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Chromatic Number of Split Graph of Cycle C_n

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Abstract:
In this paper we have discuss about how to find chromatic number of split graph and discussed about how to assign the colours. Also discussed about how to label vertices. The application of various graph coloring approaches in domains such as automated differentiation, mobile networks, optical networks, medical data mining, game theory, and radio networks.

Definition1.1 Coloring

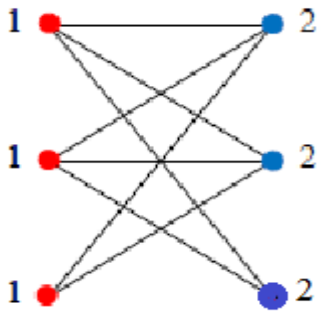
In its simplest form, it is a way of *coloring* the vertices of a *graph* such that no two adjacent vertices are of the same *color*; this is called a vertex *coloring*

Definition1.2 Split graph

A split graph is a graph whose vertices can be partitioned into a clique and an independent vertex set.

Definition1.3 Chromatic number

The chromatic number of a graph is the minimal number of colours needed to colour the vertices in such a way that no two adjacent vertices have the same colour



Definition1.4 Chromatic number for split graph

Based on the result, to determine the locating Based on the result, to determine the locating chromatic number for split graph of cycle, by dividing two cases. The first case when odd n and second case when even n. So that, obtained the locating chromatic number for split graph of cycle = 4 for odd n and 5 for even n. of cycle, by dividing two cases. The first case when odd n and second case when even n. So that, obtained the locating chromatic number for split graph of cycle Cn = 4 for odd and 5 for even n.

Definition1.5 Cycle graph

A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3, if the graph is simple) connected in a closed chain. The cycle graph with n vertices is called C_n. The number of vertices in C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.

Definition1.6 Path graph

A Path graph, also known as a linear graph, is a graph where the vertices

are arranged in a line with edges connecting vertices that are adjacent

Definition1.7 Middle graph

The Middle graph M(G) of a graph G is the graph in which the vertex set is V (G) ∪ E(G) and two vertices are adjacent if and only if either they are adjacent edges of G or one is vertex of G and the other is an edge incident with it.

Algorithm

Finding the chromatic number of split graph of cycle c_n

Step:1

Let spl(C_n) be the split graph of cycle graph c_n

Step:2

Label the vertices of spl (C_n) by using the known algorithm which is a prime graph whose vertices are $P_1, P_2, \ldots . P_m,$ and $P_1', P_2', \ldots . P_n'$

Step:3

Assign the Colour , say Red to the vertices of the spl (C_n) having even label. therefore we assign the colour Red to the verticxes $P_1', P_2', \ldots . P_n'$

Step:4

The remaining odd labels of spl(C_n) are $v_1, v_2 \ldots \ldots v_n$

Step:5

Assign the colour to the odd label vertices, fix any vertex say P_1 and assign any colour other than Red, therefore we assign the colour Green to P_1. next for the vertex P_2, The adjacent vertices are P_1', $P_3', P_1, P_3,$ But we already we assigned the colour red to P_1', P_3' and green to P_1

Therefore we should assign different colour other than Red and Green therefore we shall assign Yellow to $P_{2,}$

Step:6

Repeat step-5 to all the vertices $p_1', p_2', p_3' \ldots\ldots, p_n'$ untill all the vertices have been colourd

