Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 2s (2024)

Computational Analysis of the Framework Evaluation for Sustainable EOQ Considering Emission Tax with Capital Constraints

Ajeet Baraskar[1], Dr. Arun Thakare[2], Dr. Yogesh Deshmukh[3]
[1]G H Raisoni University, Amravati-411701, India
ajeetbaraskar@gmail.com
[2]G H Raisoni College of Engineering & Management, Pune-412207, India
arun.thakare@raisoni.net
[3]Rajiv Gandhi Institute of Technology, Andheri-400053, India
yogesh.deshmukh@mctrgit.ac.in

Article History:

Received: *15-03-2024*

Revised: *14-05-2024*

Accepted: *24-05-2024*

Abstract:

One of the key concerns in production planning has always been determining the economic lot size. Researchers have been studying this issue for a while, and numerous models have been created to meet objectives with the least amount of expense. This study's objective is to assess a more sophisticated approach to deciding lot size by taking into account the costs associated with emissions taxes (environmental impact) with capital constraints. It will be suggested to adopt a framework, called Sustainable Economic Order Quantity (SEOQ), for inventory control in the manufacturing industry.

For the purpose of assisting decision-makers and policies on inventory issues, this study included a useful numerical analysis as well as a sensitivity analysis. Finally, the experimental findings demonstrated that the suggested models would solve the issues with the lowest possible overall inventory costs.

Keywords: Sustainability, Inventory, emissions taxes, capital constraints, SEOQ.

1. INTRODUCTION

In the era of industrial evolution and rapid technological advancements, the equilibrium between economic growth and environmental sustainability has become a paramount concern. Businesses are increasingly under pressure to optimize their operations not only for cost efficiency but also for environmental responsibility. This dual objective is particularly pertinent in inventory management, where the Economic Order Quantity (EOQ) model plays a critical role. Traditionally, EOQ models have focused on minimizing the total cost of inventory, including ordering and holding costs. However, in the context of sustainable development, it is imperative to incorporate environmental considerations into these models.

The main factor that helps a business maintain its seamless functioning is its inventory. These days, environmental concerns are shared by all nations and businesses. Businesses must consider environmental factors, including carbon emissions and capital restrictions.

The SEOQ model is a lot size strategy used to establish economic ordering when dealing with inventory by taking into account environmental factors [1]. The financial and environmental perspectives must be economically balanced within this framework in order for the business community to choose the best policy to promote sustainability.

Numerous research, like Chen, et al. [2], Jaber, et al. [3], dan He, et al. [4], have examined inventory concerns that take carbon emissions into account. The studies often take effects of carbon emissions, order frequency, and storage volume into account [5].

The model was built by several researchers by including the cost of taxes on environmental consequences [6, 7]. In addition, the purchase inventory model was modified by some researchers to include environmental and tax costs [8]. Sustainable EOQ models were examined by Maulana et al. [9] while taking capital restrictions. In previous publications author Baraskar et. al. presented literature survey on inventory management [21] and studied SEOQ considering environmental factors [22].

This research paper presents a comprehensive computational analysis of a framework designed to evaluate the sustainable EOQ model considering emission taxes and capital constraints. The primary objective is to provide a decision-making tool that helps businesses optimize their inventory management practices in a manner that aligns with environmental regulations and financial limitations. By integrating these factors, the proposed framework aims to offer a more holistic approach to EOQ, promoting sustainable business practices without compromising economic viability.

The significance of this study lies in its interdisciplinary approach, merging concepts from environmental economics, operations management, and financial analysis. Traditional EOQ models are typically rooted in operational efficiency, focusing on minimizing costs associated with ordering and holding inventory. However, the incorporation of emission taxes introduces a layer of complexity that necessitates a broader analytical perspective. Emission taxes directly impact the cost structure of inventory management, influencing both the frequency and quantity of orders. Therefore, the proposed framework not only evaluates the economic aspects but also addresses the environmental impacts of inventory decisions.

Furthermore, capital constraints add another layer of complexity to the EOQ model. Limited financial resources can restrict a company's ability to purchase in bulk or invest in sustainable practices, such as adopting cleaner technologies or sourcing eco-friendly materials. This constraint must be carefully balanced with the goal of minimizing total inventory costs, including emission tax liabilities. The framework proposed in this study incorporates financial constraints into the EOQ model, ensuring that the recommendations are practical and applicable to businesses with varying financial capabilities.

2. FRAMEWORKS

In the current study, author assessed the SEOQ framework, which took into account the expenses associated with the sustainable inventory, including order cost, purchase cost, holding cost, the fixed cost of an environmental effect (carbon emission tax cost) for each cycle, and capital constraint. This study's goal is to assess a more complex approach for solving the issues of calculating lot size by taking environmental concerns, capital constraints of raw materials and purchasing of emission tax into account.

As a result, our research evaluated perspectives on inventories, particularly for SEOQ models with the capital of raw material purchase with (2) /without (1) emission tax, which is achieved by adding a constraint function i.e., capital constraint.

$$\mathrm{M} = (\mathrm{c} + \mathrm{v}) * \mathrm{Q} \qquad 1$$

$$\mathrm{M} = (\mathrm{c} + \mathrm{pv}) * \mathrm{Q} \qquad 2$$

2.1 ASSUMPTIONS

* Demand rate (λ) is predictable, constant, and uniformly dispersed over the course of the year.
* Every demand is met on schedule.
* All of the model's variables remain constant over time.
* The impact on the environment is taken into account for all costs.
* The planning horizon is infinitely long.
* A fixed cost (k + f) is incurred each and every time an order is placed. Each unit of inventory has a holding cost in inventory of (h + g). And the cost per unit of purchase is (c + v).
* Each model is applied to a single emission product, and the tax cost per item
* Tax price is per unit of emissions
* Capital used is the capital for the purchase of raw materials and purchasing of emission tax

2.2 FRAMEWORK 1: Sustainable EOQ without Emission Tax with Capital Constraint

Evaluation of the SEOQ with capital constraint and without emission tax is our aim. Study evaluated the problems of determining the lot size by considering capital constraints for purchasing raw of materials. In this study Lagrange function is used to minimize the total inventory cost against the capital constraint. The model of total inventory cost as evaluated in previous study [22] is as shown in equation (3) is added to the constraint function in equation (1). Langrange function of the proposes SEOQ model without tax and with capital constraint is shown in equation (4)

$$\mathrm{TIC} = \text{Ordering cost} + \text{Purchasing cost} + \text{Holding cost}$$

$$\mathrm{TIC(Q)} = \frac{(\mathrm{k} + \mathrm{f}) * \lambda}{\mathrm{Q}} + (\mathrm{c} + \mathrm{v}) * \lambda + \frac{(\mathrm{h} + \mathrm{g}) * \mathrm{Q}}{2} \qquad 3$$

Applying Langrange function to minimize total inventory cost against the constraint by adding the total inventory cost with capital constraint.

$$TIC(Q, \beta) = \mathrm{TIC(Q)} - \beta * constraint\ function \qquad 4$$

From equation (1), constraint function will be

$$(\mathrm{c} + \mathrm{v}) * \mathrm{Q} - \mathrm{M} = 0 \qquad 5$$

$$L(Q, \beta) = \mathrm{TIC(Q, \beta)} = \frac{(\mathrm{k} + \mathrm{f}) * \lambda}{\mathrm{Q}} + (\mathrm{c} + \mathrm{v}) * \lambda + \frac{(\mathrm{h} + \mathrm{g}) * \mathrm{Q}}{2} - \beta(Q(c + v) - M) \qquad 6$$

Now differentiate partially to Q, β and equate first differentiation to zero to get minimum cost.

