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Comparative Analysis of Fuzzy Dagum Hazard Function Approaches for Estimating Failure Rates

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Abstract

When analysing the maintenance plan for a manufacturing system, the number of failures is a crucial consideration. There are frequently some uncertainties that have an impact on this number. The category of probabilistic uncertainty is distinct from probabilistic uncertainty, which encompasses most uncertainties. By using the fuzzy theoretical framework, this uncertainty is frequently modelled. With a fuzzy shape parameter, the failure numbers of the Dagum system under study in this paper is exhausted to calculate several failures.

Here, the figure is computed via two distinct approaches. With the first approach, the quality of failure receives the exact same membership values based on fuzziness of DGM shape parameter. To determine membership, using level of α-cut of the shape parameter approach and estimate formula for failure number from the second method. For the Dagum shape parameter, we employ the Triangular Fuzzy Number (TrFN) without losing generality. We demonstrate that both approaches were successful in calculating the failure rate. When we deliberate failures as an operating time, both methodologies show that as time increases, the ambiguity (or fuzziness) of the various failures that arise rises.

The failure numbers that result as using the first method has a TrFN form. The second method's output failure rate, however, has a TrFN-like form rather than necessarily having a TrFN form. The Generalised Mean Value Defuzzification (GMDF) method is used to compare some aspects of these two approaches. The results reveal that the focal point of these fuzzy numbers of failures of corresponding DGM (Dagum distribution) are the same for any given GMDF weighting factor.

Keywords: Dagum hazard function; TrFN; Failure number; GMDF; α-cut of FN.

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1. Introduction

Almost all decision-making situations involve some level of uncertainty, and reliability and maintenance issues are no exception. This is a result of social subjectivity in a decision-making process, uncertain future events, and imprecision [7]. In every field, there are some significant variables that have a big impact on how decisions are made. The number of failures has a significant impact on the analysis of a manufacturing system's maintenance strategy in the field of reliability and preservation. Numerous uncertainties fall under probabilistic uncertainty rather than possibility of uncertainty, which is a different category. Many times, at least one of the decision function's parameters or variables have a fuzzy value rather than a crisp value.

The number of failures is a crucial factor that must be ascertained because it serves as the basis for additional decision- making in maintenance and reliability analysis. For example, this number is exhausted when creating the best maintenance plan to reduce multiple failures and maintenance costs [9,10]. It is therefore essential to understand how to estimate the failures.

Failure data are often difficult to obtain due to uncertainty, inaccuracy and complexity of the model under survey. In this context idea of fuzzy set is often used to provide a framework for controlling uncertainty and error [11]. One of the most difficulties in calculating the number of failures with probabilistic uncertainty is determining the computation of this failure numbers for a specified probability distribution with fuzzy parameters. Fuzzy number calculators are now commonly available [13].

Second, understanding how the parameters' level of uncertainty affects the failure rates that result is crucial. This is generally referred to the dissemination of fuzziness, that described as "the way in which the amount of imprecision in the model's inputs affects the changes in the model's output" [12]. In theory, model and computations use of mathematical operations leads to the spread of uncertainty.

The decision-making process will be significantly improved by understanding the number's calculation process and degree of uncertainty (see also [13,14] for examples of this in other fields). In complex engineering systems in general, fuzziness propagation can be very difficult [15].

The paper has two objectives:

- The first, step is to figure out number of failures a system with fuzzy shape parameter and failure number of DGM distribution will have, and
- secondly to understand how the number of failures that results from this shape parameters with fuzziness.
  The significance and impacts of the work performed in this paper are comprised of these two objectives.

Additionally, two different approaches are used in this paper to determine the failure rate using two different metrics. The first method propagates the membership function to failure numbers to DGM of fuzziness shape parameters giving them the same membership values. In contrast, in the method two, the shape parameter's α-cut, or level is used to compute the membership.

2. Methods

The mathematical model for describing the decline of an industrial system or piece of equipment is called the DGM distribution function, and it is the focus of this paper. DGM distribution functions, such as those found in [18], are frequently used to model the deterioration or failure data. Because of its adaptability, the DGM function has gained popularity and can be considered as a generalisation of the Rayleigh and exponential distributions, which are widely applied in the reliability studies [19].

The continuous probability density function of DGM distribution having the form.

$$f(x) = \begin{cases} \beta\delta\, x^{-(\delta+1)}\,(1+x^{-\delta})^{-(\beta+1)}, & \text{for } x>0,\ \beta,\delta>0 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

where γ, δ, β are scale and shape parameters respectively. To model wealth and income, Camilo Dagum proposed the Dagum distribution (DGM) in 1970. It is also known as the Inverse Burr distribution, or the generalised logistic Burr distribution.

