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The Markovian Batch Arrival Queue with Differentiated Vacation

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Abstract:

A Poisson Batch Arrival Queueing Model with a single server taking two different vacations at various rates is being considered for the study. The probability generating function method of the different states is considered for deriving the various performance measures of the states; numerical cases and cost studies are also extended.

Keywords: Poisson batch arrival queue; differentiated vacations; probability generating function;

1. Introduction

Queueing models where the server goes on vacation have applications in many areas, such as production and manufacturing systems, telecommunications, and service industries. In real time, during a normal time, if a customer is in the queue, the server is working at the normal service rate. When there are no customers in the system, the server leaves for a primary vacation for a certain period of time. Upon returning from a primary vacation, if the server finds no customers in the system, server leaves for another secondary vacation. After returning from the secondary vacation, the server will continue the service when customers are in the system. If there is no customers in the system, the server will wait for the arrival of the customers. Remarkable research work about server vacations on different vacations can be found in the queueing model literatures.

Since Levy and Yechiali's [1] publication, where the concept was first introduced, many academics have become interested in server vacation queuing systems. Doshi [2] did a number of great surveys on those vacation models. Many scholars spent time on the working vacation concept, and different authors have modified the original model. Baba [3] utilized the matrix analytical method for examining a G1/M/1 working queue vacations. V.M. Chandrasekaran [4] carried out research on working vacation queueing models. ANFIS and cost optimization for a Markovian queue with operation vacation were examined by Sonali Thakur [5]. Utilizing matrix-geometric and ANFIS methods, computational findings are offered to assess the depth of the adaptive neural fuzzy inference system. The fundamental ideas of queueing theory were described by Binary kumar [6]. A bulk arrival poisson with vacation was the focus of research by Borthakur, A. Choudhury [7], G.Madan and Gautam Choudhury[8]A two phase bulk arrival queueing with vacation under Bernoulli schedule was introduced by K.C.Madan. Oliver C. Ibe, [9] considered it as a multiple vacation line system with separated vacations. J.Li,W.Liu [10] , initiated steady state analysis of a discrete bulk arrival with operating vacations.A.D.Banik [11] modelled G1/M/1/N queue with various operating vacations.B.Deepa and K.Kalidass [12] viewed a single server queueing model with operating breakdowns and vacations .K.V.Vijayashree[13] considered a single server queueing line model and separated vacation and disturbance .Zhang.M [14] searched the appearance of only one server queue with operating vacations and vacation disturbance. Ganesh Sapkota [15] modelled the transient state of the M/M/C queueing line model with a standby server and reneging customers. Ammar, S. I [16] modelled transient state of a one server vacation queue with a waiting server and restless customers. Gupta [17] deals with two server model with operating vacation and reneging of customers due to impatience.V. Vijayalakshmi, B.Deepa and K.Kalidass[18] modelled cost of single server queue line with operating breakdowns using two – phase services.

The layout of the paper is described as follows, the model considered here is presented in full detail in Section 2. The analysis in steady-state of the proposed model is explained in Section 3. The reliability measures and performance measures are obtained in Sections 4 and 5. Some illustrative numerical examples to point out the effects of the server are presented in Section 6. A cost optimization problem is discussed in section 7 and a few concluding remarks are given in Section 8.

2. The Model:

Here a fixed batch-sized Markovian model with two non - identical vacation schemes is being considered. The arrival unit follows the distribution is Poisson with an arrival rate λ and server is serving the arrival unit comes the distribution of exponentially distributed independent and identically distributed random variable with a rate μ. The arrival flow of customers enters the system in the way of batches/bulks. The capacity of batches is a random variable X of positive or neutral values with a probability distribution $hi = Pr\{X = i\}, i \geq 1$. When there are zero customers in the system, the server leaves for a primary vacation at a rate r_1 for a certain period of time. Upon return from a primary vacation, if the server finds zero customers in the system, the server leaves for another vacation named as a secondary vacation at rate r_2. However, the server is from the secondary vacation then continue the service in normal state, and it will not go for vacation again. The arrival, service, and vacation periods are independent random variables of each other.

Let B(t) be state of the server at t(time),

$$B(t) = \begin{cases} 0, the\ server\ in\ normal \\ 1, the\ server\ in\ primary\ vacation\ state \\ 2, the\ server\ in\ secondary\ vacation\ state \end{cases}$$

Let X(t) be the count of consumers attending in the system at t(time).

Then $\{(B(t), X(t)), t \geq 0\}$ is considered a Continuous time MARKOV CHAIN.

Let $P_{i,n}(t) = Prob\{B(t) = i, X(t) = n\}$, $n \geq 0, i = 0,1,2$

3. Steady State Analysis

The Steady State governing equations are derived as

$$\lambda P_{2,0} = r_1 P_{1,0} \quad , n = 0 \qquad \_\_\_\_\_\_(1)$$

$$(r_2 + \lambda) P_{2,n} = \lambda \sum_{i=1}^{n} h_i P_{2,n-1} \quad , \quad n \geq 1 \qquad \_\_\_\_\_\_(2)$$

$$\therefore (\lambda + \mu) P_{0,1} = \mu P_{0,2} + r_1 P_{1,1} + r_2 P_{2,1} \quad , n = 1 \qquad \_\_\_\_\_\_(3)$$

$$\therefore (\lambda+\mu)P_{0,n} = \mu P_{0,n+1} + r_1 P_{1,n} + r_2 P_{2,n} + \lambda \sum_{i=1}^{n} h_i\, P_{0,n-1}, \quad n \geq 2 \_\_\_\_(4)$$

$$(\lambda + r_1)P_{1,0} = \mu P_{0,1} \quad , n = 0 \qquad \_\_\_\_(5)$$

$$(\lambda + r_1)P_{1,n} = \lambda \sum_{i=1}^{n} h_i\, P_{1,n-1}, n \geq 1 \qquad \_\_\_\_(6)$$

