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Optimizing Inventory Management for Perishable Goods: Managing Exponential Demand Variations, Stable Holding Charges, and Partial Backlog Handling

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Abstract:

In the ever-evolving landscape of modern business, adept inventory management stands as a cornerstone of competitiveness and profitability. This research unveils a tailored inventory optimization model designed for perishable goods, accounting for critical factors including exponential time-varying demand, consistent holding costs, and partial backlog management. The model's objective is to find an equilibrium, optimizing inventory expenses while maintaining sufficient stock to fulfil customer needs. By incorporating these key elements, businesses can enhance their inventory management strategies to adapt to changing market conditions and improve overall operational efficiency. Through analytical insights and numerical simulations, this research provides valuable guidance for decision-makers in optimizing inventory policies for perishable goods.

Keywords: Perishable goods, exponential demand variations, stable holding charge, Shortages and partial backlog.

1. Introduction

Efficient inventory management is fundamental for businesses operating in various industries to meet customer demands while minimizing costs and maximizing profitability. In particular, the management of deteriorating items presents unique challenges due to factors such as time-varying demand patterns and the potential for inventory deterioration over time. This paper focuses on developing an inventory optimization model specifically tailored to address these challenges, with a particular emphasis on items subject to exponential time-varying demand, stable holding costs, and partial backlog management.

Deteriorating items, such as perishable goods or products with limited shelf lives, require careful handling to avoid obsolescence and minimize wastage. Moreover, the demand for such items often fluctuates over time, influenced by factors such as seasonality, market trends, and promotional activities. Conventional inventory models might overlook these dynamics, potentially resulting in suboptimal inventory levels and higher expenses. In addition to demand variability, the management of deteriorating items must also consider the cost implications of holding inventory over time. Stable holding costs, representing the expenses associated with storing and maintaining inventory, play a significant role in determining the optimal inventory policy. Balancing these costs with the need to fulfil customer orders in a timely manner is essential for achieving operational efficiency and customer satisfaction.

Furthermore, the presence of partial backlog management introduces another layer of complexity to the inventory optimization problem. Partial backlogging occurs when customer demand exceeds available inventory, resulting in unfulfilled orders that may be partially satisfied in subsequent periods. Effective management of partial backlogs requires careful consideration of order fulfilment priorities and inventory replenishment strategies to minimize stockouts and associated costs.

Against this backdrop, this paper proposes an inventory optimization model that integrates these key factors to provide decision-makers with actionable insights for managing deteriorating items effectively. By capturing the interplay between exponential time-varying demand, stable holding costs, and partial backlog management, the proposed model offers a comprehensive framework for optimizing inventory policies and enhancing overall supply chain performance. Through empirical validation and numerical experiments, we demonstrate the practical applicability and the efficacy of the suggested model in real-world inventory management scenarios.

2. Literature Overview

The literature on inventory optimization for deteriorating things through various demand patterns, holding charge structures, also backlog management strategies is rich and diverse. Several notable studies have contributed to this field, offering insights into the development of effective inventory models tailored to specific contexts. Here, we review relevant literature that addresses the complexities of managing perishable goods, encompassing aspects such as time-varying demand, stable holding charges, also partial backlog management.

Su, Lin, and Tsai (1999) investigated a deterministic model for inventory management of perishable goods experiencing an exponential decline in demand. Their study aimed to provide insights into optimal inventory policies under deteriorating conditions, considering the impact of demand decline on production and inventory management decisions.

Shah and Shah (2000) carried out an extensive literature review on inventory models for perishable goods, emphasizing important trends, methodologies, and research gaps within this domain. Their analysis provided valuable insights into the progression of inventory management techniques for perishable goods throughout history.

Goyal and Giri (2001) reviewed current developments in modelling perishable inventory, providing a comprehensive overview of the methodologies and approaches employed in this field. Their study contributed to the understanding of the challenges and opportunities associated with managing deteriorating items in various industries.

Abad (2001) studied ideal pricing and order-sizing strategies for a distributor, focusing on scenarios involving partial backlog management, addressing the trade-offs between pricing decisions and inventory management strategies in a dynamic market environment.

Ouyang and Cheng (2005) presented an inventory model designed for perishable goods with an exponential decrease in demand, incorporating the concept of partial backlog management, aiming to optimize inventory policies while considering the dynamics of demand and backlog fulfilment.

Tripathy and Mishra (2010) examined inventory replenishment strategies for Weibull deteriorating items with quadratic demand and allowable payment delays, contributing to the body of literature on inventory management amidst uncertainty and credit limitations.