By varying the parameter in shape, the DGM distribution can be exhausted a variety of real-life data, for example, Rayleigh distribution function can be obtained if $\beta = 2$ and an exponential distribution can be obtained if $\beta = 2$ [19]. We will make the scale parameter δ = 1 our starting point throughout the paper for a few different reasons. For example, maintenance modelling, this choice is sufficient if assume that the equipment or system that the equipment or system under investigation, as of guarantee and heigh level eminence control, experiences its first failure on average within a unit of time, such as a month or a year.

It is possible to compute failure numbers, hazard function, preventative maintenance time and auxiliary time using the DGM distribution function. Among other places, [20,21] contains the conventional approaches for calculating these indices for both simple and complex systems. In [22,23], the DGM distribution function's theory and applications are covered in detail. The FN and its membership function, the α-cut of FN and the generalised mean value de-fuzzification (DFuZ) process are some concepts from the fuzzy theoretical framework that are presented and extended in the technique and investigation that follow.

2.1 Defining of FN and its membership function

In this section, define some terms related to FN theory that will be use innovative. One way to conceptualize FN is as generalisation of a real number because it represents uncertainty using a membership function. Membership has a defined value when it is binary. When FN is zero, it is obviously not a member of a set μ and when it is one, it is obviously a set μ.

FN $\widetilde{A}$ is defined mathematically associated with feasible values, for all values of $\widetilde{A}$ is possible. Suppose that a distinct membership in [0,1], the term "membership function" refers to this number, which is typically written as " $\xi : a \in A \rightarrow x \in [0, 1]$," and measures how likely it is for a to belong to $\widetilde{A} = (A, \xi(A))$ or $\widetilde{A} = (x, \xi_{\widetilde{A}}(x)\ |x \in X$ are two other common ways to represent this fuzzy number.

The fuzzy number can be thought of in this way as a pair of mathematical objects that are made up of a set and its membership function or grade. The fuzzy number is supposedly meant to quantify the ambiguity and inaccuracy of too much information as well as to show probabilistic uncertainty.

Numerous functional approaches may be used to identify the FN membership, that can divide into two functional forms, linear and non-linear respectively [16]. The fuzzy numbers are represented graphically in figures 1 and 2 and their functional forms and membership functions are obtained by equations (2) and (3). Observe that the TpFN in (3) is provided an increase or decrease linear curve that provides the membership function in [a, b] and [c, d]. This notion is generalised from Linear-Flat Fuzzy Number and used as a new strategy in [8,24,25].

TrFN membership function is as follows.

$$\xi_{\overline{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{a-a}, & a \leq x \leq b \\ \frac{x-c}{b-c}, & b \leq x \leq c \\ 0, & x \geq c \end{cases} \tag{2}$$

TpFN membership function is as follows.

$$\xi_{\overline{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{x-c}{c-d}, & b \leq x \leq c \\ 0, & x \geq d \\ 1, & b \leq x \leq c \end{cases} \tag{3}$$

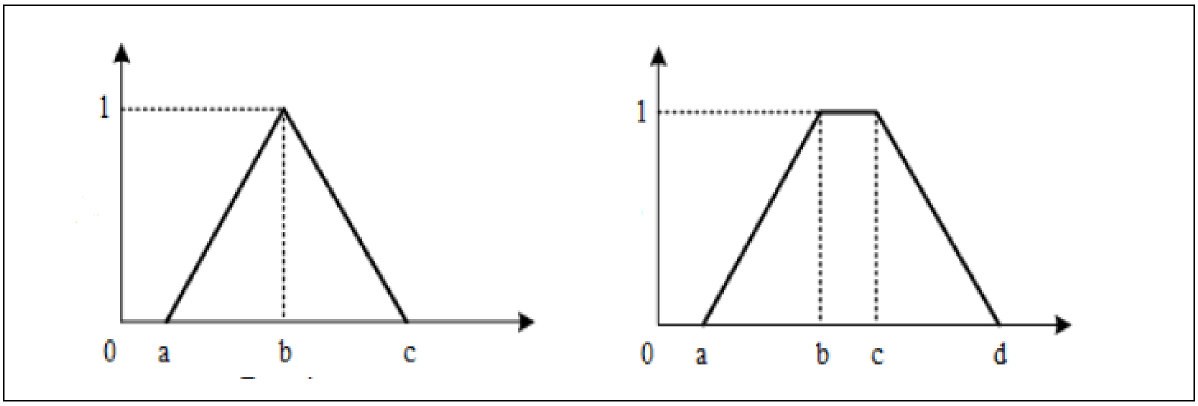


Figure1. Graphical representation of TrFN and TpFN with parameters (a,b,c)and (a,b,c,d) respectively.

