Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

Open and Closed Mappings via M-Open Sets in n-Cylindrical Neutrosophic Topological Spaces

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Article History:
Received: 21-02-2025
Revised: 22-03-2025
Accepted: 02-04-2025

Abstract:
In this article, we introduce and investigate the concepts of n-CyNM-open and n-CyNM-closed mappings within the framework of n-cylindrical neutrosophic topological spaces. These mappings are formulated within neutrosophic topology, which offers a more comprehensive approach to handling uncertainty, indeterminacy, and inconsistency. We examine several fundamental properties and structural characteristics of these mappings and establish a series of related theorems that underscore their theoretical significance. Additionally, illustrative examples are presented to validate the developed concepts and to demonstrate their applicability in abstract neutrosophic topological analysis.

Keywords and phrases:n-CyNMos, n-CyNMCts map, n-CyNM-open map, and n-CyNM-closed map.

AMS (2000) subject classification: 03E72, 54A10, 54A40

1. Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [20], Chang [3] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy set (ifs), was introduced by Atanassov in 1986 [2]. Building on this, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces (ifts) based on ifs. Jeon et al. [6] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [14, 13], a new direction in uncertainty modeling emerged. Salama and Alblowi [9] introduced neutrosophic crisp sets and neutrosophic topological spaces (Nts), extending ifts and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has since formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

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Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [1] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [19] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [10] introduced the concept of n-cylindrical neutrosophic set (abbreviated as n-CyNS), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic set (NS), n-CyNS represents the most extensive generalization of fuzzy sets (fs). In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [12] introduced the notion of n-CyN continuity for functions between two n-Cylindrical fuzzy neutrosophic topological spaces (n-CyNts). They also defined the n-CyN interior (n-CyNint) and n-CyN closure (n-CyNcl) of subsets within n-CyNts.

In a related development, El-Maghrabi and Al-Juhani [5] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [8] also found M-open sets in nano topological spaces. Vadivel et al. [15, 16, 17] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [7] and Vadivel et al. [18] introduced M-open sets in fuzzy and neutrosophic nano topological spaces.

The structure of this paper is organized as follows: Section 2 presents a concise overview of essential definitions pertaining to intuitionistic fuzzy sets (ifs's), neutrosophic sets (NS's), n-Cylindrical neutrosophic sets (n-CyNS's), and mappings on n-Cylindrical neutrosophic topological spaces (n-CyNts's). Sections 3 and 4 introduce the notions of n-CyNMO and n-CyNMC mappings within n-CyNts's, accompanied by a detailed investigation of their fundamental properties and illustrative examples. Finally, Section 5 concludes the paper with a summary of findings and potential directions for future research.

2. Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [21] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A for all $x \in X$.

Definition 2.2 [3] A fuzzy topological space (briefly, fts) is a pair (X, τ_X), where X is any set and τ_X is a family of fuzzy sets in X satisfying following axioms:

(i). $\phi, X \in \tau$,
(ii). If $A, B \in \tau$, then $A \cap B$ in τ,
(iii). If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [2] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x) / x \in X \rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of non-membership of x in A, and where α_A and γ_A satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ifs(X) denotes the set of all the ifs's on X.

Definition 2.4 [15] A neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle : x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of truth membership, $\beta_A(x)$ is the degree of indeterminacy and $\gamma_A(x)$ is the degree of non-membership. Here $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [9] A neutrosophic topology (Nt) on a non-empty set X is a family τ_X of neutrosophic subsets in X satisfying the following axioms:

(i). $0_N, 1_N \in \tau_X$,

(ii). $G_1 \cap G_2 \in \tau_X$ for any $G_1, G_2 \in \tau_X$,

(iii). $\cup G_i \in \tau_X$, for all $\{G_i : i \in J\} \subseteq \tau_X$.

In this case the pair (X, τ_X) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_X are called neutrosophic open sets. A neutrosophic set F is closed if and only if F^c is neutrosophic open.

Definition 2.6 [10] An n-Cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here α and γ are dependent neutrosophic components and β is 100% independent.
For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x) \rangle$ is called as n-Cylindrical neutrosophic number (briefly, n-CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A \rangle\}$.

Definition 2.7 [10] Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \left\{\langle x, \inf\left(\alpha_{A_i}(x)\right), \inf\left(\beta_{A_i}(x)\right), \sup\left(\gamma_{A_i}(x)\right)\rangle : x \in X\right\}$, $\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 [10] $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$

Definition 2.9 [10] **(The Basic Connectives)** Let $\tau_N(X)$ denote the family of all n-CyNs on X.

Definition 2.10 [10] Inclusion: For every two $A, B \in \tau_N(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).

Definition 2.11 [10] Union: For every two $A, B \in \tau_N(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.12 [10] Intersection: For every two $A, B \in \tau_N(X)$, the intersection of two n-CyNS's A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.13 [10] Complementary: For every $A \in \tau_N(X)$, the complement of an n-CyNS's A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Definition 2.14 [10] Sum: For every two $A, B \in \tau_N(X)$, the sum of two n-CyNS's A and B is $A \oplus B(x) = \{\langle x, (\frac{\alpha_A(x) \cdot \alpha_B(x)}{\alpha_A(x) + \alpha_B(x)}), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Definition 2.15 [10] Difference: For every two $A, B \in \tau_N(X)$, the difference of two n-CyNS's A and B is $A \ominus B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), (\frac{\gamma_A(x) \cdot \gamma_B(x)}{\gamma_A(x) + \gamma_B(x)})\rangle : x \in X\}$.

