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Domination and Independent in Complex Bipolar Fuzzy Graph

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Abstract:

With the use of a bipolar fuzzy graph, we examined the concepts of vertex and edge cardinality in complex bipolar fuzzy graphs. We also defined dominating set, independent set, and total dominating set in complex bipolar fuzzy graphs and looked at minimal domination and maximal independence. Additionally, talk about the characteristics of the neighborhood and its dominance.

Keywords: Complex Bipolar fuzzy graph, strong edge, Dominating set, Domination number, independent set, Independence number, domination number, neighborhood etc.

Introduction

The method of measuring the presence of a moment element was introduced by Zadeh [1, 2] in 1965. Its applications have spread to various fields today. It has given a new dimension and development, especially in the fields of medicine and life sciences, management sciences, social sciences, engineering, statistics, graph theory, etc.

The Fuzzy set-to-fuzzy graph was first defined by Rosenfeld [3]. And the basic principles were also introduced by him. After that, in continuation of this study, Akram introduced the presence of elements in fuzzy moments of two nodes, i.e., the quality of being an element and not an element in the moment with the intervals [0, 1] and (-1, 0], respectively, named the bipolar fuzzy set.

Introducing the bipolar fuzzy set by W.R Zhang [4, 5], we get clarity about the introduction of the bipolar fuzzy graph, its properties, and its relationship with the fuzzy graph in Akram's [6] study.

Karunambigai's [7] study of dominance in bipolar fuzzy graph marked the beginning of a chapter in the evolution of the study of bipolar fuzzy graph . Subsequently, Vinothkumar and Geetharamani's [8, 9] study on edge dominance and vertex edge dominance made me interested in the continuation of this thesis. In this study, with the above research results, I have defined its characteristics and its functions in dominance complex bipolar fuzzy and compiled some excellent results.

2. Preliminaries

Definition 2.1. Let S be a set. We recall that a fuzzy subset of S is a mapping $\sigma: S \to [0,1]$ that assigns to each element $x \in S$ a degree of membership, $0 \leq \sigma(x) \leq 1$. Similarly, a fuzzy relation on S is a fuzzy subset of $S \times S$, i.e., a mapping $\mu: S \times S \to [0,1]$ that assigns to each ordered pair of elements (x, y) a degree of membership, $0 \leq \mu(x, y) \leq 1$.

Definition 2.2. Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form $B = \left\{\left(x, \mu_B^p(x), \mu_B^n(x)\right): x \in X\right\}$, Where $\mu_B^p: X \to [0,1]$ and $\mu_B^n: X \to [-1,0]$ are mappings.

Definition 2.3. A bipolar fuzzy graph G_B on a non-empty set X is a pair $G = (A, B)$, where $A: X \to [0,1] \times [-1,0]$ is a bipolar fuzzy set on the set X and $B: X \times X \to [0,1] \times [-1,0]$ is a bipolar fuzzy relation in X such that $\mu_B^p(xy) \leq \mu_A^p(x) \wedge \mu_A^p(y)$ and $\mu_B^p(xy) \leq \mu_A^p(x) \wedge \mu_A^p(y)$, for all $x, y \in X$.

Definition 2.4. A complex bipolar fuzzy set A on a non-empty set X is an object of the form $A = \left\{\left(x, r_A^p(x)e^{iw_A^p(x)}, r_A^n(x)e^{iw_A^n(x)} : x \in X\right)\right\}$, where$i = \sqrt{-1}$, $r_A^p: X \to [0,1]$ and $r_A^n: X \to [-1,0]$ are mappings, $w_A^p(x) \in [0,\pi]$ and $w_A^n(x) \in [\pi, 2\pi]$ or $w_A^n(x) \in [-\pi, 0]$. For any element $x \in X, r_A^p(x)$ and $r_A^n(x)$ are called amplitude terms, $w_A^p(x)$ and $w_A^n(x)$ are called Phase terms.

The Complex bipolar fuzzy set can also write as $A = \left\{\left(x, r_A^p(x)e^{iw_A^p(x)}, r_A^n(x)e^{iw_A^n(x)}\right): x \in X\right\}$.

Definition 2.5. A complex bipolar fuzzy graph on a non-empty set X is a pair $G = (A, D)$, where $A = \left(r_A^p e^{w_A^p}, r_A^p e^{w_A^n}\right): X \to \{z: z \in \mathbb{C}, |z| \leq 1\}^2$ is a complex bipolar fuzzy set on X and $D = \left(r_D^p e^{w_D^p}, r_D^n e^{w_D^n}\right): X \times X \to \{z: z \in \mathbb{C}, |z| \leq 1\}^2$ is a complex bipolar fuzzy relation in X such that, for all $x, y \in X$.