In this scenario, b is referred to as the fuzzy number's core group, while the sets [a, b) and (b, c) are referred to as FN's. Similarly, in TpFN (3), the core of the FN provided with [b, c], while supporting by the sets [a, b) and (c, d) where a, b, c, d ∈ R.

There are many different types of fuzzy numbers [24], such as piecewise quadratic [26], pentagonal [27], bell designed [28], parabolic trapezoidal [29], new bell- shaped [30], and numerous others are an excellent reference for how certain new methodologies and strategies are established to expand fuzzy number notions for current analytics. For the sake of simplicity, in this work assume TrFN to highlight the methodological component. We will briefly cover the α-cut of TrFN.

2.2 The α-Cut of FN

Each FN connected with fuzzy set that have membership values, this is a fuzzy integer represented by a clear set. By applying this idea, it is possible to show that the TrFN α-cut (1) is provided by

$\mathcal{A}_\alpha = [a_1{}^\alpha, a_2{}^\alpha] = [(b - a)\alpha + a, (b - c)\alpha + c]$, for all $\alpha \in [0,1]$ (4)

GMDF (Generalised Mean value De-Fuzzification)

Information regarding suitable way to represent a crisp number as FN is desired for a variety of reasons. The fuzzy number is de-fuzzified in this situation. Regarding a fuzzy set, it is arithmetical estimation that translates a FN into a single crisp value. The literature contains a few DFuZ, including weighted fuzzy mean, last of maxima, centre of gravity and basic DFuZ of distributions [31-33]. The GMDF technique in this work is described as

$\mu(\mathcal{A}) = \frac{a+t\,b+c}{t+2}$, where $\mathcal{A}$ = (a; b; c) is a TrFN and t be the weight of the FN (5)

Features of GMDF will be explored future and utilised to compare failures arise.

3. Results

3.1. DGMHF (DGM Hazard Function) failure numbers with Fuzzy Parameter

One parameter DGM distribution function as mentioned in the section above is adequate for our purposes of maintenance modelling if we assume that because of the system under investigation's warranty and good quality control, the average first failure will happen within a given time frame, such as a month or a year. The first technique estimates the number of failures based on the assumption $(\delta = 1)$. To calculate FN, point wise, we simply need the crisp function. The probability density function f and CDF, F using Equation (1) and its hazard function h derived by

$$f(x) = \frac{\delta\,(1+x^{-1})^{-1-\delta}}{x^2}$$

$$F(x) = (1 + x^{-1})^{-\delta} \quad (6)$$

And $h(x) = \delta\,\frac{(1+x^{-1})^{-(1+\delta)}}{x^2\,[1-(1+x^{-1})^{-\delta}]}$ (7)

The failure number of DGM is obtained by $N(x) = x^{2\delta}$ (8)

The fuzzy integer that corresponds δ the shape parameter of DGM distribution function is called as the fuzziness of DGM. Two approaches will be used to deal with the shape parameter's fuzziness:

(i) a crisp function that conducts the computation pointwise and generates the fuzziness of the independent and dependent variable correspondingly.

(ii) a crisp function with fuzzy limitations by computing using level set.

3.2 Method one (Point-by-Point):

Assume that there is a TrFN $\alpha;\ \beta;$ and γ which can be identified by three unique numbers. Equation (2) is satisfied by $\tilde{\delta} = (\alpha;\ \beta;\ \gamma)$. Estimate the failure numbers, point by point to get the crisp integers one at a time to. we get μ (α) = μ (α'), μ (β) = μ (β'), and μ (γ) = μ (γ') giving a TrFN (α'; β'; γ') for functions g(x), h(x), and N(x) [17] respectively.

3.3 Second Method (α- Cut method):

FN δ is recognized as α-cut meeting equation (4) in the second method. A series of intervals connected to the number in [0,1] can be used parameter [17]. At the interval's end points, the failure's number is computed. The stack of the end points the intervals doesn't have TrFN in this case, although it is frequently (see numerical examples for details).

We utilise the GMDF specified in Equation (5) to make it easier to compare the outcomes of two techniques.

If (α; β; γ) is a TrFN, GMDF specified by Equation (5) hold the following features.

Theorem1.

