Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

Continuous and Irresolute Maps Via $\delta\beta$-open Sets in n-Cylindrical Neutrosophic Topological Spaces

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Article History:
Received: 21-02-2025
Revised: 22-03-2025
Accepted: 02-04-2025

Abstract:
In this paper, we develop the concept of n-Cylindrical neutrosophic (resp. δ, δP, δS, δα & δβ or e^*)-continuity and irresoluteness in n-Cylindrical neutrosophic topological spaces and specialize some of their basic properties with examples.

Keywords and phrases: n-CyNδos, n-CyNδCts map, and n-CyNδIrr map.

AMS (2000) subject classification: 03E72, 54A10, 54A40

1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [22], Chang [3] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy set (ifs), was introduced by Atanassov in 1986 [2]. Building on this, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs's. Jeon et al. [7] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [16, 15], a new direction in uncertainty modeling emerged. Salama and Alblowi [10] introduced neutrosophic crisp set and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of positive membership, neutral membership, and negative membership for each element. Neutrosophic has formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [1] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [21] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [11] introduced the concept of n-Cylindrical neutrosophic set (abbreviated as n-$CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic set (NS), n-$CyNS$ represents the most extensive generalization of fuzzy sets. In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [12] introduced the notion of n-CyN continuity for functions between two n-Cylindrical neutrosophic topological spaces (n-$CyNts$). They also defined the n-CyN interior (n-$CyNint$) and n-CyN closure (n-$CyNcl$) of subsets within n-$CyNts$.

This leads to encompass the notion of n-$CyNts$ by introducing n-CyN δ (resp. $\delta\alpha$, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-continuous and discuss its properties. Also, we introduce the concept of n-CyN irresoluteness called n-CyN (resp. δ, $\delta\mathcal{P}, \delta\mathcal{S}, \delta\alpha$ and $\delta\beta$)-irresolute maps by using n-$CyN\mathcal{S}os$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-

$CyN\delta Sos$, n-$CyN\delta\alpha os$ and n-$CyN\delta\beta os$)'s and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [22] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \rightarrow [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A for all $x \in X$.

Definition 2.2 [3] A fuzzy topological space (briefly, fts) is a pair (X, τ), where X is any set and τ is a family of fuzzy sets in X satisfying following axioms:

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$ then $A \cap B$ in τ,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [2] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x): x \in X\rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ $ifs(X)$ denotes the set of all ifs's on X.

Definition 2.4 [16] An neutrosophic set (briefly, NS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle: x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral membership and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [10] An neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i: i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ_N is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_N are called neutrosophic open sets. A neutrosophic set F is closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6 [11] An n-Cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle: x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A, and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical neutrosophic number (briefly, n-CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 [11] Let $\{A_i: i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \{\langle x, \inf(\alpha_{A_i}(x)), \inf(\beta_{A_i}(x)), \sup(\gamma_{A_i}(x))\rangle: x \in X\}$.

$\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle: x \in X\}$.

Definition 2.8 [11] $0_{CyN} = \{\langle x, 0,0,1\rangle: x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle: x \in X\}$.

Definition 2.9 [11] **(The Basic Connectives)** Let $\tau_{CyN}(X)$ denote the family of all n-CyNS's on X.

Inclusion: For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \le \alpha_B(x)$ and $\beta_A(x) \le \beta_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).

Union: For every two $A, B \in \tau_{CyN}(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Intersection: For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-CyNS's A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Complementary: For every $A \in \tau_{CyN}(X)$, the complement of an n-CyNS A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Sum: For every two $A, B \in \tau_{CyN}(X)$, the sum of two n-CyNS's A and B is $A \oplus B(x) = \{\langle x, (\frac{\alpha_A(x) \cdot \alpha_B(x)}{\alpha_A(x) + \alpha_B(x)}), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Difference: For every two $A, B \in \tau_{CyN}(X)$, the difference of two n-CyNS's A and B is $A \ominus B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), (\frac{\gamma_A(x) \cdot \gamma_B(x)}{\gamma_A(x) + \gamma_B(x)})\rangle : x \in X\}$.

Product: For every two $A, B \in \tau_{CyN}(X)$, the product of two n-CyNS's A and B is $A \otimes B(x) = \{\langle x, (\alpha_A(x) \cdot \alpha_A(x)), (\beta_A(x) \cdot \beta_B(x)), (\gamma_A(x) \cdot \gamma_B(x))\rangle : x \in X\}$.

Division: For every two $A, B \in \tau_{CyN}(X)$, the division of two n-CyNS's A and B is

$A \oslash B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), (\beta_A(x) \cdot \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Remark 2.1 [11]

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ and $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,
5. $A \cap A = A$ and $A \cup A = A$,
6. De Morgan's Law for A and B ie., $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$,
7. $(A \oplus B) = (B \oplus A)$,
8. $(A \otimes B) = (B \otimes A)$.

Definition 2.10 [12] An n-Cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_{CyN}, of n-CyNS's in X which satisfies the following conditions:

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-Cylindrical neutrosophic topological Spaces (briefly, n-CyNts) and any n-CyNS belongs to τ_{CyN} is called an n-Cylindrical neutrosophic open set (briefly, n-CyNos) and the complement of n-CyNos is called n-Cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-CyNts and the topology containing all the n-CyN subsets is called discrete n-CyNts.