Definition 2.16 [10] Product: For every two $A, B \in \tau_N(X)$, the product of two n-CyNS's A and B is $A \otimes B(x) = \{\langle x, (\alpha_A(x) \cdot \alpha_A(x)), (\beta_A(x) \cdot \beta_B(x)), (\gamma_A(x) \cdot \gamma_B(x))\rangle : x \in X\}$.

Definition 2.17 [10] Division: For every two $A, B \in \tau_N(X)$, the division of two n-CyNS's A and B is $A \oslash B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), (\beta_A(x) \cdot \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

Remark 2.1 [10]

For every A, B and $C \in \tau_N(X)$,

(i). If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,

(ii). $A \cup B = B \cup A \& A \cap B = B \cap A$,

(iii). $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,

(iv). $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,

(v). $A \cap A = A \& A \cup A = A$,

(vi). De Morgan's Law for $A \& B$ ie., $(A \cup B)^c = A^c \cap B^c \& (A \cap B)^c = A^c \cup B^c$,

(vii). $(A \oplus B) = (B \oplus A)$,

(viii). $(A \otimes B) = (B \otimes A)$.

Definition 2.18 [12] An n-cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_X, of n-CyNS in X which satisfies the following conditions:

(i). $0_{CyN}, 1_{CyN} \in \tau_X$,

(ii). $A_1 \cap A_2 \in \tau_X$,

(iii). $\cup A_i \in \tau_X$, for any arbitrary family $A_i \in \tau_X, i \in I$.

The pair (X, τ_X) is called an n-cylindrical neutrosophic topological Spaces (briefly, n-CyNts) and any n-CyNS belongs to τ_X is called an n-cylindrical neutrosophic open set (briefly, n-CyNos) and the complement of n-CyNos is called n-cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-CyNts and the topology containing all the n-CyN subsets is called discrete n-CyNts.

Remark 2.2 [12] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-CyNts as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.19 [12] Let A and B be two n-cylindrical neutrosophic subsets of an n-CyNts. B is called neighbourhood of A if there exists an n-CyNos, O such that $A \subset O \subset B$.

Proposition 2.1 [12] $A \subset X$ is n-cylindrical neutrosophic open in (X, τ_X) if and only if it carries a neighbourhood of its subsets.

Definition 2.20 [12] Let (X, τ_X) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of X. ie, $\text{n-CyNint}(A) = \bigcup\{G : G \in \tau_X \text{ and } G \subseteq A\}$. Clearly, n-CyNint(A) is the biggest n-CyNos that is contained by A.

Definition 2.21 [12] Let (X, τ_X) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-CyNS in X. Then, the n-cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, $\text{n-CyNcl}(A) = \bigcap\{K : K \in \tau_X \text{ and } A \subseteq A\}$. Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

Definition 2.22 [11] Let (X, τ_1) and (Y, τ_2) be two n-CyNts and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then f said to be n-CyN continuous (briefly, n-CyNCts) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of f(A) there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subset \mathfrak{V}$.

3. n-Cylindrical Neutrosophic M-open Mappings

In this section, we introduce the concept of n-Cylindrical neutrosophic M-open mappings and examine some of their fundamental properties.

Definition 3.1 Let (X, τ_X) be an n-CyNts and A be an n-CyNs. Then, A is said to be an n-CyN

(i). regular open set (briefly, n-CyNros), if $A = \text{n-CyNint}(\text{n-CyNcl}(A))$,
(ii). regular closed set (briefly, n-CyNrcs), if $A = \text{n-CyNcl}(\text{n-CyNint}(A))$.

Definition 3.2 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNs in X. Then, the n-Cylindrical neutrosophic δ-interior of A and the n-Cylindrical neutrosophic δ-closure of A are denoted by n-CyNδint(A) and n-CyNδcl(A) are defined as follows:

(i). $\text{n-CyN}\delta\text{int}(A) = \bigcup\{G | G \text{ isan n-CyNros and } G \subseteq A\}$,
(ii). $\text{n-CyN}\delta\text{cl}(A) = \bigcap\{K | K \text{ isan n-CyNrcs and } A \subseteq K\}$.

Definition 3.3 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-Cylindrical neutrosophic θ-interior of A and the n-Cylindrical neutrosophic θ-closure of A are denoted by n-CyNθint(A) and n-CyNθcl(A) are defined as follows:

(i). n-CyNθint(A) $= \cup\{n-\text{CyNint}(B): B \subseteq A \& B \text{ isa } n-\text{CyNcs in } X\}$,
(ii). n-CyNθcl(A) $= \cap\{n-\text{CyNcl}(B): A \subseteq B \& B \text{ isa } n-\text{CyNos in } X\}$.

Definition 3.4 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN

(i). δ-open set (briefly, n-CyNδos), if A = n-CyNδint(A),

(ii). δ-pre open set (briefly, n-CyNδ$\mathcal{P}$os), if A ⊆ n-CyNint(n-CyNδcl(A)),

(iii). δ-semi open set (briefly, n-CyNδSos), if A ⊆ n-CyNcl(n-CyNδint(A)),

(iv). θ-open set (briefly, n-CyNθos), if A = n-CyNθint(A),

(v). θ-semi open set (briefly, n-CyNθ$\mathcal{S}$os), if A ⊆ n-CyNcl(n-CyNθint(A)),

(vi). e-open set (briefly, n-CyNeos), if A = n-CyNcl(n-CyNδint(A)) ∪ n-CyNint(n-CyNδcl(A)),

(vii). M-open set (briefly, n-CyNMos), if A ⊆ n-CyNcl(n-CyNθint(A)) ∪ n-CyNint(n-CyNδcl(A)).

