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Z-symmetries of a Class of Indefinite Almost Paracontact Metric Manifolds

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Abstract:

The primary objective of this paper is to examine the Z-symmetries of indefinite almost paracontact metric manifolds, with particular emphasis on three-dimensional (ϵ)-para Kenmotsu manifolds. We demonstrate that the Einstein manifolds and the stipulation of possessing distinct Z-symmetries are equivalent for three-dimensional (ϵ)-para Kenmotsu manifolds. It has also been deduced that a Z-pseudosymmetric three-dimensional (ϵ)-paraKenmotsu manifold exists. In the conclusion, we prove that all projectively Z-semisymmetric three-dimensional (ϵ)-para Kenmotsu manifolds are Einstein manifolds.

Keywords: Einstein manifolds, indefinite para Kenmotsu manifolds, Z-tensors, pseudosymmetric functions, semisymmetric manifolds, and Ricci-semisymmetric manifolds.

1. Introduction

In 1969, Takahashi [1] examined (ϵ)-Sasakian and (ϵ)-almost contact metric manifolds, introducing virtually almost contact manifolds with pseudo-Riemannian metrics, with a focus on Sasakian manifolds. De and Sarkar [2] introduced the (ϵ)-Kenmotsu manifold, which is based on a type of Riemannian manifold called a Kenmotsu manifold [3].

Sato [4] was the first person to define the idea of a almost paracontact structure in 1976. Tripathi $et\ al.$, [5] introduced the concept of an indefinite almost paracontact metric structure, also known as a (ϵ)-almost paracontact structure, by connecting an almost paracontact structure to a semi-Riemannian metric that does not have to be Lorentzian. In this case, the symbol ϵ = 1 or ϵ = -1 shows whether the vector field ξ has space-like or time-like properties. They also discussed and investigated into the properties of (ϵ)-para Sasakian [5] and (ϵ)-para Sasakian 3-manifolds [6].

In the year 1995, Sinha and Sai Prasad [7] constructed a category of almost paracontact metric manifolds that are known as para-Kenmotsu manifolds and special para-Kenmotsu manifolds and in brief, these manifolds are known as P-Kenmotsu and SP-Kenmotsu manifolds. Mona and Pankaj [8] conducted research on the Cotton tensor on para Kenmotsu 3-manifold admitting the Etta-Ricci solitons. This research was carried out quite recently. Sunitha and Sai Prasad [9] presented a novel class of indefinite almost paracontact metric manifolds, which they referred to as (ϵ)-para Kenmotsu manifolds. This class was introduced by them in the year 2025. A comparison may be made between these manifolds and the (ϵ)-para Sasakian manifolds. To be more specific, they looked into the curvature properties of the (ϵ)-para Kenmotsu 3-manifolds that were being explored. In addition, they explored these manifolds that are flexible enough to accommodate quarter-symmetric connections and discovered that the Ricci and Riemannian tensors of these manifolds share several common identities [10]. All of these manifolds are flexible enough to accommodate quarter-symmetric connections.

Mantica and Molinari [11] made a generalised $(0, 2)$ symmetric Z-tensor in 2012 by doing the following:

$$Z(X,Y) = \psi\, g(X,Y) + S(X,Y), \tag{1.1}$$

where ψ is any scalar function. Mallick and De [12] delineated several curvature conditions that the Z-Tensor satisfies on $N(k)$-Quasi-Einstein manifolds. Gatti $et\, al.$, [13] investigated the Z-symmetries of ϵ-para-Sasakian 3-manifolds in 2021, revealing some compelling findings.