Remark 2.2 [12] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-CyNts as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.11 [12] Let A and B be two n-Cylindrical neutrosophic subsets of an n-CyNts. B is called neighbourhood of A if there exists an n-CyNos, O such that $A \subset O \subset B$.

Proposition 2.1 [12] $A \subset X$ is n-Cylindrical neutrosophic open in (X, τ_{CyN}) if and only if it carries a neighbourhood of its subsets.

Definition 2.12 [12] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-Cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of X. ie, $\text{n-CyNint}(A) = \bigcup \{G : G \in \tau_{CyN} \text{ and } G \subseteq A\}$.

Clearly, n-CyNint(A) is the biggest n-CyNos that is contained by A.

Definition 2.13 [12] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-Cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, $\text{n-CyNcl}(A) = \bigcap \{K : K \in \tau_{CyN} \text{ and } A \subseteq K\}$.

Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

3 $\delta\beta$-Continuous Maps in $n\text{-}CyNts$

We will introduce n-Cylindrical neutrosophic $\delta\beta$-continuous maps and look at some of its feature in this section.

Definition 3.1 Let (X, τ_{CyN}) be an n-CyNts and A be an n-CyNS. Then, A is said to be an n-CyN

1. regular open set (briefly, n-CyNros), if A = n-CyNint(n-CyNcl(A)),
2. regular closed set (briefly, n-CyNrcs), if A = n-CyNcl(n-CyNint(A)).

Definition 3.2 Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic δ-interior of A and the n-Cylindrical neutrosophic δ-closure of A are denoted by n-CyNδint(A) and n-CyNδcl(A) are defined as follows:

1. $\text{n-CyN}\delta\text{int}(A) = \bigcup \{G | G \text{ isan n-CyNros and } G \subseteq A\}$,
2. $\text{n-CyN}\delta\text{cl}(A) = \bigcap \{K | K \text{ isan n-CyNrcs and } A \subseteq K\}$.

Definition 3.3 Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN

1. open set (briefly, n-CyNos), if A = n-CyNint(A),
2. δ-open set (briefly, n-CyNδos), if A = n-CyNδint(A),
3. δ-pre open set (briefly, n-CyNδ$\mathcal{P}$os), if A ⊆ n-CyNint(n-CyNδcl(A)),
4. δ-semi open set (briefly, n-CyNδ$\mathcal{S}$os), if A ⊆ n-CyNcl(n-CyNδint(A)),
5. δα-open set (briefly, n-CyNδαos), if A ⊆ n-CyNint(n-CyNcl(n-CyNδint(A))),
6. δβ or e^*-open set (briefly, n-CyNδβos or e^*os), if A ⊆ n-CyNcl(n-CyNint(n-CyNδcl(A))).

The complement of a n-CyNδos (resp. n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNδαos, and n-CyNδβos) is called a n-CyNδ (resp. n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNδα, and n-CyNδβ) closed set (briefly, n-CyNδcs (resp. n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$cs, n-CyNδαcs, and n-CyNδβcs)) in X.

The family of all n-CyNδos (resp. n-CyNδcs, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$os, n-CyNδ$\mathcal{S}$cs, n-

CyNδαos, n-CyNδαcs, n-CyNδβos, and n-CyNδβcs) of X is denoted by n-CyNδOS(X), (resp. n-CyNδCS(X), n-CyNδ$\mathcal{P}$OS(X), n-CyNδ$\mathcal{P}$CS(X), n-CyNδ$\mathcal{S}$OS(X), n-CyNδ$\mathcal{S}$CS(X), n-CyNδαOS(X), n-CyNδαCS(X), n-CyNδβOS(X), and n-CyNδβ CS(X)).

Definition 3.4

Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic

1. δ$\mathcal{P}$-interior (resp. n-CyNδ$\mathcal{S}$-interior, n-CyNδα-interior, and n-CyNδβ-interior or n-CyNe*-interior) of A (briefly, n-CyNδ$\mathcal{P}$int(A) (resp. n-CyNδ$\mathcal{S}$int(A), n-CyNδαint(A), and n-CyNδβint(A) or n-CyNe*int(A)) is defined by n-CyNδ$\mathcal{P}$int(A) (resp. n-CyNδ$\mathcal{S}$int(A), n-CyNδαint(A), and n-CyNδβint(A) or n-CyNe*int(A))= ⋃ {G: G ⊆ A and G is a n-CyNδ$\mathcal{P}$os (resp. n-CyNδ$\mathcal{S}$os, n-CyNδαos and n-CyNδβos or n-CyNe*os) in X}.

2. δ$\mathcal{P}$-closure (resp. n-CyNδ$\mathcal{S}$-closure, n-CyNδα-closure and n-CyNδβ-closure or n-CyNe*-closure) of A (briefly, n-CyNδ$\mathcal{P}$cl(A) (resp. n-CyNδ$\mathcal{S}$cl(A), n-CyNδαcl(A) and n-CyNδβcl(A) or n-CyNe*cl(A)) is defined by n-CyNδ$\mathcal{P}$cl(A) (resp. n-CyNδ$\mathcal{S}$ cl(A), n-CyNδαcl(A) and n-CyNδβcl(A) or n-CyNe*cl(A))= ⋂ {K: K ⊇ A and K is a n-CyNδ$\mathcal{P}$cs (resp. n-CyNδ$\mathcal{S}$cs, n-CyNδαcs and n-CyNδβcs or n-CyNe*cs) in X}.

