

Bipartite through Prescribed Median and Antimedial of a Commutative Ring with Respect to an Ideal

Raghupatruni Sunil kumar¹, V. B. V. N. Prasad², Chudamani Ramineni³, T. Rama Rao⁴

¹Research Scholar, ^{1,2}Department of Engineering Mathematics, Koneru Lakshmaiah Education Foundation, Green Fields, Vaddeswaram, Andhra Pradesh, India-522302.

³Department of Mathematics, P.V.P Siddhartha Institute of technology, Kanuru, Vijayawada, Andhra Pradesh, India-520007.

⁴Professor of Mathematics, Vikas college of engineering and technology, Vijayawada, Andhra Pradesh, India-521212.

Article History:

Received: 10-04-2024

Revised: 20-05-2024

Accepted: 02-06-2024

Abstract:

Introduction: There are plenty of ways of partners with arithmetical constructions. Some of them to make reference to are bipartite from gatherings, median and anti – median from commutative rings with reference to an ideal. Partnering a median and anti – median of a commutative ring was presented by Beck in 1988. Similarly, Beck has researched the exchange between the ring theoretic properties of a commutative ring and related median and anti – median. Further Anderson and Badawi presented the idea absolute of commutative rings with median and anti – median in the year 2008. The absolute of a commutative ring R is the undirected with R as the vertex set and two particular vertices in R are nearby if and provided that their total is a median and anti – median of R . As of late Anderson and Badawi presented and concentrated on the summed up all out the of commutative rings concerning the multiplicatively prime subset H of R . The summed up complete of a commutative ring is the undirected with all components of R as vertices, and for two unmistakable vertices in R are nearby if and provided that their total is in H .

Objectives: In this article, an endeavour has been made to learn about in hypothetical properties and different control boundaries of summed up absolute of commutative rings of median and anti – median and its supplement.

Methods: Theorem 2.1. For a determinate commutative semigroup S , the set $V(\phi(\Gamma(S))) \cup \{0\}$ is an median of S .

Remark 2.1. A subgraph H of a graph G is a crossing subgraph of G if $V(H) = V(G)$. On the off chance that U is a bunch of edges of a graph G , $G \setminus U$ is the crossing subgraph of G acquired by erasing the edges in U from $E(G)$. A subset U of the edge set of an associated chart G is an edge set of G if $G \setminus U$ is separated. An edge set of G is negligible assuming no appropriate subset of U is edge set. Assuming e is an edge of G , with the end goal that $G \setminus \{e\}$ is detached, then e is known as an extension. Note that on the off chance that U is an insignificant edge set, $G \setminus U$ has precisely two associated median parts.

Corollary 2.1. Let T be the minimal edge set of $\Gamma(S)$, and G_1, G_2 are two median parts of $G \setminus T$. Then the following hold.

- (i) For any $i = 1, 2$, $(V(G_i) \cap V(T)) \cup \{0\}$ is ideal of S provided G_i has at least two vertices.
- (ii) $V(T) \cup \{0\}$ is an ideal if G_1 or G_2 has only one vertex. A commutative semigroup is called reduced if for any $x \in S$, $xn = 0$ implies $x = 0$. The annihilator of $x \in S$ is denoted by $\text{Ann}(x)$ and it is defined as $\text{Ann}(x) = \{a \in S \mid ax = 0\}$.

Results: Theorem 2.3. Let S be a commutative ring of median and anti – median. Then the subsequent results hold:

(i) If $|\text{Ass}(S)| \geq 3$ and $\emptyset = \text{Ann}(x)$, $\chi = \text{Ann}(y)$ are two distinct elements of $\text{Ass}(S)$, then $xy = 0$.

(ii) If $|\text{Ass}(S)| \geq 3$, then $\text{girth}(\Gamma(S)) = 4$.

(iii) If $|\text{Ass}(S)| \geq 6$, then $\Gamma(S)$ is not planar (A graph G is planar if it can be drawn in the plane in

such a way that no two edges meet except at vertex with which they are both incident).

Conclusions: Theorem 2.4. Let S be a commutative semigroup, then the median graph of a bipartite graph is induced by the vertices of maximum degree in G .