The complement of a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os&n-CyNeos) is called a n-CyNM (resp. n-CyNδ, n-CyNδP, n-CyNδS, n-CyNθ, n-CyNθS&n-CyNe) closed set (briefly, n-CyNMcs (resp. n-CyNδcs, n-CyNδPcs, n-CyNδScs, n-CyNθcs, n-CyNθScs & n-CyNecs)) in X.

The family of all n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os&n-CyNeos) of X is denoted by n-CyNMOS(X), (resp. n-CyNMCS(X), n-CyNδOS(X), n-CyNδCS(X), n-CyNδ$\mathcal{P}$OS(X), n-CyNδPCS(X), n-CyNδSOS(X), n-CyNδSCS(X), n-CyNθOS(X), n-CyNθCS(X), n-CyNθSOS(X), n-CyNθSCS(X), n-CyNeOS(X) & n-CyNeCS(X)).

Definition 3.5 Let (X, τ_X) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-CyN (i). M-interior (resp. n-CyNδ-interior, n-CyNδP-interior, n-CyNδS-interior, n-CyNθ-interior, n-CyNθS-interior & n-CyNe-interior) of A (briefly, n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδPint(A), n-CyNδSint(A), n-CyNθint(A), n-CyNθSint(A)&n-CyNeint(A)) is defined by n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδPint(A), n-CyNδSint(A), n-CyNθint(A), n-CyNθSint(A) & n-CyNeint(A)) $= \bigcup\{G: G \subseteq A$ and G is a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδSos, n-CyNθos, n-CyNθ$\mathcal{S}$os & n-CyNeos) in X}.

(ii). M-closure (resp. n-CyNδ-closure, n-CyNδP-closure, n-CyNδS-closure, n-CyNθ-closure, n-CyNθS-closure & n-CyNe-closure) of A (briefly, n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδPcl(A), n-CyNδScl(A), n-CyNθcl(A), n-CyNθScl(A)&n-CyNecl(A)) is defined by n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδPcl(A), n-CyNδScl(A), n-CyNθcl(A), n-CyNθScl(A) & n-CyNecl(A)) $= \bigcap\{K: K \subseteq A$ and K is a n-CyNMcs (resp. n-CyNδcs, n-CyNδPcs, n-CyNδScs, n-CyNθcs, n-CyNθScs & n-CyNecs) in X}.

Definition 3.6 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN

(i). δ-continuous map (briefly, n-CyNδCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδos in (X, τ_1),

(ii). δ-pre continuous map (briefly, n-CyNδ$\mathcal{P}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{P}$os in (X, τ_1),

(iii). δ-semi continuous map (briefly, n-CyNδ$\mathcal{S}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{S}$os in (X, τ_1),

(iv). θ-continuous map (briefly, n-CyNθCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNθos in (X, τ_1),

(v). θ$\mathcal{S}$-continuous map (briefly, n-CyNθ$\mathcal{S}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNθ$\mathcal{S}$os in (X, τ_1),

(vi). e-continuous map (briefly, n-CyNeCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNeos in (X, τ_1),

(vii). M-continuous map (briefly, n-CyNMCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNMos in (X, τ_1).

Definition 3.7 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN

(i). δ-irresolute map (briefly, n-CyNδIrr map), if $f^{-1}(B)$ is a n-CyNδos in (X, τ_1) for every n-CyNδosB of (Y, τ_2),

(ii). δ-pre irresolute map (briefly, n-CyN$\delta\mathcal{P}$Irr map), if $f^{-1}(B)$ is a n-CyN$\delta\mathcal{P}$os in (X, τ_1) for every n-CyN$\delta\mathcal{P}$osB of (Y, τ_2),

(iii). δ-semi irresolute map (briefly, n-CyN$\delta\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyN$\delta\mathcal{S}$os in (X, τ_1) for every n-CyN$\delta\mathcal{S}$osB of (Y, τ_2),

(iv). θ-irresolute map (briefly, n-CyNθIrr map), if $f^{-1}(B)$ is a n-CyNθos in (X, τ_1) for every n-CyNθosB of (Y, τ_2),

(v). θ-semi irresolute map (briefly, n-CyN$\theta\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyN$\theta\mathcal{S}$os in (X, τ_1) for every n-CyN$\theta\mathcal{S}$osB of (Y, τ_2),

(vi). e-irresolute map (briefly, n-CyNeIrr map), if $f^{-1}(B)$ is a n-CyNeos in (X, τ_1) for every n-CyNeosB of (Y, τ_2),

(vii). M-irresolute map (briefly, n-CyNMIrr map), if $f^{-1}(B)$ is a n-CyNMos in (X, τ_1) for every n-CyNMosB of (Y, τ_2).

