Autism Detection Using Fuzzy $\beta$-Neighborhood Based Soft Rough Sets

Praba B$^1$, Balambal Suryanarayanan$^2$

$^1$Department of Mathematics, Sri Siva Subramaniya Nadar College of Engineering, Kalavakkam- 603110; prabab@ssn.edu.in

$^2$Sri Siva Subramaniya Nadar College of Engineering, Kalavakkam- 603110; balambal19013@ece.ssn.edu.in

Abstract:
Autism Spectrum Disorder refers to a variety of conditions represented by complications in social skills, limited interests and communication that’s majoritarily non-verbal. The early signs and symptoms of the disorder were found to be noticeable at a young age. However, the clinical tests take longer to diagnose and come at a higher cost. As a result, in recent years, several studies have been made to enable autism detection through early intervention. Rough Set theory is one such efficient mathematical tool for this application. This paper introduces the concept of fuzzy $\beta$-neighborhood based soft rough sets ($\beta - RS$). The major advantage of introducing this concept lies in the construction of fuzzy $\beta$-neighborhood of each object obtained through its $\beta$-reduct. This neighborhood captures those objects having similar characteristics with respect to the significant attributes. The application of this model is highlighted through autism detection by obtaining the neighborhood of the given image. The proposed model also efficiently captures the various levels of autism with great accuracy. The validation is carried out by taking real time data.

Keywords: Autism, Fuzzy Sets, Rough Sets, $\beta$ -neighborhood, Lower Approximation, Upper Approximation.

1. Introduction
The uniqueness of human beings as contrasted to other primates can be attributed to the fascinating cognitive functionalities and mysteries of the human brain. At times, damage inflicted to a characteristic part of the brain [18] adds to the brain’s enigma, resulting in intriguing disorders where a person sees sounds, hears shapes, touches emotions, and feels limbs [15]. The difficulties in cognitive, social, and emotional functioning, influenced by genetic and environmental factors give rise to a collection of disorders called neuro-developmental disorders. The most known neuro-developmental disorders include ADHD, autism, mental retardation, cerebral palsy, and control disorders. Of these, autism has continued to enthrall and puzzle modern medicine, becoming an unknown frontier like the brain itself [5].

As a spectrum disorder where every person with autism possesses a unique set of strengths and challenges, the prospect of autism going undiagnosed due to mild symptoms or suppressed exhibition through childhood and getting misdiagnosed as post-traumatic stress disorder in cases of trauma-induced autism add on to its mystery. Increasing fascination and research towards the subject have presented a multitude of possibilities to consider in the diagnosis of autism. Autistic individuals exhibit facial signs that are often intuitive [8, 20, 21], resulting in the spilling of expressions inappropriate to
a situation they are interacting with. This inability of autistic individuals to convey appropriate signals and emotions in a conversation tends to hinder their capability to appropriately communicate with the external environment, resulting in misunderstandings to arise in a relationship. However, the face markers that engage in the formation of such facial expressions hold the key to understand how an autistic individual differs extensively from their normal counterparts [1], thus offering an insight into understanding the nature of the cues adopted by an autistic person.

Studies have been done to upgrade and accelerate the diagnosis of autism spectrum disorder (ASD) using the techniques of Machine Learning. The ultimate objective of all these studies lies in enabling a timely intervention and treatment by aiding in early ASD diagnosis. While Praveena et al. [14] focused on using facial image processing to recognize emotions and predict ASD in children, Vakadkar et al. [17] used Logistic Regression to identify ASD in children during early developmental stages. Mujeeb Rahman et al. [11] used the static features extracted from the photographs of autistic children to identify ASD by utilizing CNN and DNN models. Several studies have also employed facial features for ASD in automated ML models while other studies leveraged data from brain neuroimaging. Boughattas et al. [2] utilized the ABIDE dataset containing functional Magnetic Resonance Images (fMRI) to detect ASD using convolutional neural networks. Moridian et al. [10] reviews several Computer-Aided Diagnosis Systems developed for automated diagnosis of ASD using MRI modalities. Lamani et al. [7] employs a graph convolutional network classifier to determine whether an image depicts normal or autistic characteristics and the research of Emel Koc et al. [3] aims to improve the diagnosis accuracy of ASD by using functional magnetic resonance imaging data. These studies show that an appropriate mathematical model is significant in deriving a crucial conclusion. Specifically, in a growing area of research where one utilizes facial features for the diagnosis of autism, the need for building a model that’s accurate and efficient is imminent [12, 13]. Building on these trends, this paper constructs a fuzzy $\beta$–neighborhood based soft rough set to detect autism by utilizing crucial face landmarks.

