

$(1, 2)^* \text{-} D^{**} \text{Sp}$ Open Sets in Bitopological Spaces

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Abstract

This article explains the concept of $(1, 2)^* \text{-} D^{**}$ semi- pre-open sets based on the concepts of semi- preopen sets and semi- pre- continuity in topological space. In addition to that the concept of $(1, 2)^* \text{-} D^{**} \text{Sp}$ generalized continuous maps and generalized homeomorphisms are also discussed.

Keywords: $(1, 2)^*$ -Open map, $(1, 2)^* \text{-} D^{**} \text{SpOS}$, $(1, 2)^* \text{-} D^{**} \text{Sp}$ Continuous map.

1. Introduction

Bhattacharya and Lahiri [1] introduced a new class of semi generalized open sets by means of semi open sets introduced by Levine [5]. In view of that we introduce a new class of open sets namely $(1, 2)^* \text{-} D^{**} \text{Sp}$ open sets and their properties are also studied.. Also $(1, 2)^* \text{-} D^{**} \text{Sp}$ Continuous maps, irresolute maps, $(1, 2)^* \text{-} D^{**} \text{Sp}$ Connected sets, homeomorphism are also studied with their characterizations.

2. Preliminaries

Entire area of this paper, (X, τ_1, τ_2) X will denote bitopological space (briefly, BTPS).

Definition 2.1: Let H be a subset of X . Then H is said to be $\tau_{1,2}$ -open [7] if $H = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$

The complement of $\tau_{1,2}$ -open set is called $\tau_{1,2}$ -closed.

Notice that $\tau_{1,2}$ -open sets need not necessarily form a topology

Note; $\tau_{1,2}$ -open sets need not necessarily form a topology.

Definition 2.2 [7]: Let H be a subset of a bitopological space X . Then

- (i) the $\tau_{1,2}$ -closure of H , denoted by $\tau_{1,2}\text{-cl}(H)$, is defined as $\{F : H \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$
- (ii) the $\tau_{1,2}$ -interior of H , denoted by $\tau_{1,2}\text{-int}(H)$, is defined as $\{F : F \subseteq H \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}$

Definition 2.3: A subset H of a BTPS X is called:

- (i) $(1, 2)^*$ -semi-open set [8] if $H \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(H))$;
- (ii) $(1, 2)^*$ -preopen set [6] if $H \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(H))$;
- (iii) $(1, 2)^*$ - α -open set [3] if $H \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(H)))$;
- (iv) regular $(1, 2)^*$ -open set [6] if $H = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(H))$.
- (v) $(1, 2)^*$ -gsp-closed [10, 11] if $(1, 2)^*\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^*$ -open. Then complement of $(1, 2)^*$ -gsp-closed set is called $(1, 2)^*$ -gsp-open set.

The complements of the above-mentioned open sets are called their respective closed sets.

3. Definition 2.4: [2] A subset H of a space (X, τ) is called ω -cld if it contains all its condensation points. The complement of ω -cld set is called ω -open.

4. Definition 2.5. A bijection $f: X \rightarrow Y$ is called $(1, 2)^*$ -homeomorphism [4] if f is bijection, $(1, 2)^*$ -continuous and $(1, 2)^*$ -open.

5. Definition 2.6: A subset A of X is called $(1, 2)^*\text{-D}^*\text{-cld}$ (briefly, $(1, 2)^*\text{-D}^*\text{-cld}$) if $(1, 2)^*\text{-scl}^*(A) \subseteq (1, 2)^*\text{-int}(U)$ whenever $A \subseteq U$ and U is $(1, 2)^*\text{-}\omega$ -open. The complement of $(1, 2)^*\text{-D}^*\text{-cld}$ set is called $(1, 2)^*\text{-D}^*\text{-open}$.

6. Definition 2.7: $(1, 2)^*\text{-D}^{**}\text{-closed}$ (briefly, $(1, 2)^*\text{-D}^{**}\text{-cld}$) if $(1, 2)^*\text{-spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^*\text{-D}^*\text{-open}$. The complement of $(1, 2)^*\text{-D}^{**}\text{-closed}$ set is called $(1, 2)^*\text{-D}^{**}\text{-open}$. The class of all $(1, 2)^*\text{-D}^{**}\text{-cld}$ in X is denoted by $(1, 2)^*\text{-D}^{**}\text{-C}(X)$.

