Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 3s (2024)

MHD with Viscosity Variation and Velocity Slip between Rough Porous Conical Bearings using Couple Stress Fluid

S Sangeetha[1], T Lawanya[2], Pragya Pandey[3], D Kavitha[4,*]

[1,3]Department of Mathematics, SRM Institute of Science and Technology, Ramapuram, Chennai, India.

[2,4]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, SIMATS, Thandalam, Chennai, India.

[*]Corresponding author email: soundarkavitha@gmail.com

Article History:

Received: *07-04-2024*

Revised: *30-05-2024*

Accepted: *16-06-2024*

Abstract:

In this study, the effect of velocity slip with viscosity variation using MHD couple stress fluid between rough porous conical bearings is analysed. The modified Reynolds equation is solved with appropriate boundary conditions in dimensionless form to find the pressure distribution which is then used to obtain the expression for load carrying capacity paving the way for the calculation of response time. Numerical computations of the results shows that the MHD on the conical bearings indicate that the slip velocity enhances the pressure distribution and the load carrying capacity of the conical bearings. Also, the effect of roughness parameter is to increase(decrease) the load carrying capacity and the response time for azimuthal (radial) roughness.

Keywords: Viscosity variation, Velocity slip, Couple Stress Fluid, Conical Bearing, MHD, Surface Roughness.

1. Introduction:

In the classical case of hydrodynamic lubrication theory, the viscosity variation of the lubricant was ignored. The relationship between viscosity variation with temperature is significant in many real-world situations. There is no fundamental mathematical relationship that will accurately predict the variation in the viscosity with temperature. The proposed formulae for determining the relationship between viscosity and temperature are entirely empirical. Barus [1] was the first to study about the viscosity variation in hydrodynamic lubrication of solid bearings. The effect of MHD and couple stress fluid in conical bearings is studied by Hanumagowda et.al [6] and observed that the presence of Hartmann number increases the squeeze film pressure and time height relationship. However, in general the viscosity of all liquids decreases with increasing temperature. Dass et.al [4] considered the wide rectangular plate bearing and noted that the slip parameter enhances the load carrying capacity and pressure of the bearing. Naduvinamani et.al [10] analysed the load carrying capacity of the journal bearings which increases with increasing values of eccentricity ratio parameter for both type of roughness patterns. Naduvinamani et.al [11,12] used fourth order Runge-Kutta method to solve time height relationship. From the results the effect of variation in viscosity is to decrease the squeeze film time.

Stokes [16] coupe stress fluid adequately describes the flow behaviour of fluids containing a substructure such as lubricants with polymer additives. Several investigators [7-9] have used the stokes couple stress fluid theory to analysed the performance characteristics of various types of bearings with surface roughness. Sangeetha et.al [10] have embraced the concept and applied this approach to study the effect of viscosity variation. Bilal Booussaha [2] analysed the squeezing behaviour of porous circular disc with sealed boundary and noted that the side leakage flow calculated in the sealed case remains constant in comparison to that of open end (unsealed) porous disc for all values of couple stress parameter. Naduvinamani et.al [13] used small perturbation technique to analysed squeeze film characteristics of parallel plates. He noted that the effect of pressure dependent viscosity and couple stress fluid is to increase pressure in both case as compared to non-viscous couple stress fluid.

All these research works showed that, the presence of couple stress fluid has a notable role in working of a bearing system as compared to Newtonian case. The problem of MHD porous conical bearings lubricated with couple stress fluid has not been studied. Hence in the paper an endeavour has been made to analysed magnetic field with velocity slip in conical bearings lubricated with couple stress fluid.

2. Mathematical Formulation:

Figure 1 demonstrates the configuration of conical bearings in the existence of a transverse magnetic field. In this figure h is the thickness of fluid film between the bearing. The magnetic field B_o is applied along the z-direction. In the film area, couple stress fluid is taken into consideration as the lubricant. The interaction between viscosity and temperature can be replaced between viscosity and film thickness.