2. Fuzzy $\beta$–Neighborhood Based Soft Rough Sets

In this section, the concepts of $\beta$-reduct, fuzzy $\beta$-neighborhood, and fuzzy $\beta$-neighbourhood-based soft rough set of an object are introduced, along with an illustrative example for each.

**Definition 2.1.** Let $I = (U, A, F)$ represent a covering-based information system where $U$ is a non-empty finite set of objects called the universal set. $A$ is the non-empty finite set of attributes with membership function defined by $\mu_a: U \rightarrow [0,1] \forall a \in A$ and $F: A \rightarrow \mathcal{P}(U)$ defined by

$$F(a) = \{x \in U | \mu_a(x) \leq \delta_a\} \forall a \in A$$

$F(a)$ is the set of objects in $U$ possessing the attribute $a$ with respect to a threshold $\delta_a$ such that $\bigcup_{a \in A} F(a) = U$.

The choice of $\delta_a$ is made following expert insights, thorough experimenting, and the observation of the system under consideration.

**Definition 2.2.** For $\beta \in [0,1]$, the $\beta$-reduct, $N^\beta(x)$ of an object $x$ is defined as

$$N^\beta(x) = \{a \in A | \mu_a(x) \geq \beta\}$$
which represents the set of attributes playing a significant role on \( x \) with respect to \( \beta \).

**Definition 2.3.** For \( \beta \in [0,1] \), the fuzzy \( \beta \)-neighborhood, \( \bar{N}_\beta(x) \) of an object \( x \) in \( U \) is defined as
\[
\bar{N}_\beta(x) = \{ y \in U \mid \mu_a(y) \geq \beta \}
\]
which represents the set of objects that are in the neighborhood of \( x \) with respect to the \( \beta \)-reduct of \( x \).

**Definition 2.4.** The fuzzy \( \beta \)-Neighborhood based soft-rough set, \( \beta - RS(x) \) of the object \( x \) belonging to the universe \( U \) is defined by
\[
\beta - RS(x) = (\bar{N}_\beta(x)_-, \bar{N}_\beta(x)^-)
\]
where,
\[
\bar{N}_\beta(x)_- = \{ F(a) \mid F(a) \subseteq \bar{N}_\beta(x), \ a \in N_\beta(x) \}
\]
\[
\bar{N}_\beta(x)^- = \{ F(a) \mid F(a) \cap \bar{N}_\beta(x) \neq \emptyset, \ a \in N_\beta(x) \}
\]
represent the lower approximation and upper approximation of the fuzzy \( \beta \)-neighborhood based soft rough set of the object \( x \).

The lower approximation space, \( \bar{N}_\beta(x)_- \) of the object \( x \) contain those objects of \( F(a) \) that are completely contained in the fuzzy \( \beta \)-neighborhood of \( x \) for every \( a \) that belongs to \( N_\beta(x) \) and the upper approximation space, \( \bar{N}_\beta(x)^- \) of the object \( x \) contains those objects of \( F(a) \) whose intersection with the fuzzy \( \beta \)-neighborhood is non empty for every \( a \) in \( N_\beta(x) \).

**Example 2.5.** Consider \( I = (U, A, F) \), the covering-based information system. Let \( U = \{x_1, x_2, x_3, x_4\} \) be the universal set and let \( A = \{a_1, a_2, a_3, a_4, a_5\} \) be the set of attributes. \( F: A \rightarrow \mathcal{P}(U) \) be defined by
\[
F(a_1) = \{x_1, x_4\} \quad F(a_2) = \{x_1, x_3\} \quad F(a_3) = \{x_1, x_2, x_4\}
\]
\[
F(a_4) = \{x_3, x_4\} \quad F(a_5) = \{x_2, x_4\}
\]
By taking \( \delta_{a_1} = 0.2, \delta_{a_2} = 0.1, \delta_{a_3} = 0.3, \delta_{a_4} = 0.2, \) and \( \delta_{a_5} = 0.1 \) respectively. The \( \beta \)-reduct and fuzzy \( \beta \)-neighborhood for \( \beta = 0.2 \) as given by Table 2. The fuzzy \( \beta \)-neighborhood based soft rough set of an object \( x \), \( 0.2 - RS(x) \) is given as in Table 3.