Probability generating function is

$$\sum_{n=0}^{\infty} P_{2,n} z^n = P_2(z), \sum_{n=1}^{\infty} P_{0,n} z^n = P_0(z)\,,$$

$$\textstyle\sum_{n=0}^{\infty} P_{1,n} z^n = P_1(z) \qquad H(z) = \sum_{n=1}^{\infty} h_n z^n$$

$$(\lambda + r_2)P_2(z) = r_2 P_{2,0} + r_1 P_{1,0} + \lambda H(z) P_2(z) \qquad \_\_\_\_(7)$$

$$P_2(z) = \frac{r_1 P_{1,0} + r_2 P_{2,0}}{\lambda\big(1 - H(z)\big) + r_2} \qquad \_\_\_\_(8)$$

$$P_2(1) = \frac{r_1 P_{1,0} + r_2 P_{2,0}}{r_2} \qquad \_\_\_\_(9)$$

$$\lambda P_0(z) + \mu P_0\,(z) = -\mu P_{0,1} + \frac{1}{2}\mu P_0(z) - r_1 P_{1,0} + r_1 P_1(z) + \lambda H(z) P_0(z) - r_2 P_{2,0} + r_2 P_2(z)$$

$$\lambda P_0(z) - \lambda H(z) P_0(z) - \frac{1}{2}\mu P_0(z) + \mu P_0\,(z) - r_1 P_1(z) - r_2 P_2(z) = -\mu P_{0,1} - r_1 P_{1,0} - r_2 P_{2,0}$$

$$(\lambda\big(1 - \lambda H(z)\big) P_0(z) + \mu\,\left(1 - \frac{1}{z}\right) P_0(z)) - r_1 P_1(z) - r_2 P_2(z) = -\mu P_{0,1} - r_1 P_{1,0} - r_2 P_{2,0}$$

$$\left[\lambda\,\big(1 - \lambda H(z)\big) + \mu(1 - \frac{1}{z})\right] P_0(z) - r_1 P_1(z) - r_2 P_2(z) = -\mu P_{0,1} - r_1 P_{1,0} - r_2 P_{2,0} \_\_\_\_(10)$$

$$\big[\lambda\big(1 - H(z)\big) + r_1\big] P_1(z) = \mu P_{0,1} \qquad \_\_\_\_(11)$$

$$\therefore P_1(z) = \frac{\mu P_{0,1}}{\lambda\big(1 - \lambda H(z)\big) + r_1} \qquad \_\_\_\_(12)$$

$$P_1(1) = \frac{\mu P_{0,1}}{r_1}$$

From (10),

$$P_0(z) = \frac{r_1 P_1(z) + r_2 P_2(z) - \mu P_{0,1} - r_1 P_{1,0} - r_2 P_{2,0}}{\big(1 - \lambda H(z)\big) + \mu(1 - \frac{1}{z})}$$

$$P_0(z) = \frac{\begin{array}{c}(-\mu P_{0,1})\left[\lambda^2\big(1 - H(z)\big)^2 + r_2\,\lambda\big(1 - \lambda H(z)\big)\right] + \\ \left[-r_1 P_{1,0} - r_2 P_{2,0}\right]\left[\lambda^2\big(1 - H(z)\big)^2 + r_1\,\big(1 - H(z)\big)\right]\end{array}}{\begin{array}{c}\lambda^3\big(1 - H(z)\big)^3 + (r_1 + r_2)\lambda^2\big(1 - H(z)\big)^2 + r_1\, r_2\,\big(1 - H(z)\big) \\ + \mu\left(1 - \frac{1}{z}\right)\lambda^2\big(1 - H(z)\big) \\ + (r_1 + r_2)\mu\left(1 - \frac{1}{z}\right)\lambda\big(1 - H(z)\big) + r_1\, r_2 \mu\left(1 - \frac{1}{z}\right)\end{array}} \qquad \_\_\_\_(13)$$

When z=1

$$P_0(1) = \frac{0}{0}$$

Using L'Hospitals rule, we get

$$P_0(z) = \frac{(-\mu P_{0,1})[-2\lambda^2(1-H(z))H'(Z) - r_2\,\lambda H'(z)] + [-r_1 P_{1,0} - r_2 P_{2,0}][-2\lambda^2 1 - H(z))H'(Z) - r_1\lambda H'(z)\,] - \theta_1}{\begin{array}{c}-3\lambda^3\left(1-H(z)\right)^2 H'(z) - (r_1+r_2)2\lambda^2 1 - H(z))H'(Z) - r_1\,r_2\,H'(z) \\ +\mu\frac{1}{z^2}\lambda^2\left(1-H(z)\right)^2 - 2\left(1-\frac{1}{z}\right)\mu\lambda^2\left(1-H(z)\right)H'(z) \\ +(r_1+r_2)\mu\lambda\frac{1}{z^2}\left(1-H(z)\right) - (r_1+r_2)\mu\left(1-\frac{1}{z}\right)\lambda H'(z) + r_1\,r_2\mu\frac{1}{z^2} - \theta_2\end{array}} \quad \_\_\_\_\_\_(14)$$

$$P_0(1) = \frac{\mu r_2\,\lambda H'(1)P_{0,1} + {r_1}^2\,\lambda H'(1)P_{1,0} + r_1\,r_2\lambda H'(1)P_{2,0}}{r_1\,r_2\left(\mu - \lambda H'(1)\right)} = \frac{Q_1(1)}{Q_2(1)} \quad \_\_\_\_\_\_(15)$$

