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d^{dpM}-DISTANCE IN SNAKE RELATED GRAPHS

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Abstract:

For two vertices u and v of a graph G, the usual distance d(u,v), is the length of the shortest path between u and v. In this paper we study the concept of d^{dpM}- distance in snake related graph. We study some properties of snake related graph with this new distance. We define the eccentricities of vertices, radius and diameter of snake related graph with respect to the d^{dpM}-distance. We compare the usual, geodesic and d^{dpM}-distances of two vertices u,v of V.

Keywords:

Geodesic distance, d^{dpM} - distance, d^{dpM} - Eccentricity, d^{dpM} - Radius and d^{dpM} - Diameter.

1. Introduction

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notations (for any unexplained notation and terminology we refer [2]) V(G) or V is the vertex set of G and E(G) or E is the edge set of G = G(V, E). Let u, v be two vertices of G. The standard or usual distance d(u,v) between u and v is the length of the shortest u - v path in G. Chartrand et al [3] introduced the concept of detour distance in graphs as follows: For two vertices u, v in a graph G, the detour distance D(u,v) is defined as the length of the longest u - v path in G. In this article we study the new distance, which was introduced by us ie) d^{dpM} - distance and study some of the properties of snake related graphs. Chartand et al introduced the concept of detour distance by considering the length of the longest path between u and v. Kathiresan et al [4] introduced the concept of superior distance and signal distance. Goldern Ebenezer et al [13] introduced the concept of d^d - distance. In some of these distances only the lengths of various paths were considered. We study the concept of product mean d^{dpM}- distance in snake related graphs .

2. d^{dpM}- DISTANCE IN GRAPHS

Definition 2.1

Let *u*, *v* be two vertices of a connected graph *G*. Then the product mean d^{dpM} - *length* of a *u* - *v* path defined as $d^{dpM}(u,v) = d(u, v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)\deg(v)}{2} \right\rceil$ or $\left\lfloor \frac{\deg(u)\deg(v)}{2} \right\rfloor$, where $d(u, v)$ is the shortest distance between the vertices *u* and *v*. The d^{dpM} - *distance* between two vertices *u* and *v* is defined as the d^{dpM} - length of a *u* - *v* path.

Definition 2.2

The d^{dpM} - eccentricity of any vertex v, $e^{d^{dpM}}(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^{d^{dpM}}(v) = \max\{d^{dpM}(u,v): u, v \in V(G)\}$

Definition 2.3

Any vertex u for which $d^{dpM}(u, v) = e^{d^{dpM}}(v)$ is called d^{dpM} - eccentric vertex of v. Further, a vertex u is said to be d^{dpM} - eccentric vertex of G if it is the d^{dpM} - eccentric vertex of some vertex.

Definition 2.4

The d^{dpM} - radius, denoted by $r^{d^{dpM}}(G)$, is the minimum d^{dpM} - eccentricity among all vertices of G i.e., $r^{d^{dpM}}(G) = \min\{e^{d^{dpM}}(v) : v \in V(G)\}$. Similarly the d^{dpM} - diameter, $d^{dpM}(G)$, is the maximum d^{dpM} - eccentricity among all vertices of G.

Definition 2.5

A graph is called d^{dpM} self centered if radius is same as diameter, i.e., $r^{d^{dpM}} = d^{d^{dpM}}$.

Remark

If *G* is any connected graph, then the d^{dpM} - distance is a metric on the set of vertices of *G*.

Theorem 2.6

In a **Triangular Snake C(T$_n$)**, with 2n - 1 vertices. Then the product mean d^{dpM} is given bolow.

Diameter $d^{d^{dpM}}\left(C(T_n)\right) = n + 13$.

$$\text{Radius } r^{d^{dpM}}\left(C(T_n)\right) = \begin{cases} \left(\frac{n+1}{2}\right) + 9 & n \text{ is odd} \\ \left(\frac{n}{2}\right) + 9 & n \text{ is even} \end{cases}$$

Proof :

Diameter

Let $d\ (u, v) = n - 3$, $\deg(u) = 4, \deg(v) = 4$

$$d^{dpM}\ (u, v) = d\ (u, v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)\ \deg(v)}{2} \right\rceil$$

$$= n - 3 + 4 + 4 + \left\lceil \frac{4 \times 4}{2} \right\rceil$$

$$= n + 5 + 8$$

$$= n + 13$$

Radius

Let $d\ (u, v) = \frac{n-2}{2}\ and\ \frac{n-1}{2}$, $\deg(u) = 4, \deg(v) = 2$

Case i: n is odd

$$r^{d^{dpM}}(u, v) = d(u, v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u) \deg(v)}{2} \right\rceil$$

$$= \frac{n-1}{2} + 4 + 2 + \left\lceil \frac{4 \times 2}{2} \right\rceil$$

$$= \frac{n-1}{2} + 6 + 4$$

$$= \frac{n+1}{2} + 9$$

Case ii: n is even

$$r^{d^{dpM}}(u, v) = d(u, v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u) \deg(v)}{2} \right\rceil$$

$$= \frac{n-2}{2} + 4 + 2 + \left\lceil \frac{4 \times 2}{2} \right\rceil$$

$$= \frac{n-2}{2} + 6 + 4$$

$$=\frac{n}{2}+9$$

Theorem 2.7

In a **Double Triangular Snake D(T_n)** , with 3n - 2 vertices. Then the product mean d^{dpM} of any vertex is given below

Diameter $d^{d^{dpM}}\left(D(T_n)\right) = n + 27.$

Radius $r^{d^{dpM}}\left(D(T_n)\right) = \begin{cases} \left(\frac{n+1}{2}\right) + 13 & n \text{ is odd} \\ \left(\frac{n}{2}\right) + 13 & n \text{ is even} \end{cases}$

Theorem 2.8

In a **Triple Triangular Snake T(T_n)**, with 4n - 3 vertices. Then the product mean d^{dpM} is given bolow.

