Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

Anisotropic Bianchi Type-VI Cosmological Models with Cosmic Strings and Domain Walls in $f(R,T)$ Gravity

Shakeel Khan[1]*, Rishikumar Agrawal[2],
[1]Department of Mathematics, R T M Nagpur University, Nagpur, Maharashtra 440033, India.
[2]Department of Mathematics, Hislop College, Nagpur, Maharashtra 440001, India.

Article History:
Received: 15-10-2025
Revised: 19-11-2025
Accepted: 10-12-2025

Abstract:

In this work, we investigate anisotropic cosmological models of Bianchi type-VI in the framework of modified gravity described by the function F(R,T), where R is the Ricci scalar and T is the trace of the energy–momentum tensor. The cosmic matter content is modelled in two physically motivated forms, namely a cloud of cosmic strings with attached particles and thick domain walls. To obtain exact and tractable solutions of the field equations, we assume that the scalar expansion is proportional to the shear scalar and adopt Berman's law for the variation of the Hubble parameter, which leads to a constant deceleration parameter.

Explicit solutions for the metric functions are derived, and the dynamical behavior of key physical quantities such as the expansion scalar, shear scalar, anisotropy parameter, energy density, string tension density, and pressure are examined. The resulting models exhibit accelerated expansion at late times and are free from initial singularities. It is also found that the universe remains anisotropic throughout its evolution due to the non-vanishing anisotropy parameter. Furthermore, both the energy density and the string tension (or domain wall pressure) decrease with cosmic time and tend toward negligible values in the far future.

These results indicate that Bianchi type-VI cosmological models with cosmic strings and domain walls in F(R,T) gravity provide a consistent theoretical framework for describing an anisotropic, accelerating universe and offer useful insights into the possible role of topological defects in cosmic evolution.

Keywords: Bianchi type-VI cosmology, F(R,T) gravity , cosmic strings, domain walls.

1. Introduction

Several independent astronomical observations show that the universe is currently undergoing an accelerated expansion. The first strong evidence for this behavior came from the study of distant type Ia supernovae, which indicated that the universe is expanding faster today than in the past [1,2]. This conclusion was later confirmed by measurements of the cosmic microwave background radiation from satellite missions such as WMAP and Planck [3,4], baryon acoustic oscillation observations from galaxy surveys [5], and large-scale structure data [6]. More recent datasets, including the Pantheon+ supernova compilation and the Dark Energy Survey, have further improved the precision of cosmological parameters and strengthened the evidence for late-time acceleration [7–9]. These observations strongly suggest that a dominant component with negative pressure, commonly called dark energy, governs the present dynamics of the universe.

Although the cosmological constant provides a simple explanation for dark energy, it faces conceptual problems such as fine-tuning and coincidence. This has motivated the study of modified theories of gravity, where the geometrical part of Einstein's equations is changed instead of introducing unknown matter components [10]. Examples include f(R) gravity [11,12], teleparallel $f(T)$ gravity [13], and scalar–tensor theories [14]. These models have been extensively studied and shown to produce accelerated expansion in many cosmological settings.

The $f(R,T)$ theory of gravity, proposed by Harko et al. [15], extends the idea of modified gravity by introducing an explicit coupling between matter and geometry. In this theory, the gravitational action depends on both the Ricci scalar R and the trace T of the energy–momentum tensor [15]. The gravitational field equations have been derived from the Helbert-Einstein type variational principal by taking the action [15]

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + L_m\sqrt{-g}d^4x \tag{1}$$

Here $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the trace of energy momentum tensor of the matter T_{ij}, L_m is the matter Lagrangian.