Our presentation and research of the Z-symmetries of the freshly defined (ϵ)-para Kenmotsu 3-manifolds are prompted by the work that has been carried out up until this point. As part of Section 2, there are a few conditions that must be met for (ϵ)-para Kenmotsu manifolds to exist in three dimensions. We present evidence in the third part that demonstrates that the conditions for having separate Z-symmetries and Einstein manifolds are same for the (ϵ)-para Kenmotsu 3-manifold. Furthermore, that Ricci symmetry is present for all Z-symmetric (ϵ)-para Kenmotsu 3-manifolds has been demonstrated by empirical evidence. In spite of the fact that the Ricci curvature tensor of (ϵ)-para Sasakian 3-manifolds is significantly different from that of (ϵ)-para Kenmotsu 3-manifolds, there exists a result that is in consistent with this occurrence [13].

2. Preliminaries

A manifold has numerous dimensions that can be specified in a variety of ways. Assuming that M_n is capable of accepting a (1, 1) tensor field ϕ, a vector field ξ and a 1-form η that satisfy the requirements, then it is considered to have a structure that is almost paracontact.

$$\phi^2 X = X - \eta(X)\,\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$\eta(\phi X) = 0, \quad \phi\,\xi = 0. \tag{2.3}$$

By taking into account the semi-Riemannian metric $g(X,Y)$, which has an index of $g(X,Y) = \nu$, it is possible to assert that

$$g(\phi X, \phi Y) = g(X,Y) - \epsilon\,\eta(X)\,\eta(Y), \tag{2.4}$$

for any X, Y belongs to the set $\chi(M_n)$, which is a group of vector fields on M_n, and (ϵ) is either 1 or -1. Then M_n is considered as (ϵ)-almost paracontact metric manifold endowed with (ϵ)-almost paracontact metric structure $(\phi, \xi, \eta, g, \epsilon)$.

According to the information provided in reference [14], the manifold M_n is regarded to be a Lorentzian manifold that is almost paracontact if the index of the function $g(X,Y)$ is one. Due to the fact that the metric $g(X,Y)$ is positive definite, the manifold M_n is regarded as almost paracontact metric manifold [4]. A clear and convincing demonstration of the assertion is provided by equation (2.4) that

$$g(\phi X, Y) = g(X, \phi Y) \;\; and \;\; g(X,\xi) = \epsilon\,\eta(X). \tag{2.5}$$

In addition, the equations (2.1), (2.2), (2.3) and (2.5) give us the following:

$$g(\xi,\xi) = \epsilon. \tag{2.6}$$

The structure of the (ϵ)-paracontact metric is also known as the (ϵ)-para Kenmotsu metric if

$$(\nabla_X\phi)\,Y = g(X,\phi Y)\xi - \epsilon\,\eta(Y)\,\phi X. \tag{2.7}$$

Moreover, the manifold M_n that contains the structure is understood to be the (ϵ)-para Kenmotsu manifold [9]. The symbol ∇ is used to denote the Levi-Civita connection for the indefinite metric $g(X,Y)$. In a particular instance, for the Riemannian metric $g(X,Y)$ and ϵ = 1, the manifold is para Kenmotsu [7]. We may change the manifold M_n into a Lorentzian para-Kenmotsu manifold by substituting $-\xi$ for ξ and ϵ = -1. This manifold is characterised by the Lorentzian metric $g(X,Y)$ [14].

Among the features that the Riemannian Christoffel curvature tensor $R(X,Y)$ possesses in a (ϵ)-para Kenmotsu manifold are as follows [9]:

$$R(X,Y)\xi = \eta(X)\,Y - \eta(Y)\,X, \tag{2.8}$$

$$R(\xi,X)\,Y = \epsilon\,\eta(Y)X - g(X,Y)\,\xi, \tag{2.9}$$

$$R(\xi,X)\xi = \epsilon X - \epsilon\,\eta(X)\,\xi, \tag{2.10}$$

$$S(Y,\xi) = -(n-1)\,\eta(Y), \tag{2.11}$$

$$\nabla_Y\xi = \epsilon\,(Y - \eta(Y)\,\xi), \tag{2.12}$$

for any vector fields X, Y and Z, where the Ricci tensor is denoted by the symbol $S(X,Y)$.