$$P_0{}'(z) = \frac{Q_2 Q_1{}' - Q_1 Q_2{}'}{{Q_2}^2}$$

$$P_0{}'(1) = \frac{Q_2(1)Q_1{}'(1) - Q_1(1)Q_2{}'(1)}{\{Q_2(1)\}^2}$$

$$Q_1'(z) = -\mu P_{0,1}\left[-2\lambda^2\left(1-\lambda H(z)\right)H''(z) + 2\lambda^2 H'(z) - r_2\lambda H''(z)\right] + \left[-r_1 P_{1,0} - r_2 P_{2,0}\right]\left[-2\lambda^2\left(1-\lambda H(z)\right)H''(z) + 2\lambda^2 H'(z)^2 - r_1\lambda H''(z)\right]$$

$$Q_1'(1) = -\mu P_{0,1}\,[2\lambda^2 H'(1)^2 - r_2\lambda H''(1)] - \left[r_1 P_{1,0} + r_2 P_{2,0}\right][2\lambda^2 H'(1)^2 - r_1\lambda H''(1)]$$

$$Q_2'(1) = 2\lambda^2(r_1+r_2)H'(1)^2 - r_1\,r_2\lambda H''(1) - 2(r_1+r_2\,)\lambda H'(1) - 2r_1\,r_2\mu$$

$$P_0'(1) = \frac{Q+R+ST}{U} \quad \_\_\_\_\_\_(16)$$

Q=$r_1\,r_2\mu - r_1\,r_2\lambda H'(1) - \mu P_{0,1}\left(2\lambda^2 H'(1)^2 - r_2\lambda H''(1)\right)$

R=$\left[-r_1 P_{1,0} - r_2 P_{2,0}\right]2\lambda^2 H'(1)^2 - r_1\lambda H''(1) - (\mu r_2\,\lambda H'(1)P_{0,1}$

S=${r_1}^2\,\lambda H'(1)P_{1,0} + r_1\,r_2\lambda H'(1)P_{2,0}$

T=$[2\lambda^2(r_1+r_2)H'(1)^2 - r_1\,r_2\lambda H''(1) - 2(r_1+r_2\,)\lambda H'(1) - 2r_1\,r_2\mu]$

U=${r_1}^2\,{r_2}^2\left(\mu - \lambda H'(1)\right)^2$

We know

$$P_0(1) + P_1(1) + P_2(1) = 1$$

$${r_1}^2\mu P_{1,0} + r_1\,r_2\mu P_{2,0} + r_2\mu^2 P_{0,1} = r_1\,r_2\left(\mu - \lambda H'(1)\right) \quad \_\_\_\_\_\_(17)$$

From (1)

$$P_{2,0} = \frac{r_1}{\lambda}P_{1,0} \quad \_\_\_\_\_\_(18)$$

From (5)

$$P_{0,1} = \frac{\lambda + r_1}{\mu}P_{1,0} \quad \_\_\_\_\_\_(19)$$

Sub (18) & (19) in (17)

$$P_{1,0} = \frac{r_1\, r_2 \lambda[\mu - \lambda H'(1)]}{\mu[r_1{}^2\, \lambda + r_1{}^2\, r_2{}^2 + \lambda^2 r_2 + r_1\, r_2 \lambda]} \text{\_\_\_\_\_\_\_\_\_\_\_\_}(20)$$

$$P_{0,1} = \frac{(\lambda + r_1)\rho r_1\, r_2\big(\mu - \lambda H(1)\big)}{\mu[\, \lambda r_1{}^2 + r_1\, r_2{}^2 + \lambda^2 r_2 + r_1\, r_2 \lambda]} \text{\_\_\_\_\_\_\_\_\_\_\_\_}(21)$$

equation (16) $P_0{}'(1)$ and

$$P_1{}'(z) = \mu P_{0,1}\big[\lambda\big(1 - H(z)\big) + r_1\big]^{-2} H'(z)$$

$P_1'(1) = \frac{\mu P_{0,1} H'(1)}{r_1{}^2}$ and

$$P_2'(z) = \big[r_1 P_{1,0} + r_2 P_{2,0}\big]\big[\lambda\big(1 - H(z)\big) + r_2\big]^{-2} H'(z)$$

$$P_2'(1) = \frac{r_1 P_{1,0} + r_2 P_{2,0}}{r_2{}^2} H'(1)$$

Now, customers count in the system=$E(X) = P_0'(1) + P_1'(1) + P_2'(1)$

4. Reliability Measures:

Here, we discuss few reliability pointers of this model under deliberation. Specifically, we examine the accessibility of the server, the failure of the server. Accessibility is probability of the system operating rightly when it is asked to use it and server failure rate is probability of system will not work correctly. Let P(t) be probability of the server either serving to a customer or idle or on vacation. Steady state availability defined by $\rho < 1$.Figure 1 illustrate the availability of the server against service rate μ. We observe that the availability of the server under steady state decreases with increase in the arrival rate λ for the three different values of λ = 2, 2.1 and 2.2 as expected. Figure 2. depicts variation of the failure frequency of the server against the arrival rate λ for three different values of the primary vacation rate r_1=0.1,0.12 and 0.13. As we expect the failure frequency of the server increases with increasing in the arrival rate λ.

The server accessibility is, Availability A= $P_0(1)$ it is shown in Figure 1.

The server failure frequency is, Failure F=$P_1(1)$ +$P_2(1)$ it is shown in Figure 2.