The energy–momentum tensor of matter is defined by [15]

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2}$$

and its trace is

$$T = g^{ij}T_{ij} \tag{3}$$

Assuming that the matter Lagrangian L_m depends only on the metric tensor and not on its derivatives, one obtains

$$T_{ij} = g_{ij}\, L_m - 2\frac{\partial\, L_m}{\partial g^{ij}} \tag{4}$$

Variation of the action with respect to the metric tensor yields the field equations of $f(R,T)$ gravity

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\,\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij}$$
(5)

Where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2\, L_m}{\partial g^{ij}\partial g^{lm}} \tag{6}$$

Here $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, $\Box = \nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m

For a perfect fluid, the energy–momentum tensor is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{7}$$

Where $u^i = (1,0,0,0)$ is the four-velocity vector which satisfies the condition $u^i u_i = 1$ and $u^i\nabla_j u_i = 0$. Here ρ and p are energy density and pressure of the fluid respectively.

and the corresponding variation leads to

$$\theta_{ij} = -2T_{ij} - pg_{ij} \tag{8}$$

If one chooses the functional form

$$f(R,T) = R + 2f(T) \tag{9}$$

Then the field equations in (5) reduce to

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{10}$$

Where the overhead prime denotes the derivative with respect to the argument.

These equations show explicitly how matter and geometry are coupled in this theory.

Since its introduction, $f(R,T)$ gravity has been applied to a wide variety of cosmological models. Harko et al. [15] studied homogeneous cosmologies and basic theoretical aspects of the model. Adhav [16] investigated anisotropic LRS Bianchi type-I cosmologies, Singh and Sharma [17] examined Bianchi type-II models with dark energy, and Ram and Chandel [18] studied magnetized and string cosmologies. More recently, observational constraints and statistical analyses have been performed to test the viability of $f(R,T)$ models using modern data [19,20].

Cosmic strings and domain walls are topological defects that may have formed during symmetry-breaking phase transitions in the early universe. The formation mechanism was first described by Kibble [21], and the cosmological consequences of domain walls were discussed by Zel'dovich et al. [22]. Cosmic strings were studied in detail by Vilenkin and Shellard [23], Gott [24], and Letelier [25], while domain wall dynamics were analyzed by Ipser and Sikivie [26] and Mukherjee [27]. These defects can influence cosmic expansion and anisotropy and therefore provide physically interesting matter sources for anisotropic cosmological models.

Motivated by the strong observational evidence for cosmic acceleration, the theoretical flexibility of modified gravity, the matter–geometry coupling in $f(R,T)$ gravity, and the physical relevance of cosmic strings and domain walls, we investigate Bianchi type-VI cosmological models with these defect sources in the framework of $f(R,T)$ gravity. We adopt physically reasonable assumptions such as a proportionality relation between shear and expansion [28] and Berman's law for the Hubble parameter [29] to obtain exact solutions and analyze their physical behavior.

2. Metric and field equations

In this work we consider a spatially homogeneous but anisotropic Bianchi type-VI space-time, which generalizes the standard isotropic cosmological models by allowing different expansion rates along different spatial directions. Such anisotropic models are useful for studying possible deviations from isotropy in the early universe and for investigating the role of anisotropic matter sources such as cosmic strings and domain walls [30,31].

The line element for the Bianchi type-VI metric is taken in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2hx}dy^2 + C^2(t)e^{2hx}dz^2 \tag{11}$$

Where $A(t)$, $B(t)$, and $C(t)$ are directional scale factors depending only on cosmic time t, and h is a constant parameter that characterizes the anisotropy of the spatial sections [30]. The exponential terms in the metric reflect the non-trivial spatial curvature of the Bianchi type-VI geometry and distinguish it from other Bianchi classes.

This metric is spatially homogeneous, since the metric functions depend only on time, but it is anisotropic because the expansion along the x, y, and z directions is governed by different scale factors. The Bianchi type-VI space-time reduces to simpler Bianchi models for special choices of the parameter h and the scale factors, and therefore provides a flexible framework for studying anisotropic cosmological evolution in modified gravity theories [31,32]. For these reasons, the Bianchi type-VI metric is a suitable choice for analyzing cosmological models with anisotropic matter sources such as cosmic strings and domain walls in the framework of $f(R,T)$ gravity.