$r_D^p(xy) \leq r_A^p(x) \wedge r_A^p(y)$ and $r_D^n(xy) \geq r_A^n(x) \vee r_A^n(y)$,

$w_D^p(xy) \leq w_A^p(x) \wedge w_A^p(y)$ and $w_D^n(xy) \geq w_A^n(x) \vee w_A^n(y$

Where $i = \sqrt{-1}, r_A^p: X \to [0,1], r_A^n: X \to [-1,0], r_D^p: X \times X \to [0,1]$ and $r_D^n: X \times X \to [-1,0]$ are mappings, $w_A^p, w_D^p \in [0,\pi]$, $w_A^n, w_D^n \in [\pi, 2\pi], or\ w_A^n, w_D^n \in [-\pi, 0]$.

Here, $E \subseteq X \times X$ is the set of edges. A is called a complex bipolar fuzzy vertex set of G and D is a complex bipolar fuzzy edge set of G.

3. Domination in Complex Bipolar Fuzzy Graph

In this chapter we take each vertex and edges in the complex bipolar fuzzy graphs as in the cartesian form like $v = (v^{rp}, v^{ip}, v^{rn}, v^{in})$ and $e = (e^{rp}, e^{ip}, e^{rn}, e^{in})$, where rp, ip, rn, in are represent respectively as real positive, imaginary positive, real negative and imaginary negative membership degree.

Definition 3.1. A vertex cardinality is denoted by $|v_k|$ is defined as $|v_k| = \frac{2+\mu_{V_C}^{rp}(v_k)+\mu_{V_C}^{ip}(v_k)+\mu_{V_C}^{rn}(v_k)+\mu_{V_C}^{in}(v_k)}{4}$, here $v_k \in V_C$ in complex bipolar fuzzy graph $G_C(V_C, E_C)$.

Definition 3.2. An edge cardinality in a complex bipolar fuzzy graph $G_C(V_C, E_C)$, is defined as $|e_k| = \frac{2+\mu_{E_C}^{rp}(e_k)+\mu_{E_C}^{ip}(e_k)+\mu_{E_C}^{rn}(e_k)+\mu_{E_C}^{in}(e_k)}{4}$, here $e_k \in E_C$.

Definition 3.3. In a complex bipolar fuzzy graph $G_C(V_C, E_C)$, graph cardinality is defined as follows

$$|G_C| = \sum_{v_k \in V_C} \left(\frac{2+\mu_{V_C}^{rp}(v_k)+\mu_{V_C}^{ip}(v_k)+\mu_{V_C}^{rn}(v_k)+\mu_{V_C}^{in}(v_k)}{4} \right) +$$
$$\sum_{(v_k,v_l) \in V_C} \left(\frac{2+\mu_{E_C}^{rp}(e_k)+\mu_{E_C}^{ip}(e_k)+\mu_{E_C}^{rn}(e_k)+\mu_{E_C}^{in}(e_k)}{4} \right)$$ for all $v_k \in V_C$ and $e_k \in E_C$.

Note:

1. The sum of all vertex cardinality in $G_C(V_C, E_C)$ is called the order of $G_C(V_C, E_C)$.
   i.e., $|G_C| = \sum_{v_k \in V_C} \left(\frac{2+\mu_{V_C}^{rp}(v_k)+\mu_{V_C}^{ip}(v_k)+\mu_{V_C}^{rn}(v_k)+\mu_{V_C}^{in}(v_k)}{4} \right)$ for all $v_k \in V_C$.
2. The sum of all edge cardinality in $G_C(V_C, E_C)$ is called the size of $G_C(V_C, E_C)$.
   i.e.,$|S(G_C)| = \sum_{e_k \in V_C} \left(\frac{2+\mu_{E_C}^{rp}(e_k)+\mu_{E_C}^{ip}(e_k)+\mu_{E_C}^{rn}(e_k)+\mu_{E_C}^{in}(e_k)}{4} \right)$ for all $e_k \in E_C$.

Definition 3.4. A Complex bipolar fuzzy graph $G_C(V_C, E_C)$ is connected if any pair of vertices in V_C linked by a path . A path $P: v_1, v_2, \dots, v_n$ is considered as μ^{rp} strength and is defined as $\min\left(\mu_{E_C}^{rp}(v_k, v_l)\right)$ for all k, l, represented in symbol by S_μ^{rp}. A path $P: v_1, v_2, \dots, v_n$ is μ^{ip} strength and is defined as $\min\left(\mu_{E_C}^{ip}(v_k, v_l)\right)$ for all k, l, represented in symbol by S_μ^{ip}. A path $P: v_1, v_2, \dots, v_n$ is defined as $\max\left(\mu_{E_C}^{rn}(v_k, v_l)\right)$ for all k, l, is called the μ^{rn} strength of P and is represented by S_μ^{rn}. A path $P: v_1, v_2, \dots, v_n$ is defined as $\max\left(\mu_{E_C}^{in}(v_k, v_l)\right)$ for all k, l, is called the μ^{in} strength of P and is represented by S_μ^{in}.