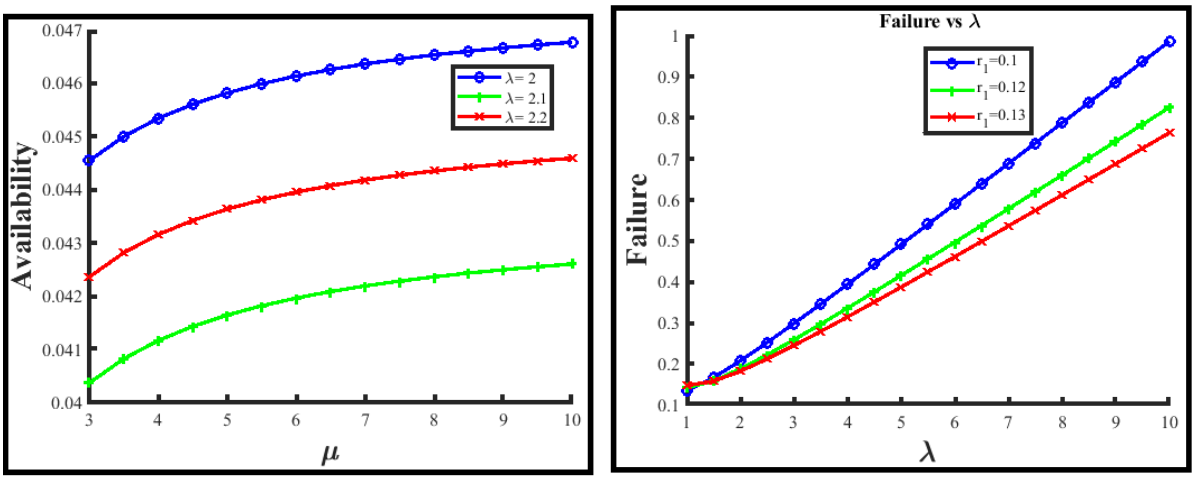

Fig.1 & 2-Availability and Failure of server

5. Performance measure:

Various measures of system performance can be developed from the steady state distribution.

1. Expected customers count in system

$$E(X) = P_0'(1) + P_1'(1) + P_2'(1) = \sum_{n=0}^{\infty} n\,P_n$$

2. Expected customers count in queue: $E(Q) = \sum_{n=1}^{\infty}(n-1)\,P_n$
3. Expected waiting time of a clients in system $E(W_s) = \frac{E(X)}{\lambda}$
4. Expected waiting t (time) of a customer in queue $E(W_q) = \frac{E(Q)}{\lambda}$
5. The fraction of t(time), the server is in normal use $Q_0 = P_0(1)$
6. The fraction of t(time), the server in primary vacation $Q_1 = P_1(1)$
7. The fraction of t(time), server being secondary vacation $Q_2 = P_2(1)$

6. Numerical Analysis:

Here is the numeric expression for different performance indices that are provided. Figure 3 is a representation of the relationship between the server's service rate μ and arrival rate λ .

Figure 4 is a representation of the relationship between the service rate μ, the primary vacation rate r_1, and the arrival rate λ. Figure 5 is a representation of the relationship between the service rate μ, secondary vacation rate r_2, and arrival rate λ.

The picture 6 is for r_1= 0.1, r_2 = 0.2, and $H'(1)$ = 0.1. Here, the value $P_{0,1}$ decreases as μ increases. It shows the effect of the service rate μ on the steady-state probability $P_{0,1}$. We can see that all three probability curves for $P_{0,1}$decrease as the probability increases the value of service rate μ. We also see a rapid decline Probability curve $P_{0,1}$for λ = 0.1, probability curve decreases from $P_{0,1}$. It is slower for λ = 0.2 and λ = 0.4 when r_1= 0.1, r_2= 0.2, $H'(1)$ = 0.1. The figure 7 is for r_2=0.2 λ =0.3, $H'(1)$ =0.1. It was observed that the probability$P_{0,1}$decreases when the arrival rate increases and μ increases. The figure 8 is for r_1= 0.5, λ = 0.3 λ =0.3 = 0.1. Observed that as $P_{0,1}$decreases, μ increases.

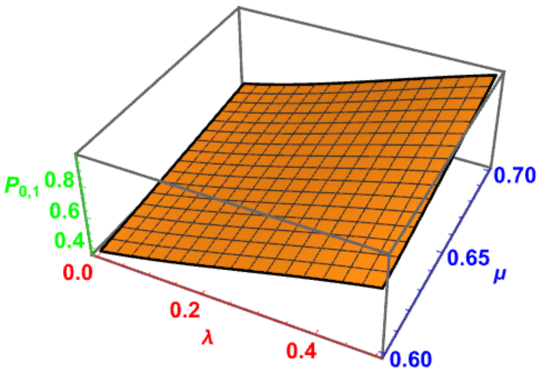

Fig.3: Correlation between $P_{0,1}$ verses λ & μ

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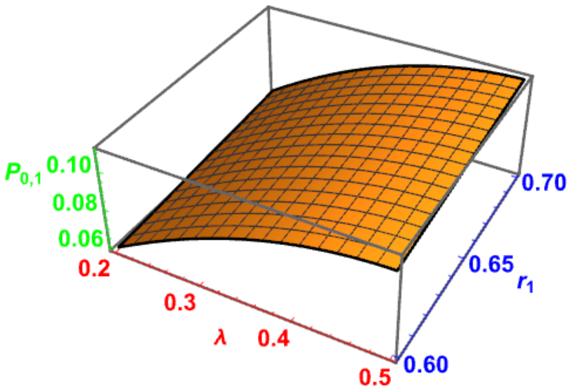