Singh and Pattnayak (2012, 2013) proposed EOQ models for perishable goods with time-dependent demand, variable deterioration, also partial backlog, offering practical insights into inventory optimization strategies under various demand and cost structures.

Amutha and Chandrasekaran (2013) investigated an economic order quantity model for perishable goods with quadratic demand and time-dependent holding charges, highlighting the importance of considering nonlinear cost functions in inventory management decisions.

Dash, Singh, and Pattnayak (2014) proposed an inventory model for perishable items with exponentially decreasing demand and holding costs that vary over time, addressing the complexities of inventory management amidst fluctuating cost structures.

Dutta and Kumar (2015) formulated a model for managing partially backlogged inventory of deteriorating items, considering fluctuating demand and holding costs over time, offering insights into inventory management strategies under uncertain demand and cost conditions.

Uthayakumar and Karuppasamy (2017) explored a model for managing inventory of variable deteriorating pharmaceutical items, considering time-dependent demand and holding costs while utilizing trade credit, addressing the unique challenges faced by the healthcare industry in managing perishable inventory.

Sekar and Uthayakumar (2018) investigated a manufacturing inventory model for coping with exponentially rising demand, incorporating preservation technology to mitigate shortages, contributing to the understanding of inventory management in dynamic production environments.

Babangida and Baraya (2020) proposed a model for managing inventory of non-instantaneously perishable items with time-dependent quadratic demand, involving two storage facilities and accounting for shortages within a trade credit policy, addressing the complexities of managing inventory with multiple constraints and cost considerations.

Overall, the reviewed literature underscores the importance of developing tailored inventory optimization models to address the challenges of managing deteriorating items effectively. By considering factors such as demand variability, cost structures, and backlog management strategies, businesses can enhance their inventory management practices and improve overall operational efficiency.

Within this investigation, we introduce an EOQ inventory model tailored for perishable goods. Our model considers an exponentially increasing demand function and incorporates backlogging dynamics tied to the dwell time for replenishment. The main objective is to reduce total inventory expenses. We illustrate the application and solution approach of the model through numerical examples.

Additionally, we conduct a sensitivity analysis on key parameters to evaluate their impact on the model's performance.

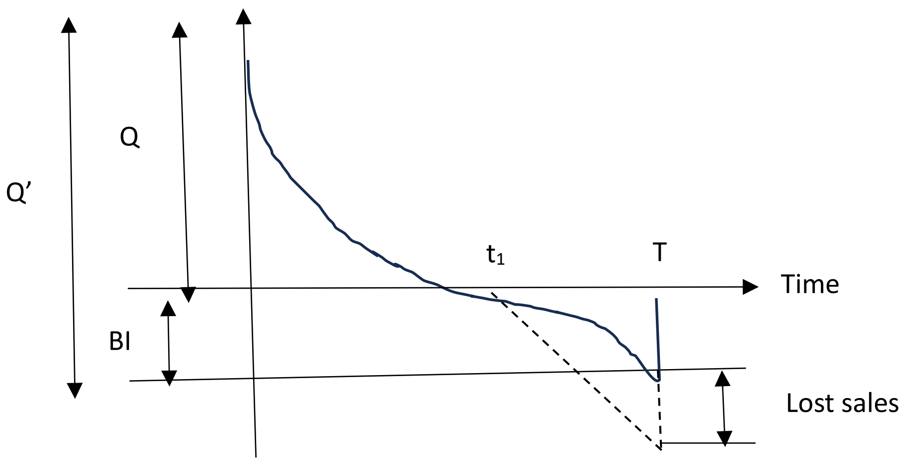
3. Assumptions and Notations

- The demand rate for the item exhibits exponential time dependence.

  (i.e.) $D(t) = e^{\mu t}, where\ \mu\ is\ constant.$

- The lead time is constant.
- Shortages are permissible, and during periods of stockouts, the backlogging rate varies and is contingent upon the length of the wait for the next restocking. Specifically, the backorder rate is defined as $B(t) = e^{-\delta(T-t)}$, where δ represents the backlogging parameter with a range of 0 to 1, and the parameter (T-t) denotes the dwell time, where $t_1 \leq t \leq T$.
- $I_1(t)$ – stock level at time t, $0 \leq t \leq t_1$.
- $I_2(t)$ – stock level at time t, $t_1 \leq t \leq T$.
- T – duration of the inventory cycle.
- t_1 – The duration of time during which the inventory remains without any shortage.
- C_O – Ordering cost (OC).
- C_p – Purchase cost (PC).
- C_h – Holding cost (HC) is constant.
- θ – The constant deterioration rates.
- C_d – The cost of deteriorated items (DC).
- C_s – The cost associated with shortages of backlogged items.
- C_L – The financial impact resulting from lost sales.
- δ – Backlogging parameter.
- $B(t)$ – Backlog inventory level
- C_T – Total cost (TC).