Equation (6) is differentiated with respect to Q to obtain the optimal Q for the SEOQ model with capital constraint without tax, and the first derivative is then set equal to zero to obtain the minimal cost.

$$\frac{d}{dQ}[TIC(Q,\beta)] = \frac{d}{dQ}\left[\frac{(k+f)*\lambda}{Q} + (c+v)*\lambda + \frac{(h+g)*Q}{2} - \beta(Q(c+v)-M)\right] = 0 \quad 7$$

$$(k+f)\lambda\frac{d}{dQ}\left[\frac{1}{Q}\right] + \frac{d}{dQ}[(c+v)\lambda] + \frac{(h+g)}{2}\frac{d}{dQ}[Q] - \beta(c+v)\frac{d}{dQ}[Q] + \frac{d}{dQ}[\beta M] = 0 \quad 8$$

$$-(k+f)\lambda\frac{\frac{d}{dQ}[Q]}{Q_{sm}^2} + 0 + \frac{(h+g)}{2}*1 - \beta(c+v)*1 + 0 = 0 \quad 9$$

$$-\frac{(k+f)\lambda}{Q_{sm}^2} + \frac{(h+g)}{2} - \beta(c+v) = 0 \quad 10$$

$$\frac{(k+f)\lambda}{Q_{sm}^2} = \frac{(h+g)}{2} - \beta(c+v) \quad 11$$

$$\frac{(k+f)\lambda}{Q_{sm}^2} = \frac{(h+g) - 2\beta(c+v)}{2} \quad 12$$

$$\frac{2(k+f)\lambda}{Q_{sm}^2} = (h+g) - 2\beta(c+v) \quad 13$$

$$Q_{sm}^2 = \frac{2\lambda(k+f)}{(h+g) - 2\beta(c+v)} \quad 14$$

$$Q_{sm} = \sqrt{\frac{2\lambda(k+f)}{(h+g) - 2\beta(c+v)}} \quad 15$$

Now differentiate equation (6) partially with respect to β and equate first differentiation to zero to get minimum cost.

$$\frac{d}{d\beta}[TIC(Q,\beta)\,] = \frac{d}{d\beta}\left[\frac{(k+f)*\lambda}{Q} + (c+v)*\lambda + \frac{(h+g)*Q}{2} - \beta(Q(c+v) - M)\right] = 0 \quad 16$$

$$\frac{d}{d\beta}\left[\frac{(k+f)\lambda}{Q}\right] + \frac{d}{d\beta}[(c+v)\lambda] + \frac{d}{d\beta}\left[\frac{(h+g)Q}{2}\right] - Q(c+v)\frac{d}{d\beta}[\beta] + M\frac{d}{d\beta}[\beta] = 0 \quad 17$$

$$0 + 0 + 0 - Q_{sm}(c+v)*1 + M*1 = 0 \quad 18$$

$$M - Q_{sm}(c+v) = 0 \quad 19$$

$$M = Q_{sm}(c+v) \quad 20$$

$$Q_{sm} = \frac{M}{(c+v)} \quad 21$$

Equating equation [15] and [21]

$$Q_{sm} = \frac{M}{(c+v)} = \sqrt{\frac{2(k+f)\lambda}{(h+g) - 2\beta(c+v)}} \quad 22$$

$$\frac{M^2}{(c+v)^2} = \frac{2\lambda(k+f)}{(h+g) - 2\beta(c+v)} \quad 23$$

$$M^2((h+g) - 2\beta(c+v)) = 2\lambda(k+f)(c+v)^2 \quad 24$$

$$M^2(h+g) - 2\beta M^2(c+v) = 2\lambda(k+f)(c+v)^2 \quad 25$$

$$2\beta M^2(c+v) = M^2(h+g) - 2\lambda(k+f)(c+v)^2 \quad 26$$

$$\beta = \frac{M^2(h+g) - 2\lambda(k+f)(c+v)^2}{2M^2(c+v)} \quad 27$$

Further, substitute equation (15) and (27) in equation (6) to calculate the total inventory cost with capital constraint without emission tax (TIC(Qsm))

From figure 2, At Optimal quantity ordering cost and holding cost are same.

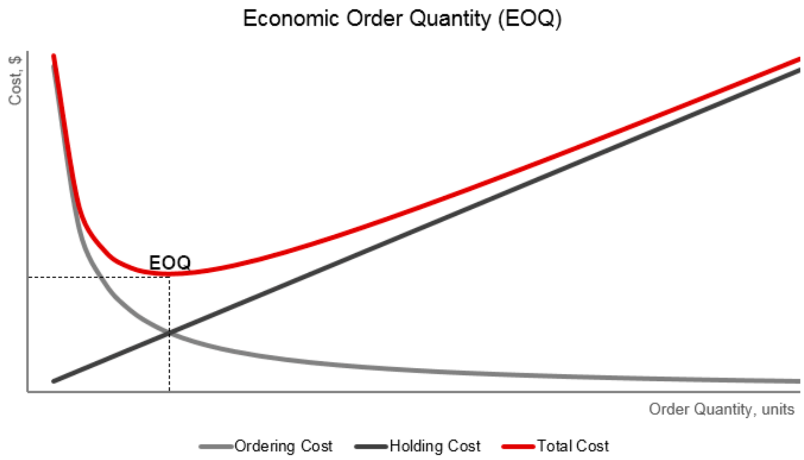

Figure 2. Ordering and holding costs as functions of the order quantity.

So equation (6) becomes

$$\mathrm{TIC(Qsm)} = \frac{2(\mathrm{k}+\mathrm{f})*\lambda}{\mathrm{Qsm}} + (\mathrm{c}+\mathrm{v})*\lambda - \beta(Qsm(c+v)-M) \tag{28}$$

Further substituting constraint equation (20) in equation (28), we will get TIC(Qsm) as

$$\mathrm{TIC(Qsm)} = \frac{2(\mathrm{k}+\mathrm{f})*\lambda}{\mathrm{Qsm}} + (\mathrm{c}+\mathrm{v})*\lambda - \beta*0 \tag{29}$$

$$\mathrm{TIC(Qsm)} = \lambda(\mathrm{c}+\mathrm{v}) + \frac{2\lambda(\mathrm{k}+\mathrm{f})}{\mathrm{Qsm}} \tag{30}$$

Further substituting equation (15) in equation (30), we will get TIC(Qsm) as

$$\mathrm{TIC(Qsm)} = \lambda(\mathrm{c}+\mathrm{v}) + \frac{2\lambda(\mathrm{k}+\mathrm{f})}{\sqrt{\frac{2\lambda(k+f)}{(h+g)-2\beta(c+v)}}} \tag{31}$$

$$\mathrm{TIC(Qsm)} = \lambda(\mathrm{c}+\mathrm{v}) + 2\lambda(\mathrm{k}+\mathrm{f})\sqrt{\frac{(h+g)-2\beta(c+v)}{2\lambda(k+f)}} \tag{32}$$

$$\mathrm{TIC(Qsm)} = \lambda(\mathrm{c}+\mathrm{v}) + \sqrt{\frac{\left(2\lambda(\mathrm{k}+\mathrm{f})\right)^2((h+g)-2\beta(c+v))}{2\lambda(k+f)}} \tag{33}$$

$$\mathrm{TIC(Qsm)} = \lambda(\mathrm{c}+\mathrm{v}) + \sqrt{2\lambda(\mathrm{k}+\mathrm{f})((h+g)-2\beta(c+v))} \tag{34}$$