Fig.4: Correlation between $P_{0,1}$ verses λ & r_1

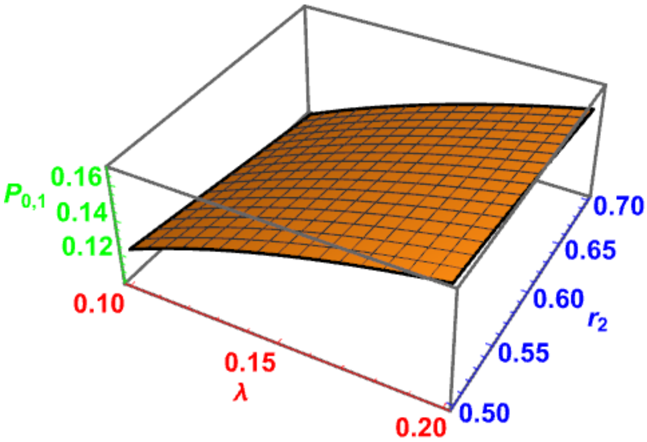

Fig.5: Correlation between $P_{0,1}$ verses λ & r_2

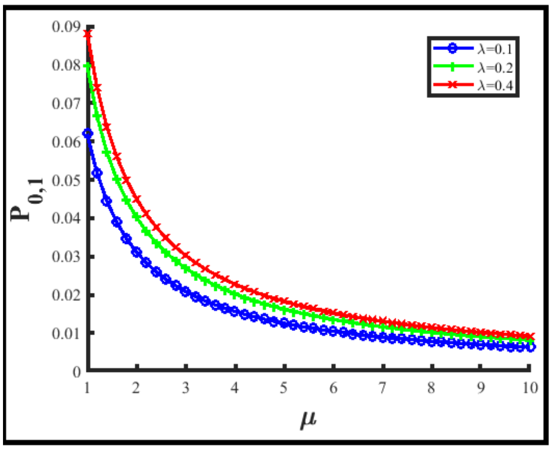

Fig.6: Correlation between $P_{0,1}$ and service rate μ based on λ

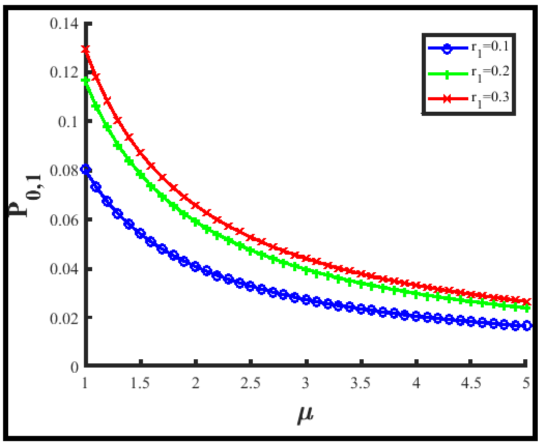

Fig.7: Correlation between $P_{0,1}$ and service rate μ based on r_1

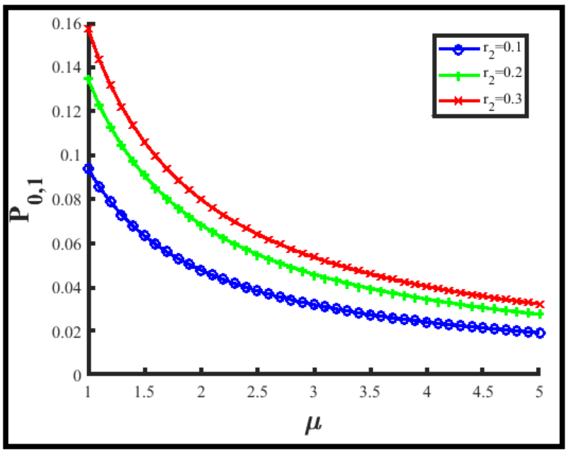

Fig.8: Correlation between $P_{0,1}$ and service rate μ based on r_2

Figure 9 is the representation of the relationship between primary vacation $P_{1,0}$ with vacation rate r_1 and arrival rate λ. Fig.10 says the correlation between $P_{1,0}$ and arrival rate λ when r_1=0.7, r_2=0.5, $H'(1)$ = 0.1

Figure 10 says, $P_{1,0}$ decreases λ increases for different values of μ =4,7,20.

Below figure 11 is for r_2=0.5, μ =6, $H'(1)$ =0.1. Here $P_{1,0}$ value decreases the value of λ increases for different values of primary vacation rate r_1.It is observed that $P_{1,0}$ value decreases the value of λ increases for different values of vacation rate r_1. It can be seen that all three probability curves for $P_{1,0}$ decrease in probability as the value of the arrival rate λ increases. Also, when r_1= 1, the probability curve $P_{1,0}$ is decreasing rapidly, and the probability curve is decreasing from $P_{1,0}$. It is slow for r_1= 0.5 and r_1= 0.7.