4. Mathematical Formulation



Let $I_1(t)$ represent the current stock at any time instant t $(0 \leq t \leq t_1)$. An stock level is Q at t = 0 and the stock level drops to zero at t = t_1. The proportion of variation of inventory level is specified by,

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -e^{\mu t}, \quad 0 \leq t \leq t_1 \longrightarrow (1)$$

through boundary conditions $I_1(0) = Q$ & $I_1(t_1)=0$.

The solution of (1) is

$$I_1(t) = \frac{(t_1-t)}{1+\theta t}\left[1 + \frac{(\mu+\theta)}{2}(t_1+t)\right]$$

$$\text{And } Q = t_1 + (\mu+\theta)\frac{t_1^2}{2}$$

During the interval [t_1,T], shortages occur also the demand is partially backlogged. Let $I_2(t)$ represent the inventory level at $t(t_1 \leq t \leq T)$ then the differential equation is

$$\frac{dI_2(t)}{dt} = -e^{\mu t}e^{-\delta(T-t)}, \quad t_1 \leq t \leq T \longrightarrow (2)$$

With boundary conditions $t = t_1, I_2(t) = 0$.

The solution of (2) is

$$I_2(t) = (t_1 - t)\left[1 - \delta T + \frac{(\mu+\delta)}{2}(t_1+t)\right]$$

The highest level of backordered inventory denoted as BI is reached at t=T.

$$BI = -I_2(t) = -(t_1 - T)\left[1 + \frac{(\mu+\delta)}{2}t_1 + \frac{(\mu-\delta)}{2}T\right]$$

Therefore, the total order quantity during the entire time interval [0,T] is

$Q' = Q + BI = t_1 + (\mu+\theta)\frac{t_1^2}{2} - (t_1 - T)\left[1 + \frac{(\mu+\delta)}{2}t_1 + \frac{(\mu-\delta)}{2}T\right]$

Holding charge $(HC) = \int_0^{t_1} C_h I_1(t)dt$

$$HC = \frac{C_h t_1^2}{24}[12 + 4t_1(\theta + 2\mu) - 3\theta t_1^2(\mu+\theta)]$$

Ordering charge (OC) = C_0

Purchase charge (PC) = C_p Q'

$$PC = C_p\left[t_1 + (\mu+\theta)\frac{t_1^2}{2} - (t_1 - T)\left[1 + \frac{(\mu+\delta)}{2}t_1 + \frac{(\mu-\delta)}{2}T\right]\right]$$

Deterioration charge (DC) = $C_d\left\{Q - \int_0^{t_1} D(t)dt\right\}$

$$= C_d\left[\frac{\theta t_1^2}{2}\right]$$

Shortage Charge = $-C_s\int_{t_1}^{T} I_2(t)dt$

$$= C_s(t_1 - T)\left[\left(\frac{1-\delta T}{2}\right)(t_1 - T) - \frac{(\mu+\delta)}{6}(T^2 + Tt_1 - 2t_1^2)\right]$$

Lost sales charge = $C_L \int_{t_1}^{T} \left[1 - e^{-\delta(T-t)}\right] e^{\mu t} dt$

$$= C_L \frac{\delta}{2}(T - t_1)^2$$

Total cost = Ordering charge + Holding charge + Purchase charge + Deterioration charge + Shortage charge + Lost sales charge

$$C_T = C_0 + \frac{C_h t_1^2}{24}[12 + 4t_1(\theta + 2\mu) - 3\theta t_1^2(\mu + \theta)]$$

$$+C_p\left[t_1 + (\mu + \theta)\frac{t_1^2}{2} - (t_1 - T)\left[1 + \frac{(\mu+\delta)}{2}t_1 + \frac{(\mu-\delta)}{2}T\right]\right] + C_d\left[\frac{\theta t_1^2}{2}\right]$$

$$+C_s(t_1 - T)\left[\left(\frac{1-\delta T}{2}\right)(t_1 - T) - \frac{(\mu+\delta)}{6}(T^2 + Tt_1 - 2t_1^2)\right] + C_L\frac{\delta}{2}(T - t_1)^2$$

Our goal is to minimize the overall cost.