2.1 Cosmic strings

Cosmic strings are one–dimensional topological defects that may have formed during symmetry-breaking phase transitions in the early universe and can influence the large-scale dynamics and anisotropy of the universe [33,34]. Their gravitational properties and cosmological implications have been widely studied in both general relativity and modified gravity theories [35,36]. In cosmological modelling, a cloud of cosmic strings with particles attached is described by an anisotropic energy–momentum tensor that reflects the fact that strings possess tension along a preferred spatial direction.

The energy–momentum tensor for a cloud of strings is taken as [35,37]

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{12}$$

Where ρ is the total energy density of the string cloud, λ is the string tension density, u_i is the four-velocity vector of the matter distribution, and x_i is a unit space-like vector indicating the direction of the strings.

The total energy density is written as $\rho = \rho_p + \lambda$

Where ρ_p represents the energy density of the particles attached to the strings. The vectors u_i and x_i satisfy the normalization and orthogonality conditions [35]

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0 \tag{13}$$

Ensuring that u_i is time-like, x_i is space-like, and the string direction is orthogonal to the matter flow.

In the co-moving coordinate system, and by choosing the strings to be aligned along the xxx-direction, the non-zero components of the energy–momentum tensor reduce to [37]

$$T_1^1 = \lambda,\ T_2^2 = 0,\ T_3^3 = 0,\ T_0^0 = \rho \tag{14}$$

This form clearly shows that the pressure (tension) is present only along the string direction, while the transverse directions are pressure less, highlighting the anisotropic character of the string cloud.

Using the Bianchi type-VI metric (11), the cosmic string energy–momentum tensor (12)–(14), and the choice $f(T) = \mu T$ where μ is a constant, the modified field equations of the $f(R,T)$ gravity given by (10) reduce to the following system of coupled differential equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{h^2}{A^2} = -[(8\pi+2\mu)\lambda + 2p\mu + \mu(\rho+\lambda)] \tag{15}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2} = -[2p\mu + \mu(\rho+\lambda)] \tag{16}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{h^2}{A^2} = -[2p\mu + \mu(\rho+\lambda)] \tag{17}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2} = -[(8\pi+2\mu)\rho + 2p\mu + \mu(\rho+\lambda)] \tag{18}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C} = 0 \tag{19}$$

Here, an over dot denotes differentiation with respect to cosmic time t, and for this string-dominated model the trace of the energy–momentum tensor takes the form $T = \rho + \lambda$

2.1.1 Solutions of field equations

From (19), we obtain a direct relation between the metric functions $B(t)$, and $C(t)$. On integrating this equation, we get $C = kB$

where k is a constant of integration. Without loss of generality, we choose $k = 1$, which leads to the simple relation

$$C = B \tag{20}$$

With this condition, the field equations (15)– (18) reduce to a system of two independent equations involving the functions $A(t)$, $B(t)$, and the physical quantities ρ and λ.

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\left(\frac{\dot{B}}{B}\right)^2 - 2\frac{h^2}{A^2} = (8\pi+2\mu)\lambda \tag{21}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-\left(\frac{\dot{B}}{B}\right)^2 = (8\pi+2\mu)\rho \tag{22}$$

To close the system and obtain a determinate solution, we introduce the following physically motivated assumptions.

First, we assume that the scalar expansion θ is proportional to the shear scalar σ^2. This condition, which is commonly used in anisotropic cosmology, leads to a simple relation between the scale factors [28],

$B = A^m,$ where m is non zero. (23)

Second, we adopt Berman's special law for the variation of the Hubble parameter, which gives a constant deceleration parameter q [29]. This law is expressed as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \quad (24)$$

Integration of this relation yields the average scale factor in the form

$$R = (AB^2)^{\frac{1}{3}} = (a_0 t + b)^{\frac{1}{1+q}} \quad (25)$$

Where $a \neq 0$ and b are constants of integration. The condition for accelerated expansion in this model is therefore $1 + q > 0$.