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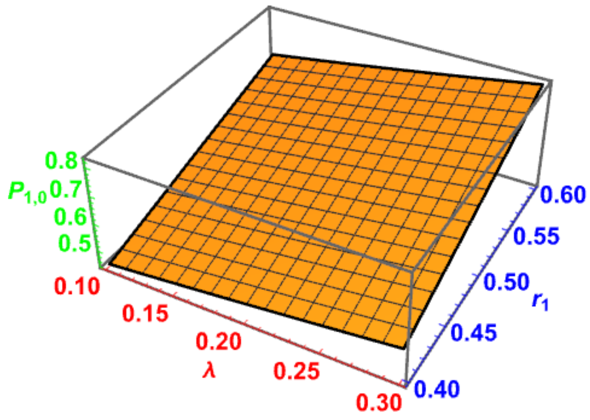

Fig.9: Correlation between $P_{1,0}$ and arrival rate λ based on r_1

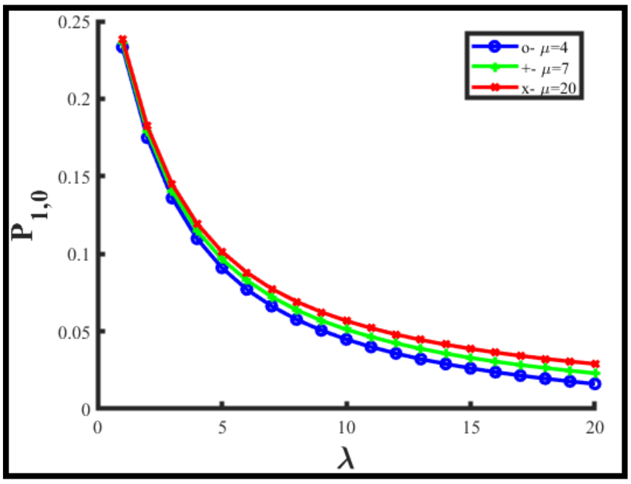

Fig.10: Correlation between $P_{1,0}$ and arrival rate λ based on μ

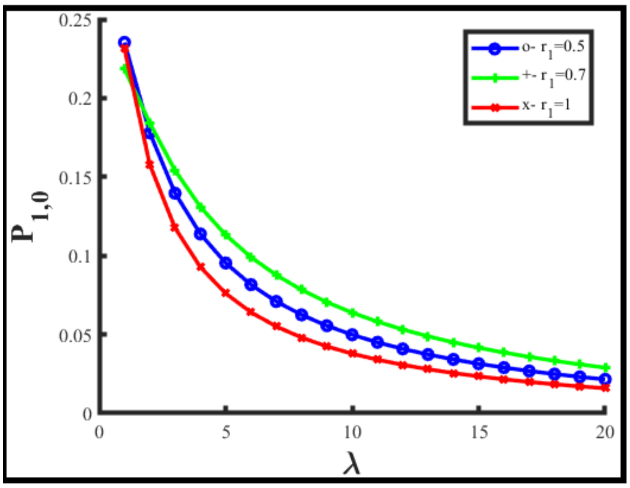

Fig.11: Correlation between $P_{1,0}$ verses λ & r_1

Figure 12 is the representation of the relationship between secondary vacation $P_{2,0}$ with vacation rate r_2 and arrival rate λ .The figure 13 is for r_2=0.5, μ=2, H' (1) =0.1. It is noticed that λ increases $P_{2,0}$

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decreases. Figure13 is the representation of the Correlation between $P_{2,0}$ and arrival rate λ based on r_1 values.

Figure 14 says, $P_{2,0}$ decreases when λ increases. It is noticed that $P_{2,0}$ decreases when λ increases for different values of r_2 when r_1=2, μ =2, H' (1) =0.1. We see that as the value of the arrival rate λ increases, the probabilities of all three probability curves for $P_{2,0}$ decrease. Even when r_2= 0.2, the probability curve $P_{2,0}$ decreases rapidly, and from $P_{2,0}$ the probability curve decreases. It is slow for r_2= 0.4 and r_2= 0.6.