The necessary condition is $\frac{\partial C_T}{\partial t_1} = 0 \; and \; \frac{\partial^2 C_T}{\partial {t_1}^2} > 0 \; for \; all \; t_1 > 0$

We get

$$\frac{\partial C_T}{\partial t_1} = C_h\left[t_1 + \frac{\theta t_1^2}{2} + \mu t_1^2 - \frac{\mu\theta t_1^3}{2} - \frac{\theta^2 t_1^3}{2}\right] + C_p[\theta t_1 - \delta t_1 + \delta T] + C_d\theta t_1 + C_L[-\delta T + \delta t_1]$$
$$+ C_s\left[t_1 - T - \delta T t_1 + \delta T^2 - \frac{\mu T}{6} + \frac{2}{3}t_1\mu - \frac{\delta T}{6} + \frac{2}{3}\delta t_1\right] = 0$$

And

$$\frac{\partial^2 C_T}{\partial {t_1}^2} = C_h\left[1 + \theta t_1 + 2\mu t_1 - \frac{3}{2}\theta t_1^2(\mu + \theta)\right] + C_p[\theta - \delta] + C_d\theta + C_L\delta$$
$$+ C_s\left[1 - \delta T + \frac{2}{3}(\mu + \delta)\right] > 0$$

5. Numerical illustration 1:

Let's examine an inventory system characterized by the following parameter values, expressed in appropriate units $\left[\theta, T, \mu, \delta, C_o, C_d, C_h, C_p, C_s, C_L\right] = [0.03,5,1.5,0.001,2,0.5,0.1,30,3,1]$.

Then we get t_1 =2.8203, Q'=23.7596 and C_T=761.3449

Numerical illustration 2:

Let's examine an inventory system characterized by the following parameter values, expressed in appropriate units $\left[\theta, T, \mu, \delta, C_o, C_d, C_h, C_p, C_s, C_L\right] = [0.1,4,2.5,0.005,3,0.3,1,30,4,1.5]$.

Then we get t_1 = 1.2475, Q'=24.0589 and C_T = 824.1926

6. Sensitivity Analysis

Table – 1 : Fluctuation in deterioration rate (θ)

θ	t_1	Q'	C_T
0.01	2.8447	23.7517	760.3105
0.02	2.8324	23.7557	760.8297
0.03	2.8203	23.7596	761.3449
0.04	2.8082	23.7634	761.8560
0.05	2.7962	23.7671	762.3629

Table-1 shows that when deterioration rate θ increases, automatically quantity, total cost are increases and the duration of time during which the inventory remains without any shortage (t_1) is decreases.

Table – 2 : Fluctuation in length of the inventory cycle (T)

T	t_1	Q'	C_T
4.6	2.6076	20.4782	652.7475
4.8	2.7142	22.0889	705.9319
5.0	2.8203	23.7596	761.3449
5.2	2.9259	25.4903	819.0066
5.4	3.0310	27.2810	878.9372

Table-2 shows that when T increases, automatically the duration of time during which the inventory remains without any shortage (t_1), quantity and total cost are increases.

Table – 3 : Fluctuation in μ

μ	t_1	Q'	C_T
1.3	2.9414	21.2609	677.6937
1.4	2.8786	22.5102	719.4678
1.5	2.8203	23.7596	761.3449
1.6	2.7659	25.0090	803.3134
1.7	2.7152	26.2584	845.3630

Table-3 shows that when **μ** increases, automatically quantity, total cost are increases and the duration of time during which the inventory remains without any shortage (t_1) is decreases.

7. Conclusion

In conclusion, the development of an inventory optimization model tailored specifically for perishable goods with exponential demand variation, stable holding charges, also partial backlog management represents a notable advancement in the realm of supply chain management. By addressing the complexities inherent in managing such inventory items, this model offers valuable insights and practical guidance for businesses striving to achieve efficient inventory management practices. Through numerical examples and sensitivity analyses, the effectiveness and robustness of the proposed model have been demonstrated, highlighting its potential to enhance operational efficiency, minimize costs, and improve customer satisfaction. As businesses continue to face evolving market dynamics and supply chain challenges, the implementation of advanced inventory optimization models like the one presented here can serve as a strategic tool for driving sustainable growth and competitive advantage.

Future extensions of the inventory optimization model could include advanced demand forecasting techniques, integration of dynamic pricing strategies, implementation of real-time inventory tracking using IoT technology, and consideration of sustainability metrics for environmentally conscious inventory management.

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