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Solutions of Ordinary Differential Equations using Differential Transform Method

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Abstract:
Differential Transform Method is a semi-analytical numerical technique that depends on Taylor's series for the resolution of ordinary Differential Equations. This technique useful to obtain the series solutions of ordinary Differential Equations. Differential Transform method provides the solutions in the form of a polynomial. In this paper, we study Differential Transform Method , Its theorems, properties and examples. The Differential Transform method is a monotonous procedure for attaining the analytic solutions of ordinary Differential Equations

Keywords: Differential Transform Method,Two Dimensional Differential Transform, ordinary Differential Equations method , Series solutions, Approximate solutions

Introductions: The Differential Transform Method (DTM)[17] is a powerful mathematical technique used to solve higher-order boundary value problems in ordinary differential equations. It's particularly useful when dealing with complex problems that might not have straight forward analytical solutions. In essence, DTM involves transforming the ordinary differential equations into algebraic equations using a series of transformations based on Taylor series expansions. The method involves transforming the derivatives in the original equation into a series of algebraic expressions, which are then solved to obtain the solution. [2,11] This transformation is carried out by expressing the dependent variables and their derivatives as series in terms of a parameter. The resulting system of algebraic equations can then be solved to obtain an approximate solution to the original Differential Equation. The Two-Dimensional Differential Transform Method has been applied in various fields, including heat conduction, fluid dynamics, and elasticity, providing a numerical technique for solving two-dimensional problems in engineering and physics[4,8]. It is particularly useful when analytical solutions are challenging or impossible to obtain.[3,10]

Methodology:

Differential Transformation Method[17]:

The differential transformation of the kth derivative of a function u(x) is defined as follows:

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \qquad (1)$$

In (1), $u(x)$ is the original function and $U(k)$ is the transformed function.

and inverse differential transformation of is defined as follow:

$$u(x) = \sum_{k=0}^{\infty} U(k)(x-x_0)^k \qquad (2)$$

From (1) and (2), we obtain.

$$u(x) = \sum_{k=0}^{\infty} \frac{1}{k!}(x - x_0)^k \left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \quad (3)$$

Eq. (3) implies that the concept of differential transformation is derived from the Taylor series expansion. From the definitions of (2) and (3), it is easy to prove that the transformed functions comply with the following basic mathematics operations (Table1).

Original Function	Transformed Function
$z(x) = u(x) \pm v(x)$	$Z(k) = U(k) + V(k)$ (4)
$z(x) = \lambda u(x)$	$Z(k) = \lambda U(k)$ (5)
$z(x) = \frac{d^2 g(x)}{dx^n}$	$Z(k) = \frac{(k+n)!}{k!} G(k+n)$ (6)
$z(x) = u(x)v(x)$	$Z(k) = \sum_{l=0}^{k} U(l)V(k-l)$ (7)
$z(x) = \lambda x^m$	$Z(k) = \lambda\delta(k-m) = \begin{cases} 1, if\ k = m \\ 0, if\ k \neq m \end{cases}$ (8)

Table1

Example 1- Consider the following differential equation of second order,

$$\frac{d^2 v}{dt^2} - 2\frac{dv}{dt} + 5v = 0$$

With the conditions *v(0)= -1, v'(0) = 7*

We apply DTM, with initial conditions *V(0)= -1, V(1)=7,* we get

$$V(k+2) = \frac{1}{(k+1)(k+2)}[2(k+1)V(k+1) - 5V(k)]$$

Put *k=0,1,2,3,4,...* $V(2) = \frac{19}{2}, V(3) = \frac{1}{2}, V(4) = \frac{-89}{24} \ldots$

Therefore, the closed form of the solution can be easily written as,

$$v(t) = \sum_{k=0}^{2} V(k)t^k = -1 + 7t + \frac{19}{2}t^2 + \frac{1}{2}t^3 - \frac{89}{24}t^4$$

and the exact solution is $v(t) = -e^t cos2t + 4e^{2t} sin2t$

Example 2- Consider the following differential equation of second order,

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 4v = 6e^{-t}$$

With the conditions *v(0)= -2, v'(0)=8*

We apply DTM, with initial conditions *V(0)= -2, V(1)=8*, we get

$$V(k+2) = \frac{1}{(k+1)(k+2)}[\frac{-6}{k!} - 4(k+1)V(k+1) - 4V(k)]$$

Put *k=0,1,2,3,4,…* $V(2) = -15, V(3) = \frac{41}{3}, V(4) = \frac{-107}{12} \dots$