Note:

The same edge has all strength values like μ^{rp}, μ^{ip}, μ^{rn}, μ^{in} after that the edge say strength of the path. The strength is $\left(\mu_{E_C}^{rp}(v_k, v_l), \mu_{E_C}^{ip}(v_k, v_l), \mu_{E_C}^{rn}(v_k, v_l), \mu_{E_C}^{in}(v_k, v_l)\right) = (S_\mu^{rp}, S_\mu^{ip}, S_\mu^{rn}, S_\mu^{in})$

The strength of the strongest path P and it is denoted by

$$S_P = \left(\left(\mu_{E_C}^{rp}(v_k, v_l)\right)^\infty, \left(\mu_{E_C}^{ip}(v_k, v_l)\right)^\infty, \left(\mu_{E_C}^{rn}(v_k, v_l)\right)^\infty, \left(\mu_{E_C}^{in}(v_k, v_l)\right)^\infty\right) \text{ for all } k, l = 1, to\ n.$$

Definition 3.5. An edge $e = \left(e^{rp},\ e^{ip}, e^{rn}, e^{in}\right)$ is called strong edge in complex bipolar fuzzy graph $G_C = (V_C, E_C)$ if $\mu_{E_C}^{rp}(e^{rp}) \geq \left(\mu_{E_C}^{rp}\right)^\infty$, $\mu_{E_C}^{ip}\left(e^{ip}\right) \geq \left(\mu_{E_C}^{ip}\right)^\infty$, $\mu_{E_C}^{rn}(e^{rn}) \leq \left(\mu_{E_C}^{rn}\right)^\infty$, $\mu_{E_C}^{in}\left(e^{in}\right) \leq \left(\mu_{E_C}^{in}\right)^\infty$.

Definition 3.6. In a complex bipolar fuzzy graph $G_C = (V_C, E_C)$, degree of a vertex $v = \left(v^{rp},\ v^{ip}, v^{rn}, v^{in}\right)$ is defined as being sum of the weights of all strong edges that incident on v. It is represented by $d_{G_C}\left(v^{rp},\ v^{ip}, v^{rn}, v^{in}\right)$.

Note:

The minimum degree of $G_C(V_C, E_C)$ is

$$\delta(G_C) = \min\left\{d_{G_C}\left(v^{rp}, v^{ip}, v^{rn}, v^{in}\right): \left(v^{rp}, v^{ip}, v^{rn}, v^{in}\right) \in V_C\right\}$$

The minimum degree of $G_C(V_C, E_C)$ is

$$\Delta(G_C) = \max\left\{d_{G_C}\left(v^{rp}, v^{ip}, v^{rn}, v^{in}\right): \left(v^{rp}, v^{ip}, v^{rn}, v^{in}\right) \in V_C\right\}$$

Definition 3.7. Let $(u^{rp}, u^{ip}, u^{rn}, u^{in}), (v^{rp}, v^{ip}, v^{rn}, v^{in})$ be any two vertices in a complex bipolar fuzzy graph $G_C(V_C, E_C)$. We say that $(u^{rp}, u^{ip}, u^{rn}, u^{in})$ is dominates $(v^{rp}, v^{ip}, v^{rn}, v^{in})$ if there exists a strong edge between $(u^{rp}, u^{ip}, u^{rn}, u^{in})$ and $(v^{rp}, v^{ip}, v^{rn}, v^{in})$.

Note:

In $G_C(V_C, E_C)$, a vertex $(u^{rp}, u^{ip}, u^{rn}, u^{in})$ is isolated vertex which is not dominating any other vertex.

Definition: 3.8. A subset A of V_C is called a dominating set in complex bipolar fuzzy graph $G_C(V_C, E_C)$ if for every $(u^{rp}, u^{ip}, u^{rn}, u^{in}) \in V_C - A$ there exists a vertex $(v^{rp}, v^{ip}, v^{rn}, v^{in}) \in A$ such that $(u^{rp}, u^{ip}, u^{rn}, u^{in})$ dominates $(v^{rp}, v^{ip}, v^{rn}, v^{in})$.

Definition 3.9. A dominating set A in a complex bipolar fuzzy graph, is called minimal if there is no proper subset of A is dominating.