Using the relations (23) and (25) in the reduced field equations (21) and (22), we obtain explicit expressions for the metric functions. The scale factor $A(t)$ is given by

$$A = (a_0 t + b)^{\frac{3}{(2m+1)(1+q)}} \quad (26)$$

and the remaining scale factors become

$$B = C = (a_0 t + b)^{\frac{3m}{(2m+1)(1+q)}} \quad (27)$$

Substituting these expressions into the line element (11), the Bianchi type-VI space-time in the framework of $f(R,T)$ gravity with cosmic strings can finally be written as

$$ds^2 = -dt^2 + (a_0 t + b)^{\frac{6}{(2m+1)(1+q)}} dx^2 + (a_0 t + b)^{\frac{3m}{(2m+1)(1+q)}} e^{-2hx} dy^2 + (a_0 t + b)^{\frac{3m}{(2m+1)(1+q)}} e^{2hx} dz^2 \quad (28)$$

This metric represents an anisotropic but spatially homogeneous cosmological model whose dynamical behaviour depends on the constants m, q, a_0, b, and h.

2.1.2 Physical properties of the model

The spatial volume V is

$$V = (a_0 t + b)^{\frac{3}{1+q}} \quad (29)$$

The scalar expansion θ is

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = \frac{3a_0}{(1+q)(a_0 t + b)} \quad (30)$$

The shear scalar σ^2 is defined as

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 = \frac{3{a_0}^2(m-1)^2}{(1+q)^2(2m+1)^2(a_0 t + b)^2} \quad (31)$$

The generalized mean Hubble parameter H is given in the form

$$H = \frac{1}{3}\theta = \frac{a_0}{(1+q)(a_0 t + b)} \quad (32)$$

The mean anisotropy parameter

$$A_m = 2\frac{(m-1)^2}{(2m+1)^2} \quad (33)$$

The energy density

$$\rho = \frac{1}{(8\pi+2\mu)}\left[\frac{9{a_0}^2(1-m)-3{a_0}^2(m+1)(1+q)(2m+1)}{(2m+1)^2(1+q)^2(a_0t+b)^2}\right] \tag{34}$$

The string tension density

$$\lambda = \frac{1}{(8\pi+2\mu)}\left[\frac{3{a_0}^2(2m+1)(1+q)(m-1)-9{a_0}^2(2m^2-m-1)}{(2m+1)^2(1+q)^2(a_0t+b)^2} - 2\frac{h^2}{(a_0t+b)^{\frac{6}{(2m+1)(1+q)}}}\right] \tag{35}$$

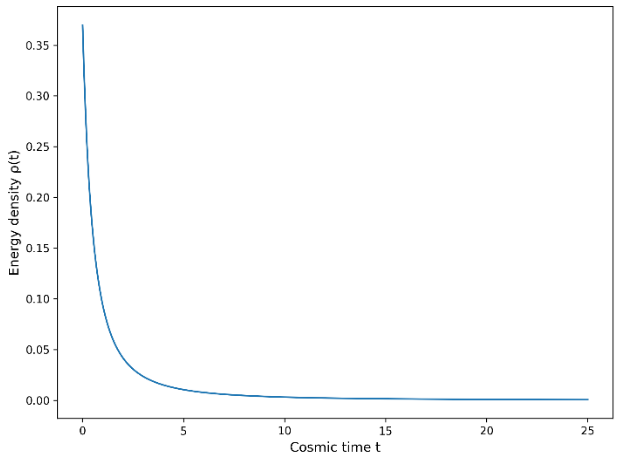

Figure 1: The Energy density's relationship to cosmic time t when the right constants are chosen