Therefore, the closed form of the solution can be easily written as,

$$v(t) = \sum_{k=0}^{2} V(k)t^k = -2 + 8t - 15t^2 + \frac{41}{3}t^3 - \frac{107}{12}t^4$$

and the exact solution is $v(t) = 6e^t - 8e^{-2t}$

Example3: consider the following system of non homogenous differential equations

$$\frac{dy_1}{dx} = y_3 - cosx\,, \quad \frac{dy_2}{dx} = y_3 - e^x, \quad \frac{dy_3}{dx} = y_1 - y_2$$

With initial conditions $y_1(0) = 1, y_2(0) = 0,\ y_3(0) = 0$

Using a basic theorems(4),(5),(6) of Differential Transform Method,

$$Y_1(k+1) = \frac{1}{k+1}\left(Y_3(k) - \frac{1}{k!}\cos\left(\frac{k\pi}{2}\right)\right), Y_1(0) = 1$$

$$Y_2(k+1) = \frac{1}{k+1}\left(Y_3(k) - \frac{1}{k!}\right), Y_2(0) = 1\ , Y_3(k+1) = \frac{1}{k+1}(Y_1(k) - Y_2(k)), Y_3(0) = 2$$

Put $k = 0,1,2,34,5$ in above equations, we get

$Y_1(1) = 1\,, Y_2(1) = 1\ ,\ Y_3(1) = 1 \quad Y_1(2) = \frac{1}{2}\ ,\ Y_2(2) = 0\,,\ Y_3(2) = 0$

$Y_1(3) = \frac{1}{6}\ ,\ Y_2(3) = -\frac{1}{6}\,,\ Y_3(3) = \frac{1}{6}\ \ Y_1(4) = \frac{1}{24}\ ,\ Y_2(4) = 0\,,\ Y_3(4) = \frac{1}{12}$

$Y_1(5) = \frac{1}{5!}\ ,\ Y_2(5) = \frac{1}{5!}\,,\ Y_3(5) = \frac{1}{5!}$

$Y_1(6) = \frac{1}{6!}\ ,\ Y_2(6) = 0\,,\ Y_3(6) = 0$

By substituting these values of $Y_1(k), Y_2(k), Y_3(k)$ to obtain $y_1(x), y_2(x), y_3(x)$ in (3.1.2)

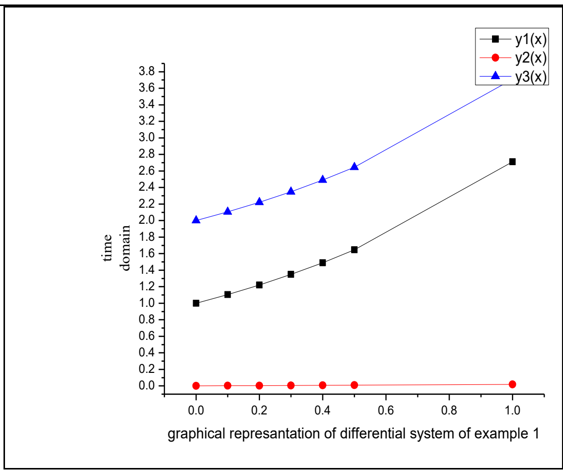
$$y_i(x) = \sum_{k=0}^{k=\infty} x^k \, Y_i(k)$$

Take i = 1, $y_1(x) = 1 + x + x^2 + . . . = e^x$

Take i = 2, $y_2(x) = x - \frac{x^3}{3!} + . . . = sinx$

Take i = 1, $y_3(x) = 2 + x(1) + x^2(0) + \frac{x^3}{6} + \frac{x^4}{12} + . . . = e^x + cosx$

X	$y_1(x)$	$y_2(x)$	$y_3(x)$
0	1	0	2
0.1	1.11	0.9983333	2.100175
0.2	1.24	1.9806666	2.204689
0.3	1.39	0.2955524	2.315648
0.4	1.56	0.3893331	2.425648
0.5	1.75	0.4791661	2.545321
1	3	0.8333312	3.251667



CONCLUSION:

From our study we can conclude that we use Differential Transform Method can use to solve various types of ordinary differential equations in physics, mathematics, mechanical problems. The DTM provides a direct, recursive approach for solving differential equations, avoiding numerical calculations or discretization techniques. The computed approximate solutions using DTM show excellent agreement with exact analytical solutions, with minimal deviation even for small truncation orders. The method efficiently handles systems of differential equations, making it applicable to real-world problems involving coupled processes. Compared to conventional numerical methods, DTM offers a simple yet powerful alternative with fewer computational steps and reduced computational cost. DTM is a convergent method and that the accuracy of the solution improves significantly with the inclusion of more terms in the transformed series. DTM proves to be a reliable and computationally effective technique for solving integral equations.

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