Proof. S be a commutative semigroup and G is a bipartite median graph, thus $d(v, u) < 2$ for any pair of vertices u, v of G . Let the degree of v in G be d . Then, these d vertices are at a distance 1 from v .

So, there are $p - 1 - d$ vertices u in G such that $d(v, u) = 2$ and $D(v) = d + 2(p - 1 - d) = 2(p - 1) - d$. Hence the vertices in G such that $D(v)$ is minimum are those for which the degree is maximum. Hence the proof.

Remark 2.3 The median graph of a bipartite graph is also a bipartite graph.

Keywords: median, anti - median, commutative ring, ideal.

1. Introduction

Let $G = (V, E)$ be a on n vertices with vertex set V and edge set E . A is bipartite if its vertex set can be partitioned into two nonempty subsets X and Y such that each edge of G has one end in X and the other in Y , and a is k -partite if its vertex set can be partitioned into k nonempty subsets such that no edge in G has its both ends in the same subset. Degree of a vertex v , $d(v)$, is the number vertices adjacent to v and by $N(v)$ we denote the neighbor set of v . The smallest and largest degrees of vertices in G are respectively denoted by $\delta(G)$ and $\Delta(G)$.

Given a G the issue of tracking down a H to such an extent that $M(H) \cong G$ is alluded to as the median issue. In [6], it is shown that any $G = (V, E)$ is the median of some associated. In [3] the thought of against median of a was presented and demonstrated that each is the counter median of some. The issue of concurrent inserting of median and hostile to median is examined in [1]. Another development, which sums up every one of the recently referenced developments, can be found in [5].

The median vertices have the base normal distance in a and subsequently the median issue is huge among the improvement issues including the position of organization servers. Nonetheless, the median developments for general can't be straightforwardly applied to a huge number as their fundamental has a place with various classes of. It tends to be seen that are bipartite to basic s of a huge number. For instance, the vast majority of the examinations in network networks are finished utilizing inclination networks [4] and they are displayed utilizing bipartite.

It is notable that the median of a tree is a vertex or an edge. This administrator was additionally read up for certain classes of in [7] and [8]. In this paper we show that any bipartite is the median of another bipartite. With an alternate development, we show that the comparative outcome additionally hold for k -partite. The undifferentiated from results for against median issue on these classes are additionally acquired. Since any is a k -partite, for some k , these developments can be applied overall. For any

remaining fundamental ideas and documentations not referenced in this paper we allude to [2].

In variation to the concept of zero divisor, few authors [8] introduced the total of a commutative ring. Let R be a commutative ring with $Nil(R)$ its ideal of nilpotent elements, $Z(R)$ its set of zero-divisors, and $Reg(R)$ its set of regular elements. The total of R , denoted by $T(R)$, is the undirected with all elements of R as vertices, and for distinct $x, y \in R$, the vertices x and y are adjacent if and only if $x + y \in Z(R)$. Also they introduced the three induce subs $Nil(R)$, $Z(R)$ and $Reg(R)$ of $T(R)$ with vertices $Nil(R)$, $Z(R)$, and $Reg(R)$.

A graph wherein each set of particular vertices is joined by an edge is known as a total graph. We use K_n for the total graph with n vertices. A r -partite graph is a graph whose vertex set can be divided into r subsets so that no edge has the two vertices in any one subset. A total I-partite graph is one in which every vertex is joined to each vertex that isn't in a similar subset as the given vertex. The total bipartite (i.e., complete 2-partite) graph is signified by $K_{m,n}$ where the arrangement of segment has sizes m and n . The circumference of a graph G is the length of a most limited cycle in G and is meant by bigness (G) . We characterize a shading of a graph G to be a task of tones (components of some set) to the vertices of G , one tone to every vertex, so nearby vertices are doled out unmistakable tones. In the event that n tones are utilized, the shading is alluded to as a n -shading. On the off chance that there exists a n -shading of a graph G , G is called n -colorable. The base n for which a graph G is n -colorable is known as the chromatic number of G , and is indicated by $\chi(G)$. A club of a graph is a maximal complete sub and the quantity of vertices in the biggest inner circle of graph G , signified by $\omega(G)$, is known as the faction number of G . Clearly $\chi(G) \geq \omega(G)$ for general graph G .