Definition 3.8 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN

(i). δ-open mapping (briefly, n-CyNδO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNδos in (Y, τ_2),

(ii). δ- pre open mapping (briefly, n-CyN$\delta\mathcal{P}$O map), if the image of every n-CyNos in (X, τ_1) is a n-CyN$\delta\mathcal{P}$os in (Y, τ_2),

(iii). δ-semi open mapping (briefly, n-CyN$\delta\mathcal{S}$O map), if the image of every n-CyNos in (X, τ_1) is a n-CyN$\delta\mathcal{S}$os in (Y, τ_2),

(iv). θ-open mapping (briefly, n-CyNθO map), if the image of every n-CyNos in (X, τ_1) is a n-CyNθos in (Y, τ_2), (v). θ- semi open mapping (briefly, n-CyNθ$\mathcal{S}$O map), if the image of every n-CyNos in (X,τ_1) is a n-CyNθ$\mathcal{S}$os in (Y,τ_2).

(vi). e-open mapping (briefly, n-CyNeO map), if the image of every n-CyNos in (X,τ_1) is a n-CyNeos in (Y,τ_2),

(vii). M-open mapping (briefly, n-CyNMO map), if the image of every n-CyNos in (X,τ_1) is a n-CyNMos in (Y,τ_2).

Example 3.1 Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and the n-Cylindrical neutrosophic sets A in (X,τ_1) and B_1, B_2, B_3, B_4 in (Y,τ_2) are defined as

$$A=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_1=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_2=\{\langle y_1,0.1150731,0.50,0.1950739\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\},$$
$$B_3=\{\langle y_1,0.1950739,0.50,0.1150731\rangle,\langle y_2,0.1750737,0.50,0.1350733\rangle\},$$
$$B_4=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\}.$$

Then, we have $\tau_1=\{0_X,\ 1_X,\ A\}$ and $\tau_2=\{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f:(X,\tau_1)\to(Y,\tau_2)$ be an identity map. Then, f is n-CyNMO map.

Theorem 3.1 The following statements are hold but the converse does not true.

(i). Every n-CyNθO map is a n-CyNO map.

(ii). Every n-CyNθO map is a n-CyNθ$\mathcal{S}$O map.

(iii). Every n-CyNθ$\mathcal{S}$O map is a n-CyNMO map.

(iv). Every n-CyNδO map is a n-CyNO map.

(v). Every n-CyNδO map is a n-CyNδ$\mathcal{P}$O map.

(vi). Every n-CyNδO map is a n-CyNδ$\mathcal{S}$O map.

(vii). Every n-CyNδ$\mathcal{S}$O map is a n-CyNeO map.

(viii). Every n-CyNδ$\mathcal{P}$O map is a n-CyNMO map.

(ix). Every n-CyNMO map is a n-CyNeO map.

Proof. Only (viii) is proven; the others are similar.

(viii). Let A be a n-CyNos in (X,τ_1). Since f is n-CyN$\delta\mathcal{P}$O map, f(A) is a n-CyN$\delta\mathcal{P}$os in (Y,τ_2). Since every n-CyN$\delta\mathcal{P}$os is a n-CyNMos, f(A) is a n-CyNMos in (Y,τ_2). Hence, f is a n-CyNMO map.

Example 3.2 Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and the n-Cylindrical neutrosophic sets A in (X,τ_1) and B_1, B_2, B_3, B_4 in (Y,τ_2) are defined as

$$A=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_1=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_2=\{\langle y_1,0.1150731,0.50,0.1950739\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\},$$
$$B_3=\{\langle y_1,0.1950739,0.50,0.1150731\rangle,\langle y_2,0.1750737,0.50,0.1350733\rangle\},$$
$$B_4=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\}.$$

Then, we have $\tau_1=\{0_X,\ 1_X,\ A\}$ and $\tau_2=\{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f:(X,\tau_1)\to(Y,\tau_2)$ be an identity map. Then, f is n-CyNO map in (X,τ_1) but not n-CyNθO map (Y,τ_2).

Example 3.3 Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and the n-Cylindrical neutrosophic sets A in (X,τ_1) and B_1, B_2, B_3, B_4, B_5 in (Y,τ_2) are defined as

$$A=\{\langle x_1,0.1850738,0.50,0.1250732\rangle,\langle x_2,0.1650736,0.50,0.1450734\rangle\},$$
$$B_1=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_2=\{\langle y_1,0.1150731,0.50,0.1950739\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\},$$
$$B_3=\{\langle y_1,0.1950739,0.50,0.1150731\rangle,\langle y_2,0.1750737,0.50,0.1350733\rangle\},$$
$$B_4=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1350733,0.50,0.1750737\rangle\},$$
$$B_5=\{\langle y_1,0.1850738,0.50,0.1250732\rangle,\langle y_2,0.1650736,0.50,0.1450734\rangle\}.$$

Then, we have $\tau_1=\{0_X,\ 1_X,\ A\}$ and $\tau_2=\{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,B_4\}$. Let $f:(X,\tau_1)\to(Y,\tau_2)$ be an identity map. Then, f is n-CyNθ$\mathcal{S}$O map in (X,τ_1) but not n-CyNθO map in (Y,τ_2).

Example 3.4 Let $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$ and the n-Cylindrical neutrosophic sets A in (X,τ_1) and B_1, B_2, B_3, B_4 in (Y,τ_2) are defined as

$$A=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$
$$B_1=\{\langle y_1,0.1250732,0.50,0.1850738\rangle,\langle y_2,0.1450734,0.50,0.1650736\rangle\},$$

$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$
$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$

Then, we have $\tau_1 = \{0_X,\ 1_X,\ A\}$ and $\tau_2 = \{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNMO map in (X, τ_1) but not n-CyN$\theta\mathcal{S}$O map in (Y, τ_2).