Example:

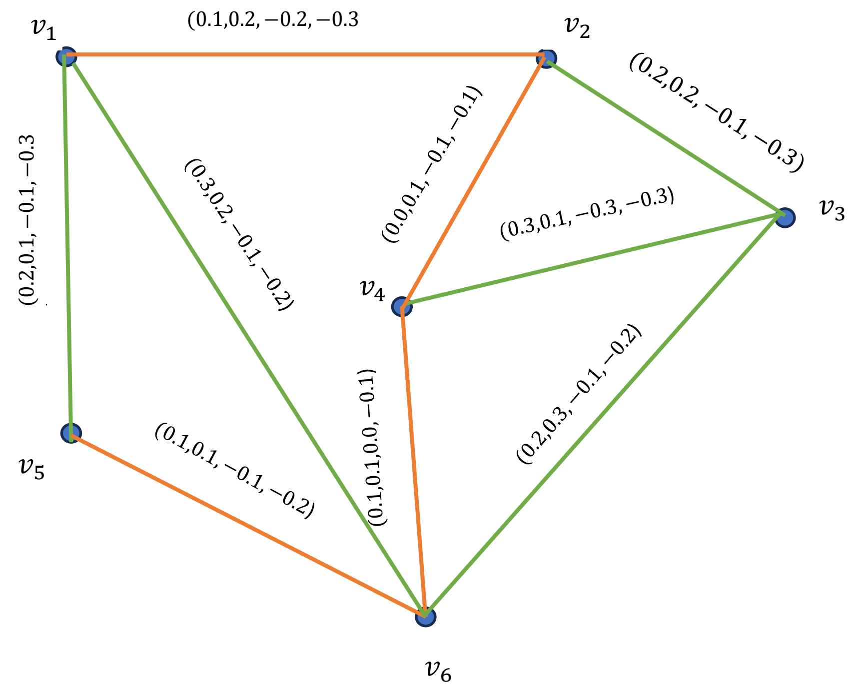

Fig. 3.1. Complex bipolar fuzzy graph

Definition 3.10. In a complex bipolar fuzzy graph $G_C(V_C, E_C)$, any two vertices in V_C are called independent to each other if there is no strong edge incident with them .

Definition 3.11. A subset B of vertex set V_C is called independent set if either no edge with any pair of vertices in B or exist an edge with the conditions

$$\mu_{E_C}^{rp}(e^{rp}) \leq \left(\mu_{E_C}^{rp}\right)^{\infty}, \mu_{E_C}^{ip}(e^{ip}) \leq \left(\mu_{E_C}^{ip}\right)^{\infty}, \mu_{E_C}^{rn}(e^{rn}) \geq \left(\mu_{E_C}^{rn}\right)^{\infty}, \mu_{E_C}^{in}(e^{in}) \geq \left(\mu_{E_C}^{in}\right)^{\infty}.$$

for all pair of vertices in B.

Definition 3.12. A subset B of vertex set V_C is called maximal independent if for every vertex $v = (v^{rp}, v^{ip}, v^{rn}, v^{in}) \in V_C - B$, the set $B \cup \{v\}$ is not independent .

Table 3.1. Edge domination and independent for complex bipolar fuzzy graph in fig.3.1.

Edge	**Path between two vertices**	**Min-max**	**Max-min**	**Strong or not**	**Domination**	**Independent**
$v_1 - v_2$	$v_1 - v_2$ $v_1 - v_5 - v_6 - v_4 - v_2$ $v_1 - v_5 - v_6 - v_3 - v_4 - v_2$ $v_1 - v_5 - v_6 - v_3 - v_2$	$(.1, .2, -.2, -.3)$ $(.0, .1, .0, -.1)$ $(.0, .1, -.1, -.1)$ $(.1, .1, -.1, -.2)$	$(.1, .2, -.2, -.3)$ $(.2, .1, -.1, -.3)$ $(.3, .3, -.3, -.3)$ $(.2, .3, -.1, -.3)$	Not Strong	Not Dominate	Independent