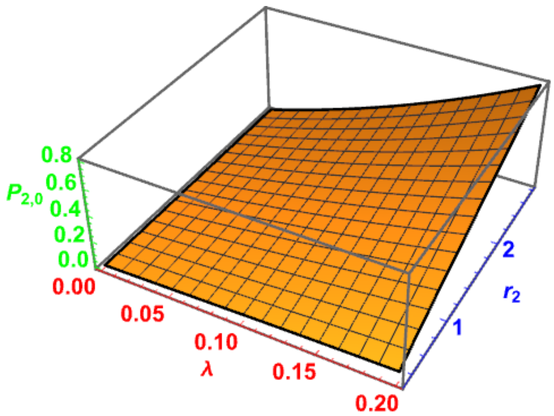

Fig.12: Correlation between $P_{2,0}$ verses λ & r_2

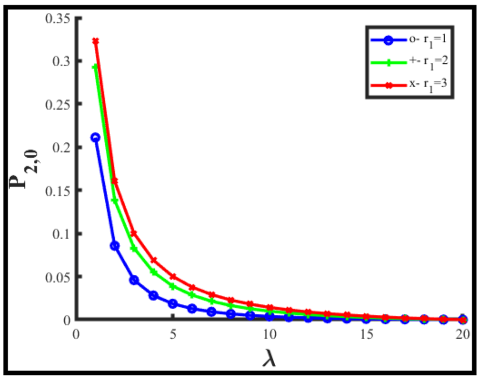

Fig.13 Correlation of $P_{2,0}$ and arrival rate λ based on r_1

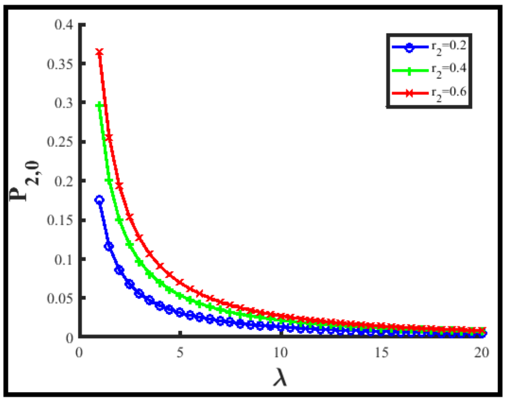

Fig.14: Correlation between $P_{2,0}$ and arrival rate λ based on r_2

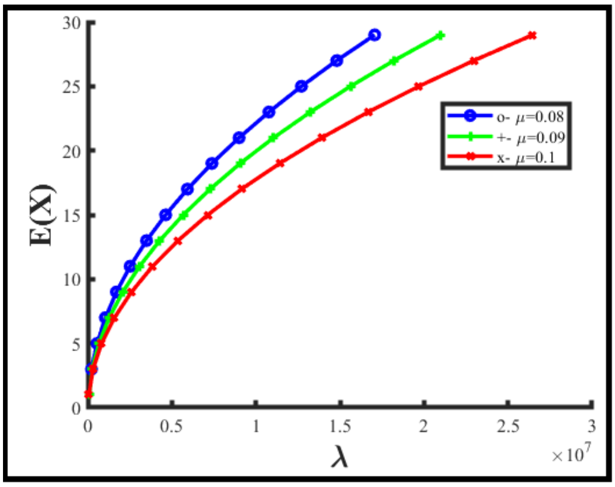

Fig.15: Correlation between E(X) (customers count in the system) and arrival rate λ

Figure 15 says the correlation between E(X) and arrival

rate λ for $\mu = .08, .09, .1$ Here E(X) increasing with growing of λ

7.1 COST - BENEFIT ANALYSIS:

We have created an anticipated cost function F / unit time for Markovian vacation queue and batch arrival size. Our aim is to compute the optimal values for μ to minimize function cost.

Describe the cost C_i , $i = 1$ *to* 6 elements as follows

In this sub section, we have developed an anticipated cost function F to find the optimum normal μ^* and total expected optimum cost, $F(\mu^*)$.We define the cost elements are

C_1= service cost / unit time when the server is on

C_2= holding cost /unit time when the server is on

C_3= cost / unit time when a customer connects queue in normal time

C_4= cost / unit time when a customer connects queue in primary vacation time

C_5= cost /unit time when a customer connects the queue in secondary vacation time

C_6= cost / unit time when a customer waiting time in the system

Based on the interpretations of every cost element and the system performance metrics, an expected total cost function / unit time obtained

$$F(\mu^*) = C_1\mu + C_2E(X) + C_3P_{0,1} + C_4P_{1,0} + C_5P_{2,0} + C_6E(W_s)$$

The aim is calculating the optimal rate μ^* to minimize the function cost F. We have followed to calculate the convexity of the total cost by the method of direct search

7.2 Direct search method

The following are the cost parameter by using assumptions $C_1 = 5, C_2 = 15, C_3 = 3, C_4 = 6, C_5 = 4,\ C_6 = 10$

Here the Numerical examples to define the μ *value* using direct search method.

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From figure16, cost function and tables here that for different values of $\lambda = 0.5,0.8,1$ found **75.3865, 70.51219, 78.25883** these are minimum expected cost.

The figure 17 of cost output and tables shows that for different values of $r_1 = 0.5,0.6,0.7$ received **132.707, 112.815, 96.3627** which are minimum expected cost.

From figure18, cost function and tables here that for different values of $r_2 = 1,3,7$ found **84.53188, 49.66225, 49.66225** these are minimum expected cost.

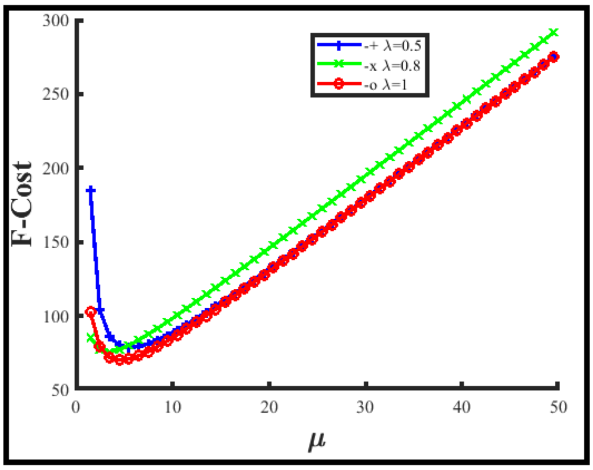

Fig.16: Correlation between the cost F and service rate μ based on λ

Figure 16. The result of μ on the cost function F. Convexity of cost graph is exhibited, Minimum cost is found for this queue model when the different values of arrival rate λ.

$r_2 = 1.2, r_1 = 1, H'(1) = 1, H''(1) = 0.3$ and

$C_1 = 5, C_2 = 15, C_3 = 3, C_4 = 6, C_5 = 4, C_6 = 10$.The table is

μ	$\lambda = 0.5$			$\lambda = 0.8$			$\lambda = 1$		
	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost
2.5	0.117834	2.185464	76.53421	0.132146	2.290377	78.8957	0.130909	3.186639	103.6493
3.5	0.090179	1.96252	**75.3865**	0.107081	1.83848	71.41864	0.111317	2.363622	85.83763
4.5	0.072737	1.838736	76.96695	0.088769	1.595472	69.69908	0.094276	1.965868	79.81138
5.5	0.060865	1.760273	79.79719	0.075484	1.444192	**70.51219**	0.081142	1.729574	**78.25883**
6.5	0.052293	1.706152	83.29956	0.065544	1.341025	72.65519	0.071006	1.572616	78.90495
7.5	0.045824	1.666583	87.20413	0.057868	1.266187	75.58179	0.06303	1.460662	80.79865
8.5	0.040773	1.6364	91.36823	0.051777	1.209424	79.00865	0.056622	1.376739	83.4696