Definition 3.5 *Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-Cylindrical neutrosophic*

1. continuous mapping (briefly, *n-CyNCts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyN$\mathcal{S}$os* in (X, τ_1).
2. δ-continuous mapping (briefly, *n-CyNδCts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyNδos* in (X, τ_1).
3. δ-pre continuous mapping (briefly, *n-CyNδ$\mathcal{P}$Cts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyNδ$\mathcal{P}$os* in (X, τ_1).
4. δ-semi continuous mapping (briefly, *n-CyNδ$\mathcal{S}$Cts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyNδ$\mathcal{S}$os* in (X, τ_1).
5. $\delta\alpha$-continuous mapping (briefly, *n-CyNδαCts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyNδαos* in (X, τ_1).
6. $\delta\beta$-continuous mapping (briefly, *n-CyNδβCts* map), if the inverse image of every *n-CyNos* in (Y, τ_2) is a *n-CyNδβos* in (X, τ_1).

Proposition 3.1

The following statements are true, but the converse need not be true.

1. Every *n-CyNδCts* map is a *n-CyNCts* map,
2. Every *n-CyNδCts* map is a *n-CyNδ$\mathcal{S}$Cts* map,
3. Every *n-CyNδCts* map is a *n-CyNδ$\mathcal{P}$Cts* map,
4. Every *n-CyNδ$\mathcal{S}$Cts* map is a *n-CyNδβCts* map,
5. Every *n-CyNδ$\mathcal{P}$Cts* map is a *n-CyNδβCts* map,
6. Every *n-CyNδαCts* map is a *n-CyNδ$\mathcal{S}$Cts* map,
7. Every *n-CyNδαCts* map is a *n-CyNδ$\mathcal{P}$Cts* map.

Proof.

1. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta os$ in (X,τ_1). Since, every $n\text{-}CyN\delta os$ are $n\text{-}CyNos$, $f^{-1}(B)$ is $n\text{-}CyNos$ in (X,τ_1). Hence, f is a $n\text{-}CyNCts$ map.

2. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta os$ in (X,τ_1). Since, every $n\text{-}CyN\delta os$ are $n\text{-}CyN\delta\mathcal{S}os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\mathcal{S}Cts$ map.

3. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta os$ in (X,τ_1). Since, every $n\text{-}CyN\delta os$ are $n\text{-}CyN\delta\mathcal{P}os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\mathcal{P}Cts$ map.

4. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta\mathcal{S}Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1). Since, every $n\text{-}CyN\delta\mathcal{S}os$ are $n\text{-}CyN\delta\beta os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\beta os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\beta Cts$ map.

5. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta\mathcal{P}Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1). Since, every $n\text{-}CyN\delta\mathcal{P}os$ are $n\text{-}CyN\delta\beta os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\beta os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\beta Cts$ map.

6. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta\alpha Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta\alpha os$ in (X,τ_1). Since, every $n\text{-}CyN\delta\alpha os$ are $n\text{-}CyN\delta\mathcal{S}os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\mathcal{S}Cts$ map.

7. Let B be a $n\text{-}CyNos$ in (Y,τ_2). Since, f is $n\text{-}CyN\delta\alpha Cts$ map, $f^{-1}(B)$ is $n\text{-}CyN\delta\alpha os$ in (X,τ_1). Since, every $n\text{-}CyN\delta\alpha os$ are $n\text{-}CyN\delta\mathcal{P}os$, $f^{-1}(B)$ is $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1). Hence, f is a $n\text{-}CyN\delta\mathcal{P}Cts$ map. It is also true for their respective closed sets.

Example 3.1 *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ *and the* n*-Cylindrical neutrosophic sets* A_1,A_2,A_3,A_4 *in* (X,τ_1) *and* B *in* (Y,τ_2) *are defined as*

$$A_1=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$

$$A_2=\{\langle x_1,0.1150731,0.50,0.1950739\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\},$$

$$A_3=\{\langle x_1,0.1950739,0.50,0.1150731\rangle,\langle x_2,0.1750737,0.50,0.1350733\rangle\},$$

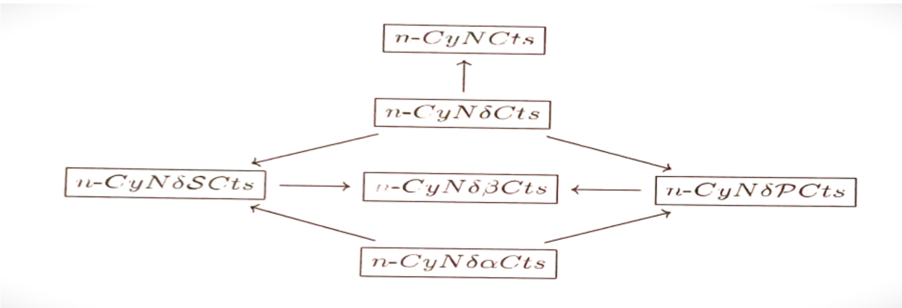
$$A_4=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\},$$

$$B=\{\langle y_1,0.1150731,0.50,0.1950739\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\}.$$

Then, we have $\tau_1=\{0_{CyN},1_{CyN},A_1,A_2,A_3,A_4\}$ and $\tau_2=\{0_Y,1_Y,B\}$. Let $f:(X,\tau_1)\to(Y,\tau_2)$ be an identity map. Then, f is $n\text{-}CyN\delta Cts$ map.