Assume that S is a commutative semigroup with nothing. For ideal hypothesis in commutative semigroup, we allude to the overview of median and anti – median [3] (additionally see [2]). Here we simply review a portion of the ideas. A non-void subset I of S is called ideal if $xS \subseteq I$ for any $x \in I$. An optimal p of a commutative semigroup is known as an excellent ideal of S if for any two component $x, y \in S$, $xy \in p$ infers $x \in p$ or $y \in p$. Let $Z(S)$ be its arrangement of zero-divisors of S . All together that $\Gamma(S)$ be non void, we generally expect S generally contains somewhere around one nonzero zero divisor. In [14] we can view that $\Gamma(S)$ (as in the ring case) is generally associated, and the breadth of $\Gamma(S) \leq 3$. In the event that $\Gamma(S)$ has a cycle bigness $(\Gamma(S)) \leq 4$. They additionally show that the quantity of insignificant beliefs of S gives a lower bound to the coterie number of S . In [26] authors concentrated on a graph $\Gamma(S)$ where the vertex set of this chart is $Z(S) \setminus \{0\}$ and for particular components $x, y \in Z(S) \setminus \{0\}$, in the event that $xy = 0$, there is an edge interfacing x and y . Note that $\Gamma(S)$ is a subgraph of $\Gamma(S)$. As of late, several authors concentrated on additional the graph $\Gamma(S)$ and its augmentation to a simplicial complex, cf. [13]. Obviously for any superb ideal p in the event that x and y are nearby in $\Gamma(S)$, $x \in p$ or $y \in p$. So, for each superb ideal p and each edge e , one of the end points of e has a place with p .

Building graphs from commutative rings was started by Ivan Beck through his work on zero-divisor charts and from that point a few graphs developments were made by a few creators. Through the development of charts from commutative rings, exchange between mathematical properties of commutative rings and graphs hypothetical properties of determined charts are contemplated. A portion of the charts characterized out of gatherings are Cayley graphs from bunches [25], non-commutating chart of a gathering [2], power chart of a limited gathering [29]. A chart is characterized out of non no divisors of a ring and is called zero-divisor graphs of a ring [12]. Intriguing varieties are likewise characterized like all out graphs [8], unit charts [15] and co maximal charts [33] related with

rings. Additionally, charts are characterized out of standards of a ring, to be specific obliterating ideal graphs of a ring [23], convergence chart of beliefs of rings [30, 31] and so forth.

Connecting a graphs with zero-divisors of a commutative ring was presented [24] in 1988, where the creator discussed shading of such graphs. Subsequent to presenting zero-divisor graphs, I. Beck made a guess that the faction number and chromatic number of the zero-divisor graphs are equivalent. In 1993, few authors settled Beck's guess in negative by giving a counter model [11]. Additionally, they have explored the interchange between the ring hypothetical properties of a commutative ring and graphs hypothetical properties of the zero-divisor chart. The definition alongside name for zero-divisor chart (R) was first presented in 1999, subsequent to altering the meaning of D. D. Anderson [11, 12].

Example of a zero-divisor graph for $R = Z_2X \frac{Z_2(x)}{\langle x^2 \rangle}$ is shown in fig 1.1

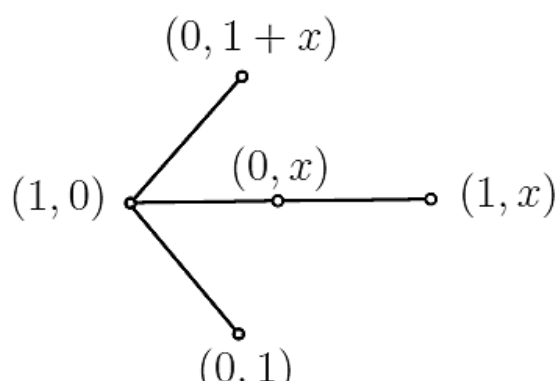


Figure 1.1: $R = Z_2X \frac{Z_2(x)}{\langle x^2 \rangle}$

2. Objectives

Definition 1.1. [2] A ring $(R, +, \cdot)$ is a nonempty set R together with binary operations ‘+’ and ‘ \cdot ’ defined on R , which satisfy the following conditions:

- (i) $(R, +)$ is an abelian group
- (ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c, \forall a, b, c \in R$
- (iii) $a \cdot (b + c) = a \cdot b + a \cdot c, \forall a, b, c \in R$
- (iv) $(a + b) \cdot c = a \cdot c + b \cdot c, \forall a, b, c \in R$.

