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Composition of an Algebraic Structure from Fuzzy Set Through Binary operation

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Abstract:

In this paper, we first defined binary operation on the set of membership function values. Then, we developed algebraic structure called group through binary operation. We determined the order of an element of the fuzzy set as a group.

KEYWORDS: Binary Operation, Algebraic structure, cyclic group, Fuzzy Set, Fuzzy group.

1. INTRODUTION

The concept of fuzzy set was introduced by Zadeh **[1]**, and the notion of fuzzy subgroups was introduced by Rosenfeld **[2]**, who showed how some basic notions of group theory could be extended in an elementary manner to fuzzy groups. The purpose of this paper is to introduce some basic concepts of fuzzy algebra, some examples of fuzzy set as a group using binary operation on set of membership function values **[3]**.

2. PRELIMINARIES

2.1 Definition 1. Let $(G,*)$ be an algebraic system, where $*$ is a binary operation. $(G,*)$ is called a group if the following conditions are satisfied:

(i) $*$ is an associative operation.

(ii) There is an identity.

(ii) Every element in G have inverse.

2.2 Definition 2. Let X be a non empty set, called universal set ,then a fuzzy set on X is defined as a collection of order of pairs $A = \{(x, \mu_A(x)) | x \in X\}$ where μ_A is a function$\mu_A(x): X \to [0,1]$,called the membership function.

2.3 Definition 3. Let G be a group. A fuzzy set A of G is said to be fuzzy subgroup if it is satisfying the following axioms:

$(i)\ \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

$(ii)\mu_A(x^{-1}) \geq \mu_A(x)$ for all $x, y \in G$

2.4 Definition 4. Let A fuzzy set of a set X, For $t \in [0,1]$, the level subset of A is the set

$A_t = \{x \in X \mid \mu_A(x) \geq t\}$ **.**This is called a fuzzy level subset of A.

2.5 Definition 5. Let A be a fuzzy subgroup of a group G . The subgroup A_t of G for $t \in [0,1]$ such that $t \leq \mu_A(e)$ is called a level subgroup of G .

2.6 Definition 6. A fuzzy subgroup A of a group G is called fuzzy normal if $\mu_A(xy) = \mu_A(yx)$.

3. FUZZY SET AS A GROUP.

3.1 EXAMPLE 1. Let $A = \{(a, 0), (b, 0.5), (c, 1)\}$,Define binary operation $*$ on A as following

$(x, \mu(x)) * (y, \mu(y)) = (p, \mu(p))$ Where $(p, \mu(p)) \in A$ such that

$\mu(p) = \{\mu(x) + \mu(y) \quad if\ \mu(x) + \mu(y) \leq 1\ \mu(x) + \mu(y) - 1 \quad if\ \mu(x) + \mu(y) > 1$

Table

$*$	$(a, 0)$	$(b, 0.5)$	$(c, 1)$
$(a, 0)$	$(a, 0)$	$(b, 0.5)$	$(c, 1)$
$(b, 0.5)$	$(b, 0.5)$	$(b, 0.5)$	$(b, 0.5)$
$(c, 1)$	$(c, 1)$	$(b, 0.5)$	$(c, 1)$

Here $(x, \mu(x)) * (y, \mu(y)) = (y, \mu(y)) * (x, \mu(x))$,

For each $(x, \mu(x)), (y, \mu(y)), (z, \mu(z)) \in A$

$\{(x, \mu(x)) * (y, \mu(y))\} * (z, \mu(z)) = (x, \mu(x)) * \{(y, \mu(y)) * (z, \mu(z))\}$,

Also we have,

$(a, 0) * (a, 0) = (a, 0)$

$(a, 0) * (b, 0.5) = (b, 0.5)$

$(a, 0) * (c, 1) = (c, 1)$

$(a, 0)$ is an identity element

Thus $(A\ ,\ *)$ is monoid.