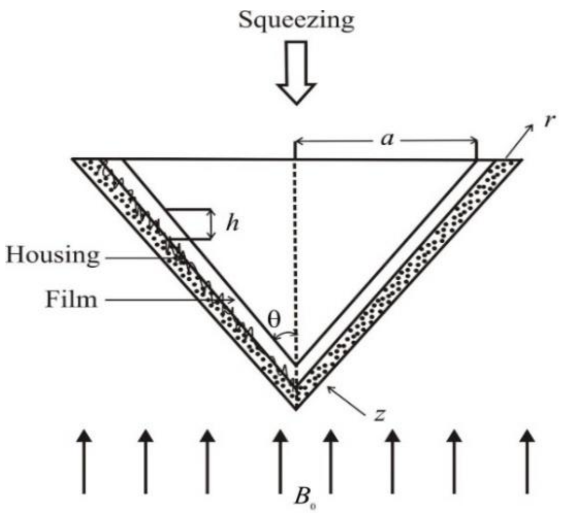

Figure1: Schematic diagram of conical bearings

where p is the hydrodynamic film pressure, u and w are the velocity components in the r and z directions respectively, μ is the lubricant viscosity, σ denotes the electrical conductivity of the lubricant and η represents a material constant responsible for the non-Newtonian couple stress fluid

$$\mu\frac{\partial^2 u}{\partial z^2}-\eta\frac{\partial^4 u}{\partial z^4}-\sigma B_o^2 u=\frac{\partial p}{\partial r} \tag{1}$$

$$\frac{\partial p}{\partial z}=0 \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}=0 \tag{3}$$

The relevant boundary conditions for the velocity components are

(i) at the upper surface $z=h\sin\theta$

$$u=0,\ \frac{\partial^2 u}{\partial z^2}=0 \tag{4a}$$

$$w=\sin\theta\frac{\partial h}{\partial t} \tag{4b}$$

(ii) At the lower surface $z=0$

$$\frac{\alpha}{\sqrt{k}}\left(u-u^*\right)=\frac{\partial u}{\partial z} \tag{5a}$$

$$\frac{\partial^2 u}{\partial z^2}=0 \tag{5b}$$

$$w=w^* \tag{5c}$$

In the porous region the pressure equation is of the form,

$$u^*=\frac{-k}{\mu\left(1-\beta+\left(k\sigma B_o^2\Big/\mu M'\right)\right)}\frac{\partial p^*}{\partial r^*} \tag{6}$$

$$w^*=\frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z^*} \tag{7}$$

$$\frac{\partial u^*}{\partial r}+\frac{\partial w^*}{\partial z^*}=0 \tag{8}$$

$$\frac{\partial^2 p^*}{\partial r^2}+\left(\frac{1-\beta+\left(k\sigma B_o^2/\mu M'\right)}{1-\beta}\right)^{1/2}\frac{\partial^2 p^*}{\partial z^2}=0 \tag{9}$$

These are the usual no-slip condition on velocity together with the condition for the elimination of couple stress at the plates. Where the film height is *h* in the direction of the cone axis, cone angle of 2θ and a radius of *a*. The inner cone moves towards the housing with a squeezing velocity, $v_{sq}=\sin\theta\frac{\partial h}{\partial t}$.

The solution of equations (1) subject to the conditions (4) - (5) are

$$u=-\frac{h_o^2}{\mu_o e^{\beta p}M^2}\frac{\partial p}{\partial r}\frac{1}{\left(E_1^2-E_2^2\right)}\left\{g_1\left[\left(\frac{E_2^2\sinh E_1\left(\left(z-h\sin\theta\right)/l\right)}{\sin\left(E_1 h\sin\theta/l\right)}\right)-\left(\frac{E_1^2\sinh E_2\left(\left(z-h\sin\theta\right)/l\right)}{\sin\left(E_2 h\sin\theta/l\right)}\right)\right]-g_2\right\}$$