Remark 3.1 *The diagram shows that* $n\text{-}CyN\delta os$*'s in* $n\text{-}CyNts$.

Figure 3.1 *From the above Proposition 3.1 and Example 3.1, the following implications are hold.*



Note: $A\to B$ denotes A implies B, but not conversely.

Example 3.2 *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ *and the* n-*Cylindrical neutrosophic sets* A_1, A_2, A_3, A_4 *in* (X, τ_1) *and* B *in* (Y, τ_2) *are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$B = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is $n\text{-}CyNCts$ map but not $n\text{-}CyN\delta Cts$ map, because the set $f^{-1}(B) = A_4$ is a $n\text{-}CyNos$ but not $n\text{-}CyN\delta os$.

Example 3.3 *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ *and the* n-*Cylindrical neutrosophic sets* A_1, A_2, A_3, A_4 *in* (X, τ_1) *and* B *in* (Y, τ_2) *are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$B = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is $n\text{-}CyN\delta\mathcal{P}Cts$ map but not $n\text{-}CyN\delta Cts$ map, because the set $f^{-1}(B) = A_4$ is a $n\text{-}CyN\delta\mathcal{P}os$ but not $n\text{-}CyN\delta os$.

Example 3.4 *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ *and the* n-*Cylindrical neutrosophic sets* A_1, A_2, A_3, A_4 *in* (X, τ_1) *and* B *in* (Y, τ_2) *are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_5 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\},$$

$$B = \{\langle y_1, 0.1850738, 0.50, 0.1250732\rangle, \langle y_2, 0.1650736, 0.50, 0.1450734\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is $n\text{-}CyN\delta\mathcal{S}Cts$ map but not $n\text{-}CyN\delta Cts$ map, because the set $f^{-1}(B) = A_5$ is a $n\text{-}CyN\delta\mathcal{S}os$ but not $n\text{-}CyN\delta os$.

Example 3.5 *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ *and the* n-*Cylindrical neutrosophic sets* A_1, A_2, A_3, A_4 *in* (X, τ_1) *and* B *in* (Y, τ_2) *are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}.$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}.$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

$$B = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\beta Cts$ map but not n-$CyN\delta\mathcal{S}Cts$ map, because the set $f^{-1}(B) = A_4$ is a n-$CyN\delta\beta os$ but not n-$CyN\delta\mathcal{S}os$.

Example 3.6 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 in (X, τ_1) and B in (Y, τ_2) are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_5 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\},$$

$$B = \{\langle y_1, 0.1850738, 0.50, 0.1250732\rangle, \langle y_2, 0.1650736, 0.50, 0.1450734\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\beta Cts$ map but not n-$CyN\delta\mathcal{P}Cts$ map, because the set $f^{-1}(B) = A_5$ is a n-$CyN\delta\beta os$ but not n-$CyN\delta\mathcal{P}os$.

Example 3.7 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 in (X, τ_1) and B in (Y, τ_2) are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$B = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\mathcal{P}Cts$ map but not n-$CyN\delta\alpha Cts$ map, because the set $f^{-1}(B) = A_4$ is a n-$CyN\delta\mathcal{P}os$ but not n-$CyN\delta\alpha os$.

Example 3.8 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 in (X, τ_1) and B in (Y, τ_2) are defined as*

$$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\},$$

$$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\},$$

$$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$

$$A_5 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\},$$

$$B = \{\langle y_1, 0.1850738, 0.50, 0.1250732\rangle, \langle y_2, 0.1650736, 0.50, 0.1450734\rangle\}.$$

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\mathcal{S}Cts$ map but not n-$CyN\delta\alpha Cts$ map, because the set $f^{-1}(B) = A_5$ is a n-$CyN\delta\mathcal{S}os$ but not n-$CyN\delta\alpha os$.

Theorem 3.1 *Let (X, τ_1) and (Y, τ_2) be a n-$CyNts$'s. A map $f: (X, \tau_1) \to (Y, \tau_2)$ satisfies the following conditions are equivalent. [(i)]*

1. f is $n\text{-}CyN\delta\beta Cts$,
2. The inverse $f^{-1}(B)$ of all $n\text{-}CyN\delta os\ B$ in Y is $n\text{-}CyN\delta\beta os$ in X.

Proof. The proof is directly, from $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ for all $n\text{-}CyN\delta os\ B$ of Y.

Theorem 3.2 *Let (X,τ_1) and (Y,τ_2) be a $n\text{-}CyNts$'s. A map $f:(X,\tau_1) \to (Y,\tau_2)$ satisfies the following conditions are hold. [(i)]*

1. $f(n\text{-}CyN\delta\beta cl(A)) \subseteq n\text{-}CyN\delta cl(f(A))$, for all $n\text{-}CyNcs\ A$ in X,
2. $n\text{-}CyN\delta\beta cl(f^{-1}(B)) \subseteq f^{-1}(n\text{-}CyN\delta cl(B))$, for all $n\text{-}CyNcs\ B$ in Y.