3.2 EXAMPLE 2. Let X be a non empty set, called universe of discourse then a fuzzy set B on X is

$B = \{x \in X,\ \mu_B(x): X \to [0,1]\ be\ a\ one-one\ function\}$

Define binary operation $*$ on B as following

$\left(x, \mu_B(x)\right) * \left(y, \mu_B(y)\right) = \left(p, \mu_B(p)\right)$ Where $\left(p, \mu_B(p)\right) \in B$ such that $\mu_B(p) = \frac{\mu_B(x)+\mu_B(y)}{1+\mu_B(x)\mu_B(y)}$

$i.e., x * y = p$ Where $p \in B$ such that $\mu_B(p) = \mu_B(x * y) = \frac{\mu_B(x)+\mu_B(y)}{1+\mu_B(x)\mu_B(y)}$

$$\mu_B\{(x * y) * z\} = \frac{\mu_B(x*y)+\mu_B(z)}{1+\mu_B(x*y)\mu_B(z)} = \frac{\frac{\mu_B(x)+\mu_B(y)}{1+\mu_B(x)\mu_B(y)}+\mu_B(z)}{1+\frac{\mu_B(x)+\mu_B(y)}{1+\mu_B(x)\mu_B(y)}\mu_B(z)} = \frac{\mu_B(x)+\mu_B(y)+\mu_B(z)+\mu_B(x)\mu_B(y)\mu_B(z)}{\mu_B(x)\mu_B(y)+\mu_B(y)\mu_B(z)+\mu_B(x)\mu_B(z)+1}$$

Similarly $\mu_B\{x * (y * z)\} = \frac{\mu_B(x)+\mu_B(y)+\mu_B(z)+\mu_B(x)\mu_B(y)\mu_B(z)}{\mu_B(x)\mu_B(y)+\mu_B(y)\mu_B(z)+\mu_B(x)\mu_B(z)+1}$

$\therefore \mu_B\{(x * y) * z\} = \ \mu_B\{x * (y * z)\}$

Let $e \in X$ such that $\mu_B(e) = 0$

For any $x \in X$, $\mu_B(x * e) = \frac{\mu_B(x)+\mu_B(e)}{1+\mu_B(x)\mu_B(e)} = \frac{\mu(x)+0}{1+0} = \mu(x)$

$x * e = x \quad (\because \mu_B(x): X \to [0,1]\ is\ one-one\ function)$

$\therefore \left(e, \mu_B(e)\right) = (e, 0)$ is an identity element.

Thus $(B\ ,\ *)$ is monoid

3.3 EXAMPLE 3. Let X be a non-empty set , called universe of discourse then a fuzzy set A on X is $C = \{x \in X,\ \mu_C(x): X \to [0,1)\ be\ a\ one-one\ function\}$

$(x, \mu_C(x)\ ) * \left(y, \mu_C(y)\right) = \left(p, \mu_C(p)\right)$ Where $\left(p, \mu_C(p)\right) \in B$ such that

$\mu_C(p) = \mu_C(x) + \mu_C(y) - [\mu_C(x) + \mu_C(y)]$ Where $[a]$ is the greatest integer less then or equal to a

$i.e., x * y = p$ Where $p \in C$ such that $\mu_C(p) = \mu_C(x * y) = \mu_C(x) + \mu_C(y) - [\mu_C(x) + \mu_C(y)]$

$\mu_c\{(x * y) * z\} = \ \mu_c\{x * (y * z)\}$

Let $e \in X$ such that $\mu_B(e) = 0$

For any $x \in X$,

$\mu_c(x * e) = \mu_C(x) + \mu_C(e) - [\mu_C(x) + \mu_C(e)] = \mu_c(x) + 0 - [\mu_C(x) + 0] = \mu_c(x)$

$x * e = x\ (\because \mu_c(x): X \rightarrow [0,1)\ is\ one - one\ function)$

And take $y \in X$ Such that $\mu_c(y) = 1 - \mu_c(x)$

$\mu_C(x) + \mu_C(y) - [\mu_C(x) + \mu_C(y)] = \mu_C(x) + 1 - \mu_c(x) - [\mu_C(x) + 1 - \mu_c(x)] = 0 = \mu_C(e)$