Definition 1.2. [9] A ring R is called commutative if for every $a, b \in R$,

$$\exists a \cdot b = b \cdot a.$$

Definition 1.3. [10] Let R be a ring. An element $e \in R$ is called an identity element if $ea = ae = a \forall a \in R$. The identity element of a ring R is denoted by ‘ 1 ’.

Definition 1.4. [16] Let R be a ring with identity. An element $u \in R$ is called a unit element if there exists $v \in R$ such that $uv = 1 = vu$ and the inverse of u is often denoted by u^{-1} . The collection of all units in R is denoted by $U(R)$ or R^\times . It is easy to check that $U(R)$ is a group under multiplication and is called multiplicative group of R .

Definition 1.5. [17] A ring R with identity is called a division ring if every nonzero element of R is a unit. A commutative division ring R is called a field.

Definition 1.6. [18] An element $x \in R$ is said to be a zero-divisor if there exists $0 \neq y \in R \ni xy = 0$ where 0 is the additive identity. The set of all zero-divisors in R is denoted by $Z(R)$.

Definition 1.7. [19] An ideal P of a ring R is called a prime ideal if $P \neq R$ and $\forall a, b \in R, ab \in P$ implies $a \in P$ or $b \in P$.

Definition 1.8. [20] A commutative ring R is called an integral domain if R has no non-zero zero-divisors.

Definition 1.9. [21] Let R be a ring. The characteristic of R is the least positive integer n such that $na = 0 \forall a \in R$. If no such positive integer exists, then R is said to be of characteristic zero.

Let us collect some basic definitions and results on s:

Definition 1.9. [5] Given a bipartite G of n vertices, there exists a connected bipartite H' such that G is an induced sub of H' and all the vertices of G in H' have equal status in H' .

Definition 1.10. [22] Given a bipartite G there exists a bipartite H such that $M(H) \cong G$.

Definition 1.11. [27] Given a bipartite G there exists a bipartite H such that $AM(H) \cong G$

Definition 1.12. [28] Given a k -partite G there exists a k -partite H such that $M(H) \cong G$.

Definition 1.13. [32] Given a k -partite G there exists a k -partite H' such that $AM(H') \cong G$.

3. Methods

Theorem 2.1. For a determinate commutative semigroup S , the set $V(\varphi(\Gamma(S))) \cup \{0\}$ is a median of S .

Proof. Let $x \in V(\varphi(\Gamma(S)))$, and $r \in S$. Suppose that $rx \neq 0$.

Then $e(rx) = \max\{d(u, rx) | u \in V(G)\} \leq \max\{d(u, x) | u \in V(G)\} = e(x)$.

Thus $e(rx) = e(x)$.

Hence, $rx \in V(\varphi(\Gamma(S))) \cup \{0\}$.

Remark 2.1. A subgraph H of a graph G is a crossing subgraph of G if $V(H) = V(G)$. On the off chance that U is a bunch of edges of a graph G , $G \setminus U$ is the crossing subgraph of G acquired by erasing the edges in U from $E(G)$. A subset U of the edge set of an associated chart G is an edge set of G if $G \setminus U$ is separated. An edge set of G is negligible assuming no appropriate subset of U is edge set. Assuming e is an edge of G , with the end goal that $G \setminus \{e\}$ is detached, then e is known as an extension. Note that on the off chance that U is an insignificant edge set, $G \setminus U$ has precisely two associated median parts.

Corollary 2.1. Let T be the minimal edge set of $\Gamma(S)$, and G_1, G_2 are two median parts of $G \setminus T$. Then the following hold.

(i) For any $i = 1, 2$, $(V(G_i) \cap V(T)) \cup \{0\}$ is ideal of S provided G_i has at least two vertices.

(ii) $V(T) \cup \{0\}$ is an ideal if G_1 or G_2 has only one vertex. A commutative semigroup is called reduced if for any $x \in S$, $x_n = 0$ implies $x = 0$. The annihilator of $x \in S$ is denoted by $\text{Ann}(x)$ and it is defined as $\text{Ann}(x) = \{a \in S | ax = 0\}$.