Proof. (i) Since $n\text{-}CyN\delta cl(f(A))$ is a $n\text{-}CyN\delta cs$ in Y and f is $n\text{-}CyN\delta\beta Cts$, then $f^{-1}(n\text{-}CyN\delta cl(f(A)))$ is $n\text{-}CyN\delta\beta cs$ in X. Now, since $A \subseteq f^{-1}(n\text{-}CyN\delta cl(f(A)))$, $n\text{-}CyN\delta\beta cl(A) \subseteq f^{-1}(n\text{-}CyN\delta cl(f(A)))$. Therefore, $f(n\text{-}CyN\delta\beta cl(A)) \subseteq n\text{-}CyN\delta cl(f(A))$.

(ii) By replacing A with B in (i), we obtain $f(n\text{-}CyN\delta\beta cl(f^{-1}(B))) \subseteq n\text{-}CyN\delta cl(f\ (f^{-1}(B))) \subseteq n\text{-}CyN\delta cl(B)$. Hence, $n\text{-}CyN\delta\beta cl(f^{-1}(B)) \subseteq f^{-1}(n\text{-}CyN\delta cl(B))$.

Remark 3.2 *Let (X,τ_1) and (Y,τ_2) be a $n\text{-}CyNts$'s. Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping. If f is $n\text{-}CyN\delta\beta Cts$, then [(i)]*

1. $f(n\text{-}CyN\delta\beta cl(A))$ is not necessarily equal to $n\text{-}CyN\delta cl(f(A))$ where $A \in X$,
2. $n\text{-}CyN\delta\beta cl(f^{-1}(B))$ is not necessarily equal to $f^{-1}(n\text{-}CyN\delta cl(B))$ where $B \in Y$.

Example 3.9 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the $n\text{-}CyNS$'s A is defined as*

$$A = B = \{\langle x_1, 0.1850738, 0.50, 0.1350733\rangle, \langle x_2, 0.1950739, 0.50, 0.1350733\rangle\}$$

Here we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A\}$ is $n\text{-}CyNts$ on X. Let $f:(X,\tau_2) \to (Y,\tau_2)$ be an identity map. Then, f is $n\text{-}CyN\delta\beta Cts$. [(i)]

1. $f(n\text{-}CyN\delta\beta cl(A)) = A$. But $n\text{-}CyN\delta cl(f(A)) = 1$. Thus, $f(n\text{-}CyN\delta\beta cl(A)) \neq n\text{-}CyN\ \delta cl(f(A))$,
2. $n\text{-}CyN\delta\beta cl(f^{-1}(A)) = A$. But $f^{-1}(n\text{-}CyN\delta cl(A)) = 1$. Thus, $n\text{-}CyN\delta\beta cl(f^{-1}(A)) \neq f^{-1}(n\text{-}CyN\delta cl(A))$.

Theorem 3.3 *Let (X,τ_1) and (Y,τ_2) be a $n\text{-}CyNts$'s. Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a map. If f is $n\text{-}CyN\delta\beta Cts$, then $f^{-1}(n\text{-}CyN\delta int(A)) \subseteq n\text{-}CyN\delta\beta int(f^{-1}(A))$, for all $n\text{-}CyNS\ A$ in Y.*

Proof. If f is $n\text{-}CyN\delta\beta Cts$ and $A \subseteq Y$. $n\text{-}CyN\delta int(A)$ is $n\text{-}CyN\delta os$ in Y and hence, $f^{-1}(n\text{-}CyN\delta int(A))$ is $n\text{-}CyN\delta\beta os$ in X. Therefore, $n\text{-}CyN\delta\beta int(f^{-1}(n\text{-}CyN\delta int(A))) = f^{-1}(n\text{-}CyN\delta int(A))$. Also, $n\text{-}CyN\delta int(A) \subseteq A$, implies that $f^{-1}(n\text{-}CyN\delta int(A)) \subseteq f^{-1}(A)$. Therefore $n\text{-}CyN\delta\beta int(f^{-1}(n\text{-}CyN\delta int(A))) \subseteq n\text{-}CyN\delta\beta int(f^{-1}(A))$. That is $f^{-1}(n\text{-}CyN\delta int\ (A)) \subseteq n\text{-}CyN\delta\beta int\ (f^{-1}(A))$.

Conversely, let $f^{-1}(n\text{-}CyN\delta int(A)) \subseteq n\text{-}CyN\delta\beta int(f^{-1}(A))$ for all subset A of Y. If A is $n\text{-}CyN\delta o$ in Y, then $n\text{-}CyN\delta int(A) = A$. By assumption, $f^{-1}(n\text{-}CyN\delta int(A)) \subseteq n\text{-}CyN\delta\beta int(f^{-1}(A))$. Thus $f^{-1}(A) \subseteq n\text{-}CyN\delta\beta int(f^{-1}(A))$. But $n\text{-}CyN\delta\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $n\text{-}CyN\delta\beta int(f^{-1}(A)) = f^{-1}(A)$. That is, $f^{-1}(A)$ is $n\text{-}CyN\delta\beta o$ in X, for all $n\text{-}CyN\delta os\ A$ in Y. Therefore f is $n\text{-}CyN\delta\beta Cts$ in X.