$\mu_c(x * y) = \mu_c(e)$

$x * y = e\ (\because \mu_c(x): X \rightarrow [0,1)\ is\ one - one\ function)$

Thus $(C\ ,\ *)$ is a group

3.4 EXAMPLE 4. Let c be a positive integer, define $*: (-c, c) \times (-c, c) \rightarrow (-c, c)$ as $x * y = \frac{x+y}{1+\left(\frac{xy}{c^2}\right)}$ then $((-c, c), *)$ is a group and define $f: (0,1) \rightarrow (-c, c)$ as $f(x) = 2cx - c$,

$D = \{x \in X,\ \mu_D(x): X \rightarrow (0,1)\ be\ a\ one - one\ and\ onto\ function\}$

Also define binary operation $\cdot$ on D as

$(x, \mu_D(x)) \cdot (y, \mu_D(y)) = (p, \mu_D(p))$ Where $(p, \mu_D(p)) \in D$ such that

$\mu_D(p) = f^{-1}\{f(\mu_D(x)) * f(\mu_D(y))\}$

$i.e., x * y = p$ Where $p \in D$ such that $\mu_D(p) = \mu_D(x \cdot y) = f^{-1}\{f(\mu_D(x)) * f(\mu_D(y))\}$

Now we show that $(D, \cdot)$ is agroup.

$$\mu_D\{(x \cdot y) \cdot z\} = f^{-1}\{f(\mu_D(x \cdot y)) * f(\mu_D(z))\}$$

$$= f^{-1}\left\{f\left(f^{-1}\left(f(\mu_D(x)) * f(\mu_D(y))\right)\right) * f(\mu_D(z))\right\}$$

$$= f^{-1}\left\{\left(f(\mu_D(x)) * f(\mu_D(y))\right) * f(\mu_D(z))\right\}$$

$$= f^{-1}\left\{f(\mu_D(x)) * \left(f(\mu_D(y)) * f(\mu_D(z))\right)\right\}$$

$$= f^{-1}\left\{f(\mu_D(x)) * f\left(f^{-1}\left(f(\mu_D(y)) * f(\mu_D(z))\right)\right)\right\}$$

$$= f^{-1}\{f(\mu_D(x)) * f(\mu_D(y \cdot z))\}$$

$\mu_D\{(x \cdot y) \cdot z\} = \mu_D\{x \cdot (y \cdot z)\}$

$(x \cdot y) \cdot z = x \cdot (y \cdot z)$

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Let $e \in X$ such that $\mu_D(e) = \frac{1}{2}$

For any $x \in X$,

$$\mu_D(x \cdot e) = f^{-1}\left(f\big(\mu_D(x)\big) * f\big(\mu_D(e)\big)\right)$$

$$= f^{-1}\left(f\big(\mu_D(x)\big) * f\left(\tfrac{1}{2}\right)\right)$$

$$= f^{-1}\big(f\big(\mu_D(x)\big) * 0\big)$$

$$= f^{-1}\left(f\big(\mu_D(x)\big)\right) \qquad (\because 0\ is\ identity\ element\ of\ the\ group\big((-c,c),*\big)\ )$$

$$= \mu_D(x)$$

$x \cdot e = x$

Now $\mu_D\left[x \cdot \mu^{-1}\left\{f^{-1}\left(-f\big(\mu_D(x)\big)\right)\right\}\right] = f^{-1}\left[f\big(\mu_D(x)\big) * f\left\{\mu\left(\mu^{-1}f^{-1}\left(-f\big(\mu_D(x)\big)\right)\right)\right\}\right]$

$$= f^{-1}\left[f\big(\mu_D(x)\big) * f\left\{f^{-1}\left(-f\big(\mu_D(x)\big)\right)\right\}\right]$$

$$= f^{-1}\big[f\big(\mu_D(x)\big) * -f\big(\mu_D(x)\big)\big] = f^{-1}(0) = \tfrac{1}{2} = \mu_D(e)$$

$$x \cdot \mu^{-1}\left\{f^{-1}\left(-f\big(\mu_D(x)\big)\right)\right\} = e$$

$$x^{-1} = \mu^{-1}\left\{f^{-1}\left(-f\big(\mu_D(x)\big)\right)\right\}$$

$(D,\cdot)$ is a group.