4. Results

Theorem 2.1. For a determinate commutative semigroup S , the set $V(\varphi(\Gamma(S))) \cup \{0\}$ is a median of S .

Remark 2.1. A subgraph H of a graph G is a crossing subgraph of G if $V(H) = V(G)$. On the off chance that U is a bunch of edges of a graph G , $G \setminus U$ is the crossing subgraph of G acquired by erasing the edges in U from $E(G)$. A subset U of the edge set of an associated chart G is an edge set of G if $G \setminus U$ is separated. An edge set of G is negligible assuming no appropriate subset of U is edge set. Assuming e is an edge of G , with the end goal that $G \setminus \{e\}$ is detached, then e is known as an extension. Note that on the off chance that U is an insignificant edge set, $G \setminus U$ has precisely two associated median parts.

Corollary 2.1. Let T be the minimal edge set of $\Gamma(S)$, and G_1, G_2 are two median parts of $G \setminus T$. Then the following hold.

- (i) For any $i = 1, 2$, $(V(G_i) \cap V(T)) \cup \{0\}$ is ideal of S provided G_i has at least two vertices.
- (ii) $V(T) \cup \{0\}$ is an ideal if G_1 or G_2 has only one vertex. A commutative semigroup is called reduced if for any $x \in S$, $x_n = 0$ implies $x = 0$. The annihilator of $x \in S$ is denoted by $\text{Ann}(x)$ and it is defined as $\text{Ann}(x) = \{a \in S \mid ax = 0\}$.

Theorem 2.2. Let S be a commutative and reduced semigroup in which $\Gamma(S)$ does not contain a median and anti – median clique. Then S satisfies the ascending chain conditions (a.c.c) on annihilators.

Theorem 2.3. Let S be a commutative rings of median and anti – median. Then the subsequent results hold:

- (i) If $|\text{Ass}(S)| \geq 3$ and $\emptyset = \text{Ann}(x)$, $\chi = \text{Ann}(y)$ are two distinct elements of $\text{Ass}(S)$, then $xy = 0$.
- (ii) If $|\text{Ass}(S)| \geq 3$, then $\text{girth}(\Gamma(S)) = 4$.
- (iii) If $|\text{Ass}(S)| \geq 6$, then $\Gamma(S)$ is not planar (A graph G is planar if it can be drawn in the plane in such a way that no two edges meet except at vertex with which they are both incident).

Corollary 2.2. Let $\emptyset_1 = \text{Ann}(x_1)$, $\emptyset_2 = \text{Ann}(x_2)$, $\emptyset_3 = \text{Ann}(x_3)$ and $\emptyset_4 = \text{Ann}(x_4)$, \dots $\emptyset_n = \text{Ann}(x_n)$ belong to $\text{Ass}(S)$ with reference to commutative rings of median and anti – median. Then $x_1 - x_2 - x_3 - x_4 - \dots - x_n$ is a cycle of length n .

Remark 2.2. Let S be a commutative semigroup and let $\text{Ann } a$ be a maximal element of $\{\text{Ann } x : 0 \neq x \in S\}$. Then $\text{Ann } a$ is a prime ideal.

Theorem 2.4. Let S be a commutative semigroup, then the median graph of a bipartite graph is induced by the vertices of maximum degree in G .

Remark 2.3 The median graph of a bipartite graph is also a bipartite graph.

5. Discussion

Theorem 2.2. Let S be a commutative and reduced semigroup in which $\Gamma(S)$ does not contain a median and anti – median clique. Then S satisfies the ascending chain conditions (a.c.c) on annihilators.

Proof. Suppose that $\text{Ann } x_1 < \text{Ann } x_2 < \dots < \text{Ann } x_n$ be a cumulative chain of ideals. For each $i \geq 2$, select $a_i \in \text{Ann } x_i \setminus \text{Ann } x_{i-1}$. Then every a_i is nonzero, for $i = 2, 3, \dots$. Also $y_i y_j$ for any $i \neq j$. Since S is a commutative and condensed semigroup, we have $y_i \neq y_j$ when $i \neq j$. Thus, one can obtain a median and anti – median in S . This is a contradiction and so the assertion holds.