Remark 3.3 *Let (X,τ_1) and (Y,τ_2) be a $n\text{-}CyNts$'s. Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a map. If f is $n\text{-}CyN\delta\beta Cts$, then $n\text{-}CyN\delta\beta int(f^{-1}(B))$ is not necessarily equal to $f^{-1}(n\text{-}CyN\delta int(B))$ where $B \in Y$.*

Example 3.10

In Example 3.9, f is a $n\text{-}CyN\delta\beta Cts$.

Then $n\text{-}CyN\delta\beta int(f^{-1}(A)) = A$. But $f^{-1}(n\text{-}CyN\delta int(A)) = 0$. Thus $n\text{-}CyN\delta\beta int(f^{-1}(B)) \neq$

$f^{-1}(n\text{-}CyN\delta int(B))$.

Remark 3.4 *Theorems 3.1, 3.2, 3.3, and Remarks 3.2, 3.3 are true for* $n\text{-}CyN\delta\mathcal{P}os, n\text{-}CyN\delta\mathcal{S}os$, *and* $n\text{-}CyN\delta\alpha os$.

4 n-Cylindrical Neutrosophic δ-irresolute maps

We introduce n-Cylindrical neutrosophic δ-irresolute mapping and look at some of its feature in this section.

Definition 4.1 *Let* (X,τ_1) *and* (Y,τ_2) *be any two* $n\text{-}CyNts$*'s. A map* $f:(X,\tau_1)\to(Y,\tau_2)$ *is said to be a* n*-Cylindrical neutrosophic*

1. irresolute mapping (briefly, $n\text{-}CyNIrr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\mathcal{S}os$ B of (Y,τ_2),
2. δ-irresolute mapping (briefly, $n\text{-}CyN\delta Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta os$ in (X,τ_1) for every $n\text{-}CyN\delta os$ B of (Y,τ_2),
3. δ-pre irresolute mapping (briefly, $n\text{-}CyN\delta\mathcal{P}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{P}os$ B of (Y,τ_2),
4. δ-semi irresolute mapping (briefly, $n\text{-}CyN\delta\mathcal{S}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{S}os$ B of (Y,τ_2),
5. $\delta\alpha$-irresolute mapping (briefly, $n\text{-}CyN\delta\alpha Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\alpha os$ in (X,τ_1) for every $n\text{-}CyN\delta\alpha os$ B of (Y,τ_2),
6. $\delta\beta$-irresolute mapping (briefly, $n\text{-}CyN\delta\beta Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\alpha os$ in (X,τ_1) for every $n\text{-}CyN\delta\beta os$ B of (Y,τ_2).

Theorem 4.1 *Let* (X,τ_1) *and* (Y,τ_2) *be a* $n\text{-}CyNts$*'s. Let* $f:(X,\tau_1)\to(Y,\tau_2)$ *be a map. Then, the following statements are hold for* $n\text{-}CyNts$*, but not conversely.*

1. Every $n\text{-}CyNIrr$ map is a $n\text{-}CyN\mathcal{S}Cts$ map,
2. Every $n\text{-}CyN\delta\mathcal{P}Irr$ map is a $n\text{-}CyN\delta\mathcal{P}Cts$ map,
3. Every $n\text{-}CyN\delta\mathcal{S}Irr$ map is a $n\text{-}CyN\delta\mathcal{S}Cts$ map.

But the converse is not true.

Proof.

1. Let $n\text{-}CyNIrr$ map f and a $n\text{-}CyNos$ B in (Y,τ_2). As each $n\text{-}CyNos$ is a $n\text{-}CyN\mathcal{S}os$, B is a $n\text{-}CyN\mathcal{S}os$ in (Y,τ_2). By presumption, $f^{-1}(B)$ is a $n\text{-}CyN\mathcal{S}os$ in (X,τ_1). Thus, f is a $n\text{-}CyN\mathcal{S}Cts$ map.
2. Let $n\text{-}CyN\delta\mathcal{P}Irr$ map f and a $n\text{-}CyN\delta os$ B in (Y,τ_2). As each $n\text{-}CyN\delta os$ is a $n\text{-}CyNos$ and $n\text{-}CyN\delta\mathcal{P}os$, B is a $n\text{-}CyN\delta os$ and $n\text{-}CyN\delta\mathcal{P}os$ in (Y,τ_2). By presumption, $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1). Thus, f is a $n\text{-}CyN\delta\mathcal{P}Cts$ map.
3. Let $n\text{-}CyN\delta\mathcal{S}Irr$ map f and a $n\text{-}CyN\delta os$ B in (Y,τ_2). As each $n\text{-}CyN\delta os$ is a $n\text{-}CyNos$ and $n\text{-}CyN\delta\mathcal{S}os$, B is a $n\text{-}CyN\delta os$ and $n\text{-}CyN\delta\mathcal{S}os$ in (Y,τ_2). By presumption, $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1). Thus, f is a $n\text{-}CyN\delta\mathcal{S}Cts$ map. width 0.22 true cm height 0.22 true cm depth 0pt

Example 4.1 *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ *and the* n*-Cylindrical neutrosophic sets* A_1,A_2,A_3,A_4 *in* (X,τ_1) *and* B *in* (Y,τ_2) *are defined as*

$$A_1=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$

$$A_2=\{\langle x_1,0.1150731,0.50,0.1950739\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\}$$

$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$B = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}$.

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\mathcal{S}Cts$ map but not n-$CyNIrr$ map, because the set $f^{-1}(B) = A_2$ is a n-$CyN\mathcal{S}os$ in (Y, τ_2) but not n-$CyN\mathcal{S}os$ in (X, τ_1).

