Fuzzy Soft Set Applications Explored in W-Algebras

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Abstract
Fuzzy soft set theory in Wajsberg algebras (W-algebras) is presented in this paper and its application to a problem of decision making is illustrated. We first introduce the notion of fuzzy soft ideals (fs-ideals) and then analyse some of the associated characteristics.

Keywords: W-algebras, soft set, fs-set, fs-ideals, SOFT ENGINE.

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1. INTRODUCTION

Research on the soft set theory is now progressing at a rapid pace. The application of soft set theory to a decision-making issue was covered by Maji et al. [12]. Maji and [13] additionally investigated some techniques related to soft set theory. A new definition of soft set parametrization reduction was developed by Chen et al. [4], who also compared it to the same attribute reduction idea in rough set theory. The algebraic structure of uncertainty-aware set theories has been studied by a few authors. The best theory for dealing with uncertainty is Zadeh's notion of fuzzy sets. M. Wajsberg introduced the ideas of W-algebra [15]. LPW-Algebras were first presented by Ceterchi Rodica [2]. A generalisation of regular soft sets, known as fs-sets, is presented in [11]. The use of fs-sets to a decision-making scenario is then demonstrated. In this work, we use the notion of fs-ideals and fs-W-algebras and then derive their basic characteristics. In the future, this paper's approach will be expanded upon to analyse many kinds of ideals in W-algebras.

2. BASIC RESULTS ON SOFT SETS

The following is a description of Molodtsov's [14] definition of the soft set. Suppose that ℳ is an initial universe set and that ℋ is a set of parameters. Let ℐ ∈ ℋ and ℰ(ℳ) represent the power set of ℳ and ℐ ⊂ ℋ.

Definition 2.1. [14] [17] A pair (S, ℐ) is called a soft set over ℳ, where S is a mapping given by S: ℐ → ℰ(ℳ).
A soft set over $\mathcal{M}$, stated differently, defines a parameterized family of universe $\mathcal{M}$ subsets. It is possible to consider elements of the soft set $(\mathcal{S}, I)$ that are $\alpha$–approximate if $\alpha \in I$. It is evident that a soft set is not a set.

**Definition 2.2.** [11] Let $\mathcal{K}$ be a set of parameters and let $\mathcal{M}$ be the initial universe set. $\mathcal{T}(\mathcal{M})$ denote the set of fuzzy sets within $\mathcal{M}$. Where $I \subseteq \mathcal{K}$ and $\tilde{\mathcal{P}} : I \rightarrow \mathcal{T}(\mathcal{M})$.

$\tilde{\mathcal{P}}[a]$ is often referred to as the fuzzy value set of parameter $a$. It is a fuzzy set in $\mathcal{M}$ for every $a \in I$. $(\tilde{\mathcal{P}}, I)$ degenerates to the usual soft set. If $\tilde{\mathcal{P}}[a]$ is a crisp subset of $\mathcal{M}$ for every $a$ in $I$, fs-sets are therefore a generalization of standard soft sets due to the above concept.

### 3. MAIN RESULT (FUZZY SOFT Wajsberg-Algebras)

Unless otherwise noted, let $\mathcal{K}$ be a collection of the following text’s parameters. The expression “Soft engine” will be used, referring to the fact that it generates a $W$-algebras.

**Definition 3.1.** Let $(\tilde{\mathcal{P}}, I)$ be a fs-set over a W-algebra $\mathcal{X}$, because $I$ is a subset of $\mathcal{K}$. We say that $(\tilde{\mathcal{P}}, I)$ is a fs-set set on $\mathcal{M}$ over $\mathcal{X}$, if there exist $m \in I$ such that $\tilde{\mathcal{P}}(m)$ is a fuzzy W-algebra in $\mathcal{X}$. It may be stated that $(\tilde{\mathcal{P}}, I)$ is a fs-W-algebra over $\mathcal{X}$. If $(\tilde{\mathcal{P}}, I)$ is based on parameter $m$ over $\mathcal{X}$ for all $m \in I$.

**Example 3.2.** Assuming the universe $\mathcal{M}$ has five colours, that is $\mathcal{M} = \{pink, gray, red, olive, tan\}$.

Assume the role of $\circ$ a Soft engine that combines two colours in the prescribed order to get the desired results.