The complements of the above mentioned open sets are called their respective closed sets.

3 $(1, 2)^*\text{-D}^{**}\text{spOpen Sets}$

Definition 3.1

For $S \subseteq X$, $(1, 2)^*\text{-spcl}^{**}(S) = \cap \{K/S \subseteq K, K \text{ is } (1, 2)^*\text{gspClosed}\}$.

Remark 3.2

$(1, 2)^*\text{-spcl}^{**}(S) =$ Kuratowski closure operator on X .

Definition 3.3

$S \subseteq X$, $(1, 2)^*\text{-D}^{**}\text{spOpen}$ iff there exists an $\tau_{1,2}\text{OS}$ U Such that $U \subseteq S \subseteq (1, 2)^*\text{-spcl}^{**}(U)$. in X .

Example 3.4

Let $G = \{1, 2, 3\}$, $\tau_1 = \{G, \varphi, \{1\}\}$ and $\tau_2 = \{G, \varphi, \{1\}, \{1, 2\}\}$. Then $(1, 2)^*\text{-D}^{**}\text{spOS}$ of (X, τ_1, τ_2) are $X, \varphi, \{1\}, \{1, 2\}$ and $\{1, 3\}$.

Remark 3.5

If $C \subseteq X, D \subseteq X \ni C \subseteq D$, then we have $(1, 2)^*\text{-spcl}^{**}(C) \subseteq (1, 2)^*\text{-spcl}^{**}(D)$.

Theorem 3.6

For $S \subseteq X$. we have S is $(1, 2)^*$ - $D^{**}spOpen$
 iff $S \subseteq (1, 2)^*spcl^{**}(\tau_{1,2}Int(S))$.

Proof.

Assume S is $(1, 2)^*$ - $D^{**}spOpen$ in X . $U \subseteq S$ implies $U \subseteq \tau_{1,2}int(S)$. hence from Remark 3.5 and by defn 3.3, $(1, 2)^*spcl^{**}(U) \subseteq (1, 2)^*spcl^{**}(\tau_{1,2}Int(S))$. So $S \subseteq (1, 2)^*spcl^{**}(\tau_{1,2}Int(S))$.

To prove the converse let $S \subseteq (1, 2)^*spcl^{**}(\tau_{1,2}Int(S))$. substitute $U = \tau_{1,2}Int(S)$. finally by defn 3.3. S in X is $(1, 2)^*$ - $D^{**}spOS$.

Theorem 3.7 In BTS X

$$\tau_{1,2}OS(R) \Rightarrow (1, 2)^*D^{**}spOS(R)$$

Proof.

Assume R be a $\tau_{1,2}OS$ in X let
 $R = \tau_{1,2}Int(R) \subseteq (1, 2)^*spcl^{**}(\tau_{1,2}Int(R))$. We have S is $(1, 2)^*$ - $D^{**}spOS$ in X .

Example 3.8

Reverse of theorem 3.7 is proved by this example
 $S = \{1, 3\}$ is a $(1, 2)^*$ - $D^{**}spOS$ in X but not an $\tau_{1,2}OS$ in X .

Definition 3.9 In BTS X

$(1, 2)^*D^{**}spT_{1/2}$ space for every $(1, 2)^*D^{**}spOS$ is $\tau_{1,2}OS$

Remark 3.10 In

$(1, 2)^*T_{1/2}$ space, Every $(1, 2)^*SpOS$ is $(1, 2)^*$ - $D^{**}spOS$.

Remark 3.11

$(1, 2)^*spcl^{**}(A) \subseteq \tau_{1,2}spcl(A)$ for a subset A in X .

Theorem 3.12 In BTS X.

S is $(1, 2)^*$ - $D^{**}spOS \Rightarrow S$ is $(1, 2)^*$ - $spOS$.

Proof In X , Suppose S is $(1, 2)^*$ - $D^{**}spOS$ and By defn 3.3 and. By Remark 3.11, $(1, 2)^*spcl^{**}(U) \subseteq \tau_{1,2}spcl(U)$. Hence $U \subseteq S \subseteq \tau_{1,2}spcl(U) \Rightarrow S$ is $(1, 2)^*$ - $spOS$.

Example 3.13

To prove the reverse of thm 3.12 is not true assume $G = \{1, 2, 3\}$, $t_1 = \{G, \phi, \{2\}\}$ and $t_2 = \{G, \phi\}$.