2.3 FRAMEWORK 2: Sustainable EOQ with Emission Tax with Capital Constraints

Evaluation of the SEOQ with capital constraint and emission tax is our aim. Study evaluated the problems of determining the lot size by considering sustainability and capital constraints for purchasing raw of materials. In this study Lagrange function is used to minimize the total inventory cost against the capital constraint. The model of total inventory cost as evaluated in previous study [22] is as shown in equation (35) is added to the constraint function in equation (2). Langrange function of the proposes SEOQ model with tax and capital constraint is shown in equation (36)

$$\mathrm{TIC} = \text{Ordering cost} + \text{Purchasing cost} + \text{Holding cost}$$

$$\mathrm{TIC(Q)} = \frac{(\mathrm{k}+\mathrm{pf})*\lambda}{\mathrm{Q}} + (\mathrm{c}+\mathrm{pv})*\lambda + \frac{(\mathrm{h}+\mathrm{pg})*\mathrm{Q}}{2} \qquad 35$$

Applying Langrange function to minimize total inventory cost against the constraint by adding the total inventory cost with capital constraint.

$$TIC(Q,\beta) = \mathrm{TIC(Q)} - \beta * constraint\ function \qquad 36$$

From equation (2), constraint function will be

$$(\mathrm{c}+\mathrm{pv})*\mathrm{Q} - \mathrm{M} = 0 \qquad 37$$

$$L(Q,\beta) = \mathrm{TIC(Q,\beta)} = \frac{(\mathrm{k}+\mathrm{pf})*\lambda}{\mathrm{Q}} + (\mathrm{c}+\mathrm{pv})*\lambda + \frac{(\mathrm{h}+\mathrm{pg})*\mathrm{Q}}{2} - \beta(Q(c+pv)-M) \qquad 38$$

Now differentiate partially to Q, β and equate first differentiation to zero to get minimum cost.

Equation (38) is differentiated with respect to Q to obtain the optimal Q for the SEOQ model with capital constraint with tax, and the first derivative is then set equal to zero to obtain the minimal cost.

$$\frac{\mathrm{d}}{\mathrm{dQ}}[\mathrm{TIC(Q,\beta)}] = \frac{\mathrm{d}}{\mathrm{dQ}}\left[\frac{(\mathrm{k}+\mathrm{pf})*\lambda}{\mathrm{Q}} + (\mathrm{c}+\mathrm{pv})*\lambda + \frac{(\mathrm{h}+\mathrm{pg})*\mathrm{Q}}{2} - \beta(Q(c+pv)-M)\right] = 0 \qquad 39$$

$$(\mathrm{k}+\mathrm{pf})\lambda\frac{\mathrm{d}}{\mathrm{dQ}}\left[\frac{1}{\mathrm{Q}}\right] + \frac{\mathrm{d}}{\mathrm{dQ}}[(\mathrm{c}+\mathrm{pv})\lambda] + \frac{(\mathrm{h}+\mathrm{pg})}{2}\frac{\mathrm{d}}{\mathrm{dQ}}[\mathrm{Q}] - \beta(c+pv)\frac{\mathrm{d}}{\mathrm{dQ}}[Q] + \frac{\mathrm{d}}{\mathrm{dQ}}[\beta M] = 0 \qquad 40$$

$$-(k+pf)\lambda\frac{\frac{\mathrm{d}}{\mathrm{dQ}}[Q]}{\mathrm{Q}_{spm}^{2}}+0+\frac{(\mathrm{h}+\mathrm{pg})}{2}*1-\beta(c+pv)*1+0=0 \quad 41$$

$$-\frac{(k+pf)\lambda}{\mathrm{Q}_{spm}^{2}}+\frac{(\mathrm{h}+\mathrm{pg})}{2}-\beta(c+pv)\ =0 \quad 42$$

$$\frac{(k+pf)\lambda}{\mathrm{Q}_{spm}^{2}}=\frac{(\mathrm{h}+\mathrm{pg})}{2}-\beta(c+pv) \quad 43$$

$$\frac{(k+pf)\lambda}{\mathrm{Q}_{spm}^{2}}=\frac{(\mathrm{h}+\mathrm{pg})-2\beta(c+pv)}{2} \quad 44$$

$$\frac{2(k+pf)\lambda}{\mathrm{Q}_{spm}^{2}}=(h+pg)-2\beta(c+pv) \quad 45$$

$$\mathrm{Q}_{spm}^{2}=\frac{2\lambda(k+pf)}{(h+pg)-2\beta(c+pv)} \quad 46$$

$$Q_{spm}=\sqrt{\frac{2\lambda(k+pf)}{(h+pg)-2\beta(c+pv)}} \quad 47$$

Now differentiate equation (38) partially with respect to β and equate first differentiation to zero to get minimum cost.

$$\frac{\mathrm{d}}{\mathrm{d}\beta}[\mathrm{TIC}(\mathrm{Q},\beta)\ ] = \frac{\mathrm{d}}{\mathrm{d}\beta}\left[\frac{(\mathrm{k}+\mathrm{pf})*\lambda}{\mathrm{Q}}+\ (\mathrm{c}+\mathrm{pv})*\lambda+\frac{(\mathrm{h}+\mathrm{pg})*\mathrm{Q}}{2} -\beta(Q(c+pv)-M)\right]=0 \quad 48$$

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\left[\frac{(\mathrm{k}+\mathrm{pf})\lambda}{\mathrm{Q}}\right]+\frac{\mathrm{d}}{\mathrm{d}\beta}[(\mathrm{c}+\mathrm{pv})\lambda]+\frac{\mathrm{d}}{\mathrm{d}\beta}\left[\frac{(\mathrm{h}+\mathrm{pg})Q}{2}\right]-Q(c+pv)\frac{\mathrm{d}}{\mathrm{d}\beta}[\beta] +\mathrm{M}\frac{\mathrm{d}}{\mathrm{d}\beta}[\beta]=0 \quad 49$$

$$0+0+0-Q_{spm}(c+pv)*1+\mathrm{M}*1=0 \quad 50$$

$$M-Q_{spm}(c+pv)\ =0 \quad 51$$

$$M=Q_{spm}(c+pv) \quad 52$$

$$Q_{spm} = \frac{M}{(c + pv)} \quad 53$$

Equating equation [47] and [53]

$$Q_{spm} = \frac{M}{(c + pv)} = \sqrt{\frac{2(k + pf)\lambda}{(h + pg) - 2\beta(c + pv)}} \quad 54$$

$$\frac{M^2}{(c + pv)^2} = \frac{2\lambda(k + pf)}{(h + pg) - 2\beta(c + pv)} \quad 55$$

$$M^2((h + pg) - 2\beta(c + pv)) = 2\lambda(k + pf)(c + pv)^2 \quad 56$$

$$M^2(h + pg) - 2\beta M^2(c + pv) = 2\lambda(k + pf)(c + pv)^2 \quad 57$$

$$2\beta M^2(c + pv) = M^2(h + pg) - 2\lambda(k + pf)(c + pv)^2 \quad 58$$

$$\beta = \frac{M^2(h + pg) - 2\lambda(k + pf)(c + pv)^2}{2M^2(c + pv)} \quad 59$$

Further, substitute equation (47) and (59) in equation (38) to calculate the total inventory cost with capital constraint with emission tax (TIC(Qspm))

At Optimal quantity ordering cost and holding cost are same.