Example 4.2 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 in (X, τ_1) and B in (Y, τ_2) are defined as*

$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$,

$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$B = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\}$.

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\mathcal{P}Cts$ map but not n-$CyN\delta\mathcal{P}Irr$ map, because the set $f^{-1}(B) = A_3$ is a n-$CyN\delta\mathcal{P}os$ in (Y, τ_2) but not n-$CyN\delta\mathcal{P}os$ in (X, τ_1).

Example 4.3 *Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A_1, A_2, A_3, A_4 in (X, τ_1) and B in (Y, τ_2) are defined as*

$A_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$,

$A_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$A_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$A_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$B = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}$.

Then, we have $\tau_1 = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ and $\tau_2 = \{0_Y, 1_Y, B\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-$CyN\delta\mathcal{S}Cts$ map but not n-$CyN\delta\mathcal{S}Irr$ map, because the set $f^{-1}(B) = A_2$ is a n-$CyN\delta\mathcal{S}os$ in (Y, τ_2) but not n-$CyN\delta\mathcal{S}os$ in (X, τ_1).

Definition 4.2 *A n-$CyNts$ (X, τ_1) is known as a n-Cylindrical neutrosophic $\delta\mathcal{P}U_{\frac{1}{2}}$ (resp. $\delta\mathcal{S}U_{\frac{1}{2}}$, $\delta\alpha U_{\frac{1}{2}}$ and $\delta\beta U_{\frac{1}{2}}$) (briefly, n-$CyN\delta\mathcal{P}U_{\frac{1}{2}}$ (resp. n-$CyN\delta\mathcal{S}U_{\frac{1}{2}}$, n-$CyN\delta\alpha U_{\frac{1}{2}}$, and n-$CyN\delta\beta\ U_{\frac{1}{2}}$))-space, if each n-$CyN\delta\mathcal{P}os$ (resp. n-$CyN\delta\mathcal{S}os$, n-$CyN\delta\alpha os$ and n-$CyN\delta\beta os$) in (X, τ_1) is n-$CyNos$ in (X, τ_1).*

Theorem 4.2 *Let $f: (X, \tau_1) \to (Y, \tau_2)$ and $g: (Y, \tau_2) \to (Z, \tau_3)$ be n-$CyN\delta Irr$ (resp. n-$CyN\delta\mathcal{P}Irr$, n-$CyN\delta\mathcal{S}Irr$, n-$CyN\delta\alpha Irr$, and n-$CyN\delta\beta Irr$) maps, then $g \circ f: (X, \tau_1) \to (Z, \tau_3)$ is a n-$CyN\delta Irr$ (resp. n-$CyN\delta\mathcal{P}Irr$, n-$CyN\delta\mathcal{S}Irr$, n-$CyN\delta\alpha Irr$, and n-$CyN\delta\beta Irr$) map.*

Proof. Let n-$CyN\delta\beta os$ B in (Z, τ_3). So $g^{-1}(B)$ is a n-$CyN\delta\beta os$ in (Y, τ_2). As f is a n-$CyN\delta\beta Irr$ map, $f^{-1}(g^{-1}(B))$ is a n-$CyN\delta\beta os$ in (X, τ_1). Thus, $g \circ f$ is a n-$CyN\delta\beta Irr$ map. The other cases are similar.

Theorem 4.3 *Let n-$CyN\delta Irr$ (resp. n-$CyN\delta\mathcal{P}Irr$, n-$CyN\delta\mathcal{S}Irr$, n-$CyN\delta\alpha Irr$, and n-$CyN\delta\beta Irr$) map $f: (X, \tau_1) \to (Y, \tau_2)$ and a n-$CyN\delta Cts$ (resp. n-$CyN\delta\mathcal{P}Cts$, n-$CyN\delta\mathcal{S}Cts$, n-$CyN\delta\alpha Cts$, and n-$CyN\delta\beta Cts$) map $g: (Y, \tau_2) \to (Z, \tau_3)$. Then, $g \circ f: (X, \tau_1) \to (Z, \tau_3)$ is a n-$CyN\delta Cts$ (resp. n-$CyN\delta\mathcal{P}Cts$, n-$CyN\delta\mathcal{S}Cts$, n-$CyN\delta\alpha Cts$, and n-$CyN\delta\beta Cts$) map.*

Proof. Let $n\text{-}CyNos\ B$ in (Z,τ_3). So, $g^{-1}(B)$ is a $n\text{-}CyN\delta\beta os$ in (Y,τ_2). As, f is a $n\text{-}CyN\delta\beta Irr$ map, $f^{-1}(g^{-1}(U))$ is a $n\text{-}CyN\delta\beta os$ in (X,τ_1). Thus, $g\circ f$ is a $n\text{-}CyN\delta\beta Cts$ map. The other cases are similar.