Then $S = \{2, 3\}$ is $(1, 2)$ -spOS but not a $(1, 2)$ -D**spOS.

Remark 3.14

In BTS X

Consider

$$C \subseteq X, D \subseteq X$$

we have $(1, 2)$ -spcl** $(C \cup D) = (1, 2)$ -spcl** $(C) \cup (1, 2)$ -spcl** (D) .

Theorem 3.15

In BTS X Consider

$$C \subseteq X, D \subseteq X$$

We have

$$(1, 2)$$
-D**spOS

\Rightarrow $C \cup D$ is also a $(1, 2)$ -D**spOS .

Proof.

In BTS X

Suppose C and D are $(1, 2)$ -D**spOS in X . By defn & Remark3.14, $(1, 2)$ -spcl** $(S) \subseteq (1, 2)$ -spcl** (T)

$\Rightarrow (1, 2)$ -spcl** $(S \cup T)$.

$\Rightarrow S \cup T$ is also $(1, 2)$ -D**spOS .

Example 3.16 In BTS X

Suppose C, D are $(1, 2)$ -D**spOS \Rightarrow

$C \cap D$ may not $(1, 2)$ -D**spOS .

Let $X = \{1, 2, 3, 4\}$, $t_1 = \{X, \emptyset, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}\}$, $t_2 = \{X, \emptyset, \{1, 2\}, \{3, 4\}\}$.

Then the set $A = \{1, 2, 3\}$ and $B = \{3, 4\}$ are $(1, 2)$ -D**spOS in X and $A \cap B = \{3\}$ is not a $(1, 2)$ -D**spOS.

Theorem 3.17

In BTS X

Assume B be a $(1, 2)$ -D**spOS, $B \subseteq C$ and $B \subseteq C \subseteq ((1, 2)$ -spcl** $(\tau_{1,2} \text{Int}(A)))$. we have C is a $(1, 2)$ -D**spOS

Proof

From statement B is $(1, 2)$ -D**spOS and By Thm 3.16 $B \subseteq (1, 2)$ -spcl** $(\tau_{1,2} \text{Int}(B))$, Also $B \subseteq C$

$\Rightarrow \tau_{1,2}\text{-Int}(A) \subseteq \tau_{1,2}\text{-Int}(B)$. Hence, $(1, 2)\text{-spcl}^{**}(\tau_{1,2}\text{Int}(B)) \subseteq (1, 2)\text{-spcl}^{**}(\tau_{1,2}\text{Int}(C))$. We have $C \subseteq (1, 2)\text{-spcl}^{**}(\tau_{1,2}\text{Int}(B)) \subseteq (1, 2)\text{-spcl}^{**}(\tau_{1,2}\text{Int}(C))$ it proves C is $(1, 2)\text{-D}^{**}\text{spOS}$.

Remark 3.18

A Map

$h : L \rightarrow M$ is $(1, 2)\text{gspContinuous}$

$\Rightarrow f((1, 2)\text{-spcl}^{**}(A)) \subseteq 1,2\text{-spCl}(f(A))$.

Theorem 3.19

In BTS X

Suppose a map $h : L \rightarrow M$ be $(1, 2)\text{gspContinuous}$ and $(1, 2)\text{Open} \Rightarrow B$ is $(1, 2)\text{-D}^{**}\text{spOS}$

$\Rightarrow f(B)$ in Y is $(1, 2)\text{-spOS}$.

Proof

From statement B is $(1, 2)\text{-D}^{**}\text{spOS}$ in L . By defn 3.3 and. Remark 3.18,

$h((1, 2)\text{-$

$\text{spCl}^{**}(V)) \subseteq \sigma_{1,2}\text{-spCl}(h(B))$. We have $h(B) \subseteq h((1, 2)\text{-spCl}^{**}(V)) \subseteq \sigma_{1,2}\text{-spCl}(h(V))$. Also given h is $(1, 2)\text{Open Map}$ $h(V)$ in M is $\sigma_{1,2}\text{-Open}$. it follows that $h(B)$ in M is $(1, 2)\text{-spOS}$.

Theorem 3.20 in BTS X consider

A Map $h : L \rightarrow M$ be a $(1, 2)\text{homeomorphism}$. If B in L is $(1, 2)\text{-D}^{**}\text{spOS}$, then $h(B)$ is $(1, 2)\text{-D}^{**}\text{spOS}$ in M .