So equation (38) becomes

$$\mathrm{TIC(Qspm)} = \frac{2(\mathrm{k + pf}) * \lambda}{\mathrm{Qspm}} + (\mathrm{c + pv}) * \lambda - \beta(Qspm(c + pv) - M) \quad 60$$

Further substituting constraint equation (52) in equation (60), we will get TIC(Qspm) as

$$\mathrm{TIC(Qspm)} = \frac{2(\mathrm{k + pf}) * \lambda}{\mathrm{Qspm}} + (\mathrm{c + pv}) * \lambda - \beta * 0 \quad 61$$

$$\mathrm{TIC(Qspm)} = \lambda(\mathrm{c + pv}) + \frac{2\lambda(\mathrm{k + pf})}{\mathrm{Qspm}} \quad 62$$

Further substituting equation (47) in equation (62), we will get TIC(Qspm) as

$$\mathrm{TIC(Qspm)} = \lambda(\mathrm{c + pv}) + \frac{2\lambda(\mathrm{k + pf})}{\sqrt{\frac{2\lambda(k + pf)}{(h + pg) - 2\beta(c + pv)}}} \quad 63$$

$$TIC(Qspm) = \lambda(c + pv) + 2\lambda(k + pf)\sqrt{\frac{(h + pg) - 2\beta(c + pv)}{2\lambda(k + pf)}} \quad 64$$

$$TIC(Qspm) = \lambda(c + pv) + \sqrt{\frac{\left(2\lambda(k + pf)\right)^2((h + pg) - 2\beta(c + pv))}{2\lambda(k + pf)}} \quad 65$$

$$TIC(Qspm) = \lambda(c + pv) + \sqrt{2\lambda(k + pf)((h + pg) - 2\beta(c + pv))} \quad 66$$

2.4 NUMERICAL EXAMPLE

This section shows the numerical experiment procedure on the proposed SEOQ models. The experiment was carried out to test the sensitivity of the proposed models. A case study data is presented in Table 1.

Table 1. Experimental Data Variables

	Data Variables	Unit	Value
Demand	λ	qty	50
Cost per order	k	$/order	40
Purchasing cost per unit	c	$/qty	20
Emission tax cost	p	$/qty	2
Holding cost per unit	h	$/qty	10
Total emissions from ordering	f	$/qty CO2	60
Total emissions from purchasing	v	$/qty CO2	5
Total emissions from holding	g	$/qty CO2	1
Capital	M	$	1000

2.5 RESULTS AND DISCUSSION

The jamovi project (2022). jamovi. (Version 2.3) Software is used for solving frameworks and checking numerical sensitivity and sustainability.

2.5.1 Sustainable EOQ without Emission Tax with Capital Constraints

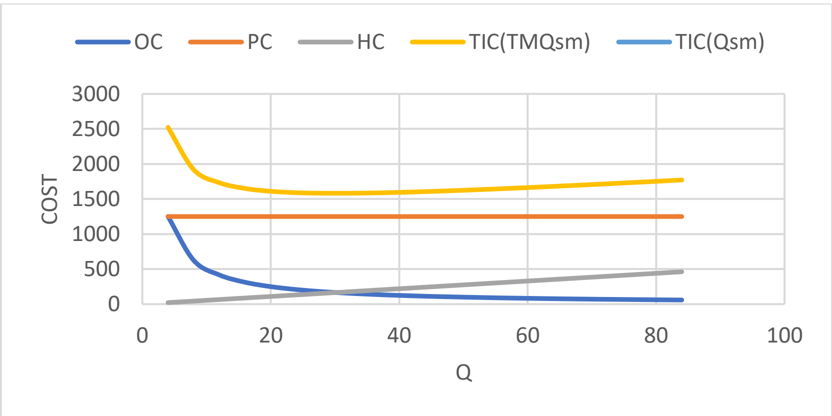

Figure 3. Ordering and holding costs as functions of the order quantity.

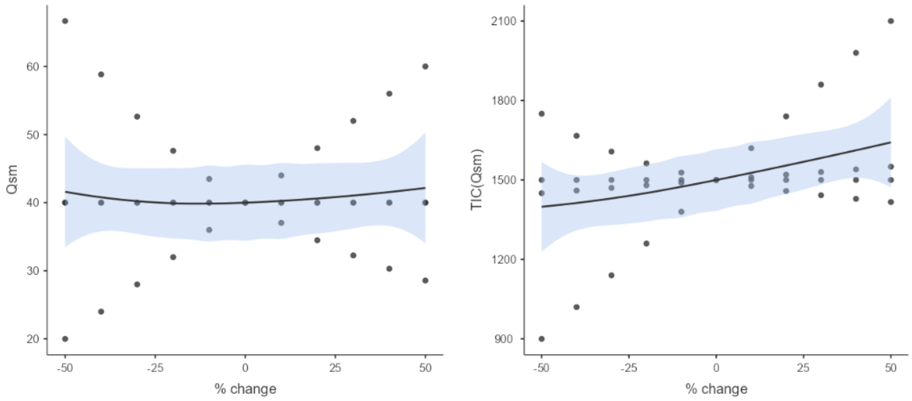

Figure 4. Effect of % Change in k, c, h on Qsm and TIC(Qsm) along with standard error (Std. Err.).

2.5.2 Sustainable EOQ with Emission Tax with Capital Constraints

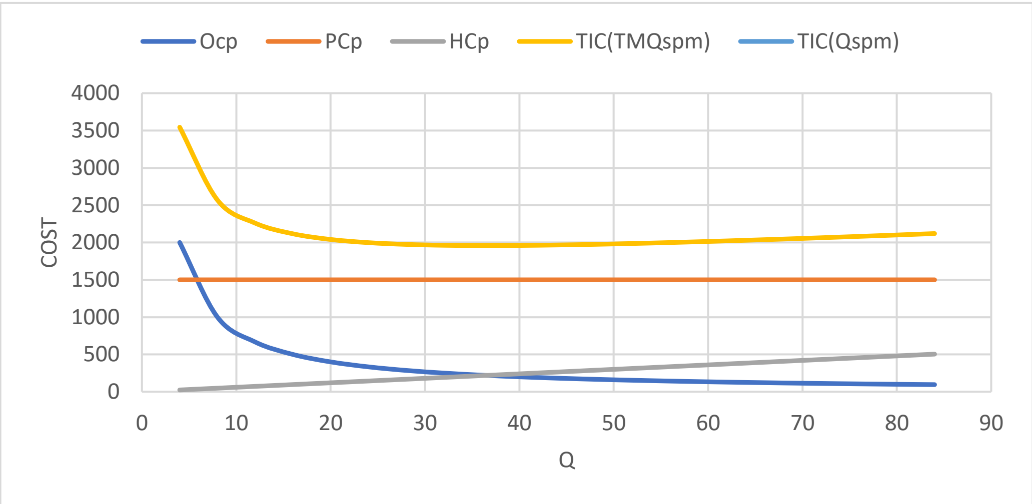

Figure 5. Ordering and holding costs as functions of the order quantity.

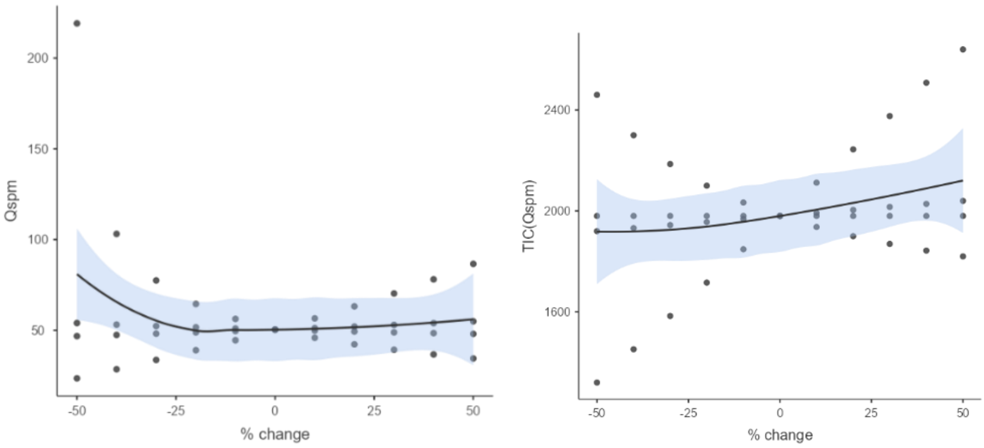

Figure 6. Effect of % Change in k, c, h on Qspm and TIC(Qspm) along with standard error (Std. Err.).