Let a TrFN is given by (α; β; γ), then the GMDF obtained by (5) holds given:

1. In a symmetrical case, $\beta - \alpha = \gamma - \lambda = \Delta$ then $\mu(\mathcal{A}) = \beta$
2. For an asymmetrical case $\beta - \alpha = \Delta_\alpha \neq \gamma - \lambda = \Delta_\gamma$,
   a. if $\Delta_\alpha < \Delta_\gamma$ then $\mu(\mathcal{A}) > \beta$
   b. if $\Delta_\alpha > \Delta_\gamma$ then $\mu(\mathcal{A}) < \beta$
3. If $\mu(\mathcal{A}) = \beta$, where k→ ∞ however, value α and β

Proof :

Symmetrical case :

$\beta - \alpha = \gamma - \lambda = \Delta$ then

$$\mu(\mathcal{A}) = \frac{\alpha+k\beta+\alpha+2\Delta}{k+2} = \frac{2\alpha+k\beta+2\Delta}{k+2}$$

$$= \frac{2(\alpha+\Delta)+k\beta}{k+2} = \frac{2(\beta)+k\beta}{k+2} = \frac{(2+k)\beta}{k+2} = \beta$$

Hence $\mu(\mathcal{A}) = \beta$

1. Asymmetrical case:

   $\beta - \alpha = \Delta_\alpha$ is not equal to $\gamma - \lambda = \Delta_\gamma$ then

   a. $\mu(\mathcal{A}) > \beta$ if $\Delta_\alpha < \Delta_\gamma$

   $$\mu(\mathcal{A}) = \frac{\alpha+k\beta+\alpha+\Delta_\alpha+\Delta_r}{k+2} > \frac{\alpha+k\beta+(\alpha+2\Delta_\alpha)}{k+2}$$

   $$= \frac{2(\alpha) + k\beta + 2\Delta_\alpha}{k+2}$$

   Hence $\mu(\mathcal{A}) > \beta$

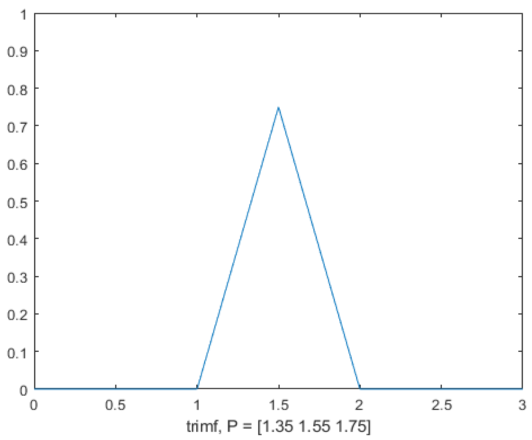
   b. $\mu(\mathcal{A}) < \beta$ if $\Delta_\alpha > \Delta_r$

   $$\mu(\mathcal{A}) = \frac{\alpha+k\beta+\alpha+\Delta_\alpha+\Delta_r}{k+2} < \frac{\alpha+k\beta+(\alpha+2\Delta_\alpha)}{k+2}$$

   $$= \frac{2(\alpha) + k\beta + 2\Delta_\alpha}{k+2}$$

   Hence $\mu(\mathcal{A}) < \beta$.

2. If k tends to ∞ then $\lim_{k\to\infty} \mu(\mathcal{A}) = \lim_{k\to\infty} \mu\left(\frac{k\beta+\alpha+\gamma}{k+2}\right) = \beta$