Proof

B is $(1, 2)\text{-D}^{**}\text{spOS}$ in L . By definition 3.3 $h(V) \subseteq h(B) \subseteq h((1, 2)\text{-spCl}^{**}(V))$. Also given h is $(1, 2)\text{homeomorphism}$ we have $h((1, 2)\text{-spCl}^{**}(V)) \subseteq (1, 2)\text{-spCl}^{**}h(V)$. it follows $h(V) \subseteq h(B) \subseteq (1, 2)\text{-spCl}^{**}(h(V))$. so that $h(B)$ in M is $(1, 2)\text{-D}^{**}\text{spOS}$

Theorem 3.21

in BTS X Consider A Map $h : L \rightarrow M$ if

h is $(1, 2)\text{homeomorphism}$.and B in M is $(1, 2)\text{-D}^{**}\text{spOS}$

then $\exists \tau_{1,2}\text{-OS}$ such that $h^{-1}(B)$ in L is $(1, 2)\text{-D}^{**}\text{spOS}$

Proof

From statement B in L is $(1, 2)\text{-D}^{**}\text{spOS}$. By defn 3.3 we have $h^{-1}(V) \subseteq h^{-1}(B) \subseteq h^{-1}((1,$

$(1, 2)$ - σ Cl ** (V)). and Since h is

$(1, 2)$ -homeomorphism $\Rightarrow h^{-1}((1, 2)$ - σ Cl ** (V)) $\subseteq (1, 2)$ - σ Cl ** (h^{-1} (V)). hence we have $h^{-1}(V) \subseteq h^{-1}(B) \subseteq (1, 2)$ - σ Cl ** (h^{-1} (V)) thus $h^{-1}(B)$ is $(1, 2)$ - σ OS

4. $(1, 2)$ - σ Closed and $(1, 2)$ - σ Open Mappings

Definition 4.1

A Map $h : L \rightarrow M$ is $(1, 2)$ - σ O-map if $h(V)$ in M is $(1, 2)$ - σ OS $\forall \tau_{1,2}$ OS V in L .

Theorem 4.2 A map

$h : L \rightarrow M$ is $(1, 2)$ -Open map $\Rightarrow (1, 2)$ - σ Open-Map

Proof.

From statement $h : L \rightarrow M$ is $(1, 2)$ -Open map .and G is $\tau_{1,2}$ -OS in L . we have $h(G)$ in M is $\sigma_{1,2}$ Open. From Theorem3.7, $h(G)$ is

$(1, 2)$ - σ OS in M .. Henceforth h is $(1, 2)$ - σ Open .

Example 4.3

Reverse of Theorem 4.2 is not true. by this example

Let $L = M = \{a_1, b_1, c_1\}$, $\tau_1 = \{L, \phi, \{a_1\}\}$, $\tau_2 = \{L, \phi, \{a_1, c_1\}\}$. Let $\sigma_1 = \{L, \phi, \{a_1\}\}$, $\sigma_2 = \{L, \phi, \{a_1, b_1\}\}$.

Let $h : L \rightarrow M$ is an identity map. we have h is

$(1, 2)$ - σ Open but h is not $(1, 2)$ -Open. map

Definition 4.4 The map

$h : L \rightarrow M$ is $(1, 2)$ - σ Closed Map if For every $\tau_{1,2}$ -CS V in L , $h(V)$ in M is $(1, 2)$ - σ Closed

Remark 4.5

$h : L \rightarrow M$ is $(1, 2)$ -Closed $\Rightarrow h$ is $(1, 2)$ - σ Closed but conversely not true

Proof.

From Theorem 4.2. proof is clear

5. $(1, 2)$ - σ Continuous Mappings

Definition 5.1 A map

$h : K \rightarrow L$ is called $(1, 2)$ - σ Continuous $\forall \sigma_{1,2}$ -OS in L its inverse image of h is $(1, 2)$ - σ Open in K ..

Theorem 5.2

A map

$$h : K \rightarrow L$$

h is $(1, 2)^*$ Continuous $\Rightarrow h$ is $(1, 2)^*$ -D**spContinuous.