$pink \circ q = pink$ for all $q \in \mathcal{M}$

$gray \circ \zeta = \left\{ \begin{array}{l}
pink : \zeta \in \{gray, olive, tan\} \\
gray : \zeta \in \{pink, red\} \\end{array} \right\}$

$red \circ z = \left\{ \begin{array}{l}
pink : z \in \{red, tan\} \\
gray : z \in \{pink, gray, olive\} \\end{array} \right\}$

$olive \circ u = \left\{ \begin{array}{l}
pink : u \in \{olive, tan\} \\
olive : u \in \{pink, gray, red\} \\end{array} \right\}$

$\tan \circ v = \left\{ \begin{array}{l}
pink : v = tan \\
red : v = olive \\
olive : v = red \\
tan : v = \{pink, gray\} \\end{array} \right\}$

Then $(\mathcal{M}, \circ, pink)$ is a W-algebra.
(i) Let \((\mathcal{P}, \mathcal{K})\) be a \(fs\)-set over \(\mathcal{M}\), then \(\mathcal{P}[\text{glorious}], \mathcal{P}[\text{elegant}]\) & \(\mathcal{P}[\text{modest}]\) are fuzzy sets in \(\mathcal{M}\). Here is how we define them:

<table>
<thead>
<tr>
<th>(\mathcal{P})</th>
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<th>olive</th>
<th>tan</th>
</tr>
</thead>
<tbody>
<tr>
<td>glorious</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>elegant</td>
<td>0.8</td>
<td>0.7</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>modest</td>
<td>0.6</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Then, \(fs\)-sets \(W\)-algebras \(\mathcal{P}[\text{glorious}], \mathcal{P}[\text{elegant}]\) and \(\mathcal{P}[\text{modest}]\) are based on parameters “glorious”, “elegant” and “modest” over \(\mathcal{M}\), respectively. Thus, \((\mathcal{P}, \mathcal{K})\) is a \(fs\)-\(W\)-algebras over \(\mathcal{M}\).

(ii) \(\mathcal{Q}[\text{glorious}], \mathcal{Q}[\text{elegant}]\) and \(\mathcal{Q}[\text{modest}]\) are fuzzy sets in \(\mathcal{M}\). Let \((\mathcal{Q}, \mathcal{K})\) be \(fs\)-set over \(\mathcal{M}\). We define them as listed below,

<table>
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<td>0.3</td>
</tr>
<tr>
<td>modest</td>
<td>0.9</td>
<td>0.1</td>
<td>0.7</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\((\mathcal{Q}, \mathcal{K})\) is not a \(fs\)-\(W\)-algebras over \(\mathcal{M}\). As \((\mathcal{Q}, \mathcal{K})\) is not a \(fs\)-\(W\)-algebras. It is based on a parameter “elegant” over \(\mathcal{M}\).

Infect \(\mathcal{Q}[\text{elegant}]\) (gray \(\odot\) olive) = \(\mathcal{Q}[\text{elegant}]\) (pink) = 0.4 \(\geq 0.5\)

\[= \min \{\mathcal{Q}[\text{elegant}](\text{gray}), \mathcal{Q}[\text{elegant}](\text{olive})\}\]

A \(fs\)-\(W\)-algebras based on both a parameter “glorious” and a parameter “modest” over \(\mathcal{M}\), we are able to confirm that \((\mathcal{Q}, \mathcal{K})\).

**Example 3.3:** Think about the universe \(\mathcal{M} = \{\text{pink, gray, red, olive, tan}\}\) and examine the soft engine \(\Delta\) that generates the products listed below

\[\text{pink} \triangle \varphi = \{\text{pink} : \varphi \in \{\text{pink, gray, red}\}\}
\]

\[\text{gray} \triangle \varsigma = \begin{cases} \\
\text{pink} : \varsigma = \text{gray} \\
\text{red} : \varsigma = \{\text{pink, red}\} \\
\text{olive} : \varsigma = \text{tan} \\
\text{tan} : \varsigma = \text{olive} \\
\end{cases}
\]

\[\text{red} \triangle \zeta = \begin{cases} \\
\text{pink} : \zeta = \text{red} \\
\text{red} : \zeta \in \{\text{pink, gray}\} \\
\text{olive} : \zeta \in \{\text{olive, tan}\} \\
\end{cases}
\]
\[
\text{olive} \triangle u = \begin{cases} 
\text{pink} : u \in \{\text{olive, tan}\} \\
\text{olive} : u \in \{\text{pink, gray, red}\}
\end{cases}
\]
\[
\tan \triangle v = \begin{cases} 
\text{pink} : v = \tan \\
\text{gray} : v = \text{olive} \\
\text{olive} : v = \text{gray} \\
\tan : v = \{\text{pink, red}\}
\end{cases}
\]

Then \((\mathcal{M}, \triangle, \text{pink})\) is a W-algebra.

\[\mathcal{K} = \{\text{glorious, elegant, modest}\}.\] Let \((\mathcal{P}, \mathcal{K})\) be a fs-set over \(\mathcal{M}\), then \(\mathcal{P}[\text{glorious}], \mathcal{P}[\text{elegant}]\) & \(\mathcal{P}[\text{modest}]\) are fuzzy sets in \(\mathcal{M}\). We define,

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Then \((\mathcal{P}, \mathcal{K})\) is a fs-W-algebras over \(\mathcal{M}\).