**Table 1:** Information System \( I = (U, A, F) \)

<table>
<thead>
<tr>
<th>U/A</th>
<th>a_1</th>
<th>a_2</th>
<th>a_3</th>
<th>a_4</th>
<th>a_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>x_2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>x_3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>x_4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Remark 2.6.

- The 0.2-reduct of \( x_1 \), \( N^{0.2}(x_1) = \{a_1, a_3, a_4, a_5\} \) consists those significant attributes of \( x_1 \) with respect to \( \beta = 0.2 \).
- The fuzzy 0.2-neighborhood of \( x_1 \), \( \{x_1, x_3\} \) contains those objects that are in the neighborhood of \( x_1 \) with respect to the 0.2-reduct of \( x_1 \).
- The fuzzy 0.2-neighborhood based soft rough set of \( x_1 \), \( 0.2 - RS(x_1) = (F(a_2), U - F(a_5)) \) consists of the lower and upper approximation spaces of the object \( x_1 \). The lower approximation space, \( \tilde{N}^{0.2}(x_1) \), contains \( F(a_2) \) which is completely contained in the fuzzy 0.2 neighborhood of \( x_1 \) where \( \{a_1, a_3, a_4, a_5\} \) are in \( N^{0.2}(x_1) \).
- The upper approximation space, \( \tilde{N}^{0.2}(x_1)^- \) contains all the objects in \( \{F(a_1), F(a_2), F(a_3), F(a_4)\} \) whose intersection with the fuzzy 0.2 neighborhood is non-empty where \( \{a_1, a_2, a_3, a_4\} \) are all in \( N^{0.2}(x_1) \).

From the given example, it can be observed that the \( \beta \)-reduct is the set of attributes that primarily characterize the chosen object \( x \), the fuzzy \( \beta \)-neighborhood [19] embodies those neighborhoods of \( x \) that resemble the chosen object in terms of the attributes that characterize it. Thus, by making an appropriate choice for \( \beta \), one can regulate the lower approximation space to record the neighborhood that closely identifies with the pivotal qualities represented by the chosen object with excellent precision.

3. **Implementation of Fuzzy \( \beta \)-Neighborhood Based Soft Rough Sets to Detect Autism**

**Objective:**
- To determine whether an object is autistic or otherwise using the \( \beta \)-RS model by studying the facial features of the object through collected face landmark points.

### Table 2: 0.2-reduct and fuzzy 0.2-neighbourhood of the objects

<table>
<thead>
<tr>
<th>( U )</th>
<th>( N^{0.2}(x) )</th>
<th>( \tilde{N}^{0.2}(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>{( a_1, a_3, a_4, a_5 )}</td>
<td>{( x_1, x_3 )}</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>{( a_1, a_2, a_3, a_4 )}</td>
<td>{( x_2 )}</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>{( a_1, a_3, a_4, a_5 )}</td>
<td>{( x_1, x_3 )}</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>{( a_1, a_2, a_4 )}</td>
<td>{( x_2, x_4 )}</td>
</tr>
</tbody>
</table>

### Table 3: Fuzzy 0.2-Neighbourhood Based Soft Rough Set of The Objects

<table>
<thead>
<tr>
<th>( U )</th>
<th>( 0.2 - RS(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( (F(a_2), U - F(a_5)) )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( (\emptyset, F(a_3) \cup F(a_5)) )</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>( (F(a_2), U - F(a_5)) )</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>( (F(a_3), F(a_1) \cup F(a_3) \cup F(a_5)) )</td>
</tr>
</tbody>
</table>

https://internationalpubls.com
To classify the different levels of autism using the defined $\beta$–RS model.