Example 3.5 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and $B_1,\ B_2,\ B_3,\ B_4$ in (Y, τ_2) are defined as

$A = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$
$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$
$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$

Then, we have $\tau_1 = \{0_X,\ 1_X,\ A\}$ and $\tau_2 = \{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNO map in (X, τ_1) but not n-CyNδO map in (Y, τ_2).

Example 3.6 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and $B_1,\ B_2,\ B_3$ in (Y, τ_2) are defined as

$A = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$
$B_2 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$
$B_3 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\}.$

Then, we have $\tau_1 = \{0_X,\ 1_X,\ A\}$ and $\tau_2 = \{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyN$\delta\mathcal{P}$O map in (X, τ_1) but not n-CyNδO map in (Y, τ_2).

Example 3.7 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and $B_1,\ B_2,\ B_3,\ B_4,\ B_5$ in (Y, τ_2) are defined as

$A = \{\langle x_1, 0.1450734, 0.50, 0.1250732\rangle, \langle x_2, 0.1450734, 0.50, 0.1450734\rangle\},$
$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$
$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$
$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$
$B_5 = \{\langle y_1, 0.1450734, 0.50, 0.1250732\rangle, \langle y_2, 0.1450734, 0.50, 0.1450734\rangle\}.$

Then, we have $\tau_1 = \{0_X,\ 1_X,\ A\}$ and $\tau_2 = \{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyN$\delta\mathcal{S}$O map in (X, τ_1) but not n-CyNδO map in (Y, τ_2).

Example 3.8 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and B_1, B_2, B_3, B_4, B_5 in (Y, τ_2) are defined as

$$A = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1650736\rangle\},$$
$$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$$
$$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$$
$$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_5 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1650736\rangle\}.$$

Then, we have $\tau_1 = \{0_X,\ 1_X,\ A\}$ and $\tau_2 = \{0_Y,\ 1_Y,\ B_1,\ B_2,\ B_3,\ B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNeO map in (X, τ_1) but not n-CyN$\delta\mathcal{S}$O map in (Y, τ_2).

Example 3.9 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and B_1, B_2, B_3, B_4, B_5 in (Y, τ_2) are defined as

$$A = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\},$$
$$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$$
$$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$$
$$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_5 = \{\langle y_1, 0.1850738, 0.50, 0.1250732\rangle, \langle y_2, 0.1650736, 0.50, 0.1450734\rangle\}.$$

Then, we have $\tau_1 = \{0_X, 1_X, A\}$ and $\tau_2 = \{0_Y, 1_Y, B_1, B_2, B_3, B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNMO map in (X, τ_1) but not n-CyN$\delta\mathcal{P}$O map in (Y, τ_2).

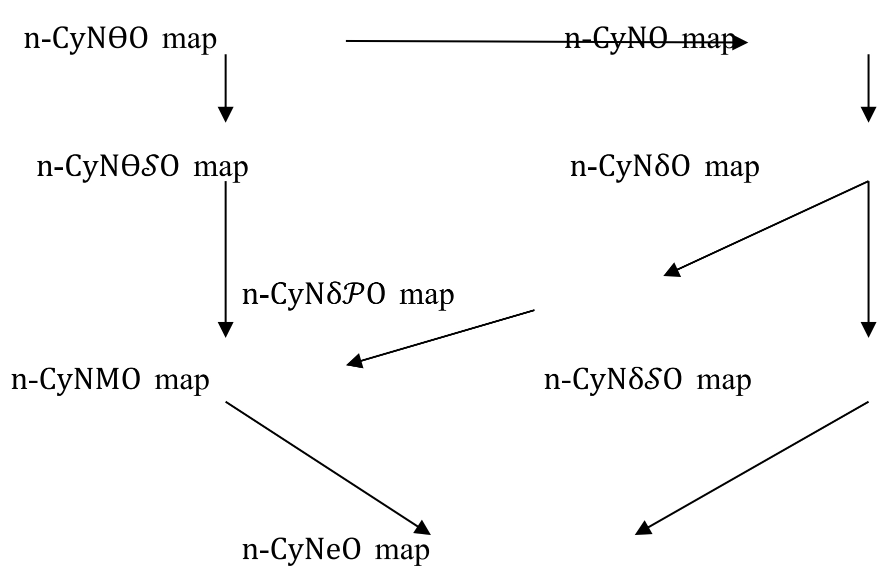
Example 3.10 Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and the n-Cylindrical neutrosophic sets A in (X, τ_1) and B_1, B_2, B_3, B_4, B_5, B_6 in (Y, τ_2) are defined as

$$A = \{\langle x_1, 0.1450734, 0.50, 0.1250732\rangle, \langle x_2, 0.1450734, 0.50, 0.1450734\rangle\},$$
$$B_1 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1450734, 0.50, 0.1650736\rangle\},$$
$$B_2 = \{\langle y_1, 0.1150731, 0.50, 0.1950739\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_3 = \{\langle y_1, 0.1950739, 0.50, 0.1150731\rangle, \langle y_2, 0.1750737, 0.50, 0.1350733\rangle\},$$
$$B_4 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$B_5 = \{\langle y_1, 0.1250732, 0.50, 0.1850738\rangle, \langle y_2, 0.1350733, 0.50, 0.1650736\rangle\},$$

$B_6 = \{\langle y_1, 0.1450734, 0.50, 0.1250732\rangle, \langle y_2, 0.1450734, 0.50, 0.1450734\rangle\}$.