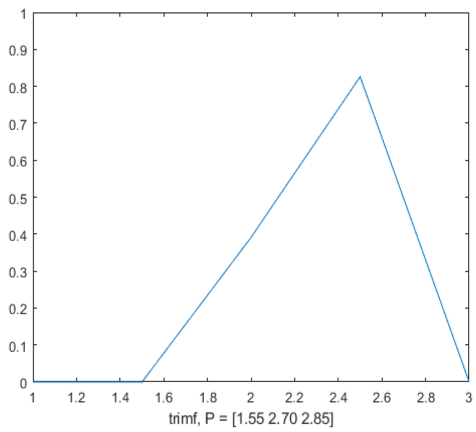


Figure 2. Fuzzy membership for Triangular function with DGM shape parameter

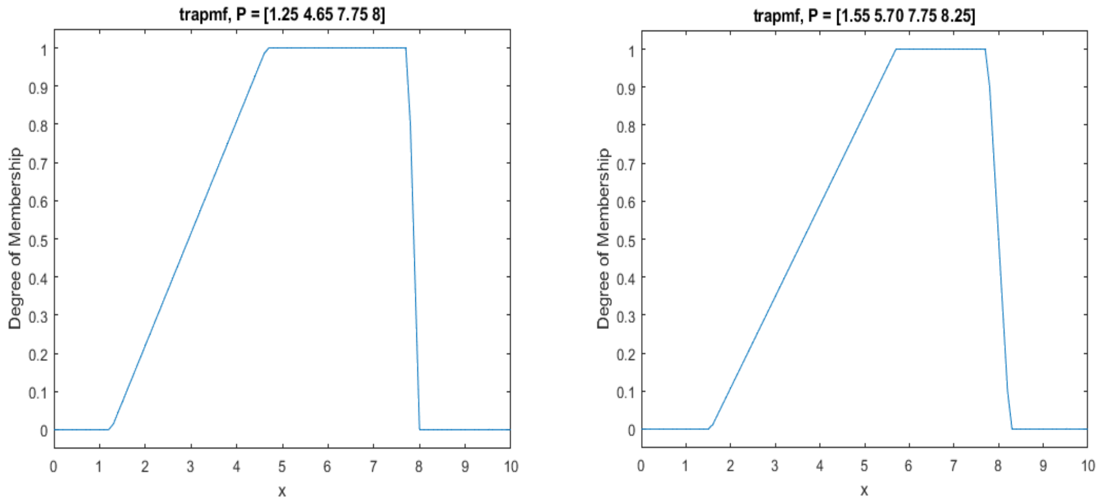


Figure 3. Fuzzy membership for Trapezoidal function with DGM shape parameter

As demonstrated by the theorem, the GMDF has a unique property in that it can discover a crisp number that is near to the group of a TrFN. A symmetric TrFN is denoted by the equation $\bar{\delta}$ = (α = 1.35; β = 1.55; γ = 1.75). If k=1 then GMDF = 1.55; If k = 1000 then GMDF = 1.55. GMDF are the same TrFN core for all k since it is symmetrical. On the other hand, with non- symmetrical TrFN, skewed such as left TrFN $\bar{\delta}$ = (α = 2.55; β = 2.70; γ = 2.85), If k=1 and k= 1000 then GMDF = 2.6833 and 2.7498 respectively. In this case, k is closer to the TFN's core, or 2.75, the larger k. The comparatively small shape parameter $\bar{\delta}$ = (α = 1.35; β = 1.55; γ = 1.75, while the quite big shape parameter $\bar{\delta}$ = (α = 1.55; β = 2.70; γ = 2.85) is presented on the right figure.

The fuzzy failure numbers generated by the DGM distribution is then examined using α -cut technique. Let us evoke that α -cut of TrFN μ($\mathcal{A}$) = (α; β; γ) is μ($\mathcal{A}$)= [a 1, a 2] = [(β α) + α, (β γ) + γ]. The PDF, CDF and Hazard function for DGM distribution with shape parameter represented in α-cut obtained by,

$$\delta_\alpha = [t_1 + t_3\,\alpha\,, t_2 - t_3\,\alpha,] \tag{9}$$

R for some t_1, t_2, t_3. By analysing the α -cut in Equation (9) and applying fuzzy arithmetic to replace it in Equations (6) and (7), the cumulative distribution may be determined.

$$f(t)_\alpha = [\,(t_1 + t_3\,\alpha)\;x^{-2}\,(1 + x^{-1})^{-(1+(t_1+t_3\,\alpha)\,)},\;(t_2 + t_3\,\alpha)\;x^{-2}\,(1 + x^{-1})^{-(1+t_2+t_3\,\alpha)}$$

$$F(t)_\alpha = [\,(1 + x^{-1})^{-(t_1+t_3\,\alpha)}\,, (1 + x^{-1})^{-(t_2-t_3\,\alpha)}\,], \text{ where } t_1, t_2, t_3 \in R \tag{10}$$

$$h(t) = [\,(y_1 + y_3{}^{\alpha})x^{-(2)}(1 + x^{-1})^{-(1+(y_1+y_3{}^{\alpha}))}\left[1 - (1 + x^{-1})^{-(y_1+y_3{}^{\alpha})}\right]^{-1}, (y_2 - y_3{}^{\alpha})x^{-(2)}(1 + x^{-1})^{-(1+(y_2-y^{\alpha}))}[1 - (1 + x^{-1})^{-(y_2-y^{\alpha})}]^{-1} \tag{11}$$

R. for some y_1, y_2, y_3, y_4, y_5, y_6 ∈ R. So, integrate Equation (11) on both sides we get the number of failures, which is provided by.

The failure's number given by $N(t) = [t^{2(s_1+s_3\,\alpha\,)}, t^{2(s_2\,-s_3\,\alpha\,)}]$, s_1, s_2, s_3 ∈ R (12)

Theorem 2 proves GMDF of failures raises and supports to expand. As a result, the level of uncertainty rises.

Theorem2.

For $\Delta\lambda > 0$ let $N(\lambda)_\alpha$ and $N(\lambda+\Delta\lambda)_\alpha$ be the fuzzy failures at time λ and $\lambda+\Delta\lambda$ respectively then:

(a) $N(\lambda)_\alpha = (\lambda^{c\alpha}, \lambda^{d\alpha})$ and $N(\lambda+\Delta\lambda)_\alpha = (\lambda+\Delta\lambda)^{c\alpha}, (\lambda+\Delta\lambda)^{d\alpha})$
(b) GMDF $N(\lambda+\Delta\lambda)_\alpha \geq$ GMDF $N(\lambda)_\alpha \; \forall \lambda \in R^+$
(c) $[(\lambda+\Delta\lambda)^{c\alpha} - (\lambda+\Delta\lambda)^{d\alpha}] - [(\lambda^{c\alpha} - \lambda^{d\alpha})] \geq 0 \; \forall \lambda \in Z^+$.