Proof

Assume R as a $\sigma_{1,2}$ OS in L . Also h is $(1, 2)^*$ Continuous, $h^{-1}(R)$ is $\tau_{1,2}$ Open in X . From Theorem 3.7, $h^{-1}(R)$ is $(1, 2)^*$ -D**spOpen in X . Thus h is $(1, 2)^*$ -D**spContinuous.

Example 5.3 This example proves reverse of thm 5.2 Consider A map

$h : K \rightarrow L$ let the two sets $L = K = \{a_1, a_2, a_3\}$, $\tau_1 = \{L, \phi, \{a_1\}\}$, $\tau_2 = \{L, \phi, \{a_1\}, \{a_1, a_2\}\}$, $\sigma_1 = \{K, \phi, \{a_1\}\}$ and $\sigma_2 = \{K, \phi, \{a_1, a_3\}\}$. Suppose $h : K \rightarrow L$ be the identity map we have h is $(1, 2)^*$ -D**spContinuous but h is not $(1, 2)^*$ Continuous.

Remark 5.4

In $(1, 2)^*$ -D**sp-T1/2 space, every $(1, 2)^*$ -D**spContinuous map is $(1, 2)^*$ Continuous.

Theorem 5.5

$h : K \rightarrow L$ is a map. we have the below implications are true.

- 1 h is $(1, 2)^*$ -D**spContinuous.
- 2. For each $\sigma_{1,2}$ CS in L its inverse image is $(1, 2)^*$ -D**spClosed in K .

Proof.

(1) \Rightarrow (2) Let R is $\sigma_{1,2}$ CS in L . Then $L - R$ is $\sigma_{1,2}$ Open in L . Also h is $(1, 2)^*$ -D**spContinuous, $f^{-1}(L - R)$ is $(1, 2)^*$ -D**spOpen in K . so that we have $K / f^{-1}(R)$ is $(1, 2)^*$ -D**spOpen in $K \Rightarrow f^{-1}(R)$ is $(1, 2)^*$ -D**spClosed in K .

(ii) \Rightarrow (i) Let S is a $\sigma_{1,2}$ OS in L . Then $L - S$ is $\sigma_{1,2}$ Open in $L \Rightarrow f^{-1}(L \setminus S)$ is $(1, 2)^*$ -D**spClosed in $K, \Rightarrow L \setminus f^{-1}(S)$ is $(1, 2)^*$ -D**spClosed in L . So that $h^{-1}(S)$ is $(1, 2)^*$ -D**spOpen in L . Hence h is $(1, 2)^*$ -D**spContinuous.

Theorem 5.6

Assume $h : K \rightarrow L$ is a map

If h is $(1, 2)^*$ -D**spContinuous Map, Then $h(\tau_{1,2}\text{-D**spcl}(B)) \subseteq \sigma_{1,2}\text{-spcl}(h(B))$.

Proof.

Given $h(B) \subseteq \sigma_{1,2}\text{-spCl}(h(B)), \Rightarrow B \subseteq h^{-1}(\sigma_{1,2}\text{-spCl}(h(B)))$.

Then $\sigma_{1,2}\text{-spCl}(h(B))$ is a $\sigma_{1,2}$ CS in L and

h is $(1, 2)$ - $D^{**}sp$ Continuous map $\Rightarrow h^{-1}(\sigma_{1,2}\text{-}spCl(h(B)))$ is $(1, 2)$ - $D^{**}sp$ Closed in L . Hence $\tau_{1,2}\text{-}D^{**}spcl(B) \subseteq h^{-1}(\sigma_{1,2}\text{-}scl(f(B)))$. it proves $h(\tau_{1,2}\text{-}D^{**}spcl(B)) \subseteq \sigma_{1,2}\text{-}spcl(h(B))$.

$(1, 2)$ - $D^{}sp$ Continuous and $(1, 2)$ - $D^{**}sp$ Irresolute Mappings Definition 6.1** Consider a map $h : K \rightarrow L$

H is $(1, 2)$ - $D^{**}sp$ irresolute if for every $(1, 2)$ - $D^{**}sp$ OS of L its inverse image of h is $(1, 2)$ - $D^{**}sp$ Open in K

Remark 6.2

Consider a map $h : K \rightarrow L$

For every $(1, 2)$ - $D^{**}sp$ CS of L by defn of 6.1 $(1, 2)$ - $D^{**}sp$ Closed in K .

Theorem 6.3

Consider a map

Proof.