Then, we have $\tau_1 = \{0_X, 1_X, A\}$ and $\tau_2 = \{0_Y, 1_Y, B_1, B_2, B_3, B_4\}$. Let $f: (X, \tau_1) \to (Y, \tau_2)$ be an identity map. Then, f is n-CyNeO map in (X, τ_1) but not n-CyNMO map in (Y, τ_2).

Remark 3.1 From the above mentioned results. We get the diagram below.



Note: A → B denotes A implies B, but not conversely.

Theorem 3.2 A map $f: (X, \tau_1) \to (Y, \tau_2)$ is n-CyNMO map iff for every n-CyNS's A of (X, τ_1). f(n-CyNint(A)) ⊆ n-CyNMint(f(A)).

Proof. Necessity: Let f be a n-CyNMO map and A be a n-CyNos in (X, τ_1). Now, n-CyNint(A) ⊆ A implies f(n-CyNint(A)) ⊆ f(A). Since, f is a n-CyNMO map. f(n-CyNint(A)) is n-CyNMos in (Y, τ_2) such that f(n-CyNint(A)) ⊆ f(A). Therefore, f(n-CyNint(A)) ⊆ n-CyNMint(f(A)).

Sufficiency: Assume A is a n-CyNos of (X, τ_1). Then, f(A) = f(n - CyNint(A)) ⊆ n - CyNMint(f(A)). But n-CyNMint(f(A)) ⊆ f(A). So, f(A) = n-CyNMint(A) which implies f(A) is a n-CyNMos of (Y, τ_2) and hence, f is a n-CyNMO map.

Theorem 3.3 If $f: (X, \tau_1) \to (Y, \tau_2)$ is a n-CyNMO map then n-CyNint(f^{-1}(n-CyNMint(A)) for every n-CyNS's A of (Y, τ_2).

Proof. Let A be a n-CyNS's of (Y, τ_2). Then, n-CyNint(f^{-1}(A)) is a n-CyNos in (X, τ_1). Since, f is n-CyNMO map. f(n-CyNint(f^{-1}(A))) is n-CyNMO map in (Y, τ_2) and hence, f(n-CyNint(f^{-1}(A))) ⊆ n - CyNMint(f(f^{-1}(A))) ⊆ n - CyNMint(A) . Thus, n - CyNint(f^{-1}(A)) ⊆ f^{-1}(n-CyNMint(A)).

Theorem 3.4 A map f: $(X, \tau_1) \to (Y, \tau_2)$ is n-CyNMO map iff for each n-CyNS's B of (Y, τ_2) and for each n-CyNcs A of (X, τ_1) containing f^{-1}(B) there is a n-CyNMcs C of (Y, τ_2) such that B ⊆ A and f^{-1}(C) ⊆ A.

Proof. Necessity: Assume f is a n-CyNMO map. Let B be the n-CyNcs of (Y, τ_2) and A is a n-CyNcs of (X, τ_1) such that f^{-1}(B) ⊆ A. Then, C = $(f^{-1}(A)^c)^c$ is n-CyNMcs of (Y, τ_2) such that f^{-1}(C) ⊆ A.

Sufficiency: Assume A is a n-CyNos of (X, τ_1). Then, $f^{-1}((f(A))^c) \subseteq A^c$ and A^c is n-CyNcs in (X, τ_1). By hypothesis, there is a n-CyNMcs B of (Y, τ_2) such that $(f(A))^c \subseteq B$ and $f^{-1}(B) \subseteq A^c$. Therefore, $A \subseteq (f^{-1}(B))^c$. Hence $B^c \subseteq f(A) \subseteq h((f^{-1}(B))^c) \subseteq B^c$ which implies $f(A) = B^c$. Since, B^c is n-CyNMos of (Y, τ_2). f(A) is n-CyNMO map in (Y, τ_2) and thus f is n-CyNMO map.

Theorem 3.5 A map f: $(X, \tau_1) \to (Y, \tau_2)$ is n-CyNMO map iff f^{-1}(n-CyNMcl(B)) ⊆ n-CyNcl(f^{-1}(B)) for every n-CyNS's B of (Y, τ_2).

Proof. Necessity: Assume f is a n-CyNMO map. For any n-CyNS's B of (Y, τ_2), f^{-1}(B ⊆ n-CyNcl(f^{-1}(B)). Therefore, by Theorem 3.4, there exists a n-CyNMcs(A) in (Y, τ_2) such that B ⊆ A and f^{-1}(A) ⊆ n - CyNcl(f^{-1}(B)) . Therefore, we obtain that f^{-1}(n - CyNMcl(B)) ⊆ f^{-1}(A) ⊆ n-CyNcl(f^{-1}(B)).

Sufficiency: Assume B is a n-CyNS's of (Y, τ_2) and A is a n-CyNcs of (X, τ_1) containing f^{-1}(B) . Put C = n - CyNcl(B) , then B ⊆ C and C is n - CyNMcs and f^{-1}(C) ⊆ n - CyNcl(f^{-1}(B)) ⊆ A. Then, by Theorem 3.4, f is a n-CyNMO map.