(i) by using Theorem 1, proof has done very clearly.

(ii) Equation (12) expressed for any $\alpha \in [0,1]$, then interval has defined $(t^{c\alpha}, t^{d\alpha})$ where $c\alpha, d\alpha \in R$. without loss of generality need to prove theorem.

$[(t+\Delta t)^c - (t+\Delta t)^d] - [(\lambda^c - \lambda^d)] \geq 0.$

Let consider the binomial expansion rule,

$$(t+\Delta t)^c = \sum_{n=0}^{c} \frac{c!}{(c-n)!\,n!} \Delta t^{(c-1)-n} \; t^n$$

$$(t+\Delta t)^c = \sum_{n=0}^{c-1} \frac{(c-1)!}{((c-1)-n)!\,n!} \Delta t^{(c-1)-n} \; t^c + t^c$$

Then for $c\alpha, d\alpha \in Z^+$ we have

$$(t+\Delta t)^c = \sum_{n=0}^{c-1} \frac{(c-1)!}{((c-1)-n)!\,n!} \Delta t^{(c-1)-n} \; t^c + t^c$$

$$(t+\Delta t)^d = \sum_{n=0}^{d-1} \frac{(d-1)!}{((d-1)-n)!\,n!} \Delta t^{(d-1)-n} \; t^d + t^d$$

then

$$[(t+\Delta t)^c - (t+\Delta t)^d] - [(t^c - t^d)] = \sum_{n=0}^{c-1} \frac{(c-1)!}{((c-1)-n)!\,n!} \Delta t^{(c-1)-n} \; t^c - \sum_{n=0}^{d-1} \frac{(d-1)!}{((d-1)-n)!\,n!} \Delta t^{(d-1)-n} \; t^d$$

$$= \sum_{n=d}^{c-1} \frac{(c-1)!}{((c-1)-n)!\,n!} \Delta t^{(c-1)-n} \; t^c \geq 0,$$

Hence, $[(t+\Delta t)^c - (t+\Delta t)^d] - [(t^c - t^d)] \geq 0 \;\; \forall t \in Z^+$. It can be extended to $c\alpha, d\alpha \in R^+$. Another way can be proved by using Newton's generalised Binomial [34,35] form of infinite series instead of series $\lambda \in Z^+$.

4. Illustration

To have a better understanding of the preceding section's conclusions, we demonstrate by utilizing two distinct values for different shape parameters comparatively small value δ = (α = 1.35; β = 1.45; γ = 1.75), bigger value δ = (α = 2.55; β = 2.70; γ = 2.85). Here, α, β, and λ are the TrFN components that make up the TrFN, which are defined similarly to α, β, and γ in Equation (2).

(i) In contrast to method one, shape parameter in fuzziness reflected number of failure same form of fuzzy numbers, when the number of failures is plotted against time, the curves are "exponentially" spread in place predicted by theory.

(ii) Second method is displayed. Figure 3 graph effectively depict the failure numbers with the parameter's end points and core, TrFNs Mathematics 2021, 9, 2858 10 of 19. More precisely, these plots show the number of failure bands for DGM, which can be obtained analytically from Equation (8) and is like the α - Cut's Equations (10) and (14) in that it has a power curve. This corresponds sharply parameterized curve for DGM's failure numbers [36]. The α- Cut approach, which is the second choice, likewise follows same logic, even though the graphs are not displayed here.

5. Results of DGM distribution by α -Cut Method

Plotting results of the $\alpha -$cut method, the number of failures yields following results. Remember that $\alpha -$cut of the TrFN = $\mathcal{A}_\alpha = [a_1{}^\alpha, a_2{}^\alpha]$= [(q-p) α + p, (q-r) α +r]. Consequently, we obtain the $\alpha -$cut as $\bar{\delta} = (\alpha = 1.35;\ \beta = 1.45;\ \gamma = 1.75)$ as the FN shape parameter. Then $\alpha -$cut

$$\delta_\alpha = [1.25 + 0.30\alpha,\ 1.85 - 0.30\alpha\,]$$
(13)

DDF (Dagum density function) $f(t)_\alpha$, DCF (Dagum cumulative distribution) $F(t)_\alpha$ is obtained by taking $\alpha -$cut from equation, (7) and using fuzzy arithmetic to substitute it into equations (5) and (6).