In a three-dimensional (ϵ)-para Kenmotsu manifold, the Riemannian and Ricci curvature tensors can be produced by following the steps outlined in [9]:

$$\begin{aligned} R(X,Y)Z = {} & [g(X,Z)Y - g(Y,Z)X]\left[\tfrac{r}{2} + 6\,\epsilon\right] \\ & +[g(Y,Z)\eta(X)\xi + \epsilon\,\eta(Y)\eta(Z)X - g(X,Z)\eta(Y)\xi - \epsilon\,\eta(X)\eta(Z)Y]\left[\tfrac{r}{2} + 5\epsilon\right], \end{aligned} \tag{2.13}$$

and

$$S(X,Y) = [\tfrac{r}{2} + 3\,\epsilon]\,g(X,Y) - \epsilon\,[\tfrac{r}{2} + 5\epsilon\,]\,\eta(X)\,\eta(Y). \tag{2.14}$$

In a $three$-dimensional (ϵ)-para Kenmotsu manifold, the projective curvature tensor $P(X,Y)$ is defined as

$$P(X,Y)U = R(X,Y)\,U + \tfrac{1}{2}[S(X,U)\,Y - S(Y,U)\,X]. \tag{2.15}$$

Hence, the tensor $Z(X,Y)$ is obtained in a three-dimensional (ϵ)-para Kenmotsu manifold M_3, by using the equations (1.1) and (2.14),

$$Z(X,Y) = [\psi + 3\,\epsilon + \tfrac{r}{2}]\,g(X,Y) - \epsilon\,[\tfrac{r}{2} + 5\,\epsilon]\eta(X)\eta(Y), \tag{2.16}$$

and the scalar Z as:

$$Z = [\frac{r}{2} + 3\epsilon + \psi]3 - [\frac{r}{2} + 5\epsilon] = 4\epsilon + 3\psi. \tag{2.17}$$

Further, we also have

$$Z(X,\xi) = (\epsilon\,\psi - 2)\eta(X). \tag{2.18}$$

3. Z-symmetries of three-dimensional (ϵ)-para Kenmotsu manifolds

In this section, we discuss the Z-symmetries of $three$-dimensional (ϵ)-para Kenmotsu manifolds. Let us start with the definition.

Definition 3.1: Semisymmetric manifolds, which are a valid generalization of locally symmetric manifolds, M_n, were defined as [15]:

$$R(X,Y)\cdot R = 0, \text{ for all } X \; and \; Y \in \chi(M_n).$$

Definition 3.2: A semi-Riemannian manifold M_n is considered Z-semisymmetric if

$$R(X,Y)\cdot Z = 0, \tag{3.1}$$

where $R(X,Y)$ serves as a derivation on Z, for all $X, Y \in \chi(M_n)$.

Assuming that M_3 is a three-dimensional Z-semisymmetric (ϵ)-para Kenmotsu manifold, we may get the following result for $X = \xi$ by applying the equation (3.1):

$$Z(R(\xi,Y)U,V) + Z(U,R(\xi,Y)V) = 0. \tag{3.2}$$

By using (2.9), the equation (3.2) reduces to

$$\begin{aligned} &Z(R(X,Y)U,V) \\ &= [\frac{r}{2} + 6\epsilon][g(X,U)Z(Y,V) - g(Y,U)Z(X,V)] \\ &+[\frac{r}{2} + 5\epsilon][g(Y,U)\eta(X)Z(\xi,V) - g(X,U)\eta(Y)\,Z(\xi,V) \\ &+ \epsilon\eta(Y)\,\eta(U)Z(X,V) - \epsilon\eta(X)\,\eta(U)Z(Y,V)], \end{aligned} \tag{3.3}$$

which further implies

$$Z(R(\xi,Y)U,V) \quad = \eta(U)\,Z(Y,V) - \epsilon\,g(Y,U)\,Z(\xi,V). \tag{3.4}$$