Theorem 2.3. Let S be a commutative ring of median and anti – median. Then the subsequent results hold:

- (i) If $|\text{Ass}(S)| \geq 3$ and $\emptyset = \text{Ann}(x)$, $\chi = \text{Ann}(y)$ are two distinct elements of $\text{Ass}(S)$, then $xy = 0$.
- (ii) If $|\text{Ass}(S)| \geq 3$, then $\text{girth}(\Gamma(S)) = 4$.
- (iii) If $|\text{Ass}(S)| \geq 6$, then $\Gamma(S)$ is not planar (A graph G is planar if it can be drawn in the plane in such a way that no two edges meet except at vertex with which they are both incident).

Proof. (i). We can assume that there exists $r \in \emptyset \setminus \chi$. Then $rx = 0$ and so $rsx = 0 \in \emptyset$. Since χ is a prime ideal, $x \in \chi$ and hence $xy = 0$.

(ii). Let $\emptyset_1 = \text{Ann}(x_1)$, $\emptyset_2 = \text{Ann}(x_2)$, $\emptyset_3 = \text{Ann}(x_3)$ and $\emptyset_4 = \text{Ann}(x_4)$ belong to $\text{Ass}(S)$. Then $x_1-x_2-x_3-x_4-x_1$ is a cycle of length 4.

(iii). Since $|\text{Ass}(S)| \geq 6$, k_5 is a subgraph of $\Gamma(S)$, and hence by Kuratowski's Theorem $\Gamma(S)$ is not planar.

Corollary 2.2. Let $\emptyset_1 = \text{Ann}(x_1)$, $\emptyset_2 = \text{Ann}(x_2)$, $\emptyset_3 = \text{Ann}(x_3)$ and $\emptyset_4 = \text{Ann}(x_4)$, \dots $\emptyset_n = \text{Ann}(x_n)$ belong to $\text{Ass}(S)$ with reference to commutative rings of median and anti – median. Then $x_1-x_2-x_3-x_4-\dots-x_n$ is a cycle of length n .

Remark 2.2. Let S be a commutative semigroup and let $\text{Ann } a$ be a maximal element of $\{\text{Ann } x : 0 \neq x \in S\}$. Then $\text{Ann } a$ is a prime ideal.

Theorem 2.4. Let S be a commutative semigroup, then the median graph of a bipartite graph is induced by the vertices of maximum degree in G .

Proof. S be a commutative semigroup and G is a bipartite median graph, thus $d(v, u) < 2$ for any pair of vertices u, v of G . Let the degree of v in G be d . Then, these d vertices are at a distance 1 from v . So, there are $p - 1 - d$ vertices u in G such that $d(v, u) = 2$ and $D(v) = d + 2(p - 1 - d) = 2(p - 1) - d$. Hence the vertices in G such that $D(v)$ is minimum are those for which the degree is maximum.

Hence proved.

Remark 2.3. The median graph of a bipartite graph is also a bipartite graph.

References

- [1] K. Balakrishnan, B. Brešar, M. Kovšec, M. Changat, A.R. Subhamathi and S. Klavžar, Simultaneous embeddings of s as median and antimedian 446 K. Pravas and A. Vijayakumar subs, Networks 56 (2010) 90–94. doi:10.002/net.20350
- [2] R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory, Second Edition (Heidelberg, Springer, 2012). doi:10.1007/978-1-4614-4529-6
- [3] H. Bielak and M.M. Syslo, Peripheral vertices in s , Studia Sci. Math. Hungar. 18 (1983) 269–275.
- [4] H. Kautz, B. Selman and M. Shah, Referral Web: combining social networks and collaborative filtering, Communications of the ACM 40(3) (1997) 63– 65. doi:10.1145/245108.245123
- [5] Karuvachery Pravas and Ambat Vijayakumar, The Median Problem On K -Partite s , Discussions Mathematical Theory, 35 (2015) 439–446 doi:10.7151/dmgt.1802.
- [6] K. Pravas and A. Vijayakumar, Convex median and anti-median at prescribed distance, communicated.
- [7] P.J. Slater, Medians of arbitrary s , J. Theory 4 (1980) 389–392. doi:10.1002/jgt.3190040408
- [8] S.B. Rao and A. Vijayakumar, On the median and the anti-median of a co, Int. J. Pure Appl. Math. 46 (2008) 703–710.
- [9] H.G. Yeh and G.J. Chang, Centers and medians of distance-hereditary s , Discrete Math. 265 (2003) 297–310. doi:10.1016/S0012-365X(02)00630-1.
- [10] S. Akbari, H. R. Maimani, S. Yassemi, when a Zero-Divisor Graph is Planar or a Complete r -Partite Graph, J. Algebra 270 (2003), 169 -180.
- [11] D. D. Anderson, Finitely generated multiplicative subsemigroups of rings, Semigroup Forum 55 (1997), 294-298.
- [12] D. D. Anderson, and E.W. Johnson, Ideal theory in commutative semigroups, Semigroup Forum 30 (1984), 127-158.
- [13] D. D. Anderson, M. Naseer, Beck's coloring of a commutative ring, J. Algebra 159 (1991), 500-514.
- [14] D. F. Anderson, A. Frazier, A. Lauve, P. S. Livingston, The Zero-Divisor Graph of a Commutative Ring II, Lecture Notes in Pure and Appl. Math., 220, Dekker, New York, 2001.
- [15] D. F. Anderson, P. S. Livingston, The Zero-Divisor Graph of a Commutative Ring, J. Algebra 217 (1999), 434-447.