Theorem 3.6 If f: $(X, \tau_1) \to (Y, \tau_2)$ and g: $(Y, \tau_2) \to (Z, \tau_3)$ be a n-CyN maps and $g \circ f: (X, \tau_1) \to (Z, \tau_3)$ is n-CyNMO map. If g: $(Y, \tau_2) \to (Z, \tau_3)$ is a n-CyNMO map, then f: $(X, \tau_1) \to (Y, \tau_2)$ is n-CyNMO map.

Proof. Necessity: Let A be a n-CyNos in (X, τ_1). Then, $g \circ f(A)$ is a n-CyNMos of (Z, τ_3) because $g \circ f$ is a n-CyNMO map. Since, g is a n-CyNMO map and $g \circ f(A)$ is a n-

CyNMos of (Z, τ_3), $g^{-1}(g \circ f(A)) = f(A)$ is a n-CyNMos in (Y, τ_2). Hence, f is a n-CyNMO map.

Theorem 3.7 If $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is n-CyNO map and $g: (Y, \tau_2) \rightarrow (Z, \tau_3)$ is a n-CyNMO map, then $g \circ f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is a n-CyNMO map.

Proof. Let A be a n-CyNos in (X, τ_1). Then, f(A) is a n-CyNos of (Y, τ_2) because f is a n-CyNO map. Since, g is a n-CyNMO map. $g(f(A)) = (g \circ f)(A)$ is a n-CyNMos of (Z, τ_3). Hence, $g \circ f$ is a n-CyNMO map.

4. n-Cylindrical Neutrosophic M-closed Mappings

In this section, we introduce the concept of n-Cylindrical neutrosophic M-closed mappings and examine some of their fundamental properties.

Definition 4.1 Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

(i). δ-closed mapping (briefly, n-CyNδC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNδcs in (Y, τ_2),

(ii). δ-pre closed mapping (briefly, n-CyNδ$\mathcal{P}$C map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNδ$\mathcal{P}$cs in (Y, τ_2).

(iii). δ-semi closed mapping (briefly, n-CyNδ$\mathcal{S}$C map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNδ$\mathcal{S}$cs in (Y, τ_2),

(iv). θ-closed mapping (briefly, n-CyNθC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNθcs in (Y, τ_2),

(v). θ-semi closed mapping (briefly, n-CyNθ$\mathcal{S}$C map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNθ$\mathcal{S}$cs in (Y, τ_2),

(vi). e-closed mapping (briefly, n-CyNeC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNecs in (Y, τ_2),

(vii). M-closed mapping (briefly, n-CyNMC map), if the image of every n-CyNcs in (X, τ_1) is a n-CyNMcs in (Y, τ_2).

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

Theorem 4.1 The following statements are hold but the converse does not true.

(i). Every n-CyNθC map is a n-CyNC map.

(ii). Every n-CyNθC map is a n-CyNθ$\mathcal{S}$C map.

(iii). Every n-CyNθ$\mathcal{S}$C map is a n-CyNMC map.

(iv). Every n-CyNδC map is a n-CyNC map.

(v). Every n-CyNδC map is a n-CyNδ$\mathcal{P}$C map.

(vi). Every n-CyNδC map is a n-CyNδ$\mathcal{S}$C map.

(vii). Every n-CyNδ$\mathcal{S}$C map is a n-CyNeC map.

(viii). Every n-CyNδ$\mathcal{P}$C map is a n-CyNMC map.

(ix). Every n-CyNMC map is a n-CyNeC map.

Proof. Only (viii) is proven; the others are similar.

(viii). Let A be a n-CyNcs in (X, τ_1). Since, f is n-CyNδ$\mathcal{P}$C map, f(A) is a n-CyNδ$\mathcal{P}$cs in (Y, τ_2). Since, every n-CyNδ$\mathcal{P}$cs is a n-CyNMcs. f(A) is a n-CyNMcs in (Y, τ_2). Hence, f is a n-CyNMC map.

Example 4.1 In Example 3.2, f is n-CyNC map but not n-CyNθC map, because the set f(A^c)= B_1^c is a n-CyNcs but not n-CyNθcs.

Example 4.2 In Example 3.3, f is n-CyNθ$\mathcal{S}$C map but not n-CyNθC map, because the set f(A^c)= B_5^c is a n-CyNθ$\mathcal{S}$cs but not n-CyNθcs.

Example 4.3 In Example 3.4, f is n-CyNMC map but not n-CyNθ$\mathcal{S}$C map, because the set f(A^c)= B_1^c is a n-CyNMcs but not n-CyNθ$\mathcal{S}$cs.

Example 4.4 In Example 3.5, f is n-CyNC map but not n-CyNδC map, because the set f(A^c)= B_4^c is a n-CyNcs but not n-CyNδcs.

Example 4.5 In Example 3.6, f is n-CyNδ$\mathcal{P}$C map but not n-CyNδC map, because the set $f(A^c) = B_3^c$ is a n-CyNδ$\mathcal{P}$cs but not n-CyNδcs.

Example 4.6 In Example 3.7, f is n-CyNδ$\mathcal{S}$C map but not n-CyNδC map, because the set, $f(A^c) = B_5^c$ is a n-CyNδ$\mathcal{S}$cs but not n-CyNδcs.

Example 4.7 In Example 3.8, f is n-CyNeC map but not n-CyNδ$\mathcal{S}$C map, because the set, $f(A^c) = B_5^c$ is a n-CyNecs but not n-CyNδ$\mathcal{S}$cs.