$q = -0.5, \quad a_0 = 1.0, \quad b = 1.0, \quad \mu = 0.1, \; m = 0.2.$

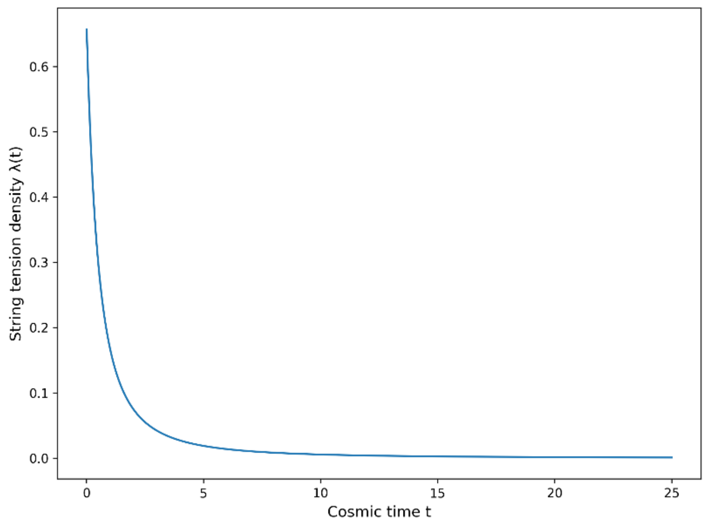

Figure 2: The String tension density's relationship to cosmic time t when the right constants are chosen

$q = -0.5, \quad a_0 = 1.0, \quad b = 1.0, \quad \mu = 0.1, \; m = 0.2, \quad h = 0.3$

The cosmological solutions obtained for the model (28) exhibit a physically viable and observationally consistent evolution. The spatial volume increases monotonically with cosmic time, implying a non-singular expanding universe. The expansion scalar θ and the mean Hubble parameter H, both inversely proportional to $(a_0 t + b)$, decrease continuously and tend to zero as $t \to \infty$, indicating accelerated expansion at early times followed by a gradual slowing down at late times. The shear scalar σ^2 remains finite throughout the evolution, and the constant non-zero value of the mean anisotropy parameter A_m confirms that anisotropy is preserved during the entire cosmic history due to the directional nature of the string source. The energy density ρ, given by equation (34), behaves as a positive, decreasing function of time, as illustrated in Figure 1, and asymptotically approaches zero for large t, which is consistent with the dilution of matter content in an expanding universe. Likewise, the string tension density λ, described by equation (35), also exhibits a monotonic decay with cosmic time, as shown in Figure 2, indicating that the dynamical influence of cosmic strings is dominant at early epochs and becomes negligible at late times. This behavior aligns well with observational expectations that topological defects, if present, should not significantly affect the present-day universe. Hence, the model successfully describes an anisotropic, accelerating universe in which cosmic strings contribute to early-time dynamics while remaining compatible with late-time cosmological observations.

2.2 Domain wall

The energy momentum tensor for thick domain walls is given by

$$T_{ij} = \rho\left(g_{ij} + \omega_i \omega_j\right) + p\omega_i \omega_j, \;\; \omega_i \omega_j = -1 \tag{36}$$

Where ρ is the density of the wall, p is the pressure in the direction of normal to the plane of the wall and ω_i is a unit space like vector in the same direction (Rahaman et al. [38]). In the co-moving co-ordinate system, from (36) we have

Here ρ denotes the energy density of the thick domain wall and p is the pressure acting along the direction normal to the wall. The space-like vector ω_i, specifies this preferred direction (Rahaman et al. [38]). Equation (36) therefore represents an anisotropic energy–momentum tensor with

$$T_2^2 = T_3^3 = T_0^0 = \rho, \; T_1^1 = -p \;\; \text{and} \; T_j^i = 0 \; for \; i \neq j. \tag{37}$$

In the co-moving frame.

Here pressure is taken in the direction of X-axis and the quantities ρ and p depend on t only.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{h^2}{A^2} = (8\pi + \mu)p - 3\mu\rho \tag{38}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = -(8\pi + 5\mu)\rho - \mu p \tag{39}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{h^2}{A^2} = -(8\pi + 5\mu)\rho - \mu p \tag{40}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = -(8\pi + 5\mu)\rho - \mu p \tag{41}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{42}$$

The trace in the considered model is given by $T=T_1^1+T_2^2+T_3^3+T_0^0=3\rho-p$

2.2.1 Solution of field equation

Integrating equation (42) we obtain $C=kB$ where k is a constant of integration without loss of any generality for its value as unity, it becomes

$$C=B \tag{43}$$

Using equation (43), the field equations (38)- (41) reduce to the following system of independent equations

$$2\frac{\ddot{B}}{B}+\left(\frac{\dot{B}}{B}\right)^2+\frac{h^2}{A^2}=(8\pi+\mu)p-3\mu\rho \tag{44}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{h^2}{A^2}=-(8\pi+5\mu)\rho-\mu p \tag{45}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\left(\frac{\dot{B}}{B}\right)^2-\frac{h^2}{A^2}=-(8\pi+5\mu)\rho-\mu p \tag{46}$$