2.6 SENSITIVITY ANALYSIS

2.6.1 Sensitivity

Table 2: Change in Qsm, Qspm, TIC(Qsm) and TIC(Qspm) due to change in cost per order(k).

% Change	k	f	λ	c	v	h	g	p	M	β	Qsm	TIC(Qsm)	B	Qspm	TIC(Qspm)
-50	20	60	50	20	5	10	1	2	1000	0.120	40	1450	-0.010	54	1920
-40	24	60	50	20	5	10	1	2	1000	0.115	40	1460	-0.016	53	1932
-30	28	60	50	20	5	10	1	2	1000	0.110	40	1470	-0.022	52	1944
-20	32	60	50	20	5	10	1	2	1000	0.105	40	1480	-0.028	52	1956
-10	36	60	50	20	5	10	1	2	1000	0.100	40	1490	-0.034	51	1968
0	40	60	50	20	5	10	1	2	1000	0.095	40	1500	-0.040	50	1980
10	44	60	50	20	5	10	1	2	1000	0.090	40	1510	-0.046	50	1992
20	48	60	50	20	5	10	1	2	1000	0.085	40	1520	-0.052	49	2004
30	52	60	50	20	5	10	1	2	1000	0.080	40	1530	-0.058	49	2016
40	56	60	50	20	5	10	1	2	1000	0.075	40	1540	-0.064	48	2028
50	60	60	50	20	5	10	1	2	1000	0.070	40	1550	-0.070	48	2040

Table 3: Change in Qsm, Qspm, TIC(Qsm) and TIC(Qspm) due to change in purchase cost per unit(c).

% Change	k	f	λ	c	v	h	g	p	M	β	Qsm	TIC(Qsm)	B	Qspm	TIC(Qspm)
-50	40	60	50	10	5	10	1	2	1000	0.292	67	900	0.140	219	1320
-40	40	60	50	12	5	10	1	2	1000	0.239	59	1020	0.097	103	1452
-30	40	60	50	14	5	10	1	2	1000	0.194	53	1140	0.058	77	1584
-20	40	60	50	16	5	10	1	2	1000	0.157	48	1260	0.023	65	1716
-10	40	60	50	18	5	10	1	2	1000	0.124	43	1380	-0.010	56	1848
0	40	60	50	20	5	10	1	2	1000	0.095	40	1500	-0.040	50	1980
10	40	60	50	22	5	10	1	2	1000	0.069	37	1620	-0.069	46	2112
20	40	60	50	24	5	10	1	2	1000	0.045	34	1740	-0.096	42	2244
30	40	60	50	26	5	10	1	2	1000	0.022	32	1860	-0.121	39	2376
40	40	60	50	28	5	10	1	2	1000	0.002	30	1980	-0.146	37	2508
50	40	60	50	30	5	10	1	2	1000	-0.018	29	2100	-0.170	35	2640

Table 4: Change in Qsm, Qspm, TIC(Qsm) and TIC(Qspm) due to change in holding cost per unit(h).

% Change	k	f	λ	c	v	h	g	p	M	β	Qsm	TIC(Qsm)	B	Qspm	TIC(Qspm)
-50	40	60	50	20	5	5	1	2	1000	-0.005	40	1500	-0.123	47	1980
-40	40	60	50	20	5	6	1	2	1000	0.015	40	1500	-0.107	47	1980
-30	40	60	50	20	5	7	1	2	1000	0.035	40	1500	-0.090	48	1980
-20	40	60	50	20	5	8	1	2	1000	0.055	40	1500	-0.073	49	1980
-10	40	60	50	20	5	9	1	2	1000	0.075	40	1500	-0.057	50	1980
0	40	60	50	20	5	10	1	2	1000	0.095	40	1500	-0.040	50	1980
10	40	60	50	20	5	11	1	2	1000	0.115	40	1500	-0.023	51	1980
20	40	60	50	20	5	12	1	2	1000	0.135	40	1500	-0.007	52	1980
30	40	60	50	20	5	13	1	2	1000	0.155	40	1500	0.010	53	1980
40	40	60	50	20	5	14	1	2	1000	0.175	40	1500	0.027	54	1980
50	40	60	50	20	5	15	1	2	1000	0.195	40	1500	0.043	55	1980

Table 5: Change in Qsm, Qspm, TIC(Qsm) and TIC(Qspm) due to change in Capital(M).

% Change	k	f	λ	c	v	h	g	p	M	β	Qsm	TIC(Qsm)	B	Qspm	TIC(Qspm)
-50	40	60	50	20	5	10	1	2	500	-0.280	20	1750	-0.760	24	2460
-40	40	60	50	20	5	10	1	2	600	-0.127	24	1667	-0.467	29	2300
-30	40	60	50	20	5	10	1	2	700	-0.035	28	1607	-0.290	34	2186
-20	40	60	50	20	5	10	1	2	800	0.025	32	1563	-0.175	39	2100
-10	40	60	50	20	5	10	1	2	900	0.066	36	1528	-0.096	45	2033
0	40	60	50	20	5	10	1	2	1000	0.095	40	1500	-0.040	50	1980
10	40	60	50	20	5	10	1	2	1100	0.117	44	1477	0.002	57	1936
20	40	60	50	20	5	10	1	2	1200	0.133	48	1458	0.033	63	1900
30	40	60	50	20	5	10	1	2	1300	0.146	52	1442	0.058	70	1869
40	40	60	50	20	5	10	1	2	1400	0.156	56	1429	0.078	78	1843
50	40	60	50	20	5	10	1	2	1500	0.164	60	1417	0.093	87	1820

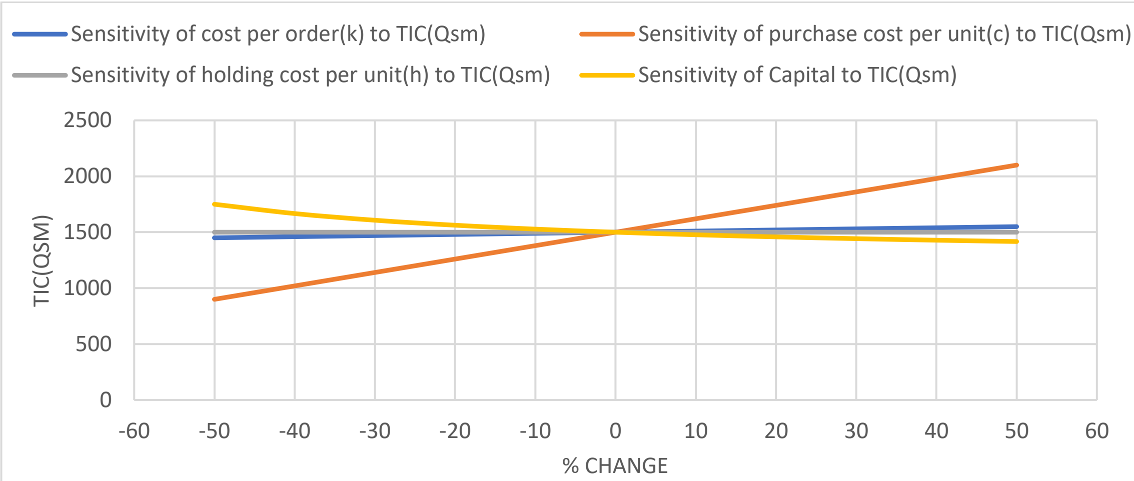

Figure 7. Change in TIC(Qsm) due to change in cost per order(k), purchase(c), holding(h) cost per unit and capital(M).