**Data:**

A public dataset taken from Kaggle has been utilized for the prediction of autism in individuals. Consider $I = (U, F, A)$ be the covering-based Information system. Let $U = \{u_1, u_2, u_3, \ldots, u_{200}\}$ be the universal set of face images, $A = \{a_1, a_2, a_3, \ldots, a_{15}\}$ be the attribute set of pivotal face landmark points of the objects where each attribute is an ordered pair of coordinates $a_i = (a_{x_i}, a_{y_i})$. Consequently, the membership value of every object with respect to each attribute is denoted as $\mu_{a_i}(u_j) = (\mu_{a_{x_i}}(u_j), \mu_{a_{y_i}}(u_j))$, $\forall i = 1, 2, \ldots, 15$ and $j = 1, 2, \ldots, 200$ and $F: A \rightarrow \mathcal{P}(U)$ is given by

$$F(a_i) = \{u \in U : |\mu_{a_i}(u) - \mu_{std}(u)| \leq \delta_{a_i}\}$$

$\mu_{std}(u)$ is the mean of $\mu_{a_i}(u)$ of neurotypical individuals and $\delta_{a_i} = 0.03$ which has been chosen following thorough experimentation and expert knowledge. In Figure 1, the attributes, i.e., the 15 facial features points and their location of an object $u_i$ is shown. The corresponding membership values of the object $u_1$ for the 15 facial features is given in Table 4.

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th>Name</th>
<th>$\mu_{a_i}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(LC_x, LC_y)$</td>
<td>Left Eye Centre</td>
<td>(0.680, 0.852)</td>
</tr>
<tr>
<td>B</td>
<td>$(RC_x, RC_y)$</td>
<td>Right Eye Centre</td>
<td>(0.577, 0.750)</td>
</tr>
<tr>
<td>C</td>
<td>$(LI_x, LI_y)$</td>
<td>Left Eye Inner Corner</td>
<td>(0.814, 0.784)</td>
</tr>
<tr>
<td>D</td>
<td>$(LO_x, LO_y)$</td>
<td>Left Eye Outer Corner</td>
<td>(0.505, 0.852)</td>
</tr>
<tr>
<td>E</td>
<td>$(RI_x, RI_y)$</td>
<td>Right Eye Inner Corner</td>
<td>(0.753, 0.739)</td>
</tr>
<tr>
<td>F</td>
<td>$(RO_x, RO_y)$</td>
<td>Right Eye Outer Corner</td>
<td>(0.423, 0.682)</td>
</tr>
<tr>
<td>G</td>
<td>$(LEI_x, LEI_y)$</td>
<td>Left Eyebrow Inner Corner</td>
<td>(0.866, 0.943)</td>
</tr>
<tr>
<td>H</td>
<td>$(LEO_x, LEO_y)$</td>
<td>Left Eyebrow Outer Corner</td>
<td>(0.330, 0.977)</td>
</tr>
<tr>
<td>I</td>
<td>$(REI_x, REI_y)$</td>
<td>Right Eyebrow Inner Corner</td>
<td>(0.742, 0.886)</td>
</tr>
<tr>
<td>J</td>
<td>$(REO_x, REO_y)$</td>
<td>Right Eyebrow Outer Corner</td>
<td>(0.278, 0.773)</td>
</tr>
<tr>
<td>K</td>
<td>$(NC_x, NC_y)$</td>
<td>Nose Tip</td>
<td>(0.979, 0.409)</td>
</tr>
<tr>
<td>L</td>
<td>$(ML_x, ML_y)$</td>
<td>Mouth Left Corner</td>
<td>(0.639, 0.205)</td>
</tr>
<tr>
<td>M</td>
<td>$(MR_x, MR_y)$</td>
<td>Mouth Right Corner</td>
<td>(0.773, 0.148)</td>
</tr>
<tr>
<td>N</td>
<td>$(MT_x, MT_y)$</td>
<td>Mouth Centre Top Lip</td>
<td>(0.907, 0.148)</td>
</tr>
<tr>
<td>O</td>
<td>$(MB_x, MB_y)$</td>
<td>Mouth Centre Bottom Lip</td>
<td>(0.907, 0.911)</td>
</tr>
</tbody>
</table>
The steps involved in the detection of whether an object \( u_j \) is autistic or otherwise are as follows.

1. Compute \( N^\beta(u_j) \) and \( \tilde{N}^\beta(u_j) \).
2. Compute \( \beta - RS(u_j) = (\tilde{N}^\beta(u_j), \tilde{N}^\beta(u_j)^-) \)
3. If \( \delta_{da}(u_j) \leq \alpha \), then \( u_j \) and \( \tilde{N}^\beta(u_j)^- \) is autistic.

Else \( u_j \) and \( \tilde{N}^\beta(u_j)^- \) is not autistic.

where \( \delta_{da}(u_j) = \text{average}(|\mu_{a_i}(u) - \mu_{stdAut_{a_i}}(u)|) \) represents the deviation of the object from autistic people, \( \mu_{stdAut_{a_i}}(u) \) is the mean of \( \mu_{a_i}(u) \) of the autistic people. Here, \( \alpha = 0.1 \) following thorough experimentation and expert insights.