Theorem 4.4 *Let map* $f:(X,\tau_1)\to(Y,\tau_2)$ *from a* $n\text{-}CyNts$ (X,τ_1) *into a* $n\text{-}CyNts$ (Y,τ_2). *The following statements are equivalent if* (X,τ_1) *and* (Y,τ_2) *are* $n\text{-}CyN\delta U_{\frac{1}{2}}$ *(resp.* $n\text{-}CyN\delta\mathcal{P}U_{\frac{1}{2}}$, $n\text{-}CyN\delta\mathcal{S}U_{\frac{1}{2}}$, $n\text{-}CyN\delta\alpha U_{\frac{1}{2}}$, *and* $n\text{-}CyN\delta\beta U_{\frac{1}{2}}$*)-spaces.*

1. f is a $n\text{-}CyN\delta Irr$ (resp. $n\text{-}CyN\delta\mathcal{P}Irr$, $n\text{-}CyN\delta\mathcal{S}Irr$, $n\text{-}CyN\delta\alpha Irr$, and $n\text{-}CyN\delta\beta Irr$) map.
2. $f^{-1}(B)$ is a $n\text{-}CyN\delta os$ (resp. $n\text{-}CyN\delta\mathcal{P}os, n\text{-}CyN\delta\mathcal{S}os, n\text{-}CyN\delta\alpha os$, and $n\text{-}CyN\delta\beta os$) in (X,τ_1) for every $n\text{-}CyN\delta os$ (resp. $n\text{-}CyN\delta\mathcal{P}os, n\text{-}CyN\delta\mathcal{S}os, n\text{-}CyN\delta\alpha os$, and $n\text{-}CyN\delta\beta os$) B in (Y,τ_2).
3. $n\text{-}CyNcl(f^{-1}(B))\subseteq f^{-1}(n\text{-}CyNcl(B))$ for every $n\text{-}CyNS\ B$ of (Y,τ_2).

Proof.

(i) → (ii): Consider a $n\text{-}CyN\delta\beta cs\ B$ in (Y,τ_2). It follows B^c is a $n\text{-}CyN\delta\beta os$ in (Y,τ_2). As f is $n\text{-}CyN\delta\beta Irr$ map, $f^{-1}((B)^c)$ is a $n\text{-}CyN\delta\beta os$ in (X,τ_1). We know that $f^{-1}((B)^c)=(f^{-1}(B))^c$. Hence, $f^{-1}(B)$ is a $n\text{-}CyN\delta\beta cs$ in (X,τ_1).

(ii) → (iii): Consider a $n\text{-}CyNS\ B$ in (Y,τ_2) and $B\subseteq n\text{-}CyN\delta\beta cl(B)$. Then, $f^{-1}(B)\subseteq f^{-1}(n\text{-}CyN\delta\beta cl(B))$. Since, $n\text{-}CyN\delta\beta cl(B))$ is a $n\text{-}CyN\delta\beta cs$ in (Y,τ_2), $n\text{-}CyN\delta\beta cl(B)$ is a $n\text{-}CyN\delta\beta cs$ in (Y,τ_2). Therefore, $(n\text{-}CyN\delta\beta cl(B))^c$ is a $n\text{-}CyN\delta\beta os$ in (Y,τ_2). By presumption, $f^{-1}((n\text{-}CyN\delta\beta cl(B))^c)$ is a $n-CyN\delta\beta os$ in (X,τ_1). We know that $f^{-1}((n-CyN\delta\beta cl(B))^c)=(f^{-1}(n-CyN\delta\beta cl\quad(B)))^c$. So, $f^{-1}(n-CyN\delta\beta cl(B))$ is a $n-CyN\delta\beta cs$ in (X,τ_1). Also, as (X,τ_1) is $n-CyN\delta\beta U_{\frac{1}{2}}$-space, $f^{-1}(n-CyN\delta\beta cl(B))$ is a $n-CyN\delta\beta cs$ in (X,τ_1).

(iii) → (i): Consider a $n-CyN\delta\beta cs\ B$ in (Y,τ_2). As (Y,τ_2) is $n-CyN\delta\beta U_{\frac{1}{2}}$-space, B is $n-CyNcs$ in (Y,τ_2) and $n-CyNcl(B)=(B)$. Thus, $f^{-1}(B)=f^{-1}(n-CyN\delta\beta cl(B))\supseteq n-CyN\delta\beta cl(f^{-1}(B))=n-CyNcl(f^{-1}(B))$. But clearly $(f^{-1}(B))\subseteq n-CyNcl(f^{-1}(B))$. Therefore, $n-CyNcl((f^{-1}(B)))=f^{-1}(B)$. It follows $f^{-1}(B)$ is a $n-CyNcs$ and so it is a $n-CyN\delta\beta cs$ in (X,τ_1). Hence, f is $n-CyN\delta\beta Irr$ map. The proof is similar for other cases.

5 Conclusion

In this paper, $n\text{-}CyN\delta Cts$, $n\text{-}CyNCts$, $n\text{-}CyN\delta\mathcal{S}Cts$, $n\text{-}CyN\delta\mathcal{P}Cts$, $n\text{-}CyN\delta\alpha Cts$, and $n\text{-}CyN\delta\beta Cts$ respective irresolute map is defined using $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\alpha o$ and $n\text{-}CyN\delta\beta o$ set and its properties are analyzed with the examples. Then $n\text{-}CyN$ continuous maps are compared with other generalized $n\text{-}CyN$ continuous maps. Also we extended the concept of $n\text{-}CyN$ irresolute maps in $n\text{-}CyN$ topological spaces using $n\text{-}CyNo$ sets. Some examples and basic relationships between the mappings were also discussed. In future, these can be extended to $n\text{-}CyN$ open, closed, homeomorphism and contra maps.

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