Additionally, from the equation (2.9), we get

$$\begin{aligned} &Z(U,R(X,Y)V) \\ &= [\frac{r}{2} + 6\epsilon][g(X,V)\,Z(U,Y) - g\,(Y,V)\,Z\,(U,X)] \\ &+ [\frac{r}{2} + 5\epsilon][g(Y,V)\,\eta(X)\,Z(U,\xi) - g(X,V)\,\eta(Y)\,Z(U,\xi) \\ &+\epsilon\,\eta(Y)\,\eta(V)\,Z(U,X) - \epsilon\,\eta(X)\,\eta(V)\,Z(U,Y)], \end{aligned} \tag{3.5}$$

which implies

$$Z(U,R(\xi,Y)V) = -\epsilon\,g(Y,V)\,Z\,(U,\xi) + \eta(V)\,Z(U,Y). \tag{3.6}$$

Then the equations (3.2), (3.4) and (3.6) together gives

$$\begin{aligned} &-\epsilon\,g(Y,U)\,Z(\xi,V) + Z(Y,V)\,\eta(U) \\ &- \epsilon\,g(Y,V)\,Z(U,\xi) + Z(U,Y)\,\eta(V) = 0. \end{aligned} \tag{3.7}$$

By inserting $V = \xi$ in (3.7) and on using (2.18), we get

$$Z(U,Y) = \psi\, g(Y,U) \;-\; 2\,\epsilon\, g(Y,U). \tag{3.8}$$

Then by using (2.16), the above equation yields

$$[\tfrac{r}{2} + 5\epsilon]\,\epsilon\,\eta(Y)\,\eta(U) = [\tfrac{r}{2} + 5\epsilon]\,g(Y,U), \tag{3.9}$$

from which we conclude that $\frac{r}{2} + 5\epsilon = 0$. This gives

$$r = -10\,\epsilon. \tag{3.10}$$

By substituting r from (3.10) in (2.14), we have

$$S(X,Y) \;+\; 2\,\epsilon\, g(X,Y) = 0, \tag{3.11}$$

This suggests that the manifold M_3 is Einstein's manifold.

Let us assume, on the other hand, that M_3 is an Einstein manifold. After that, we have the answer (3.8), which is derived from equations (1.1) and (3.11). Furthermore, we take into consideration

$$R(X,Y)\,.\,Z(U,V) \;=\; Z(U,R(X,Y)V) + \;Z(R(X,Y)U,V). \tag{3.12}$$

Then by using (3.8), the above equation (3.12) reduces to

$$R(X,Y).Z(U,\quad V) \;=\; [g(R(X,Y)U,V) + g(U,R\,(X,Y)V)]\,(\psi - 2\,\epsilon). \tag{3.13}$$

An (ϵ)-para Kenmotsu manifold of dimension 3 is reported to have [9]:

$$g(R(X,Y)U,V) = -g(U,R(X,Y)V). \tag{3.14}$$

Then, based on equations (3.13) and (3.14), we obtain

$$R(X,Y)\,.\,Z(U,V) = 0, \tag{3.15}$$

This indicates that the manifold M_3 is Z-semisymmetric. So, we may state the following:

Theorem 3.1: A three-dimensional (ϵ)-para Kenmotsu manifold is Z-semisymmetric if and only if it is an Einstein manifold.

It also makes sense that M_3 is Ricci symmetric because the manifold is Einstein. On the other hand, if M_3 is Ricci-symmetric, then $\nabla S = 0$. This means

$$(\nabla_X S)(Y,\xi) = -\epsilon\, S(Y,X) + 2\eta(X)\,\eta(Y) - 2\epsilon\, g(\phi X,\phi Y) = 0.$$

The manifold is Einstein, if we replace X with ϕX in the preceding expression. Consequently, from Theorem 3.1, we assert the following:

Theorem 3.2: An (ϵ)-para Kenmotsu manifold of dimension $three$ is considered to be Z-semisymmetric if and only if it is Ricci-symmetric.