	$v_1 - v_6 - v_4 - v_2$ $v_1 - v_6 - v_3 - v_4 - v_2$ $v_1 - v_6 - v_3 - v_2$	$(.0, .1, .0, -.1)$ $(0, .1, -.1, -.1)$ $(.2, .2, -.1, -.2)$	$(.3, .2, -.1, -.2)$ $(.3, .3, -.3, -.3)$ $(.3, .3, -.1, -.3)$			
$v_1 - v_5$	$v_1 - v_5$ $v_1 - v_6 - v_5$ $v_1 - v_2 - v_3 - v_6 - v_5$ $v_1 - v_2 - v_4 - v_6 - v_5$ $v_1 - v_2 - v_4 - v_3 - v_6 - v_5$ $v_1 - v_2 - v_3 - v_4 - v_6 - v_5$	$(.2, .1, -.1, -.3)$ $(.1, .1, -.1, -.2)$ $(.1, .1, -.1, -.2)$ $(.0, .1, .0, -.1)$ $(.0, .1, -.1, -.1)$ $(.1, .1, .0, -.1)$	$(.2, .1, -.1, -.3)$ $(.3, .2, -.1, -.2)$ $(.2, .3, -.2, -.3)$ $(.1, .2, -.2, -.3)$ $(.3, .3, -.3, -.3)$ $(.3, .2, -.3, -.3)$	Strong	Dominate	Not independent
$v_1 - v_6$	$v_1 - v_6$ $v_1 - v_5 - v_6$ $v_1 - v_2 - v_4 - v_6$ $v_1 - v_2 - v_3 - v_4 - v_6$ $v_1 - v_2 - v_3 - v_6$ $v_1 - v_2 - v_4 - v_3 - v_6$	$(.3, .2, -.1, -.2)$ $(.1, .1, -.1, -.2)$ $(.0, .1, .0, -.1)$ $(.1, .1, .0, -.1)$ $(.1, .2, -.1, -.2)$ $(.0, .1, -.1, -.1)$	$(.3, .2, -.1, -.2)$ $(.2, .1, -.1, -.3)$ $(.1, .2, -.2, -.3)$ $(.3, .2, -.3, -.3)$ $(.2, .3, -.2, -.3)$ $(.3, .3, -.3, -.3)$	Strong	Dominate	Not independent
$v_2 - v_3$	$v_2 - v_3$ $v_2 - v_4 - v_3$ $v_2 - v_4 - v_6 - v_3$ $v_2 - v_1 - v_5 - v_6 - v_3$ $v_2 - v_1 - v_6 - v_3$ $v_2 - v_1 - v_5 - v_6 - v_4 - v_3$ $v_2 - v_1 - v_6 - v_4 - v_3$	$(.2, .2, -.1, -.3)$ $(.0, .1, -.1, -.1)$ $(.0, .1, .0, -.1)$ $(.1, .1, -.1, -.2)$ $(.1, .2, -.1, -.2)$ $(.1, .1, .0, -.1)$ $(.1, .1, .0, -.1)$	$(.2, .2, -.1, -.3)$ $(.3, .1, -.3, -.3)$ $(.2, .3, -.1, -.2)$ $(.2, .3, -.2, -.3)$ $(.3, .3, -.2, -.3)$ $(.3, .2, -.3, -.3)$ $(.3, .2, -.3, -.3)$	Strong	Dominate	Not independent
$v_2 - v_4$	$v_2 - v_4$ $v_2 - v_3 - v_4$ $v_2 - v_3 - v_6 - v_4$ $v_2 - v_1 - v_6 - v_4$ $v_2 - v_1 - v_5 - v_6 - v_4$ $v_2 - v_1 - v_5 - v_6 - v_3 - v_4$ $v_2 - v_1 - v_6 - v_3 - v_4$	$(.0, .1, -.1, -.1)$ $(.2, .1, -.1, -.3)$ $(.1, .1, .0, -.1)$ $(.1, .1, .0, -.1)$ $(.1, .1, .0, -.1)$ $(.1, .1, -.1, -.2)$ $(.1, .1, -.1, -.2)$	$(.0, .1, -.1, -.1)$ $(.3, .2, -.3, -.3)$ $(.2, .3, -.1, -.3)$ $(.3, .2, -.2, -.3)$ $(.2, .2, -.2, -.3)$ $(.3, .3, -.3, -.3)$ $(.3, .3, -.3, -.3)$	Not Strong	Not Dominate	Independent
$v_3 - v_4$	$v_3 - v_4$ $v_3 - v_2 - v_4$ $v_3 - v_6 - v_4$ $v_3 - v_2 - v_1 - v_6 - v_4$ $v_3 - v_2 - v_1 - v_5 - v_6 - v_4$ $v_3 - v_6 - v_5 - v_1 - v_2 - v_4$	$(.3, .1, -.3, -.3)$ $(.0, .1, -.1, -.1)$ $(.1, .1, .0, -.1)$ $(.1, .1\ .0, -.1)$ $(.1, .1, .0, -.1)$ $(.0, .1, -.1, -.1)$	$(.3, .1, -.3, -.3)$ $(.2, .2, -.1, -.3)$ $(.2, .3, -.1, -.2)$ $(.3, .2, -.2, -.3)$ $(.2, .2, -.2, -.3)$ $(.2, .3, -.2, -.3)$	Strong	Dominate	Not independent
$v_3 - v_6$	$v_3 - v_6$ $v_3 - v_2 - v_4 - v_6$ $v_3 - v_2 - v_1 - v_6$ $v_3 - v_4 - v_6$ $v_3 - v_2 - v_1 - v_5 - v_6$	$(.2, .3, -.1, -.2)$ $(.0, .1, .0, -.1)$ $(.1, .2, -.1, -.2)$ $(.1, .1, .0, -.1)$ $(.1, .1, -.1, -.2)$	$(.2, .3, -.1, -.2)$ $(.2, .2, -.1, -.3)$ $(.3, .2, -.2, -.3)$ $(.3, .1, -.3, -.3)$ $(.\ 2, .2, -.2, -.3)$	Strong	Dominate	Not independent