**Proposition 3.4:** A fs-W-algebras \((\mathcal{P}, I)\) over \(X\) is defined as \((\mathcal{P}(m)(0) \geq \mathcal{P}(m)(q))\) for every \(q \in X\) where \(m\) is any parameter in \(I\).

**Proof:** Let \(q \in X\) and \(m \in I\), then \(\mathcal{P}(m)(0) = \mathcal{P}(m)(q \rightarrow q) \geq \min \{\mathcal{P}(m)(q), \mathcal{P}(m)(q)\} = \mathcal{P}(m)(q)\)

Hence \(\mathcal{P}(m)(0) \geq \mathcal{P}(m)(q)\) for all \(q \in X\) and any parameter \(m\) in \(I\).

**Theorem 3.5:** Let \((\mathcal{P}, I)\) be a fs-W-algebras over \(X\). If \(J\) is a subset of \(I\), then \((\mathcal{P}/J, J)\) is a fs-W-algebras over \(X\).

The example that follows demonstrates that a fs-set \((\mathcal{P}, I)\) exists over a W-algebra \(X\) in such a way that,

(i) \((\mathcal{P}, I)\) is not a fs-W-algebra over \(X\).

(ii) there exists a subset \(B\) of \(I\) such that \((\mathcal{P}/J, J)\) is a fs-W-algebra over \(X\).

**Example 3.6:** Let \((\mathcal{M}, \oslash, \text{pink})\) is a W-algebra as in example-3.2 define parameters \(I = \{\text{glorious, elegant, modest, clever, gentle}\}\). If \((\mathcal{P}, I)\) be a fs-set over \(\mathcal{M}\), then \(\mathcal{P}[\text{glorious}], \mathcal{P}[\text{elegant}], \mathcal{P}[\text{modest}], \mathcal{P}[\text{clever}]\) & \(\mathcal{P}[\text{gentle}]\) defined as

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</tr>
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</tbody>
</table>

https://internationalpubls.com
Then $(\tilde{P}, I)$ is not fs-W-algebra over $\mathcal{M}$. Since $\tilde{P}[\text{clever}]$ & $\tilde{P}[\text{gentle}]$ are not fuzzy W-algebra in $\mathcal{M}$. But $I = \{\text{glorious, elegant, modest}\}$ then $(P/I, J)$ is a fs-W-algebra over $\mathcal{M}$.

**Definition 3.7:** [1] The soft set $(\tilde{H}, \mathcal{D})$ where $\mathcal{D} = I \cup J$ and for every $t \in \mathcal{D}$ is the extended intersection (Ei) of two soft sets $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ over $\mathcal{M}$.

$$\tilde{H}[t] = \left\{ \begin{array}{ll} \tilde{P}[t] : t \in I \setminus J & \\
\tilde{Q}[t] : t \in J \setminus I & \\
\tilde{P}[t] \cap \tilde{Q}[t] : t \in I \cap J & \end{array} \right\}$$

we write $(\tilde{P}, I) \cap_t (\tilde{Q}, J) = (\tilde{H}, \mathcal{D})$.

**Definition 3.8:** Let $(\tilde{P}, I)$ & $(\tilde{Q}, J)$ be two soft sets over $\mathcal{M}$ such that $I \cap J \neq \emptyset$ the Ei of $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ is denoted by $(\tilde{P}, I) \cap_s (\tilde{Q}, J)$ and is defined as $(\tilde{P}, I) \cap_s (\tilde{Q}, J) = (\tilde{H}, \mathcal{D})$ where $\mathcal{D} = I \cap J$ and for all $u \in \mathcal{D}$, $\tilde{H}[u] = \tilde{P}[u] \cap \tilde{Q}[u]$.

**Theorem 3.9:** The Ei of $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ is a fs-W-algebra over $X$, If $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ are fs-W-algebras over $X$.