By comparing \( \delta_{da}(u_j) \) with the threshold of \( \alpha \), the system detects whether the object \( u_j \) is autistic or otherwise. If the object \( u_j \) is autistic, then the objects in the lower approximation space of \( u_j \) (\( \tilde{N}^\beta(u_j)^- \)) are also autistic since they are in the closest neighborhood of \( u_j \). The upper approximation space of the object \( u_j \) represent the objects that might possibly be present in the closest neighborhood of \( u_j \).

Hence, the level of autism might vary.

Let us check whether an object \( u_{20} \in U \) is autistic or otherwise. For \( \beta = 0.4 \), the corresponding \( \beta \)-reduct and fuzzy \( \beta \)-neighborhood are given in Table 5 and the fuzzy \( \beta \)-neighborhood based rough set of \( u_{20} \) is given in Table 6 respectively. The deviation of the object \( u_{20} \) and some of its neighbors in the lower approximation space from the standard autistic object is given in Table 7.

**Table 5: \( N^{0.4}(u_{20}) \) and \( \tilde{N}^{0.4}(u_{20}) \)**

<table>
<thead>
<tr>
<th>( U )</th>
<th>( N^{0.4}(u_{20}) )</th>
<th>( \tilde{N}^{0.4}(u_{20}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_{20} )</td>
<td>( [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}] )</td>
<td>( [u_{20}, u_{21}, u_{73}, u_{76}, u_{77}, u_{85}, u_{86}, u_{90}, u_{94}, u_{95}, u_{99}, u_{188}] )</td>
</tr>
</tbody>
</table>
Table 6: $0.4 - RS(u_{20}) = (\tilde{N}^{0.4}(u_{20}), \tilde{N}^{0.4}(u_{20})^{-})$

<table>
<thead>
<tr>
<th>$u_{20}$</th>
<th>$\tilde{N}^{0.4}(u_{20})$</th>
<th>$\tilde{N}^{0.4}(u_{20})^{-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[u_{20}, u_{73}, u_{76}, u_{77}, u_{85}, u_{86}, u_{94}, u_{188}]$</td>
<td>$[u_{2}, u_{4}, u_{6}, u_{8}, u_{10}, u_{12}, u_{18}, u_{20}, u_{23}, u_{76}, u_{77}, u_{82}, u_{83}, u_{85}, u_{86}, u_{94}, u_{96}, u_{98}, u_{188}]$</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: $0.4 - RS(u_{20}) = (\tilde{N}^{0.4}(u_{20}), \tilde{N}^{0.4}(u_{20})^{-})$

<table>
<thead>
<tr>
<th>$U$</th>
<th>$\delta_{da}(u_{j})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{20}$</td>
<td>0.0891</td>
</tr>
<tr>
<td>$u_{85}$</td>
<td>0.0862</td>
</tr>
<tr>
<td>$u_{77}$</td>
<td>0.0951</td>
</tr>
<tr>
<td>$u_{86}$</td>
<td>0.0590</td>
</tr>
<tr>
<td>$u_{188}$</td>
<td>0.0575</td>
</tr>
</tbody>
</table>

It can be inferred from Table 7 that not only $\delta_{da}(u_{20})$ is less than 0.1 but also the deviation of $\delta_{da}(u)$ for all the images in the neighborhood of $u_{20}$ is less than 0.1. Hence, it can be said that all the images in $\tilde{N}^{0.4}(u_{20})$ are autistic. Thus, it can be observed that the model has impeccably captured the characteristics of the chosen object along with its neighbors with expected precision.

For testing, we consider two unseen images. On providing these inputs, the model successfully returned its decisions in almost few seconds, the results of which are given in Figure 2. For the testobj1, as the deviation $\delta_{da} = 0.047901 \leq 0.1$, the $0.4 - RS$ model detects it to be autistic. And for the testobj2, as the deviation $\delta_{da} = 0.1930 \geq 0.1$, the $0.4 - RS$ model detects it to be non-autistic.