$$g_1=\frac{\left(1-\xi_1\operatorname{cosech}\left(E_1 h\sin\theta/l\right)+\xi_2\operatorname{cosech}\left(E_2 h\sin\theta/l\right)\right)-\left(k\sigma B_o^2/\mu\xi_3\right)}{\left(1-\xi_1\coth\left(E_1 h\sin\theta/l\right)+\xi_2\cot h\left(E_2 h\sin\theta/l\right)\right)}$$

$$g_2=\left(E_2^2\sinh\frac{E_1 z}{l}\operatorname{cosech}\frac{E_1 h\sin\theta}{l}-E_1^2\sinh\frac{E_2 z}{l}\operatorname{cosech}\frac{E_2 h\sin\theta}{l}+\left(E_1^2-E_2^2\right)\right)$$

$$\xi_1=\frac{\sigma^* E_1 E_2^2}{\left(E_1^2-E_2^2\right)l},\ \xi_2=\frac{\sigma^* E_2 E_1^2}{\left(E_1^2-E_2^2\right)l},\ \xi_3=\left(1-\beta+\frac{k\sigma B_o^2}{\mu M'}\right),\ E_1=\left\{\frac{1+\left(1-4l^2M^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}}$$

$$E_2=\left\{\frac{1-\left(1-4l^2M^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}}$$

Integration of the continuity Eq. (3) over the film thickness and the use of boundary conditions (4b) and (5c) give the modified Reynolds equation in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{h_o^2}{M^2}\left[f\left(h,\sigma^*,l,M,\theta\right)+\frac{\psi M^2}{\xi_3}\right]e^{-\beta p}r\frac{\partial p}{\partial r}\right\}=-\frac{\partial h}{\partial t}\sin\theta \tag{7}$$

For mathematical modelling of surface roughness, known as the Stochastic film thickness H which consist of two parts represented as

$$H=h(t)+h(r,\theta,\xi) \tag{8}$$

Using Christensen [13] we assume that

$$f\left(h_s\right)=\begin{cases}\dfrac{35}{32c^7}\left(c^2-h_s^2\right)^3 & -c\le h_s\le c\\ 0 & otherwise\end{cases} \tag{9}$$

where $\bar{\sigma}=\frac{c}{3}$ is the standard deviation

where the expectancy operator is denoted as $E\left(*\right)$ and defined by

$$E(*) = \int_{-\infty}^{\infty} (*) f(h_s)\, dh_s \tag{10}$$

$$F(H, \sigma^*, l, M, \theta) = \begin{cases} E\left[f(h, \sigma^*, l, M, \theta) \right] \text{ for radial} \\ E\left[1/f(h, \sigma^*, l, M, \theta) \right]^{-1} \text{ for azimuthal} \end{cases}$$

$$E\left[f(h, \sigma^*, l, M, \theta) \right] = \frac{35}{32c^7} \int_{-c}^{c} \left(c^2 - h_s^2 \right)^3 f(h, \sigma^*, l, M, \theta)\, dh_s \tag{11}$$

$$E\left[\frac{1}{f(h, \sigma^*, l, M, \theta)} \right] = \frac{35}{32c^7} \int_{-c}^{c} \frac{\left(c^2 - h_s^2 \right)^3}{f(h, \sigma^*, l, M, \theta)} dh_s \tag{12}$$

$$f(h, \sigma^*, l, M, \theta) = \frac{2l}{\left(E_1^2 - E_2^2 \right)} \begin{Bmatrix} g_1 * \left(\left[\frac{E_2^2}{E_1} \tanh\left(\frac{E_1 h \sin\theta}{2l} \right) \right] - \left[\frac{E_1^2}{E_2} \tanh\left(\frac{E_2 h \sin\theta}{2l} \right) \right] \right) \\ + \left[\left(E_1^2 - E_2^2 \right) \frac{h \sin\theta}{l} \right] \end{Bmatrix}$$

$$g_1 = \frac{\left(2 - \xi_1 \coth\left(\frac{E_1 \,\mathrm{h} \sin\theta}{2l} \right) + \xi_2 \coth\left(\frac{E_2 \,\mathrm{h} \sin\theta}{2l} \right) \right) - \left(\sigma^{*2} \alpha^2 M^2 / \xi_3 \right)}{1 - \xi_1 \coth\left(\frac{E_1 \,\mathrm{h} \sin\theta}{l} \right) + \xi_2 \coth\left(\frac{E_2 \,\mathrm{h} \sin\theta}{l} \right)}$$