$$f(x)_\alpha = [\,(1.35 + 0.30\alpha)\ x^{-2}\,(1 + x^{-1})^{-(1+1.25+0.30\alpha)},\ (1.85 - 0.30\,\alpha\,)\ x^{-2}\,(1 + x^{-1})^{-(1+1.85-0.30\,\alpha\,)}$$

$$F(x)_\alpha = [\,(1 + x^{-1})^{-(1.25+0.30\alpha)}, (1 + x^{-1})^{-(1.25+0.30\alpha)}\,]$$
(14)

And the hazard function $h(t)_\alpha$ is obtained by,

$$h(x) = [\,(1.25 + 0.30\,\alpha)x^{-(2)}(1 + x^{-1})^{-(1+(1.25+0.30\,\alpha))}\left[1 - (1 + x^{-1})^{-(1.25+0.30\,\alpha)}\right]^{-1},\ (1.85 - 0.30\alpha\,)x^{-(2)}(1 + x^{-1})^{-(1+1.85-0.30\alpha\,)}[1 - (1 + x^{-1})^{-(1.85-0.30\alpha\,)}]^{-1}$$
(15)

From the equation (9) integrating both sides we get,

$$N(x) = [x^{2(1.25+1.30\,\alpha)}, x^{2(1.85-0.30)\alpha}] \quad (16)$$

Hence by using $\alpha -$cut, we obtain the TrFN which is comparable to the FN (p; q; r) obtained by

$$\alpha = \min N(x)_{\alpha=0} = \left[x^{(1.25+1.30\,\alpha)}, x^{(1.85-0.30\,\alpha)}\right] = x^{5/4}$$

$$\beta = \min N(x)_{\alpha=1} = \left[x^{(1.25+1.30\,\alpha)}, x^{(1.85-0.30\,\alpha)}\right] = x^{31/20}$$

$$\gamma = \min N(x)_{\alpha=0} = \left[x^{(1.25+1.30\,\alpha)}, x^{(1.85-0.30\,\alpha)}\right] = x^{37/20}$$

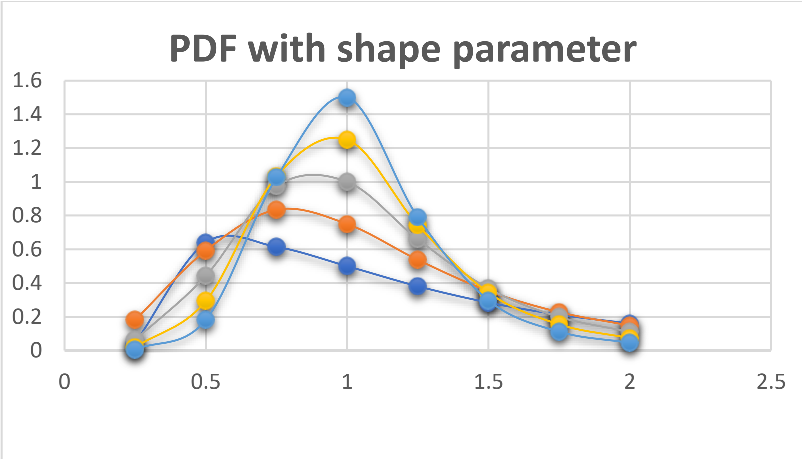


Figure 3 Density function in time series $\bar{\delta} = (\alpha = 1.35;\ \beta = 1.45;\ \gamma = 1.75)$ and $\delta = (\alpha = 2.55; \beta = 2.70; \gamma = 2.85)$

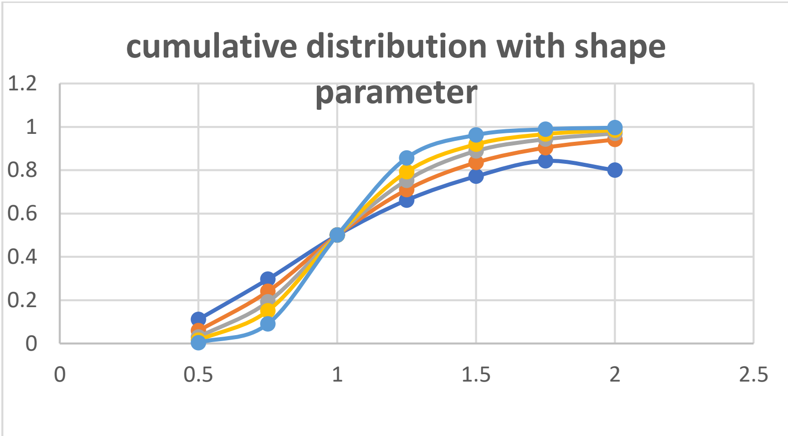


Figure 4 Cumulative distribution function in time series $\bar{\delta} = (\alpha = 1.35;\ \beta = 1.45;\ \gamma = 1.75)$ and $\delta = (\alpha = 2.55; \beta = 2.70; \gamma = 2.85)$.