Table1.The result of μ against the cost output for different points of λ

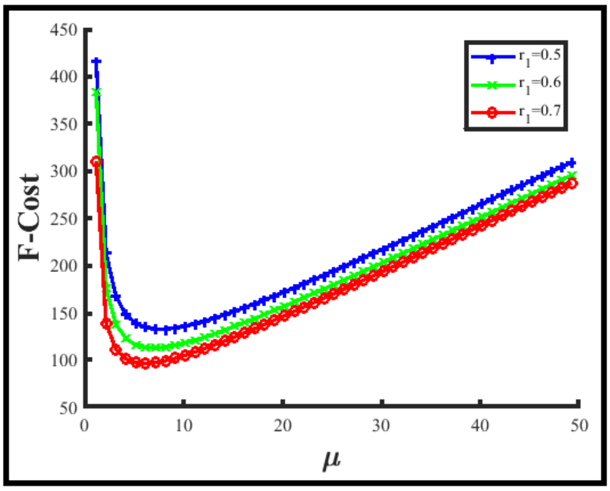

Fig.17: Correlation between the cost F and service rate μ based on r_1

Figure 17. The result of μ on the cost function F

$\lambda = 1. r_2 = 1, H'(1) = 1, H''(1) = 0.5$ and

$C_1 = 5, C_2 = 15, C_3 = 3, C_4 = 6, C_5 = 4, \; C_6 = 10$

The table is

μ	$r_1 = 0.5$			$r_1 = 0.6$			$r_1 = 0.7$		
	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost
4.2	0.074697	7.164505	148.0841	0.068027	3.153055	122.706	0.060469	3.804248	100.9624
5.2	0.063958	6.344968	138.8077	0.058247	2.780227	115.9699	0.051775	3.370196	97.13739
6.2	0.055702	5.804327	134.3884	0.050728	2.53549	113.2647	0.045092	3.086235	**96.3627**
7.2	0.049247	5.420873	**132.707**	0.04485	2.362428	**112.815**	0.039866	2.885877	97.2875
8.2	0.044091	5.134748	132.7213	0.040155	2.233551	113.7571	0.035693	2.736904	99.25701
9.2	0.039892	4.913068	133.8582	0.03633	2.133845	115.617	0.032294	2.621781	101.9149
10.2	0.036411	4.736266	135.777	0.03316	2.054407	118.1151	0.029476	2.530143	105.0503
11.2	0.033482	4.591966	138.2621	0.030493	1.989626	121.0748	0.027105	2.455465	108.5308

Table2.The result of μ against the cost output for different points of r_1.Convexity of the cost graph is exhibited; Minimum cost is found for this queue model when the different values of primary vacation rate r_1.

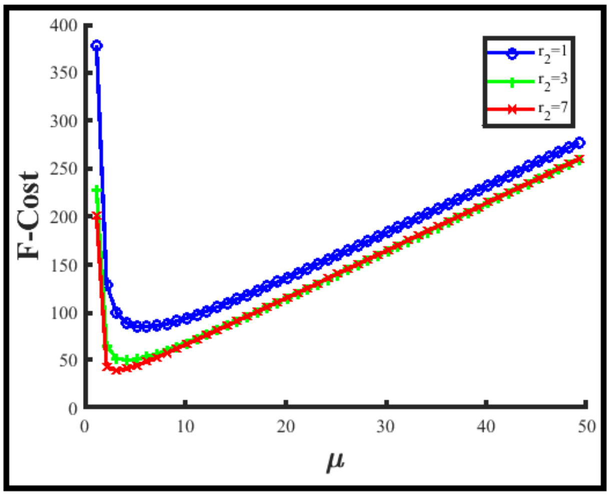

Fig.18: Correlation between the cost F and service rate μ based on r_2

Figure 18. The result of μ on the cost function F. Convexity of cost graph is exhibited, Minimum cost is found for this queue model when the different values of secondary rate r_2.

$\lambda = 1, r_1 = 1, H'(1) = 1, H''(1) = 0.5$ and

$C_1 = 5, C_2 = 15, C_3 = 3, C_4 = 6, C_5 = 4, \ C_6 = 10$

The table is

μ	$r_2 = 1$			$r_2 = 3$			$r_2 = 7$		
	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost	$P_{1,0}$	$E(W_s)$	F-Cost
2.2	0.059846	4.639807	128.7307	0.106257	0.947738	64.46707	0.123967	0.274697	43.36624
3.2	0.051859	3.232333	98.84934	0.092076	0.622104	51.78099	0.107422	0.194398	**39.26325**
4.2	0.043788	2.619898	88.67432	0.077745	0.490917	**49.66225**	0.090703	0.165711	41.02152
5.2	0.037492	2.273158	85.08117	0.066568	0.41929	50.77505	0.077663	0.150908	44.35032
6.2	0.032653	2.049288	**84.53188**	0.057975	0.374001	53.31656	0.067638	0.141854	48.32851
7.2	0.028869	1.892604	85.64727	0.051257	0.342731	56.61855	0.059799	0.135738	52.63837
8.2	0.025847	1.776737	87.77416	0.045891	0.319825	60.37437	0.05354	0.131328	57.14065
9.2	0.023385	1.687546	90.56223	0.04152	0.302315	64.42307	0.04844	0.127996	61.76458

Table3.The result of μ against the cost output for different points of r_2

8. Conclusion:

An $M^x/M/1$ queueing model with differentiated vacation is studied in this paper A probability generating function method is used to find the model's steady-state and steady-state probabilities. Further, the various system performance measures are also analyzed numerically and graphically. A cost function is also constructed to obtain the optimal (minimum) cost function corresponding to the optimal service rate through the direct search method. Mathematica is used for presenting 3Dimensional graphical pictures and MAT LAB software is also used for presenting the cost function of the above queueing model.

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