- [16] D. F. Anderson, R. Levy, J. Shapiro, Zero-Divisor Graphs, von Neumann Regular Rings, and Boolean Algebras, *J. Pure Appl. Algebra* 180 (2003), 221-241.
- [17] I. Beck, Coloring of Commutative Rings, *J. Algebra* 116 (1988), 208-226.
- [18] R. Belsho and J. Chapman, Planar zero-divisor graphs, *J. Algebra* 316 (2007), 471- 480.
- [19] G. Chartrand, O. R. Oellermann, *Applied and Algorithmic Graph Theory*, McGraw-Hill, Inc., New York, 1993.
- [20] H.-J. Chiang-Hsieh, Classification of rings with projective zero-divisor graphs, *J. Algebra* 319 (2008), 2789-2802.
- [21] A. H. Clifford and G. B. Preston, *The Algebraic Theory of Semigroups*, Mathematical Surveys, No. 7, vol 1 and 2, American Mathematical Society, Providence, RI, 1961 and 1967.
- [22] F. R. DeMeyer, L. DeMeyer, Zero-Divisor Graphs of Semigroups, *J. Algebra* 283 (2005), 190 -198.
- [23] F. R. DeMeyer, T. McKenzie, K. Schneider, The Zero-Divisor Graph of a Commutative Semigroup, *Semigroup Forum* 65 (2002), 206-214.
- [24] J. M. Howie, *An introduction to semigroup theory*, L.M.S. Monographs, No. 7. Academic Press [Harcourt Brace Jovanovich, Publishers], London-New York, 1976.
- [25] J. D. LaGrange, Complemented zero-divisor graphs and Boolean rings, *J. Algebra* 315 (2007), 600-611.
- [26] R. Levy, J. Shapiro, The Zero-Divisor Graph of von Neumann Regular Rings, *Comm. Algebra* 30 (2002), 745-750.
- [27] T. G. Lucas, The diameter of a zero-divisor graph, *J. Algebra* 301 (2006), 174-193.
- [28] H. R. Maimani, M. R. Pournaki, and S. Yassemi, Zero-divisor graph with respect to an ideal, *Comm. Algebra* 34 (2006), 923-929.
- [29] H. R. Maimani and S. Yassemi, Zero-divisor graphs of amalgamated duplication of a ring along an ideal, *J. Pure Appl. Algebra* 212 (2008), 168-174.
- [30] Y. S. Park, J. P. Kim, and M.-G. Sohn, Semiprime ideals in semigroups, *Math. Japon.* 33 (1988), 269-273.
- [31] M. Satyanarayana, Semigroups with ascending chain condition, *J. London Math. Soc.* 5 (1972), 11-14.
- [32] C. Wickham, Classification of rings with genus one zero-divisor graphs, *Comm. Algebra* 36 (2008), 325-345.
- [33] S. E. Wright, Lengths of paths and cycles in zero-divisor graphs and digraphs of semi-groups, *Comm. Algebra* 35 (2007), 1987-1991.