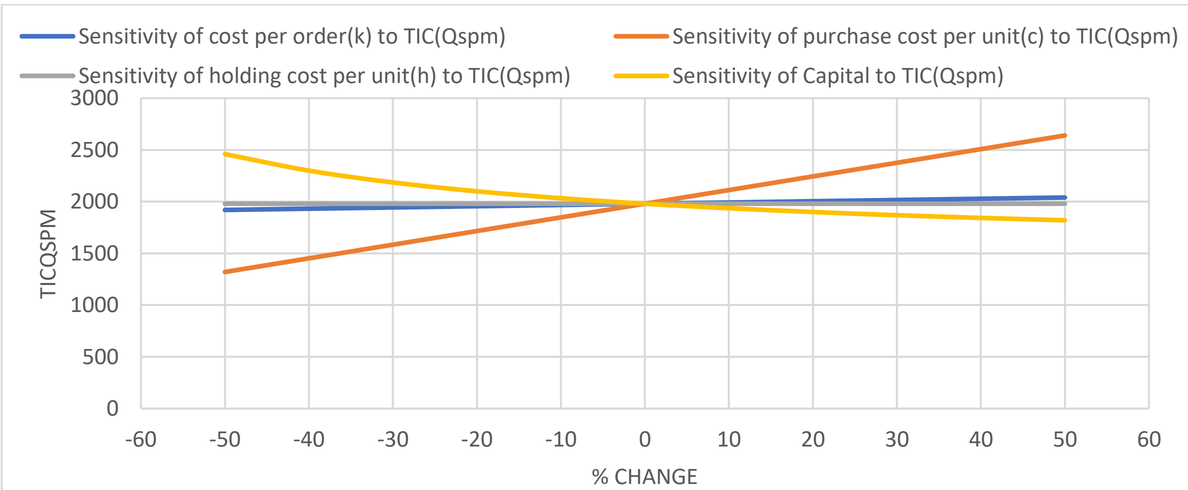

Figure 8. Change in TIC(Qspm) due to change in cost per order(k), purchase(c) and holding(h) cost per unit and capital(M).

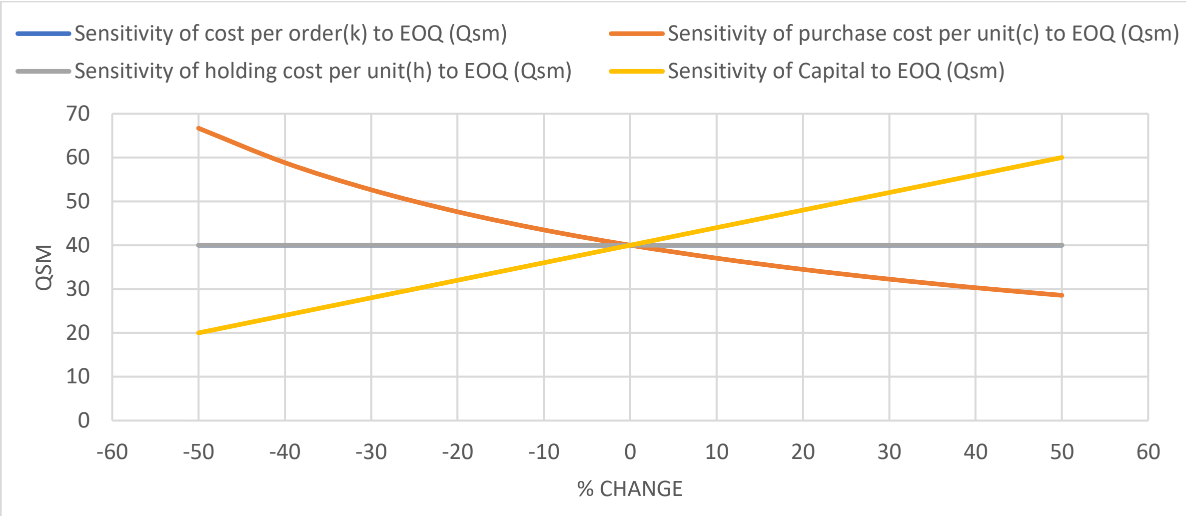

Figure 9. Change in Qsm due to change in cost per order, purchase and holding cost per unit and capital.

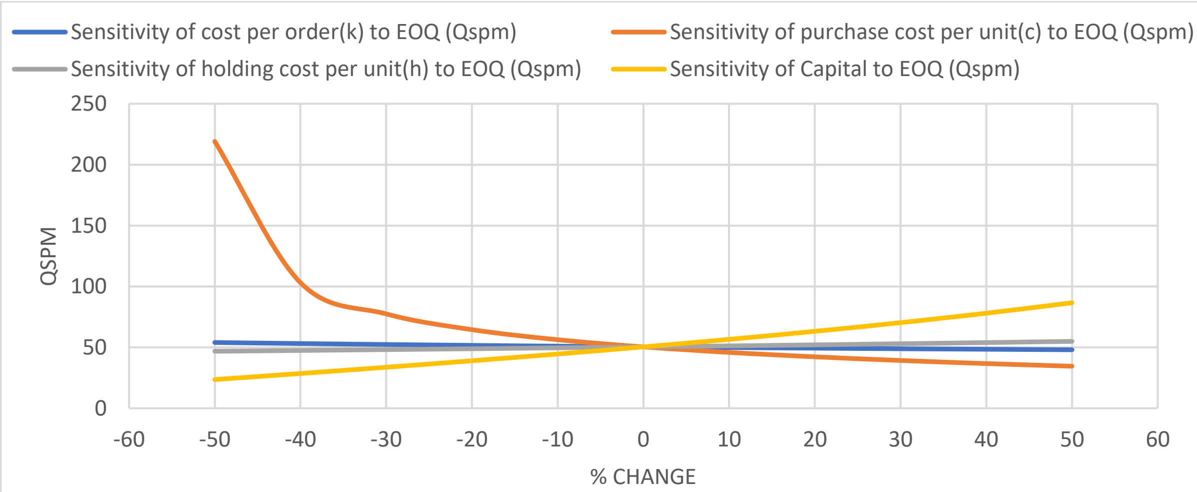

Figure 10. Change in Qspm due to change in cost per order, purchase and holding cost per unit and capital.

2.6.2 ANOVA

Table 6: ANOVA.

ANOVA – TIC(Qspm)					
	Sum of Squares	**df**	**Mean Square**	**F**	**p**
Overall Model	0	30	0	1.98	0.096
K	0	NaN			
c	0	0			
h	0	0			
k✻c	0	0			
k✻h	0	0			
c✻h	0	0			
k✻c✻h	0	0			
Residuals	427398	13	32877		

2.6.3 Assumptions Checks

Table 7: Homogeneity of Variances & Normality Test

Homogeneity of Variances Test (Levene's)			
F	**df1**	**df2**	**p**
0.444	30	13	0.967

Normality Test (Shapiro-Wilk)	
Static	**P**
0.642	<0.001

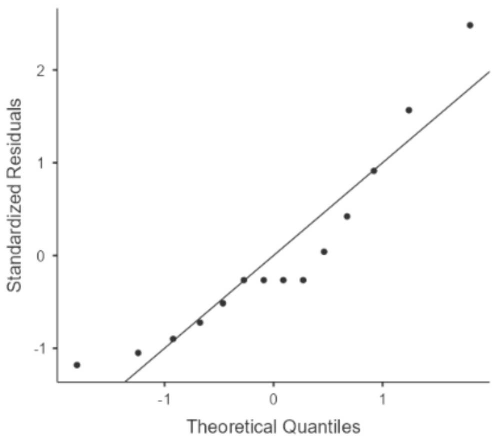

Figure 11. Q-Q Plot.