**Proof:** The Ei of $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ is $(\tilde{P}, I) \cap_t (\tilde{Q}, J) = (\tilde{H}, \mathcal{D})$ then $\mathcal{D} = I \cup J$ for any $u \in \mathcal{D}$. $\tilde{H}(u) = \tilde{P}(u)$ (resp $\tilde{H}(u) = \tilde{Q}(u)$) is a fuzzy W-algebra if $u \in I \setminus J$ (resp $\in J \setminus I$).

$$\tilde{H}[u] = \tilde{P}[u] \cap \tilde{Q}[u]$$ is a fuzzy W-algebra for all $u \in I \cap J$ if $I \cap J \neq \emptyset$. Given that two fuzzy W-algebra intersect to form a fuzzy W-algebra. Therefore, over W-algebra $X$, $(\tilde{H}, \mathcal{D})$ is a fs-W-algebra.

**Corollary 3.10:** If $(\tilde{P}, I)$ and $(\tilde{Q}, J)$ are two fs-W-algebras over $X$, then $(\tilde{P}, I) \cap (\tilde{Q}, J)$ is a fs-W-algebra over $X$ through its Ei.

**Corollary 3.11:** A fs-W-algebra is the limited Ei of two fs-W-algebras.

**Theorem 3.12:** A W-algebra $X$ has two fs-W-algebras: $(\tilde{P}, I)$ and $(\tilde{Q}, J)$. The union $(\tilde{P}, I) \cup (\tilde{Q}, J)$ is a fuzzy W-algebra over $X$, If $I \& J$ are disjoint.

**Proof:** We can write $(\tilde{P}, I) \cup (\tilde{Q}, J) = (\tilde{H}, \mathcal{D})$ where $\mathcal{D} = I \cup J$ and for all $t \in \mathcal{D}$

$$\tilde{H}[t] = \left\{ \begin{array}{ll} \tilde{P}[t] : t \in I \setminus J & \\
\tilde{Q}[t] : t \in J \setminus I & \\
\tilde{P}[t] \cup \tilde{Q}[t] : t \in I \cap J & \end{array} \right\}$$

Since $I \cap J = \emptyset$, for every $u \in \mathcal{D}$, either $u \in I \setminus J$ (or) $u \in J \setminus I$. Because $(\tilde{P}, I)$ is a fuzzy W-algebra over $X$. If $u \in I \setminus J$, then $\tilde{H}(u) = \tilde{Q}(u)$ is a fuzzy W-algebra in a $X$. $\tilde{H}(u) = \tilde{Q}(u)$ is a fuzzy W-algebra in a $X$, if $u \in J \setminus I$, for $(\tilde{Q}, J)$.

Hence $(\tilde{H}, \mathcal{D}) = (\tilde{P}, I) \cup (\tilde{Q}, J)$ is a fuzzy W-algebra over $X$.

If $I \& J$ are not disjoint, theorem 3.12 is not valid as the following example displays.

**Example 3.13:** Let $\mathcal{M} = \{ \text{pink, gray, red, olive, tan} \}$ be a universe, and determine of a soft engine $\otimes$ that generates the items listed below.
Then \((\mathcal{M}, \otimes, \text{pink})\) is a W-algebra, define \(I\) & \(J\) as,

\[
I = \{\text{glorious, elegant, modest, clever}\}
\]

\[
J = \{\text{modest, clever, gentle}\}
\]

\(I\) and \(J\) are therefore not disjoint. If \((\bar{\mathcal{P}}, I)\) be a fs-set over \(\mathcal{M}\), then fuzzy sets in \(\mathcal{M}\) include \(\bar{\mathcal{P}}[\text{glorious}], \bar{\mathcal{P}}[\text{elegant}], \bar{\mathcal{P}}[\text{modest}] \& \bar{\mathcal{P}}[\text{clever}].\) Then, define as:

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</tr>
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<td>0.3</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>

Then a fs-W-algebra over \(\mathcal{M}\) is \((\bar{\mathcal{Q}}, I)\). If \((\bar{\mathcal{Q}}, J)\) be a fs-W-algebra over \(\mathcal{M}\), then the fuzzy sets in \(\mathcal{M}\) \(\bar{\mathcal{Q}}[\text{modest}], \bar{\mathcal{Q}}[\text{clever}] \& \bar{\mathcal{Q}}[\text{gentle}].\) Here is how we define them:

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<tr>
<td>gentle</td>
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<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
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</tbody>
</table>
Hence, a fs-W-algebra over \( M \) is \((\tilde{Q}, J)\). However, the union \((\tilde{P}, I) \cup (\tilde{Q}, J)\) is not a fs-W-algebra over \( M \).