The pressure boundary conditions are:

$$\frac{dp}{dr} = 0 \text{ at } r = 0 \tag{13}$$

$$p = 0 \text{ at } r = \operatorname{cosec}\theta \tag{14}$$

Introducing the dimensionless variables and parameter as follows:

$$r^* = \frac{r}{a \csc\theta},\ H^* = \frac{h}{h_0},\ p^* = -\frac{p h_0^3}{\mu_o a^2 \left(\frac{-\partial h}{\partial t} \right) \csc\theta},\ l^* = \frac{2l}{h_0},\ G = \frac{\beta \mu_o a^2 \left(-\frac{\partial h}{\partial t} \right)}{h_0^3},\ M = B_o h_o \left(\frac{\sigma}{\mu} \right)^{\frac{1}{2}},\ l^* = \frac{2l}{h_o},$$

$$H^* = \frac{H}{h_o},\ M = B_o h_o \left(\frac{\sigma}{\mu} \right)^{\frac{1}{2}},\ k = \sigma^{*2} \alpha^2,\ s^* = \frac{\sigma^*}{h_o},\ \psi = \frac{k\delta}{h_o^3},\ \sigma^* = \sqrt{k}/\alpha$$

Let $f(h_s)$ be the probability density function of the stochastic film thickness h_s. Taking the stochastic average of (7) with respect to $f(h_s)$ the averaged modified Reynolds type equation is

obtained in the form

$$\frac{\partial}{\partial r^*}\left\{\frac{e^{-\beta p}}{M^2}\left[F\left(H^*,s^*,l^*,M,\theta\right)+\frac{\psi M^2}{\xi_3}\right]r^*\frac{\partial p^*}{\partial r^*}\right\}=r^* \tag{15}$$

$$F\left(H^*,s^*,l^*,M,\theta\right)=\frac{l^*}{2\left(E_1^{*2}-E_2^{*2}\right)}\left\{\begin{array}{l} g_1^*\left(\left[\frac{E_2^{*2}}{E_1^*}\tanh\left(\frac{AH^*\sin\theta}{l^*}\right)\right]-\left[\frac{E_1^{*2}}{E_2^*}\tanh\left(\frac{AH^*\sin\theta}{l^*}\right)\right]\right) \\ +\left[\left(E_1^{*2}-E_2^{*2}\right)\frac{2H^*\sin\theta}{l^*}\right] \end{array}\right\}$$

$$g_1^*=\frac{\left(2-\xi_1^*\coth\left(\frac{E_1^*H^*\sin\theta}{l^*}\right)+\xi_2^*\coth\left(\frac{E_2^*H^*\sin\theta}{l^*}\right)\right)-\left(s^{*2}\alpha^2M^2\Big/\xi_3^*\right)}{1-\xi_1^*\coth\left(\frac{2E_1^*H^*\sin\theta}{l^*}\right)+\xi_2^*\coth\left(\frac{2E_2^*H^*\sin\theta}{l^*}\right)}$$

$$\xi_1^*\frac{2s^*E_1^*E_2^*}{\left(E_1^{*2}-E_2^{*2}\right)l^*},\ \xi_2^*=\frac{2s^*E_1^*E_2^*}{\left(E_1^{*2}-E_2^{*2}\right)l^*},\ \xi_3^*=\left(1-\beta+\frac{s^{*2}\alpha^2M^2}{M'}\right),E_1^*=\left\{\frac{1+\left(1-l^{*2}M^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}}$$