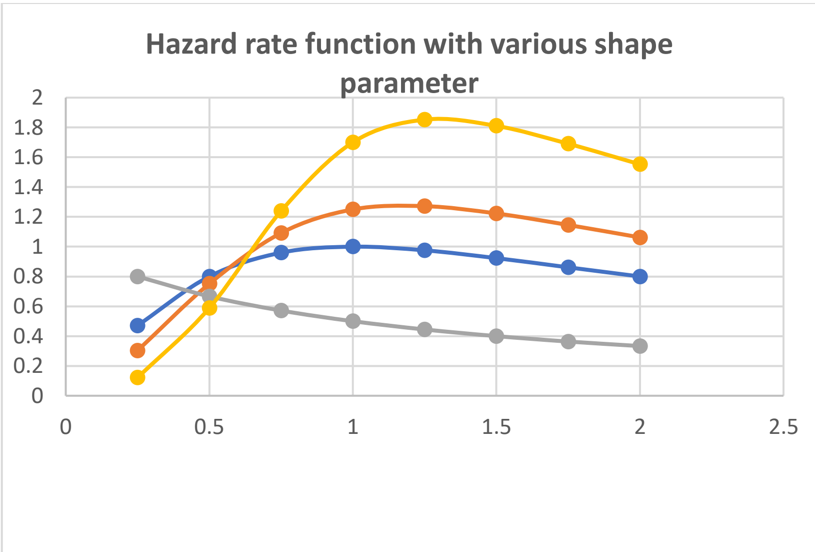


Figure 5 Hazard function and the number of failures $\bar{\delta} = (\alpha = 1.35;\ \beta = 1.45;\ \gamma = 1.75)$ and $\delta = (\alpha = 2.55; \beta = 2.70; \gamma = 2.85)$

6. Results Discussions

The results are numerically demonstrated to increase the understanding of the above conclusion by using as $\bar{\delta} = (\alpha = 1.35;\ \beta = 1.45;\ \gamma = 1.75)$ and δ = (α = 2.55; β = 2.70; γ = 2.85), which reflect a comparatively small and also large shape parameter respectively.

The TrFN is composed α, β, and γ, the TpFN parameters that are defined in the similar way as α, β, and γ in Equation (1). The plots of these TrFNs displayed Figures 2 and 3, respectively.

We suggested two methods to calculate failure numbers: the first method is straightforward and assumes that the shape parameter's fuzziness extends to failure numbers with same fuzzy number membership form. Method two uses the α-Cut method to calculate the number of failures. To broaden its application to other areas, this approach might be expanded to the DGM distribution with more parameters [37].

7. Conclusions

We examined the DGM hazard function in this study to calculate the fuzzy failure's numbers, a fuzzy shape parameter is assumed. In this research, we suggested two techniques to work out the number of failures,

(i) The first approach is simple and assumes that failure numbers with the same fuzzy number membership form is propagated by the shape parameter's fuzziness.
(ii) The second approach determines failure numbers by applying the α-cut technique.

We showed that both approaches worked well for figuring out how many times the system in question failed. Both methods show that the ambiguity (fuzziness) of the total failure numbers increased when the failures' function is examined as a function of time.

This demonstrates how form parameter uncertainty affects the total number of failures; at large x values, even a little amount of shape parameter uncertainty greatly supports the fuzzy failure numbers. In the real world, these features are crucial to consider when utilising the total failure numbers as a basis for subsequent decision-making process.

In this paper the TrFN shape parameter from the first method the resultant failure number has trFN form. Further in method two does not have TrFN form. From the comparisons between two methods using GMDV, it gives some weighting factor of GMDV. Further research can be conducted by seeing a comparison with a technique that retains uncertainty.

The TrFN and shape parameter value utilised in the DGM distribution function were assumed. This would be difficult to implement in practise. Accurately determining the fuzzy number's true shape and approximating its value should be done with readily available real facts. These concerns are between the limits of the approaches provided here, and they may potentially lead to future study directions.

Additionally, this will open a significant study avenue in the future (now crisp value application refuses the four-parameter DGM distribution). Since the scale parameter is taken to be one, we only study one-parameter DGM distributions in this section. This is enough in the context of maintenance modelling if we suppose that the equipment system under investigation has its average initial failure in a single unit of time. This might not always be the case in practice; thus, we need to expand the study to include DGM distributions with other scale parameter values.

Additional investigation may be conducted into many techniques using various kinds of fuzzy numbers, various applications of de-fuzzification techniques, and investigating the theory's applicability in several related domains, such the failure numbers in biological processes.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests

The authors have not disclosed any competing interests.

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