In addition, if M_3 is Ricci-semisymmetric, we may say that for $X \;=\; \xi$, we obtain

$$S(R(\xi,Y)U,\xi) \;+\; S(U,R(\xi,Y)\xi) \;=\; 0.$$

Substituting the values from (2.9) and (2.14) into the previous expression yields (3.11).

Contrarily, if M_3 is an Einstein manifold, it is obviously Ricci-semisymmetric. Therefore, the manifold M_3 is Ricci-semisymmetric if and only if it is Einstein. Taking into account the outcome from 3.1, we can so draw the following:

Theorem 3.3: If and only if a (ϵ)- para Kenmotsu manifold M_3 of dimension 3 is Ricci-symmetric, then it is Z-semisymmetric.

Corollary 3.4: *The following statements are similar on a (ϵ)-para Kenmotsu manifold M_3 of dimension* 3*:*

(a) M_3 is an Einstein manifold

(b) M_3 exhibits Ricci symmetry

(c) M_3 is Ricci-semisymmetric

(d) Z-semisymmetry exists in M_3.

It is obvious that, Z-symmetric ⇒ Z-semisymmetric ⇒ $\nabla S = 0$. Thus, in accordance with Corollary 3.4, we declare:

Corollary 3.5: Ricci-symmetry exists for all Z-symmetric (ϵ)-para Kenmotsu 3-manifolds.

We define $R\,.T$ and $Q(A,T)$, the $(0, k+2)$-tensor fields, as

$$(R\,.T)(Y_1,\ldots,Y_k;\, X,Y)$$
$$= -T(R(X,Y)Y_1,\ldots,Y_k) - \ldots -T(Y_1,\ldots,Y_{k-1},R(X,Y)\,Y_k), \text{ and}$$

$$Q(A,\, T)(Y_1,\ldots,Y_k;\, X,Y)$$
$$= -T((X\ \wedge_A\ Y)\,Y_1,\ldots,Y_k) - \ldots -T(Y_1,\ldots,Y_{k-1},(X\ \wedge_A\ Y)Y_k)$$

respectively. On M_n, T is a $(0,k)$ tensor field ($k \geq 1$), and A is a symmetric $(0,2)$-tensor field. Further, the endomorphism $X \wedge_A Y$ is defined by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{3.16}$$

Definition 3.3: It is said that a semi-Riemannian manifold M_n is pseudo-symmetric if [16]

$$R\,.R = L_R\,Q(g,R)$$

holds for the set $U_R = \{x \in M_n : R \neq 0\ at\ x\}$, where L_R is a function on U_R.

Definition 3.4: A semi-Riemannian manifold M_n is said to be Z-pseudosymmetric if

$$R \cdot Z = L_Z\,Q(g,Z) \tag{3.17}$$

holds for the set $U_Z = \{x \in M_n : Z \neq 0\ at\ x\}$. In this case, L_Z is a function on U_Z.

Assume that M_3 is a Z-pseudosymmetric $three$-dimensional (ϵ)-para Kenmotsu manifold. Using (3.16) and (3.17), we have:

$$(R(\xi,Y)\,.Z)(U,\xi) = L_Z\,[((\xi\ \wedge\ Y)\,.Z)(U,\xi)]. \tag{3.18}$$

Using (2.9) and (3.17), we obtain

$$R(\xi,X)Y = -\epsilon(\xi \wedge X)Y. \tag{3.19}$$

As a result, it is clear from (3.18) in (3.19) that

$$L_Z = -\epsilon. \tag{3.20}$$

Thus we can state the following:

Definition 3.5: The (ϵ)-para Kenmotsu 3-manifold M_3 is considered. Either M_3 is an Einstein manifold or $L_Z = -\epsilon$ holds on the manifold M_3 provided it is Z-pseudosymmetric.

