work will explore the intuitionistic fuzzy hesitancy threshold hypergraphs and its application in real life scenario.

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