Equations (44)-(46) are system of three independent equations in four unknowns A, B, ρ and p. Hence to find a determinate solution once again we use the conditions given by (23) and (24)

Now, the field equations (44)-(46) together with equations (23) and (24) admit the solutions

$$A=(a_0t+b)^{\frac{3}{(2m+1)(1+q)}} \tag{47}$$

$$B=C=(a_0t+b)^{\frac{3m}{(2m+1)(1+q)}} \tag{48}$$

Also, the energy density and pressure are given by

$$\rho=\frac{1}{(8\pi+5\mu)(8\pi+\mu)+3\mu^2}\left[\mu\left\{\frac{6{a_0}^2m(2m+1)(1+q)-27m^2{a_0}^2}{(2m+1)^2(1+q)^2(a_0t+b)^2}-\frac{h^2}{(a_0t+b)^{\frac{6}{(2m+1)(1+q)}}}\right\}+(8\pi+\mu)\left\{\frac{3{a_0}^2(2m+1)(1+q)(m+1)-9{a_0}^2(m^2+m+1)}{(2m+1)^2(1+q)^2(a_0t+b)^2}+\frac{h^2}{(a_0t+b)^{\frac{6}{(2m+1)(1+q)}}}\right\}\right] \tag{49}$$

$$p=\frac{1}{(8\pi+5\mu)(8\pi+\mu)+3\mu^2}\left[(8\pi+\mu)\left\{\frac{27m^2{a_0}^2-6{a_0}^2m(2m+1)(1+q)}{(2m+1)^2(1+q)^2(a_0t+b)^2}+\frac{h^2}{(a_0t+b)^{\frac{6}{(2m+1)(1+q)}}}\right\}+3\mu\left\{\frac{3{a_0}^2(2m+1)(1+q)(m+1)-9{a_0}^2(m^2+m+1)}{(2m+1)^2(1+q)^2(a_0t+b)^2}+\frac{h^2}{(a_0t+b)^{\frac{6}{(2m+1)(1+q)}}}\right\}\right] \tag{50}$$

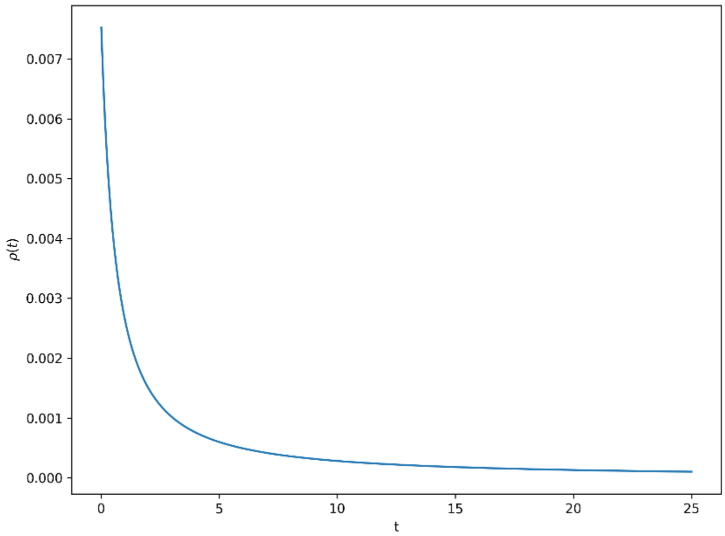

Figure 3: The Energy density's relationship to cosmic time t when the right constants are chosen

$$q = -0.5, \quad a_0 = 1.0, \quad b = 1.0, \quad \mu = 0.1, \; m = 0.2. \; h = 0.3$$