Example 4.8 In Example 3.9, f is n-CyNMC map but not n-CyNδ$\mathcal{P}$C map, because the set, $f(A^c) = B_5^c$ is a n-CyNMcs but not n-CyNδ$\mathcal{P}$cs.

Example 4.9 In Example 3.10, f is n-CyNeC map but not n-CyNMC map, because the set, $f(A^c) = B_5^c$ is a n-CyNecs but not n-CyNMcs.

Theorem 4.2 A map $f: (X, \tau_1) \to (Y, \tau_2)$ is n-CyNMC map iff for each n-CyNS's B of (Y, τ_2) and for each n-CyNos A of (X, τ_1) containing $f^{-1}(B)$ there is a n-CyNMosC map of (Y, τ_2) such that $B \subseteq A$ and $f^{-1}(C) \subseteq A$.

Proof. Necessity: Assume f is a n-CyNMC map. Let B be the n-CyNos of (Y, τ_2) and A is a n-CyNos of (X, τ_1) such that $f^{-1}(B) \subseteq A$. Then, $C = (f^{-1}(A)^c)^c$ is n-CyNMos of (Y, τ_2) such that $f^{-1}(C) \subseteq A$.

Sufficiency: Assume A is a n-CyNcs of (X, τ_1). Then, $((f(A))^c$ is a n-CyNS's of (Y, τ_2) and A^c is n-CyNos in (X, τ_1) such that$f^{-1}((f(A))^c) \subseteq A^c$. By hypothesis, there is a n-CyNMos C of (Y, τ_2) such that $(f(A))^c \subseteq C$ and $f^{-1}(C) \subseteq A^c$. Therefore, $A \subseteq (f^{-1}(C))^c$. Hence $C^c \subseteq f(C) \subseteq f((f^{-1}(C))^c) \subseteq C^c$ which implies $f(A) = C^c$. Since, C^c is a n-CyNMcs of (Y, τ_2). f(A) is a n-CyNMcs in (Y, τ_2) and thus f is a n-CyNMC map.

Theorem 4.3 If $f: (X, \tau_1) \to (Y, \tau_2)$ is a n-CyNC map and $g: (Y, \tau_2) \to (Z, \tau_3)$ is a n-CyNMC map. Then, $g \circ f: (X, \tau_1) \to (Z, \tau_3)$ is a n-CyNMC map.

Proof. Let A be a n-CyNcs in (X, τ_1). Then, f(A) is n-CyNcs of (Y, τ_2) because f is n-CyNC map. Now, $g \circ f(A) = g(f(A))$ is n-CyNMcs in (Z, τ_3) because g is a n-CyNMC map. Thus, $g \circ f$ is a n-CyNMC map

Theorem 4.4 If $f: (X, \tau_1) \to (Y, \tau_2)$ is a n-CyNMC map then n-CyNMcl(f(A)) ⊆ f(n-CyNcl(A)).

Proof. Obvious.

Theorem 4.5 Let $f: (X, \tau_1) \to (Y, \tau_2)$ and $g: (Y, \tau_2) \to (Z, \tau_3)$ are n-CyNMC map. If every n-CyNMcs of (Y, τ_2) is a n-CyNcs then $g \circ f: (X, \tau_1) \to (Z, \tau_3)$ is a n-CyNMC map.

Proof. Let A be a n-CyNcs in (X, τ_1). Then, f(A) is a n-CyNMcs of (Y, τ_2) because f is n-CyNMC map. By hypothesis, f(A) is a n-CyNcs of (Y, τ_2). Now, $g(f(A)) = g \circ f(A)$ is n-CyNMcs in (Z, τ_3) because g is a n-CyNMC map. Thus, $g \circ f$ is a n-CyNMC map.

Theorem 4.6 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a bijective map. Then, the following statements are equivalent:

(i). f is a n-CyNMO map.

(ii). f is a n-CyNMC map.

(iii). f^{-1} is a n-CyNMCts map.

Proof. (i). ⇒ (ii).: Let us assume that f is a n-CyNMO map. By definition A is a n-CyNos in (X, τ_1) then f(A) is a n-CyNMos in (Y, τ_2). Here, A is a n-CyNcs in (X, τ_1). Then, $1_{CyN} - A$ is a n-CyNos in (X, τ_1). By assumption, $f(1_{CyN} - A)$ is a n-CyNMos in (Y, τ_2). Hence, $1_{CyN} - f(1_{CyN} - A)$ is a n-CyNMcs in (Y, τ_2). Therefore, f is a n-CyNMC map.

(ii). ⇒ (iii).: Let A be a n-CyNcs in (X, τ_1). By (ii). f(A) is a n-CyNMcs in (Y, τ_2). Hence, $f(A) = (f^{-1})^{-1}(A)$. So, f^{-1} is a n-CyNMcs in (Y, τ_2). Hence, f^{-1} is a n-CyNMO map.

(iii). ⇒ (i).: Let A be a n-CyNos in (X, τ_1). By (iii). $(f^{-1})^{-1}(A) = f(A)$ is a n-CyNMO map.

5. Conclusion

In this paper, we explored the notions of n-CyNMO and n-CyNMC mappings within the framework of n-Cylindrical neutrosophic topological spaces (n-CyNts). As a direction for future research, we propose to examine n-CyNHom and n-CyNMHom mappings, along with the concepts of n-Cylindrical neutrosophic M-compactness, n-cylindrical neutrosophic M-connectedness, and n-cylindrical neutrosophic contra M-continuous functions.

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