2.6.4 Correlation Matrix

Table 8: Correlation Matrix

		Qsm	Qspm	TIC(Qsm)	TIC(Qspm)	k	c	h	M
Qsm	Pearson's r	-							
	p-value	-							
	95% CI Upper	-							
	95% CI Lower	-							
	N	-							
Qspm	Pearson's r	0.817	-						
	p-value	<0 .001	-						
	95% CI Upper	0.897	-						
	95% CI Lower	0.687	-						
	N	44	-						
TIC(Qsm)	Pearson's r	-0.811	-0.746	-					
	p-value	<0 .001	<0 .001	-					
	95% CI Upper	-0.677	-0.576	-					
	95% CI Lower	-0.893	-0.854	-					
	N	44	44	-					
TIC(Qspm)	Pearson's r	-0.883	-0.758	0.984	-				
	p-value	<0 .001	<0 .001	<0.001	-				
	95% CI Upper	-0.795	-0.594	0.991	-				
	95% CI Lower	-0.935	-0.861	0.971	-				
	N	44	44	44	-				

k	Pearson's r p-value 95% CI Upper 95% CI Lower N	0.000 1.000 0.297 -0.297 44	-0.033 0.834 0.267 -0.326 44	0.080 0.605 0.368 -0.222 44	0.082 0.599 0.370 -0.221 44	- - - - -				
c	Pearson's r p-value 95% CI Upper 95% CI Lower N	-0.656 <0 .001 -0.446 -0.797 44	-0.683 <0 .001 -0.484 -0.815 44	0.961 <0.001 0.978 0.929 44	0.897 <0.001 0.943 0.819 44	0.000 1.000 0.297 -0.297 44	- - - - -			
h	Pearson's r p-value 95% CI Upper 95% CI Lower N	0.000 1.000 0.297 -0.297 44	0.045 0.773 0.337 -0.256 44	0.000 1.000 0.297 -0.297 44	0.000 1.000 0.297 -0.297 44	0.000 1.000 0.297 -0.297 44	0.000 1.000 0.297 -0.297 44	- - - - -		
M	Pearson's r p-value 95% CI Upper 95% CI Lower N	0.724 <0.001 0.840 0.544 44	0.344 0.022 0.581 0.052 44	-0.245 0.108 0.055 -0.505 44	-0.400 0.007 -0.117 -0.623 44	0.000 1.000 0.297 -0.297 44	0.000 1.000 0.297 -0.297 44	0.000 1.000 0.297 -0.297 44	- - - - -	

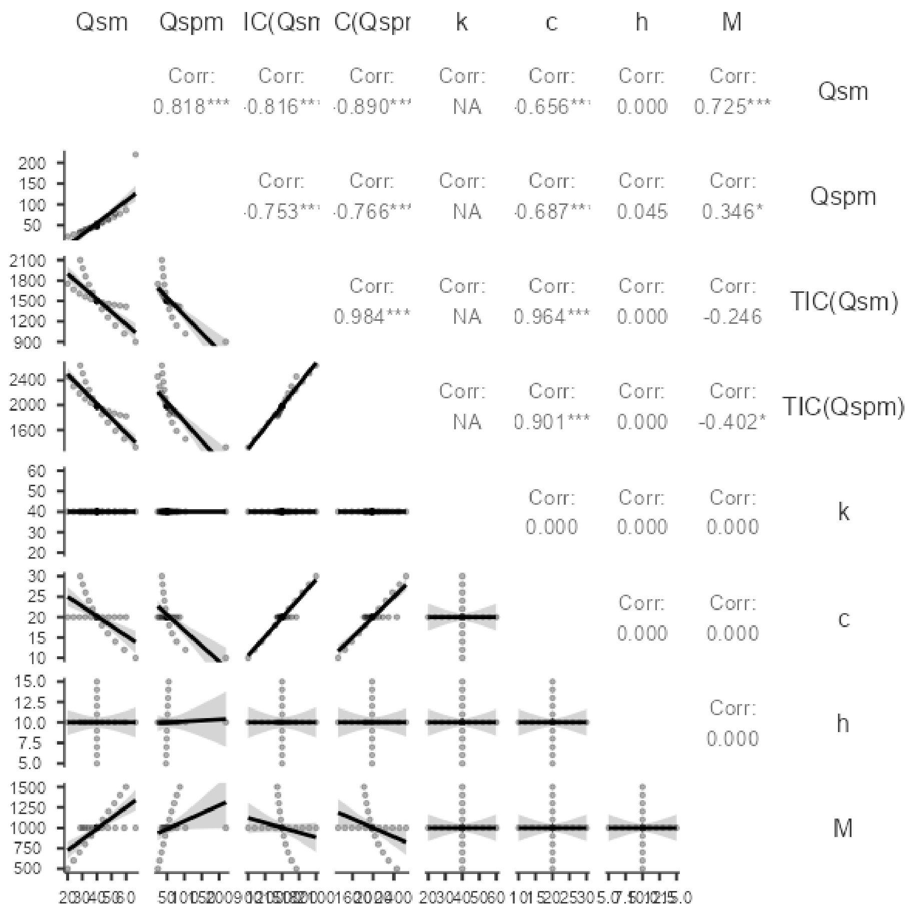

Figure 12. Correlation Matrix Plot.

Table 9: Descriptives

	k	c	h	M	Qsm	TIC(Qsm)	Qspm	TIC (Qspm)
N	44	44	44	44	44	44	44	44
Mean	40.0	20.0	10.0	1000	40.7	1508	55.9	1995
Median	40.0	20.0	10.0	1000	40.0	1500	50.4	1980
Standard Deviation	6.40	3.20	1.60	160	8.83	200	28.9	235
Minimum	20	10	5	500	20.0	900	23.6	1320
Maximum	60	30	15	1500	66.7	2100	219	2640

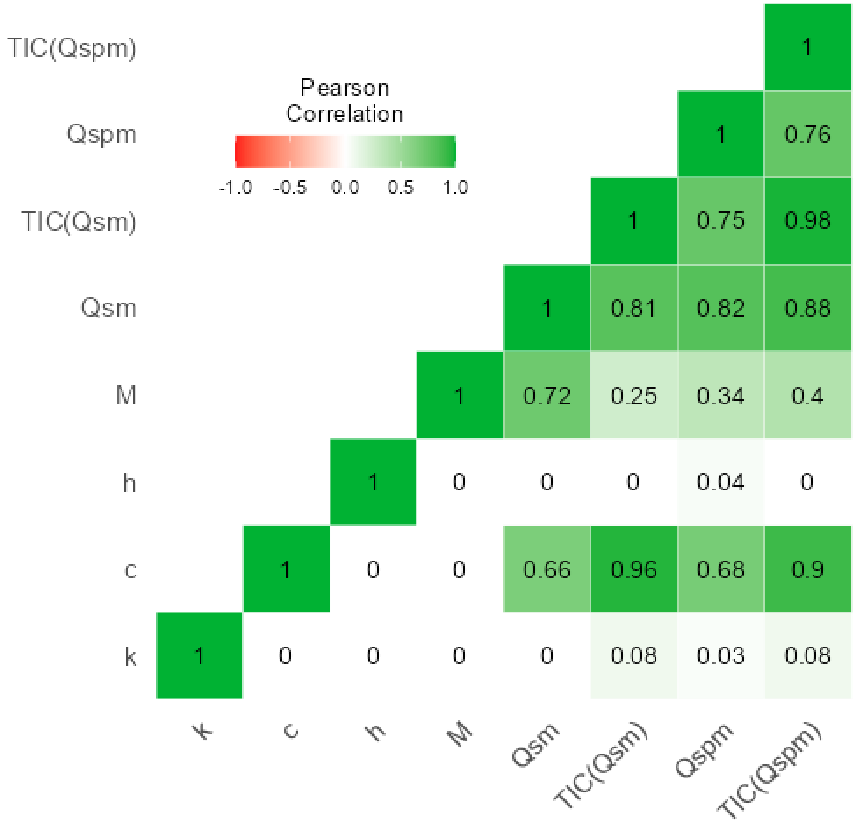

Figure 13. Correlation Heatmap.