Therefore cr(spl(c_n)) = 3

Illustration -1.1

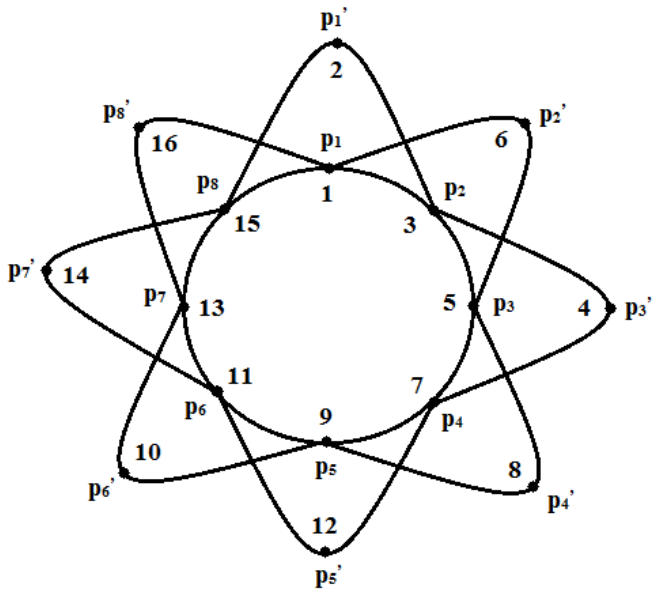


Fig. 1.1 split graph of C_8

Illustration -1.2

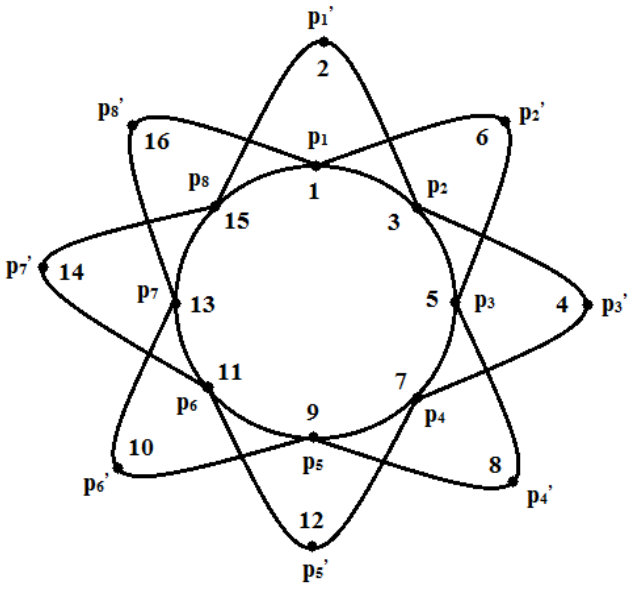


Fig:1.2.Labelled spl (C_8)

Illustration-1.3

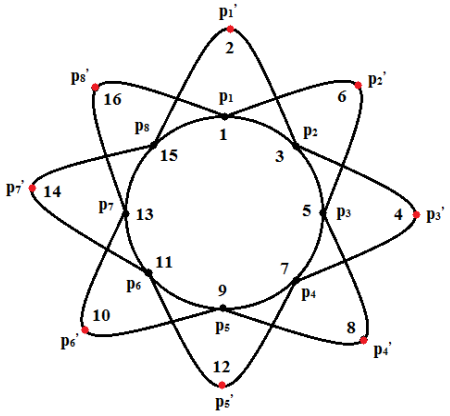


Fig: 1.3 Colouring Red for even labeled vertices

Illustration -1.4

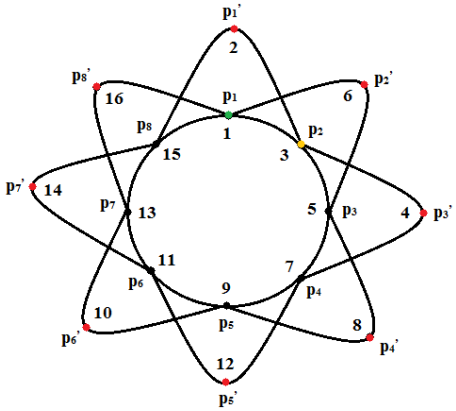


Fig: 1.4 colouring different colours for adjacent vertices P_1 (Green) P_2 (Yellow)