![Figure 2. Testing](https://internationalpubls.com)
The RS-model can also be used to determine the levels in Autism exhibited by individuals. Diagnostic and Statistical Manual of Mental Disorders describes three levels in the autism spectrum disorder. These levels depend on the severity of the differences [22] in the brain that an individual exhibits and the extent of support such individuals need to continue functioning without interference. Level-I individuals exhibit morphological features like neurotypical subjects and require minimum support when compared to their counterparts, making an attempt to engage in social conventions and create friendships in a community.

Level - II autistic subjects require more support than their Level-I counterparts, but lesser than the Level-III subjects. They exhibit an elevated amount of distress when confronted with change, even more than Level-I individuals. Level - III individuals have to rely extensively on external support to manage their activities, and it is in the second and third levels, the attributes become more prevalent. However, a timely diagnosis of the severity of the disorder [[16], [9]] in an individual can help bring about a change in their functionalities, calling for personalized help finding its way to such individuals.

To classify autistic individuals according to the severity levels, the deviation $\delta_{dn}(u_j) = \text{average}(|\mu_{a_i}(u_j) - \mu_{std_{a_i}}(u_j)|)$ of the chosen object $u_j$ from neurotypical individuals is considered. Given an image $u_j$, the $\beta$ –RS model first detects whether it is autistic or otherwise and also finds its lower approximation space. Then for every element in $\tilde{N}^\beta(u_j)$, their deviation $\delta_{dn}(u_j)$ is calculated. And according to their deviation values the level of autism of the object $u_j$ is determined. The decision taken according to values of $\delta_{dn}(u_j)$ is given in the pseudocode as follows.

```python
def findLevels(img):
    $\delta_{dn}(u_j) = \text{average}(|\mu_{a_i}(u_j) - \mu_{std_{a_i}}(u_j)|)$
    countA, countNA=0
    for i=1:size(LA):
        if $\delta_{da}$ for LA(i)>=0.1:
            countNA+=1
        else:
            countA+=1
        end
    if $\delta_{da}$ <= 0.1 & round($\delta_{da}$) = 0 & countNA > count A:
        decision = Level - I
    elseif $\delta_{da}$ <= 0.1 & $\delta_{dn} \in (0, 0.4]$ & countNA < countA:
        decision = Level - II
    elseif $\delta_{da}$ <= 0.1 & $\delta_{dn} > 0.4$ & countNA < countA:
        decision = Level – III
    end
end function
```

https://internationalpubls.com
The examples of Level-I, Level-II, Level-III autistic people along their deviation $\delta_{dn}(u_j)$ detected by the $\beta - RS$ model is given in Figure 3.

![Figure 3. Levels of Autism](image)

4. **Interpretation of Results Obtained through $\beta - RS$ model**

1. The $\beta$-reduct of the objects aids in attribute reduction and gives significant attribute with respect to $\beta$.

2. The $\beta - RS$ model can be used to detect whether the object is autistic or otherwise, continually implying that $\tilde{N}^{\beta}(u)_-$ is also autistic. For example, an image $u_{20}$ and its neighbors in $\tilde{N}^{0.4}(u_{20})_-$ detected autistic by 0.4 – RS model is shown in Figure 4.

![Figure 4. Autism Detection by $\beta$ - RS model](image)

3. By comparing the lower approximation space of an autistic and non-autistic image one can analyze how the facial features of an autistic individual differ from their normal counterparts.
This is illustrated by taking an example from the lower approximation space of an autistic individual. From the figure, it can be inferred using the bounding boxes that autistic individuals exhibit wider facial markers contrasted to neurotypical individuals.

![Autistic vs Non-Autistic](image)

Figure 5. Face Markers: Autistic Vs. Non-Autistic

(4) The lower approximation space helps find objects exhibiting similar expressions in the same class. The examples that capture a frown/frustration amongst autistic individuals are presented in Figure 6 and 7.

![Objects Frowning](image)

Figure 6. Objects Frowning

![Objects Expressing Frustration](image)

Figure 7. Objects Expressing Frustration

(5) The upper approximation space is advantageous in making decisions about mild autistic individuals who share commonalities in facial features with neurotypical individuals. By analyzing the number of neurotypical individuals in the upper approximation space, one can determine how mildly autistic an individual is. An example of the application of the upper approximation space is illustrated by taking an example from the lower approximation space of an autistic individual. From the figure, it can be inferred using the bounding boxes that autistic individuals exhibit wider facial markers contrasted to neurotypical individuals.
approximation space for an object $u_5$ along with neurotypical individuals in it is shown in Figure 8.