In contrary, if $L_Z \neq -\epsilon$, *we immediately obtain:*

Corollary 3.6: An Einstein manifold is any Z-pseudosymmetric three-dimensional (ϵ)-para Kenmotsu manifold with $L_Z \neq -\epsilon$.

However, L_Z does not always have to be zero, hence there are Z-pseudosymmetric manifolds that are not Z-semisymmetric. Therefore, if $L_Z \neq 0$, we have $R.Z = Q(g,Z)\,(-\epsilon)$ and hence L_Z value will be $-\epsilon$. Consequently, we have the following claim.

Corollary 3.7: Each three-dimensional (ϵ)-para Kenmotsu manifold is Z-pseudosymmetric of the form $R.Z = Q(g,Z)(-\epsilon)$.

Definition 3.6: A semi-Riemannian manifold M_n is termed projectively Z-semisymmetric if

$$P(X,Y)\cdot Z = 0, \tag{3.21}$$

where $P(X,Y)$ represents the projective curvature tensor.

Let M_3 be a $three$-dimensional (ϵ)-para Kenmotsu manifold that is projectively Z-semisymmetric. Then for $X = \xi$, from (3.21)

$$Z(P(\xi,Y)U,V) = -Z(U,P(\xi,Y)V). \tag{3.22}$$

We also have from the equation (2.15):

$$P(\xi,Y)U = -\frac{1}{2}\,S(Y,U)\,\xi - g(Y,U)\xi. \tag{3.23}$$

With (2.9) and (3.23), we then get

$$Z(P(\xi,Y)U,V) = -\frac{1}{2}\,S(Y,U)\,Z(\xi,V) - g(Y,U)\,Z(\xi,V); \tag{3.24}$$

and

$$Z(U,P(\xi,Y)V) + g(Y,V)\,Z(U,\xi) = -\frac{1}{2}\,S(Y,V)\,Z(U,\xi). \tag{3.25}$$

When combined, the equations (3.22), (3.24), and (3.25) yield

$$[-\frac{1}{2}\,S(Y,U) - g(Y,U)]Z(\xi,V) = [\frac{1}{2}\,S(Y,V) + g(Y,V)]\,Z(U,\xi). \tag{3.26}$$

Using (2.14) and (2.18), for $V = \xi$ in (3.26), we obtain

$$(\psi - 2\epsilon)[S(Y,U) + 2\epsilon g(Y,U)] = 0. \tag{3.27}$$

From (2.18), we derive (3.9) if $\psi = 2\epsilon$, and it shows that M_3 is Einstein. Thus the manifold M_3 is reduced to Einstein in both the situations. We therefore conclude that:

Theorem 3.8: Every projectively Z-semisymmetric three-dimensional (ϵ)-para Kenmotsu manifold is Einstein.

4. Applications

- The Z-tensor, a broad concept of the Einstein gravitational tensor, will be utilised in numerous scientific theories, including the general theory of relativity.
- The indefinite almost paracontanct metric manifolds implanted with Z-tensors have extensive use in numerous geometrical domains, including the development of super-resolution sensors in electronics and communication systems.

5. Significance of the work

- We can illustrate some significances of our research work in following points:
- In 2025, the authors presented a new class of indefinite nearly paracontact metric manifolds, which are studied in this paper.
- Obtaining the conditions for Z-symmetries of three-dimensional (ϵ)-para Kenmotsu manifolds.
- Determining the circumstances in which a three-dimensional (ϵ)-para Kenmotsu manifold is Z-pseudo symmetric.
- The research demonstrates that all projectively Z-semisymmetric three-dimensional indefinite para-Kenmotsu manifolds are Einstein manifolds.

6. Significance of the work

The investigation of Z-tensors in the context of indefinite almost paracontact metric manifolds has a wide range of applications in the sciences of geometry and physics, including an extensive application in the field of electrical engineering.

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