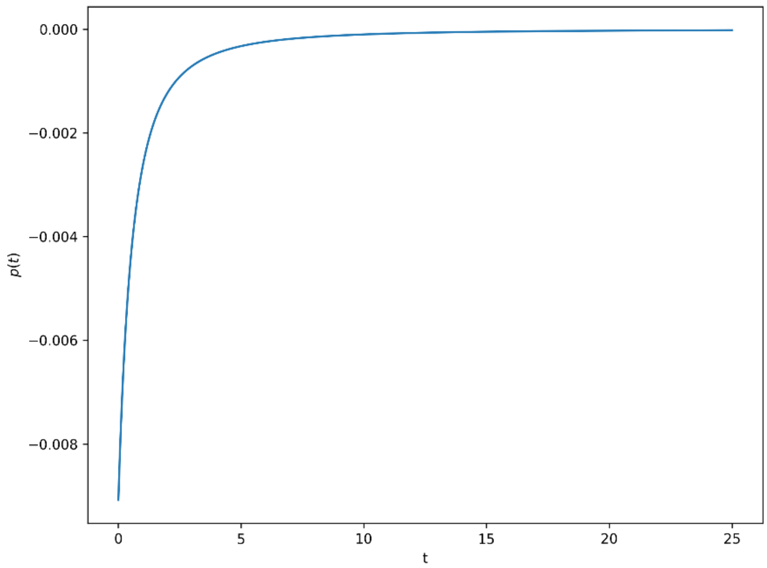

Figure 4: The Pressure relationship to cosmic time t when the right constants are chosen

$$q = -0.5, \quad a_0 = 1.0, \quad b = 1.0, \quad \mu = 0.1, \; m = 0.2. \; h = 0.3$$

The domain wall cosmological model obtained in this subsection exhibits physically meaningful behavior that is consistent with both anisotropic cosmology and observational expectations. The energy density ρ, shown in Figure 3, remains positive throughout the evolution and decreases monotonically with cosmic time, asymptotically approaching zero at late times. This decay reflects the gradual dilution of domain wall energy due to the overall expansion of the universe. Similarly, the pressure p, illustrated in Figure 4, evolves smoothly with time and remains negative over a substantial cosmic interval, indicating the repulsive gravitational effect associated with domain walls. Such negative pressure contributes to accelerated expansion and is compatible with the role of exotic matter sources in driving late-time cosmic acceleration. It is important to note that, although the matter variables ρ and p differ in form from those of the cosmic string case, all geometrical and kinematical quantities—such as the scale factors, spatial volume, expansion scalar, shear scalar, Hubble parameter, and anisotropy parameter—retain the same functional dependence as in the cosmic string model described by equation (28). Consequently, the universe remains anisotropic throughout its evolution, with a decreasing expansion rate and non-vanishing shear, while the matter content distinguishes the physical interpretation of the model. This demonstrates that both cosmic strings and domain walls can support identical background geometries in $f(R,T)$ gravity, differing only in their energy–momentum characteristics and their specific dynamical influence on the cosmic evolution.

3. Conclusion

In this paper, anisotropic Bianchi type-VI cosmological models have been investigated in the framework of $f(R,T)$ gravity by considering cosmic strings and thick domain walls as matter sources. Exact solutions of the modified field equations were obtained by assuming a proportionality between the expansion scalar and shear scalar together with Berman's law, leading to a constant deceleration parameter and late-time accelerated expansion.

The models describe a non-singular, ever-expanding universe in which the expansion rate, shear scalar, and Hubble parameter decrease with cosmic time. The persistence of a non-zero anisotropy parameter indicates that anisotropic matter sources prevent complete isotropization, even at late epochs. In the cosmic string scenario, both the energy density and string tension decay monotonically and become negligible in the far future, while in the domain wall case the energy density decreases and the negative pressure provides an effective repulsive contribution compatible with accelerated expansion. Notably, both matter configurations admit the same background geometry, differing only in their matter variables, which highlights the versatility of $f(R,T)$ gravity in accommodating distinct physical sources.

Overall, the present models provide a consistent theoretical framework for describing an anisotropic, accelerating universe and offer insight into the possible cosmological role of topological defects within modified gravity theories.