$$E_2^*=\left\{\frac{1-\left(1-l^{*2}M^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}},\ F\left(H^*,s^*,l^*,M,\theta\right)=\left\{\begin{array}{l} E\left[f\left(h,\sigma^*,l,M,\theta\right)\right]\ \textit{for radial} \\ E\left[1\Big/f\left(h,\sigma^*,l,M,\theta\right)\right]^{-1}\ \textit{for azimuthal} \end{array}\right.$$

$$E\left[f\left(h,\sigma^*,l,M,\theta\right)\right]=\frac{35}{32c^7}\int_{-c}^{c}\left(c^2-h_s^2\right)^3f\left(h,\sigma^*,l,M,\theta\right)dh_s \tag{16}$$

$$E\left[\frac{1}{f\left(h,\sigma^*,l,M,\theta\right)}\right]=\frac{35}{32c^7}\int_{-c}^{c}\frac{\left(c^2-h_s^2\right)^3}{f\left(h,\sigma^*,l,M,\theta\right)}dh_s \tag{17}$$

The pressure boundary conditions are:

$$\frac{dp^*}{dr^*}=0,\ r^*=0 \tag{18}$$

and

$$p^*=0\ ,r^*=1 \tag{19}$$

Integrating the equation (15) with respect to r^* and using the pressure boundary conditions, we can obtain the nondimensional film pressure

$$p^*=-\frac{1}{G}\ln\left\{\frac{M^2}{4}\left(\frac{G\left(r^{*2}-1\right)}{F\left(H^*,s^*,l^*,M,\theta\right)+\frac{\psi M^2}{\xi_3}}\right)+1\right\} \tag{20}$$

Substituting the expression of the film pressure and integrating the equation (20), we can obtain the load carrying capacity in the form

$$E(w) = 2\pi \int_0^1 E(p) r dr \tag{21}$$

The load carrying capacity equation in dimensionless form is acquired as

$$W^* = \left\{ {wh_0^3} \Big/ {\mu_o a^4 \left(-{dh}/{dt}\right) \csc^2 \theta} \right\} = \frac{2\pi}{G} \int_0^1 \ln \left\{ \frac{M^2}{4} \left(\frac{G\left(r^{*2}-1\right)}{F\left(H^*, s^*, l^*, M, \theta\right) + \frac{\psi M^2}{\xi_3}} \right) + 1 \right\} r^* dr^* \tag{22}$$

Now introduce the non-dimensional approaching time into the above equation

$$T^* = \left\{ {wh_0^2 dt} \Big/ {\mu_o a^4 \csc^2 \theta} \right\} = \frac{2\pi}{G} \int_{h_f}^1 \int_0^1 \ln \left\{ \frac{M^2}{4} \left(\frac{G\left(r^{*2}-1\right)}{F\left(H^*, s^*, l^*, M, \theta\right) + \frac{\psi M^2}{\xi_3}} \right) + 1 \right\} r^* dr^* dh^* \tag{23}$$

3. Results and Discussion:

A generalized form of Reynolds equation applicable to fluid film lubrication was derived by considering the variation of fluid properties along the film thickness with velocity slip at the bearing surface. The effects of velocity slip and viscosity variation in squeeze film lubrication of conical bearings has been studied. The analysis of the roughness with MHD using couple stress fluid between conical bearings is evaluated with regard to Hartmann number *M*, the non-Newtonian couple stress parameter l^*, viscosity variation parameter *G*, surface roughness parameter *c*. We have obtained a solution of conical bearings squeeze film for non-Newtonian couple stress fluid by considering the pressure-dependent viscosity.