![Figure 8. Upper Approximation of Mild Autistic Individual $u_5$](image)

(6) The $\beta - RS$ model helps to understand the effect of $\beta$ on the rough set of the chosen object in making decision. For example, the effect of $\beta$ on the $\beta$–reduct and the lower and upper approximation space of the object $u_{98}$ is shown in Table 8.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\mathcal{N}^\beta(u_{98})$</th>
<th>$\tilde{\mathcal{N}}^\beta(u_{98})$</th>
<th>$\tilde{\mathcal{N}}^\beta(u_{98})$–</th>
<th>$\mathcal{R}^\beta(u_{98})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}]$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
</tbody>
</table>
| 0.2     | $[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}]$ | $[u_2, u_3, u_4, u_5, ...
 u_{174}, u_{179}, u_{192}, u_{197}]$ | $U$ | $U$ |
| 0.4     | $[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}]$ | $[u_7, u_9, u_{11}, u_{16}, u_{21}, u_{34}, u_{35}, u_{73}, u_{76}, u_{82}, u_{84}, u_{89}, u_{94}, u_{98}, u_{109}, u_{114}, u_{140}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{21}, u_{34}, u_{35}, u_{59}, u_{34}, u_{65}, u_{73}, u_{76}, u_{77}, u_{80}, u_{82}, u_{84}, u_{89}, u_{92}, u_{94}, u_{96}, u_{98}, u_{101}, u_{109}, u_{114}, u_{139}, u_{140}]$ |
| 0.7     | $[a_5, a_{11}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{21}, u_{98}, u_{102}, u_{108}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{98}, u_{102}, u_{108}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{98}, u_{102}, u_{108}]$ |
| 0.8     | $[a_{11}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{98}, u_{102}, u_{108}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{98}, u_{102}, u_{108}]$ | $[u_6, u_7, u_9, u_{11}, u_{16}, u_{20}, u_{98}, u_{102}, u_{108}]$ |
| 0.9     | $[]$ | $[]$ | $[]$ | $[]$ |
It can be observed that for \( \beta < 0.4 \), there is overfitting of the model, as the \( \beta \)-neighborhood contains those attributes that are less significant and the fuzzy \( \beta \)-neighborhood contains those objects which have no resemblance with the chosen object. Meaning, the system does not learn any pattern in the data but understands the training data solely for the sake of model building, thus resulting in overfitting. For \( \beta \) tending to 1, there is underfitting of the model as the important characterizing attributes of the chosen object are lost and the model begins relying on assumptions about the object for decision making. Hence, an optimal choice of \( \beta \) is essential for accurate decision making.

(7) The model presented an accuracy of 92.5% for the threshold value \( \alpha = 0.1 \). The confusion matrix of the 0.4 − RS model is shown in Figure 9. In comparison to the existing models, this model is extremely advantageous owing to the fact that it makes a decision about the chosen individual by observing the characteristics shared by the individual with its neighbors, thereby improving the speed, accuracy and precision of the prediction.

![Figure 9. Accuracy of the 0.4 − RS model](https://internationalpubls.com)

Allowing the mathematical model to work with the electrical activity in the brain recorded using EEGs and focusing on the regions of the brain that contribute to giving rise to unique facial expressions in autistic individuals can help improve its prediction capability by several margins.

5. Conclusion

The \( \beta \) − reduct, fuzzy \( \beta \)-neighborhood, and fuzzy \( \beta \)-neighborhood based soft rough sets of the objects have been defined. The \( \beta \) − RS model was implemented for autism detection and the model showed an accuracy of about 92.5%. Further, the advantage of the lower approximation space in determining the levels of autism, detecting individuals with similar expression from same class, and differentiation of facial feature of an autistic individual from a neurotypical one are discussed. The significance of the upper approximation space in the case of a mild autistic people has also been shown. Consequently, the main advantage of the model has been found to lie in the fact that it requires no prior information
to make decisions about the dataset and utilizes the fuzzy $\beta$-neighborhood of the object for detection and classification of levels of autism. The $\beta$-reduct aids in understanding the behavior of individuals who express themselves similar to a chosen individual, and hence the $\beta - RS$ model executes the decision-making process in a reduced computational time frame to produce efficient results. The $\beta - RS$ model can further be tuned by working with equipment that helps understand brain activity to produce improved and reliable results.

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**References**