2.7 CONCLUSION

In the current study, SEOQ framework is evaluated, which took into account the expenses associated with the sustainable inventory, including order cost, purchase cost, holding cost, fixed cost of an environmental effect (carbon emission tax cost) and capital required for raw materials and purchasing of emission tax into account. As a result, research produced perspectives on inventories, particularly for SEOQ models with cost of the emission tax and capital constraints. Further research can be done on study of different use cases SEOQ for different industries.

REFERENCES:

[1] Battini, D., Persona, A., & Sgarbossa, F. (2014). A sustainable EOQ model: Theoretical formulation and applications. International Journal of Production Economics, 149, 145–153. doi: 10.1016/j.ijpe.2013.06.026.

[2] X. Chen, S. Benjaafar, & A. Elomri (2013). The carbon-constrained EOQ. Operations Research Letters, vol. 41, pp. 172-179. https://doi.org/10.1016/j.orl.2012.12.003.

[3] M. Y. Jaber, M. Bonney, & H. Jawad (2017). Comparison between economic order/manufacture quantity and just-in-time models from a thermodynamics point of view. Computers & Industrial Engineering, vol. 112, pp. 503-510. https://doi.org/10.1016/j.cie.2016.08.023.

[4] P. He, W. Zhang, X. Xu, and Y. Bian (2015). Production lot-sizing and carbon emissions under cap-and-trade and carbon tax regulations. Journal of Cleaner Production, vol. 103, pp. 241-248. https://doi.org/10.1016/j.jclepro.2014.08.102.

[5] A. Gurtu, M. Y. Jaber, and C. Searcy (2015), Impact of fuel price and emissions on inventory policies. Applied Mathematical Modelling, vol. 39, pp. 1202-1216. https://doi.org/10.1016/j.apm.2014.08.001.

[6] Arslan, M. C., & Turkay, M. (2013). EOQ Revisited with Sustainability Considerations. Foundations of Computing and Decision Sciences, 38(4), 223–249. doi:10.2478/fcds-2013-0011.

[7] M. Bonney and M. Y. Jaber (2011), Environmentally responsible inventory models: Non-classical models for a non-classical era. International Journal of Production Economics, vol. 133, pp. 43-53, https://doi.org/10.1016/j.ijpe.2009.10.033.

[8] D.-H. Lee, M. Dong, & W. Bian (2010). The design of sustainable logistics network under uncertainty. International Journal of Production Economics, vol. 128, pp. 159-166, 2010. https://doi.org/10.1016/j.ijpe.2010.06.009.

[9] Maulana, S. K. D. B., Utama, D. M., Asrofi, M. S., Ningrum, I. S., Alba, N., Ahfa, H. A., & Zein, T. A. (2019). The Capacitated Sustainable EOQ Models: Models Considering Tax Emissions. Jurnal Teknik Industri, 21(1), 12-21. https://doi.org/10.22219/JTIUMM.Vol21.No1.12-21.

[10] Hovelaque, V., & Bironneau, L. (2015). The carbon-constrained EOQ model with carbon emission dependent demand. International Journal of Production Economics, 164, 285–291. doi: 10.1016/j.ijpe.2014.11.022.

[11] Hua, G., Cheng, T. C. E., & Wang, S. (2011). Managing carbon footprints in inventory management. International Journal of Production Economics, 132(2), 178–185. doi: 10.1016/j.ijpe.2011.03.024.

[12] Paul, S., Wahab, M. I. M., & Cao, X. F. (2014). Supply chain coordination with energy price uncertainty, carbon emission cost, and product return. In Handbook of EOQ Inventory Problems (pp. 179-199). Springer, Boston, MA.

[13] S. Ruidas, M. R. Seikh, P.K. Nayak (2021). A production inventory model with interval-valued carbon emission parameters under price-sensitive demand. Computer & Industrial Engineering 107154. https://doi.org/10.1016/j.cie.2021.107154.

[14] X. Ma, P. Ji, W. Ho, & C. H. Yang (2018). Optimal procurement decision with a carbon tax for the manufacturing industry. Computers & Operations Research, vol. 89, pp. 360-368. https://doi.org/10.1016/j.cor.2016.02.017.

[15] N. Absi, S. Dauzère-Pérès, S. Kedad-Sidhoum, B. Penz, & C. Rapine (2016). The single-item green lot-sizing problem with fixed carbon emissions. European Journal of Operational Research, vol. 248, pp. 849-855. https://doi.org/10.1016/j.ejor.2015.07.052.

[16] The jamovi project (2022). jamovi. (Version 2.3) [Computer Software]. Retrieved from https://www.jamovi.org.

[17] R Core Team (2021). R: A Language and environment for statistical computing. (Version 4.1) [Computer software]. Retrieved from https://cran.r-project.org. (R packages retrieved from MRAN snapshot 2022-01-01).

[18] R Core Team (2018). A Language and Envionment for Statistical Computing. [Computer software]. Retrieved from https://cran.r-project.org/.

[19] Fox, J., & Weisberg, S. (2020). car: Companion to Applied Regression. [R package]. Retrieved from https://cran.rproject.org/package=car.

[20] Revelle, W. (2019). psych: Procedures for Psychological, Psychometric, and Personality Research. [R package]. Retrieved from https://cran.r-project.org/package=psych.

[21] A. Baraskar, Y. Deshmukh (2023). A literature survey on inventory management in manufacturing industry. Journal Of Aeronautical Materials. Vol. 43, Issue-1. Pp. 203-234. https://www.hkclxb.cn/article/view/2023/203.html.

[22] A. Baraskar, Y. Deshmukh, A. Thakare (2023). A framework Evolution for Sustainable EOQ Considering Environmental Factors. Journal Of Harbin Engineering University. Vol. 44, Issue-8. Pp. 56-68. https://harbinengineeringjournal.com/index.php/journal/article/view/826.

THE NOTATIONS:

: Demand

k : Cost per order

c : Purchasing cost per unit

h : Holding cost per unit

g : Total emissions from holding

v : Total emissions from purchasing

f : Total emissions from ordering

p : Emission tax cost

β : Lagrange multipliers

M : capital

Q : Number of orders

Qs : Optimal sustainable number of orders without tax

Qsp : Optimal sustainable number of orders with tax

Qsm : Optimal sustainable number of orders without tax with capital constraint

Qspm : Optimal sustainable number of orders with tax and capital constraint

TIC : Total inventory cost

TIC(Qs) : Optimum total inventory cost without tax

TIC(Qsp) : Optimum total inventory cost with tax

TIC(Qsm) : Optimum total inventory cost without tax with capital constraints

TIC(Qspm) : Optimum total inventory cost with tax and capital constraint