	$v_3 - v_4 - v_2 - v_1 - v_6$	(.0, .1, −.1, −.1)	(.3, .2, −.3, −.3)			
	$v_3 - v_4 - v_2 - v_1 - v_5 - v_6$	(.0, .1, −.1, −.1)	(.3, .2, −.3, −.3)			
$V_4 - v_6$	$v_4 - v_6$	(.1, .1, .0, −.1)	(.1, .1, .0, −.1)	Not Strong	Not Dominate	Independent
	$v_4 - v_3 - v_6$	(.2, .1, −.1, −.2)	(.3, .3, −.3, −.3)			
	$v_4 - v_2 - v_3 - v_6$	(.0, .1, −.1, −.1)	(.2, .3, −.1, −.3)			
	$v_4 - v_3 - v_2 - v_1 - v_6$	(.1, .1, −.1, −.2)	(.3, .2, −.3, −.3)			
	$v_4 - v_3 - v_2 - v_1 - v_5 - v_6$	(.1, .1, −.1, −.2)	(.3, .2, −.3, −.3)			
	$v_4 - v_2 - v_1 - v_6$	(.0, .1, −.1, −.1)	(.3, .2, −.2, −.3)			
	$v_4 - v_2 - v_1 - v_5 - v_6$	(.0, .1, −.1, −.1)	(.2, .2, −.2, −.3)			
$v_5 - v_6$	$v_5 - v_6$	(.1, .1, −.1, −.2)	(.1, .1, −.1, −.2)	Not Strong	Not Dominate	Independent
	$v_5 - v_1 - v_6$	(.2, .1, −.1, −.2)	(.3, .2, −.1, −.3)			
	$v_5 - v_1 - v_2 - v_4 - v_6$	(.0, .1, .0, −.1)	(.2, .2, −.2, −.3)			
	$v_5 - v_1 - v_2 - v_3 - v_4 - v_6$	(.1, .1, .0, −.1)	(.3, .2, −.3, −.3)			
	$v_5 - v_1 - v_2 - v_3 - v_6$	(.1, .1, −.1, −.2)	(.2, .3, −.2, −.3)			
	$v_5 - v_1 - v_2 - v_4 - v_3 - v_6$	(.0, .1, −.1, −.1)	(.3, .3, −.3, −.3)			

Table 3.1. All dominating set, minimal domination set, independent set and maximal independent set for complex bipolar fuzzy graph in fig.3.1.

Subset of V_C	**Dominating set**	**Minimal dominating**	**Independent**	**Maximal independent**
$\{v_1\}$	No	No	Yes	No
$\{v_2\}$	No	No	Yes	No
$\{v_3\}$	No	No	Yes	No
$\{v_4\}$	No	No	Yes	No
$\{v_5\}$	No	No	Yes	No
$\{v_6\}$	No	No	Yes	No
$\{v_1, v_2\}$	No	No	Yes	No
$\{v_1, v_3\}$	Yes	Yes	Yes	No
$\{v_1, v_4\}$	No	No	Yes	No
$\{v_1, v_5\}$	No	No	No	No
$\{v_1, v_6\}$	No	No	No	No
$\{v_2, v_3\}$	No	No	No	No
$\{v_2, v_4\}$	No	No	Yes	No
$\{v_2, v_5\}$	No	No	Yes	No
$\{v_2, v_6\}$	No	No	Yes	No
$\{v_3, v_4\}$	No	No	No	No
$\{v_3, v_5\}$	Yes	Yes	Yes	Yes

$\{v_3, v_6\}$	No	No	No	No
$\{v_4, v_5\}$	No	No	No	No
$\{v_4, v_6\}$	No	No	Yes	No
$\{v_5, v_6\}$	No	No	Yes	No
$\{v_1, v_2, v_3\}$	Yes	No	Yes	Yes
$\{v_1, v_2, v_4\}$	Yes	Yes	Yes	Yes
$\{v_1, v_2, v_5\}$	No	No	No	No
$\{v_1, v_2, v_6\}$	No	No	No	No
$\{v_1, v_3, v_4\}$	Yes	No	No	No
$\{v_1, v_3, v_5\}$	Yes	No	No	No
$\{v_1, v_3, v_6\}$	Yes	No	No	No
$\{v_1, v_4, v_5\}$	No	No	No	No
$\{v_1, v_4, v_6\}$	No	No	No	No
$\{v_1, v_5, v_6\}$	No	No	No	No
$\{v_2, v_3, v_4\}$	No	No	No	No
$\{v_2, v_3, v_5\}$	Yes	No	No	No
$\{v_2, v_3, v_6\}$	No	No	No	No
$\{v_2, v_4, v_5\}$	No	No	Yes	Yes
$\{v_2, v_4, v_6\}$	No	No	Yes	No
$\{v_2, v_5, v_6\}$	No	No	Yes	No
$\{v_3, v_4, v_5\}$	Yes	No	No	No
$\{v_3, v_4, v_6\}$	No	No	No	No
$\{v_3, v_5, v_6\}$	Yes	Yes	No	No
$\{v_4, v_5, v_6\}$	No	No	Yes	No
$\{v_1, v_2, v_3, v_4\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_5\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_4, v_5\}$	Yes	No	No	No
$\{v_1, v_2, v_4, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_5, v_6\}$	No	No	No	No
$\{v_1, v_3, v_4, v_5\}$	Yes	No	No	No
$\{v_1, v_3, v_4, v_6\}$	Yes	No	No	No
$\{v_1, v_3, v_5, v_6\}$	Yes	No	No	No
$\{v_1, v_4, v_5, v_6\}$	No	No	No	No
$\{v_2, v_3, v_4, v_5\}$	Yes	No	No	No
$\{v_2, v_3, v_4, v_6\}$	No	No	No	No