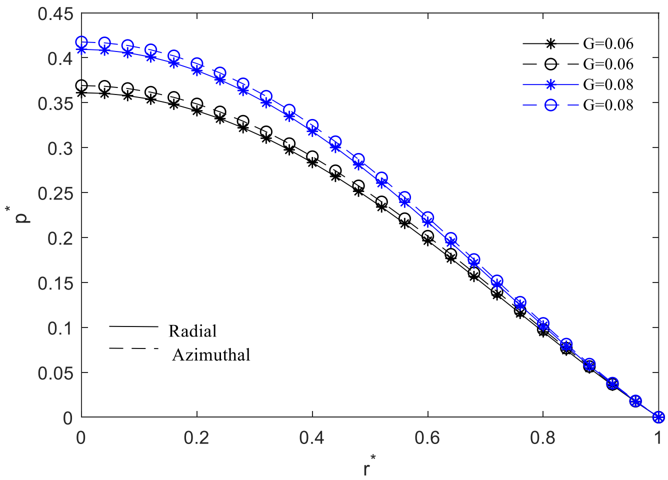

Figure 2 Variation with r^* for different values of viscosity variation parameter G on dimensionless pressure p^* with $M=5, H=1.2$, $c=0.3$, $s^*=0.7$, $l^*=0.3$, $\psi=0.01$, $\alpha=0.1$, $\beta=0.2$

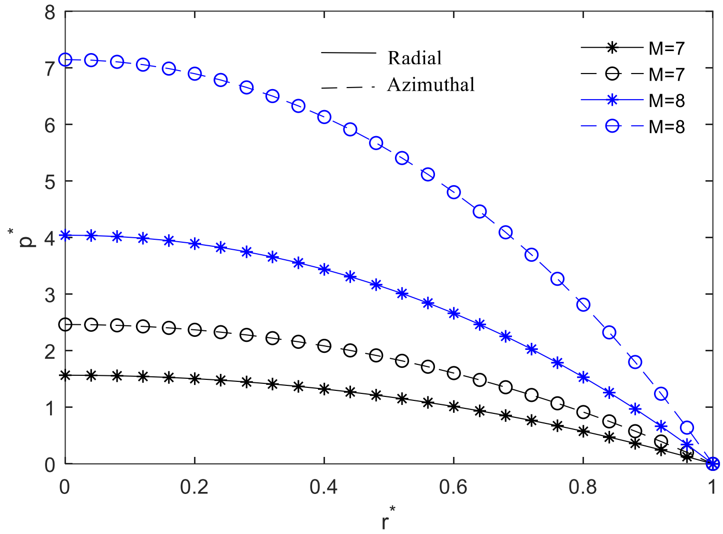

Figure 3 Variation with r^* for different values of Hartman number M on dimensionless pressure p^* with $G=0.04, H=1.2$, $c=0.3$, $s^*=0.7$, $l^*=0.3$, $\psi=0.01$, $\alpha=0.1$, $\beta=0.2$

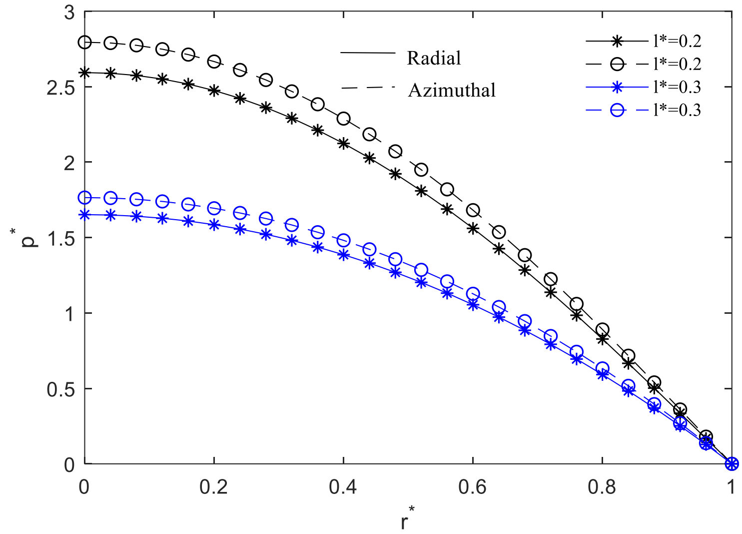

Figure 4 Variation with r^* for different values of couple stress parameter l^* on dimensionless pressure p^* with $G=0.04, H=1.2$, $c=0.3$, $s^*=0.7$, $l^*=0.3$, $\psi=0.01$, $\alpha=0.1$, $\beta=0.2$