3.5 Theorem 1 : If $([0,1],*_1)$ is a group ,fuzzy set $A = \{x \in X,\ \mu_A(x): X \to [0,1]\ be\ a\ one - one\ and\ onto\ function\}$ then $(A,*)$ is a group where $\big(x,\mu_A(x)\big) * \big(y,\mu_A(y)\big) = \big(p,\mu_A(p)\big)$ Where $\big(p,\mu_A(p)\big) \in A$ such that $\mu_A(p) = \mu_A(x) *_1 \mu_A(y)$

Proof. Here $x * y = p$ Where $p \in A$ such that $\mu_A(p) = \mu_A(x * y) = \mu_A(x) *_1 \mu_A(y)$

Let $x, y, z \in X$

$\mu_A\big((x * y) * z\big) = \mu_A(x * y) *_1 \mu_A(z)$

$= \big(\mu_A(x) *_1 \mu_A(y)\big) *_1 \mu_A(z) = \mu_A(x) *_1 (\mu_A(y) *_1 \mu_A(z)) = \mu_A(x) *_1 \mu_A(y * z) = \mu_A\big(x * (y * z)\big)$

$(x * y) * z = x * (y * z)$

Let e_1 be a identity element of the group$([0,1],*_1)$ and $e_2 \in X$ such that $\mu_A(e_2) = e_1$

$\mu_A(x * e_2) = \mu_A(x) *_1 \mu_A(e_2) = \mu_A(x) *_1 e_1 = \mu_A(x)$

$x * e_2 = x$ for any $x \in X$

Also let $x \in X$ then $\mu_A(x) \in [0,1]$

There is a $y \in [0,1]$ such that $\mu_A(x) *_1 y = e_1$ and there is $z \in X$ *such that* $\mu_A(z) = y$
$\mu_A(x) *_1 y = e_1$

$\mu_A(x) *_1 \mu_A(z) = e_1$

$\mu_A(x * z) = \mu_A(e_2)$

$x * z = e_2$

$(A,*)$ is a group

3.6 Theorem 2 . If $([0,1],*_1)$ is a group, fuzzy set $A = \{x \in X,\ \mu_A(x): X \to [0,1]$ *be a one − one and onto function*$\}$,define

$(x, \mu_A(x)) * (y, \mu_A(y)) = (p, \mu_A(p))$ Where $(p, \mu_A(p)) \in A$ such that $\mu_A(p) = \mu_A(x) *_1 \mu_A(y)$,

If m is an order of $\mu_A(x)$in the group$([0,1],*_1)$ then m is an order of x in the group$(A,*)$

Proof.

m is an order of $\mu_A(x)$in the group$([0,1],*_1)$

$\underbrace{\mu_A(x) *_1 \mu_A(x) *_1 \mu_A(x) *_1 \dots\dots\dots\dots\dots *_1 \mu_A(x)}_{m\ times} = e_1$

$\mu_A(x * x * x * \dots\dots\dots\dots\dots .. x) = \mu_A(e_2)$

$\underbrace{x * x * x * \dots\dots\dots\dots\dots .. x}_{m\ times} = e_2$

m is an order of x in the group$(A,*)$

3.7 EXAMPLE 5. Let $A = \{(a,0),(b,0.1),(c,0.2),(d,0.3),(e,0.4)\}$ and

$$\mu_A(x * y) = \frac{1}{10}\{remainder\ of\ (10\mu_A(x) + 10\mu_A(y))\ after\ divison\ by\ 5\}$$

$*$	a	b	c	d	e
a	a	b	c	d	e

b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c
e	e	a	b	c	d

Then $(A,*)$ is a group, a is an identity element

Also $A = \{(b^n, remainder\ of\ n\ after\ division\ by\ 5)\}$

$(A,*)$ is a cyclic group.

4. CONCLUSION:

This paper discusses the fundamentals of the fuzzy set and the group it presents the theorems to develop algebraic structure and to determine the order an element of the fuzzy group. Consequently, this paper developed the cyclic group $(A,*)$ from the fuzzy set A through binary operation.

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