$\{v_2, v_3, v_5, v_6\}$	Yes	No	No	No
$\{v_2, v_4, v_5, v_6\}$	Yes	Yes	Yes	Yes
$\{v_3, v_4, v_5, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_4, v_5\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_4, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_5, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_4, v_5, v_6\}$	Yes	No	No	No
$\{v_1, v_3, v_4, v_5, v_6\}$	Yes	No	No	No
$\{v_2, v_3, v_4, v_5, v_6\}$	Yes	No	No	No
$\{v_1, v_2, v_3, v_4, v_5, v_6\}$	Yes	No	No	No

Definition 3.14. The neighborhood degree of a vertex v_k in $G_C = (V_C, E_C)$ is defined as a ordered four $Ndeg(x) = (N\,deg^{rp}(v_k), N\,deg^{ip}(v_k), N\,deg^{rn}(v_k), N\,deg^{in}(v_k))$, here $N\,deg^{rp}(v_k) = \sum_{y \in N(v_k)} \mu_V^{rp}(y)$, $N\,deg^{ip}(v_k) = \sum_{y \in N(v_k)} \mu_V^{ip}(y)$, $N\,deg^{rn}(v_k) = \sum_{y \in N(v_k)} \mu_V^{rn}(y)$ and $N\,deg^{in}(v_k) = \sum_{y \in N(v_k)} \mu_V^{in}(y)$

Definition 3.15. In a complex bipolar fuzzy graph G_C, the closed neighborhood degree of a vertex v_k is defined as 4- tuples $N\,deg[v_k] = \big(N\,deg^{rp}[x], N\,deg^{ip}[x], N\,deg^{rn}[x], N\,deg^{in}[x],\big)$ such that $Ndeg^{rp}[v_k] = N\,deg^{rp}(v_k) + \mu_E^{rp}(v_k)$, $Ndeg^{ip}[v_k] = N\,deg^{ip}(v_k) + \mu_E^{ip}(v_k)$, $Ndeg^{rn}[v_k] = N\,deg^{rn}(v_k) + \mu_E^{rn}(v_k)$, $Ndeg^{in}[v_k] = N\,deg^{in}(v_k) + \mu_E^{n}(v_k)$,

Theorem 3.16. If D_C is the minimal dominating set of a complex bipolar fuzzy graph $G_C = (V_C, E_C)$ without an isolated vertex, then $V_C - D_C$ is a dominating set of G_C.

Proof. Let us consider a vertex x in D_C. We know that $N(x)$ is the neighborhood of x.Since x is in D_C. Therefore, there exist a vertex v in $N(x)$ dominated atleast one of D_C. Since D_C is minimal dominating. So $v \in V_C - D_C$. Thus, there is a strong edge between every vertex in D_C and at least one vertex in $V_C - D_C$. Therefore, $V_C - D_C$ is dominating set.

Theorem 3.17. Suppose D_C is the set with minimal dominating in a complex bipolar fuzzy graph $G_C = (V_C, E_C)$ equivalently, every $x \in D_C$, either any vertex in D_C that x is not strong neighbor or there is any vertex $v \in V_C - D_C$ such that $N(v) \cup D_C = x$.

Proof. Let us considered D_C be a set with minimal domination of a complex bipolar fuzzy graph G_C. So, we have $D_C - \{x\}$ is not a dominating set for all nodes $x \in D_C$. Hence there exist $v \in (V_C - D_C) \cup \{x\}$ which is not dominated by any nodes in $D_C - \{x\}$.

Case.1 suppose $v \neq x$, v is not dominated by $D_C - \{v\}$, but is dominated by D_C, so the node v is neighbor only to x in D_c. Hence $N(v) \cap D_C = x$.

Conversely, let us take the dominating set D_C and for every node $y \in D_C$, one of the stated conditions holds. Contradictory we take D_C is not dominating minimally, therefore, there exists a vertex $y \in D_C$. $\{y\}$ is domination G_C. Therefore, y is a strong neighbor and one of the nodes $D_C - \{d\}$, no one stated is hold. Suppose $D_C - \{y\}$ is dominating the all the nodes in $V_C - D_C$ is a neighbour to atleast one of nodes in $D_C - \{y\}$, but which contradicts stated second condition. Therefore, possible only for the set D_C is minimal dominating.