$h : K \rightarrow L$ is $(1, 2)$ - $D^{**}sp$ irresolute implies h is $(1, 2)$ - $D^{**}sp$ Continuous .

Suppose R is a $\tau_{1,2}$ OS in K . Also h is $(1, 2)$ - $D^{**}sp$ irresolute proves $h^{-1}(R)$ is $(1, 2)$ - $D^{**}sp$ Open in K . Thus h is $(1, 2)$ - $D^{**}sp$ Continuous.

Example 6.4

Reverse part of the Theorem 6.3 Can be proved by the following example to show it is not true

Let $X = Y = \{a_1, a_2, a_3\}$, $\tau_1 = \{X, \varphi, \{a_1\}, \{a_2\}, \{a_1, a_2\}\}$, $\tau_2 = \{X, \varphi, \{a_1, a_2\}\}$, $\sigma_1 = \{X, \varphi, \{a_1\}\}$, $\sigma_2 = \{X, \varphi, \{a_1\}, \{a_1, a_2\}\}$.

Let $f : X \rightarrow Y$ be the identity map. Hence f is $(1, 2)$ - $D^{**}sp$ -Continuous but f is not $(1, 2)$ - $D^{**}sp$ irresolute.

Theorem 6.5

Consider a map

$h : K \rightarrow L$

h is $(1, 2)$ -Continuous and L is $(1, 2)$ - $D^{**}sp$ - $T_{1/2}$ -space implies h is $(1, 2)$ - $D^{**}sp$ irresolute.

Proof

Assume B be $(1, 2)$ - $D^{**}sp$ OS in L . Also L is $(1, 2)$ - $D^{**}sp$ - $T_{1/2}$ -space, implies B is an $\sigma_{1,2}$ OS in L and also h is $(1, 2)$ -Continuous proves $h^{-1}(B)$ is $(1, 2)$ - $D^{**}sp$ OS

in K . Thus h is $(1, 2)$ - $D^{**}sp$ irresolute.

Theorem 6.6

Consider a map

$$h : K \rightarrow L$$

h is $(1, 2)^*$ - D^{**} spirresolute and $k : L \rightarrow M$ be an $(1, 2)^*$ - D^{**} spirresolute maps. Then $h \circ k : K \rightarrow M$ is $(1, 2)^*$ - D^{**} spirresolute.

Proof.

Suppose V be a $(1, 2)^*$ - D^{**} spOS in M . so $h^{-1}(V)$ is $(1, 2)^*$ - D^{**} spOpen in L implies $h^{-1}(k^{-1}(V))$ is $(1, 2)^*$ - D^{**} spOpen in K . Thus $(h \circ k)^{-1}(V)$ is $(1, 2)^*$ - D^{**} spOpen in K . Hence $h \circ k$ is $(1, 2)^*$ - D^{**} spirresolute.

7. $(1, 2)^*$ - D^{} spConnected Sets**

Definition 7.1

A space X is $(1, 2)^*$ - D^{**} spdisConnected if it is the Union of two disjoint non empty $(1, 2)^*$ - D^{**} spOS otherwise it is said to be $(1, 2)^*$ - D^{**} spConnected

Theorem 7.2

IN BTS X , the following statements are true.

- X is $(1, 2)^*$ - D^{**} spConnected.
- φ, X are the subsets which are both $(1, 2)^*$ - D^{**} spOpen and $(1, 2)^*$ - D^{**} spClosed .

Proof.

$i \Rightarrow ii$ Let $U \subseteq X$ which is $(1, 2)^*$ - D^{**} spOpen & $(1, 2)^*$ - D^{**} spClosed.

Then X/U is also $(1, 2)^*$ - D^{**} spOpen & $(1, 2)^*$ - D^{**} spClosed by defn of 7.1 .

U and X/U implies either

$$U = \varphi \text{ or } X/U = \varphi.$$

$ii \Rightarrow i$ Suppose A, B in X such that $A \cup B = X$ where A, B not equal to empty $(1, 2)^*$ - D^{**} spOS .

So $A - X/B$ is $(1, 2)^*$ - D^{**} spCS $\Rightarrow A$ is $(1, 2)^*$ - D^{**} spO $\subseteq X$ and $(1, 2)^*$ - D^{**} spCS $\subseteq X$

as we assumed $A = \varphi$ or X proves X is $(1, 2)^*$ - D^{**} spConnected.