References

[1] Riess, A. G., et al., *Astron. J.*, **116**, 1009 (1998).

[2] Perlmutter, S., et al., *Astrophys. J.*, **517**, 565 (1999).

[3] Spergel, D. N., et al., *Astrophys. J. Suppl.*, **148**, 175 (2003).

[4] Planck Collaboration, *Astron. Astrophys.*, **641**, A6 (2020).

[5] Eisenstein, D. J., et al., *Astrophys. J.*, **633**, 560 (2005).

[6] Tegmark, M., et al., *Phys. Rev. D*, **69**, 103501 (2004).

[7] Brout, D., et al., *Astrophys. J.*, **938**, 110 (2022).

[8] Scolnic, D., et al., *Astrophys. J.*, **859**, 101 (2018).

[9] Abbott, T. M. C., et al., *Phys. Rev. D*, **105**, 023520 (2022).

[10] Clifton, T., et al., *Phys. Rep.*, **513**, 1 (2012).

[11] Nojiri, S., Odintsov, S. D., *Phys. Rep.*, **505**, 59 (2011).

[12] De Felice, A., Tsujikawa, S., *Living Rev. Relativ.*, **13**, 3 (2010).

[13] Cai, Y. F., et al., *Rep. Prog. Phys.*, **79**, 106901 (2016).

[14] Faraoni, V., *Cosmology in Scalar–Tensor Gravity*, Springer (2004).

[15] Harko, T., et al., *Phys. Rev. D*, **84**, 024020 (2011).

[16] Adhav, K. S., *Astrophys. Space Sci.*, **339**, 365 (2012).

[17] Singh, J. K., Sharma, N. K., *Int. J. Theor. Phys.*, **53**, 1433 (2014).

[18] Ram, S., Chandel, S., *Astrophys. Space Sci.*, **355**, 202 (2015).

[19] Bouali, A., et al., *Mon. Not. R. Astron. Soc.*, **526**, 4192 (2023).

[20] Myrzakulov, N., et al., *Universe*, **11**, 45 (2025).

[21] Kibble, T. W. B., *J. Phys. A*, **9**, 1387 (1976).

[22] Zel’dovich, Ya. B., et al., *ZhETF*, **67**, 3 (1974).

[23] Vilenkin, A., Shellard, E. P. S., *Cosmic Strings and Other Topological Defects*, Cambridge Univ. Press (1994).

[24] Gott, J. R., *Astrophys. J.*, **288**, 422 (1985).

[25] Letelier, P. S., *Phys. Rev. D*, **28**, 2414 (1983).
[26] Ipser, J., Sikivie, P., *Phys. Rev. D*, **30**, 712 (1984).

[27] Mukherjee, M., *Class. Quantum Grav.*, **10**, 131 (1993).

[28] Collins, C. B., et al., *Gen. Relativ. Gravit.*, **12**, 805 (1980).

[29] Berman, M. S., *Nuovo Cimento B*, **74**, 182 (1983).

[30] Ellis, G. F. R., MacCallum, M. A. H., *Commun. Math. Phys.*, **12**, 108 (1969).

[31] Ryan, M. P., Shepley, L. C., *Homogeneous Relativistic Cosmologies*, Princeton Univ. Press (1975).

[32] Collins, C. B., *J. Math. Phys.*, **18**, 2116 (1977).

[33] Kibble, T. W. B., *J. Phys. A*, **9**, 1387 (1976).

[34] Zel’dovich, Ya. B., Kobzarev, I. Yu., Okun, L. B., *ZhETF*, **67**, 3 (1974).

[35] Vilenkin, A., *Phys. Rev. D*, **23**, 852 (1981).

[36] Vilenkin, A., Shellard, E. P. S., *Cosmic Strings and Other Topological Defects*, Cambridge Univ. Press (1994).

[37] Letelier, P. S., *Phys. Rev. D*, **28**, 2414 (1983).

[38] Rahaman, F., Chakraborthy, S., Kalam, M.: Int. J. Mod. Phys. D **10**, 735 (2001)