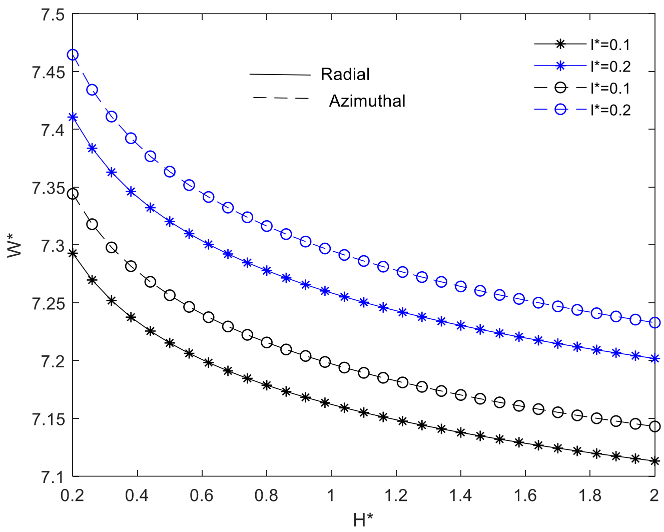

Figure 5 Variation with H^* for different values of couple stress parameter l^* on dimensionless pressure W^* with $G = 0.4, M = 2$, $M' = 0.15$, $s^* = 0.7$, $l^* = 0.3$, $\psi = 0.01$, $\alpha = 0.01$, $\beta = 0.02$, $c = 0.3$

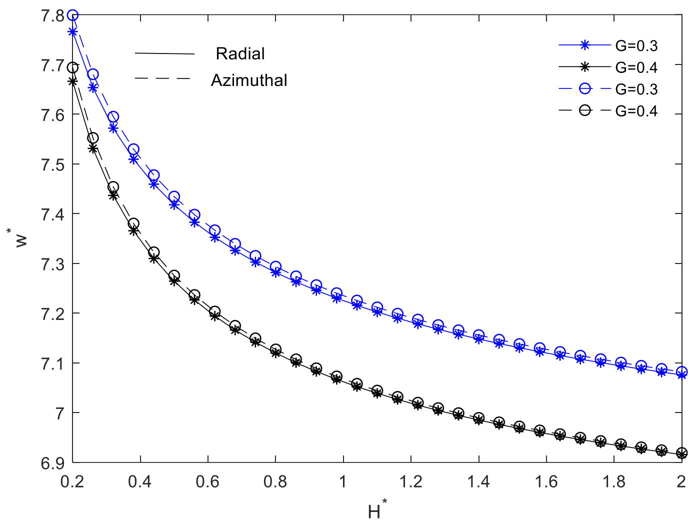

Figure 6 Variation with H^* for different values of Viscosity Variation parameter G on dimensionless pressure W^* with $c = 0.4, M = 2$, $M' = 0.15$, $s^* = 0.7$, $l^* = 0.3$, $\psi = 0.01$, $\alpha = 0.01$, $\beta = 0.2$

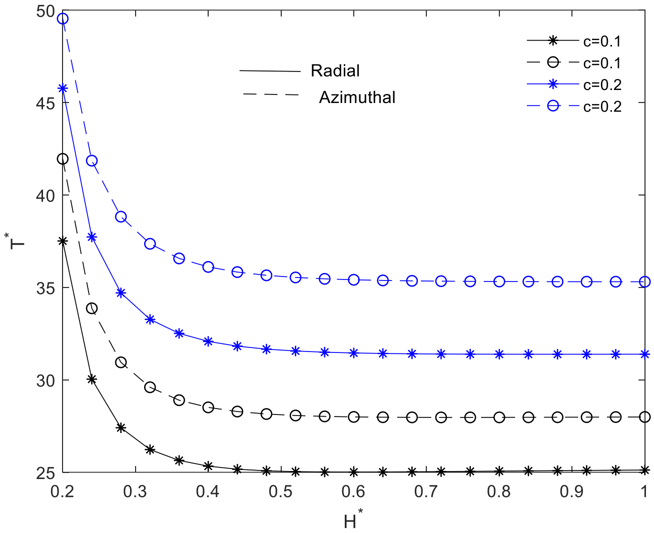

Figure 7 Variation with H^* for different values of roughness parameter c on dimensionless pressure T^* with $G=0.4, M=2$, $M'=0.15$, $s^*=0.7$, $l^*=0.3$, $\psi=0.01$, $\alpha=0.01$, $\beta=0.2$.