Diameter $d^{d^{dpM}}\left(T(T_n)\right) = n + 45.$

Radius $r^{d^{dpM}}\left(T(T_n)\right) = \begin{cases} \left(\frac{n+1}{2}\right) + 17 & n \text{ is odd} \\ \left(\frac{n}{2}\right) + 17 & n \text{ is even} \end{cases}$

Theorem 2.9

In a **Quadrilateral Snake C(Q_n)**, with 3n - 2 vertices. Then the product mean d^{dpM} is given below.

Diameter $d^{d^{dpM}}\left(C(Q_n)\right) = n + 13.$

Radius $r^{d^{dpM}}\left(C(Q_n)\right) = \begin{cases} \left(\frac{n+1}{2}\right) + 9 & n \text{ is odd} \\ \left(\frac{n}{2}\right) + 10 & n \text{ is even} \end{cases}$

Proof :

Diameter

Let $d\,(u,v) = n-3$, $\deg(u) = 4, \deg(v) = 4$

$$d^{dpM}\,(u,v) = d\,(u,v) + \deg(u) + \deg(v) + \left\lceil\frac{\deg(u)\deg(v)}{2}\right\rceil$$

$$= \boldsymbol{n - 3 + 4 + 4 +} \left\lceil \frac{\mathbf{4 \times 4}}{\mathbf{2}} \right\rceil$$

$$= \boldsymbol{n + 5 + 8}$$

$$= \boldsymbol{n + 13}$$

Radius

Let $\boldsymbol{d\ (u, v) = \frac{n}{2}\ and\ \frac{n-1}{2}}$, $\boldsymbol{\deg(u) = 4, \deg(v) = 2}$

Case i: n is odd

$$\boldsymbol{r^{d^{dpM}}(u, v) = d(u, v) + \deg(u) + \deg(v) +} \left\lceil \frac{\mathbf{deg\,(u)deg\,(v)}}{\mathbf{2}} \right\rceil$$

$$= \boldsymbol{\frac{n-1}{2} + 4 + 2 +} \left\lceil \frac{\mathbf{4 \times 2}}{\mathbf{2}} \right\rceil$$

$$= \boldsymbol{\frac{n-1}{2} + 6 + 4}$$

$$= \boldsymbol{\frac{n+1}{2} + 9}$$

Case ii: n is even

$$\boldsymbol{r^{d^{dpM}}(u, v) = d(u, v) + \deg(u) + \deg(v) +} \left\lceil \frac{\mathbf{deg\,(u)deg\,(v)}}{\mathbf{2}} \right\rceil$$

$$= \boldsymbol{\frac{n}{2} + 4 + 2 +} \left\lceil \frac{\mathbf{4 \times 2}}{\mathbf{2}} \right\rceil$$

$$= \boldsymbol{\frac{n}{2} + 6 + 4}$$

$$= \boldsymbol{\frac{n}{2} + 10}$$

Theorem 2.10

In a **Double Quadrilateral Snake D(Q_n)** , with 5n - 4 vertices. Then the product mean $\boldsymbol{d^{dpM}}$ of any vertex is given below

Diameter $\boldsymbol{d^{d^{dpM}}\left(D(Q_n)\right) = n + 27}$.

Radius $\boldsymbol{r^{d^{dpM}}\left(D(Q_n)\right) =} \begin{cases} \boldsymbol{\left(\frac{n+1}{2}\right) + 13} & \boldsymbol{n\ is\ odd} \\ \boldsymbol{\left(\frac{n}{2}\right) + 14} & \boldsymbol{n\ is\ even} \end{cases}$

Theorem 2.8

In a **Triple Quadrilateral Snake T(Q$_n$)**, with 7n - 6 vertices. Then the product mean $\boldsymbol{d^{dpM}}$ is given bolow.

Diameter $\boldsymbol{d^{d^{dpM}}(T(Q_n)) = n + 45}$.

Radius $\boldsymbol{r^{d^{dpM}}(T(Q_n))} = \begin{cases} \left(\frac{n+1}{2}\right) + 17 & n \text{ is odd} \\ \left(\frac{n}{2}\right) + 18 & n \text{ is even} \end{cases}$

Conclusion:

Many researchers are concentrating various distance concepts in graphs. In this paper we have studied about d^{dpM} - distance in snake related graphs. Then we have defined d^{dpM} - eccentricity, d^{dpM} - radius and d^{dpM} – diameter. Many results have been found.

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