\[
\tilde{P}[\text{modest}] \cup \tilde{Q}[\text{modest}](\text{olive} \odot \text{red}) = (\tilde{P}[\text{modest}] \cup \tilde{Q}[\text{modest}])(\text{tan})
\]

\[
= \max\{\tilde{P}[\text{modest}](\text{tan}), \tilde{Q}[\text{modest}](\text{tan})\} = 0.2 \quad \text{and}
\]

\[
\min\{\tilde{P}[\text{modest}] \cup \tilde{Q}[\text{modest}](\text{olive}), \tilde{P}[\text{modest}] \cup \tilde{Q}[\text{modest}](\text{red})\}
\]

\[
= \min\{\max\{\tilde{P}[\text{modest]}(\text{olive}), \tilde{Q}[\text{modest}](\text{olive})\}, \max\{\tilde{P}[\text{modest}](\text{red}), \tilde{Q}[\text{modest}](\text{red})\}\}
\]

\[
= \min\{\max\{0.4, 0.2\}, \max\{0.2, 0.4\}\} = 0.4.
\]

**Definition 3.14.** A fs-set \((\tilde{P}, I)\) over a W-algebra \( M \). \((\tilde{P}, I)\) is a W-ideal of \( M \) if there exists a parameter \( u \in I \) such that \( \tilde{P}[u] \) is a fuzzy ideal of \( M \). We say that \((\tilde{P}, I)\) is a fs-ideal of \( M \) if and only if it is based on all parameters.

**Example 3.15.** Let \( M = \{\text{guava, mango, beetroot, plum, turnip}\} \) be a universe, and look at a soft machine that produces the items listed below.

\[
guava \triangleleft q = \text{guava for all } q \in M,
\]

\[
mango \triangleleft \varsigma = \begin{cases} \text{guava} : \varsigma \in \{\text{mango, plum, turnip}\} \\ \text{banana} : \varsigma \in \{\text{guava, beetroot}\} \end{cases}
\]

\[
carrot \triangleleft z = \begin{cases} \text{beetroot} : z \in \{\text{guava, mango, turnip}\} \\ \text{guava} : z \in \{\text{beetroot, plum}\} \end{cases}
\]

\[
\text{plum} \triangleleft u = \begin{cases} \text{plum} : u = \text{guava}, \\ \text{mango} : u = \text{beetroot}, \\ \text{guava} : u = \text{plum}, \\ \text{beetroot}: u \in \{\text{mango, turnip}\} \end{cases}
\]

\[
\text{turnip} \triangleleft v = \begin{cases} \text{guava} : v = \text{turnip}, \\ \text{turnip} : v \in \{\text{guava, beetroot}\} \\ \text{mango} : v \in \{\text{mango, plum}\} \end{cases}
\]

then \( (M, \triangleleft, \text{guava}) \) is a W-algebra. Define \( \mathcal{K} = \{\text{puss, goat, yak, hen, sheep, rabbit}\} \).

Let \((\tilde{P}, \mathcal{K})\) be a fs-set over \( M \). Then \( \tilde{P}[\text{puss}], \tilde{P}[\text{goat}], \tilde{P}[\text{yak}], \tilde{P}[\text{hen}], \tilde{P}[\text{sheep}] \& \tilde{P}[\text{rabbit}] \) are fuzzy sets in \( M \).

<table>
<thead>
<tr>
<th>( \tilde{P} )</th>
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</thead>
<tbody>
<tr>
<td>puss</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>goat</td>
<td>0.7</td>
<td>0.5</td>
<td>0.7</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>yak</td>
<td>0.8</td>
<td>0.8</td>
<td>0.3</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>hen</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>sheep</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>rabbit</td>
<td>0.6</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

https://internationalpubls.com
A fs-ideal that is based on the parameters “puss”, “goat”, “yak”, “hen” and “rabbit”, then $(\mathcal{P}, \mathcal{K})$. However, based on the parameters “sheep”, $(\mathcal{P}, \mathcal{K})$ is not a fs-ideal of $\mathcal{M}$, as $\mathcal{P}[$sheep$](\text{turnip}) = 0.4 < 0.5 = \min \{\mathcal{P}[$sheep$](\text{plum}), \mathcal{P}[$sheep$](\text{plum})\}.$

We know for the most part, that sheeps prefer beetroot. Based on the parameter “sheep”, we may determine that $(\mathcal{P}, \mathcal{K})$ in the case above is not a fs-ideal of $\mathcal{M}$. This implies that $(\mathcal{P}, \mathcal{K})$ cannot be a fuzzy ideal of $\mathcal{M}$, just as sheep prefers a beetroot over another.

4. CONCLUSION

In this paper, we combine the notion of fs-sets with the theory of W-algebras. Fs-ideals and fs-W-algebras were introduced, and then some of their associated features were investigated. We shall investigate the applicability of fs-sets to different W-algebraic ideals on the basis of these results.

REFERENCES