4. Squeeze film pressure

Figure 2 presents the profile of mean pressure p^* along dimensionless co-ordinate axial r^* as a function of viscosity variation parameter G for both roughness configurations and clearly shows that for larger values of viscosity variation parameter G the mean pressure inclines (declines) for azimuthal (radial) roughness structures. The results indicate that as the viscosity variation parameter increases the pressure also increases. Figure 3 presents the profile of mean pressure p^* against r^* as a function of Hartmann number M is presented and can clearly be seen that for increasing Hartmann number M the mean pressure increases for azimuthal roughness rather than radial roughness structure. Further, it is observed that for each value of Hartmann number with the pressure dependent viscosity is significantly increases. However, the relative variation of pressure with pressure-dependent viscosity effect increases with the effect of couple stress parameter. It proves the significance of viscosity-pressure variation for couple stress lubricants. Figure 4 shows the distinctive values of couple stress parameter l^* the pressure p^* against r^* is presented and shows that for increasing couple stress parameter values the pressure increases.

5. Load supporting capacity

The load carrying capacity W^* against height H^* as a function of couple stress parameter l^* is displayed in figure 5 for both roughness configurations. It is found that the impact of couple stress fluid in the existence of applied magnetic field is to substantially enhance the load supporting capacity. This is due to the use of magnetic field normal to the flow which results in a lower velocity of the lubricant in the fluid film region. Thus, a large amount of the fluid is retained in the film region and this yields a rise in load carrying capacity. In figure 6 the profile of W^* against height H^* as a function of viscosity variation parameter G is depicted and shows that the mean load carrying capacity declines for larger values of viscosity variation parameter G.

6. Time Height Relationship

Figure 7 illustrated the combined influence of viscosity variation and roughness factors on time height relation T^* for both roughness patterns and noticed that time height relation T^* enhances as couple stress fluid characteristic enhances the fluid's resistance. The presence of couple stress fluid is responsible to strengthen the response time

7. Conclusions:

The study discussed in this paper determines the influence of the viscosity variation on the performance of squeezing action between conical bearings with couple stress fluid. Enchantments in the squeeze film pressure and load carrying capacity are observed for larger values of MHD with slip parameter. Further the time height relationship increases by increasing values roughness parameter.

- The purpose of the couple stress fluid parameter is to increase the efficiency of the load carrying capacity and pressure. When the viscosity variation parameter increases the load carrying capacity decreases and enhances when the height increases.
- The velocity slip at the porous-fluid interface is directly evaluated by means of modified Darcy's law considering viscosity variation with MHD.

8. Nomenclatures

l	–	Couple stress parameter = $\left(\dfrac{\eta}{\mu}\right)^{\frac{1}{2}}$
l^*	–	Dimensionless couple stress parameter
MHD	–	Magneto-hydrodynamic
B_0	–	Magnetic field
ψ	–	Permeability parameter = $\dfrac{k\delta}{h_0^3}$
μ	–	Viscosity co-efficient
M	–	Hartmann number = $\left(B_0 h_0 \left(\dfrac{\sigma}{\mu}\right)^{1/2}\right)$
δ	–	Porous layer thickness
M'	–	Porosity
α	–	Dimensionless slip constant
s^*	–	Slip parameter $\dfrac{\sigma^*}{h_0}$
u^*, w^*	–	Dimensionless velocity components in r and z directions

p^*	–	Dimensionless pressure
w^*	–	Dimensionless work load
T^*	–	Dimensionless time

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