Theorem 6.3 suppose a mapping $j : K \rightarrow L$ is

$i)$ $(1, 2)^*$ - D^{**} spContinuous and onto , K is $(1, 2)^*$ - D^{**} spConnected

$\Rightarrow L$ is $(1, 2)^*$ Connected.

(ii) If $j : K \rightarrow L$ is $(1, 2)^*$ - D^{**} - σ irresolute surjection map and K is $(1, 2)^*$ - D^{**} - σ Connected $\Rightarrow L$ is $(1, 2)^*$ - D^{**} - σ Connected.

Proof.

Assume L is not $(1, 2)^*$ - σ Connected. We have $L = C \cup D$ is not empty where C and D are disjoint $\sigma_{1,2}$ -OS in L .

Also j is $(1, 2)^*$ - D^{**} - σ Continuous, onto $K = f^{-1}(C) \cup f^{-1}(D)$ where

$f^{-1}(C)$ and $f^{-1}(D)$ are disjoint but not empty $(1, 2)^*$ - D^{**} - σ OSs contradicts X is $(1, 2)^*$ - D^{**} - σ Connected. So that L is $(1, 2)^*$ - σ Connected.

ii proof obvious from 7.1.

8. $(1, 2)^*$ - D^{} - σ Homeomorphisms**

Definition 8.1 $f : X \rightarrow Y$ is a bijection map called $(1, 2)^*$ - D^{**} - σ homeomorphism if the mapping is $(1, 2)^*$ - D^{**} - σ Continuous, $(1, 2)^*$ - D^{**} - σ Open.

Remark 8.2

Every $(1, 2)^*$ - σ homeomorphism is $(1, 2)^*$ - D^{**} - σ homeomorphism but conversely not true

Theorem 8.3 consider

The mapping $h : X \rightarrow Y$, is 1-1 and onto we have the following statements are true.

- (i) $h^{-1} : Y \rightarrow X$ is $(1, 2)^*$ - D^{**} - σ Continuous.
- (ii) The mapping is $(1, 2)^*$ - D^{**} - σ Open.
- (iii) the mapping is $(1, 2)^*$ - D^{**} - σ Closed.

Proof.

• (i) \Rightarrow (ii) Let K be any $\tau_{1,2}$ -OS in X . Since f^{-1} is $(1, 2)^*$ - D^{**} - σ Continuous, $f(K)$ in Y is $(1, 2)^*$ - D^{**} - σ Open. Thus the mapping is $(1, 2)^*$ - D^{**} - σ Open.

• (ii) Implies (iii) In X

suppose F is $\tau_{1,2}$ -CS, Then X/F is $\tau_{1,2}$ -OS

and Also the mapping is $(1, 2)^*$ - D^{**} - σ Open,

$f(X/F)$ in Y is $(1, 2)^*$ - D^{**} - σ Open in Y . But in Y , $f(X/F) = Y/f(F)$

where $f(F)$ is $(1, 2)^*$ - D^{**} - σ Closed. Thus the mapping is $(1, 2)^*$ - D^{**} - σ Closed Map

(iii) implies (i)

Suppose R is $\tau_{1,2}$ -CS in X , We have $f(R)$ in Y is $(1, 2)^*$ - D^{**} - σ Closed. Also the mapping f is $(1, 2)^*$ - D^{**} - σ Closed its inverse mapping is $(1, 2)^*$ - D^{**} - σ Continuous.

Theorem 8.4

Suppose the mapping f is 1-1, onto and $(1, 2)^*$ -D**spContinuous. Then the implications are true. To prove the mapping f is

- i) $(1, 2)^*$ -D**spOpen .
- ii) $(1, 2)^*$ -D**spHomeomorphism .
- iii) $(1, 2)^*$ -D**spClosed

Proof

Assume i) f is $(1, 2)^*$ -D**spOpen also the mapping is 1-1 and onto, $(1, 2)^*$ -D**spContinuous. From the definition 8.1, the mapping is $(1, 2)^*$ -D**spHomeomorphism. (ii) is proved.

Assume (ii) the mapping is $(1, 2)^*$ -D**spOpen, 1-1 and onto it is $(1, 2)^*$ -

D**spClosed from thm 7.8. (iii) proved

Assume iii) f is $(1, 2)^*$ -D**spClosed and bijective. f is $(1, 2)^*$ -D**spOpen map. By Theorem 8.3 (i), proved

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