Result:

Suppose the complex bipolar fuzzy graph $G_C = (V_C, E_C)$ has no isolated nodes and D_C is dominating minimally. Then the subset $V_c - D_C$ of V_C is dominating complex bipolar fuzzy graph G_C.

Definition 3.18. The Complement of $G_C = (V_C, E_C)$ is denoted by $\overline{G_C} = \overline{G_C}(\bar{V}, \bar{E})$, here

1. $\overline{V_C} = V_C$,
2. $\overline{r^p_{V_C}(v_i)} = r^p_{V_C}(v_i)$ and $\overline{r^n_{V_C}(v_i)} = r^n_{V_C}(v_i)$
3. $\overline{r^p_{E_C}(v_i, v_j)} = \min(r^p_{V_C}(v_i, v_j))$

Theorem 3.19. In complex bipolar fuzzy graph, the inequality $d_C(G_C) + d_C(\overline{G_C}) < 2\big(O(G_C)\big)$ is hold, where $d_C(\overline{G_C})$ is noted as lower domination number of complement complex bipolar fuzzy graph $\overline{G_C}$ if and only if $0 < r^p_{E_C}(v_j, v_k) < \left(r^p_{E_C}\right)^\infty (v_j, v_k)$ and $0 > r^n_{E_C}(v_j, v_k) > \left(r^n_{E_C}\right)^\infty (v_j, v_k)$ for all the nodes in V_C in G_C.

Proof. Let us consider $d_C(G_C) + d_C(\overline{G_C}) < 2(O(G_C)$ holds for all nodes $v_j, v_k \in V_C$.
Case i. Suppose $d_C(G_C) = o(G_C)$ implies $r^p_{E_C}(v_j, v_k) < \left(r^p_{E_C}\right)^\infty (v_j, v_k)$ and $r^n_{E_C}(v_j, v_k) > \left(r^n_{E_C}\right)^\infty (v_j, v_k)$ for all nodes $v_j, v_k \in V_C$.

Case ii. Suppose $d_C(\overline{G_C}) = O(G_C)$ implies $\overline{r^p_{E_C}(v_j, v_k)} - r^p_{E_C}(v_j, v_k) < \left(r^p_{E_C}\right)^\infty (v_j, v_k)$ and $\overline{r^n_{E_C}(v_j, v_k)} - r^n_{E_C}(v_j, v_k) > \left(r^n_{E_C}\right)^\infty (v_j, v_k)$ for all $v_j, v_k \in V_C$. Hence, we have from the above $r^p_{E_C}(v_j, v_k) > 0$ and $r^n_{E_C}(v_j, v_k) < 0$. Furthermore, $d_C(G_C) - d_B(\overline{G_C}) \leq 2O(G_C)$.

4. Conclusion

An overview of the meanings and characteristics of dominance and independence of complicated bipolar fuzzy graphs is given in this study. We anticipate that more intricate bipolar fuzzy graph operations will follow. We'll be interested in utilising its qualities in a variety of applications in the future and observing how the outcomes alter.

References

[1] L.A. Zadeh, Fuzzy sets, Inf. Compute. 8 (1965) 338–353.

[2] L.A. Zadeh, Similarity relations and fuzzy orderings, Inf. Sci. 3 (1971) 177–200.

[3] A. Rosenfeld, L.A. Zadeh, K.S. Fu, M. Shimura, Fuzzy graphs, Fuzzy Sets and their Applications, Academic Press, New York, 1975, pp. 77–95. https://doi.org/10.1016/B978-0-12-775260- 0.50008-6

[4] W.R. Zhang, Bipolar fuzzy sets and relations, a computational framework for cognitive modeling and multiagent decision analysis, Proc. IEEE Conf. (1994) 305–309.http://dx.doi.org/10.1109/IJCF.1994.375115

[5] W.R. Zhang, Bipolar fuzzy sets, in: Proceedings of FUZZ-IEEE, 1998, pp. 835– 840. http://doi:10.1007/s00521- 011-0733-0

[6] Akram, M. (2011). Bipolar fuzzy graphs. Information sciences 181, 5548-5564. https://doi.org/10.1016/j.ins.2011.07. 037

[7] M.G. Karunambigai, M. Akram, K. Palanivel, S. Sivasankar, Domination in Bipolar Fuzzy Graphs, in: FUZZ-IEEE International Conference on Fuzzy Systems, Hyderabad, India, 2013, pp. 1–6. https://doi: 10.1109/FUZZ-IEEE.2013.6622326

[8] N. Vinothkumar, G. Geetharamani, Edge domination in fuzzy graphs, Pensee Multi-Discip. J. 76 (2014) 61–70.

[9] N. Vinothkumar, G. Geetharamani, Vertex edge domination in operation of fuzzy graphs, International Journal of Advanced, Eng. Technol. 7 (2016) 401– 405.