Illustration -1.5

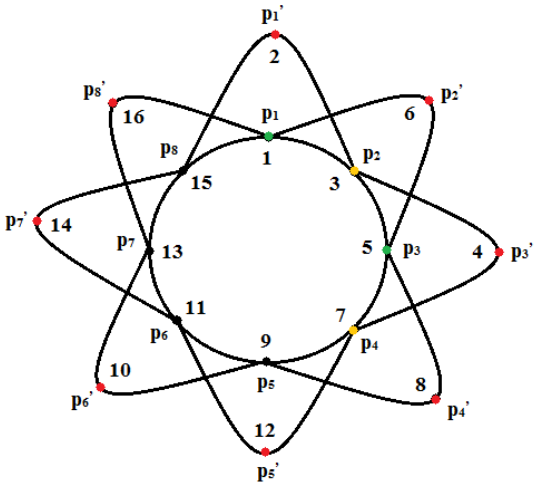


Fig: 1.5 Colouring Green for P_3 and Yellow for P_4

Illustration -1.6

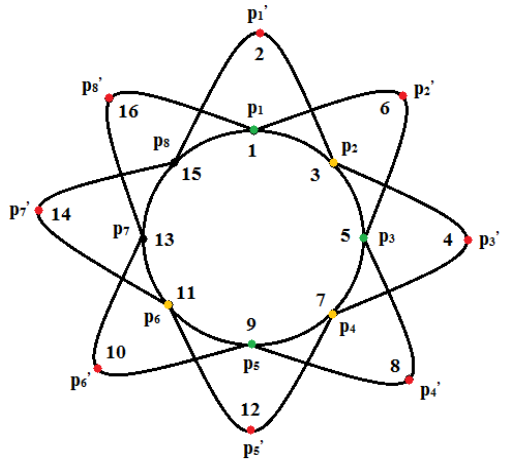


Fig:1.6 Colouring Green for P_5 and Yellow for P_6

Illustration -1.7

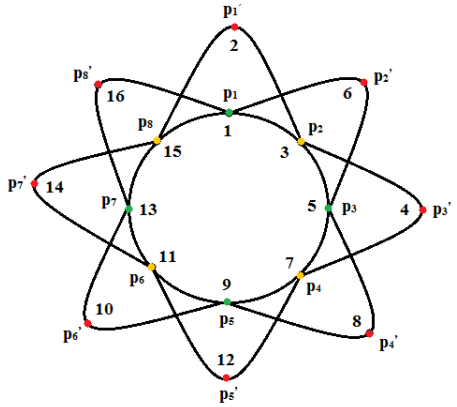


Fig:1.7 Colouring Green for $P_5$and Yellow for P_6

Illustration -1.8

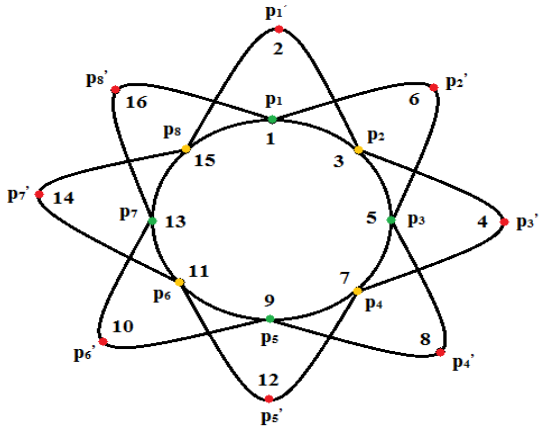


Fig:1.8 Colouring Green for P_7 and Yellow for P_8

Chromatic number of middle graph of path graph P_n

Algorithm

Finding the chromatic number of middle graph of path graph P_n

Step:1

Let $MI(P_n)$ be the middle graph of path graph P_n

Step:2

We have to label the vertices of middle graph of path graph whose vertices are $P_1,P_2,P_3….P_n$ and $P_1',P_2',….P_n'$

Step:3

Assign any colour say red to the vertices of the $MI(P_n)$ having even labels. Therefore we assign the colour red to the vertices $P_1,P_2,….P_n$

Step:4

The remaining odd labels of $MI(P_n)$ are $P_1',P_2',….P_n'$

Step:5

Fix any vertex say P_1' and assign any colour other than red therefore we assign the colour green to P_1'. Next for the vertex P_2' The adjacent vertices are P_2,P_1',P_3,P_3' but already we assigned the colour red to P_2,P_3 and green to P_1'

Therefore we should assign different colour other than red and green, therefore we shall assign yellow to P_2',P_4'

Step:6

Repeat step-5 to all the vertices $P_1',P_2',…..P_n'$ until all the vertices have been coloured

Therefore $Cr(MI(P_n))=3$

Illustration-2.1

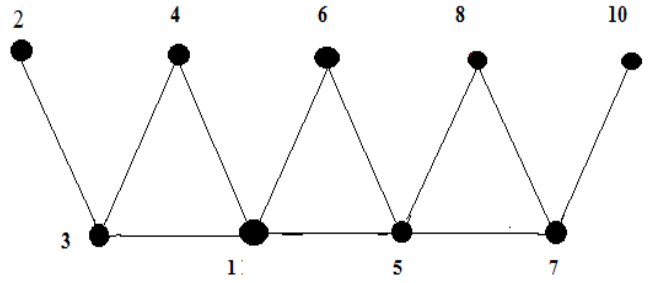


Figure 2.1 middle graph P_5

Illustration-2.2

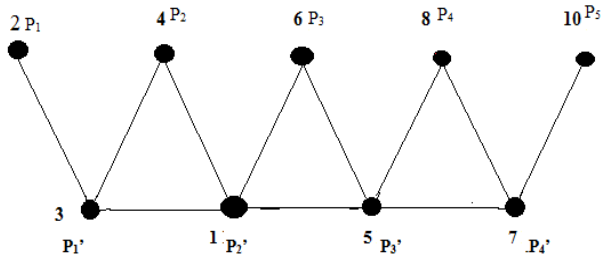


Figure 2.2 Labeled MI(P_5)

Illustration-2.3

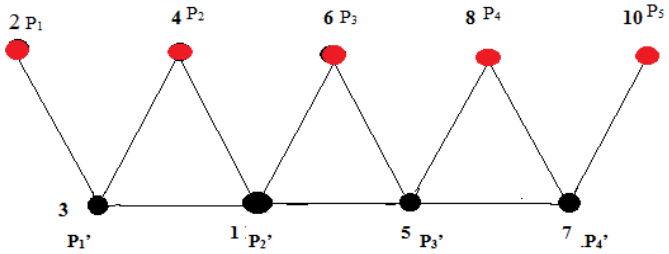


Figure 2.3 Colouring for even labeled vertices

Illustration-2.4

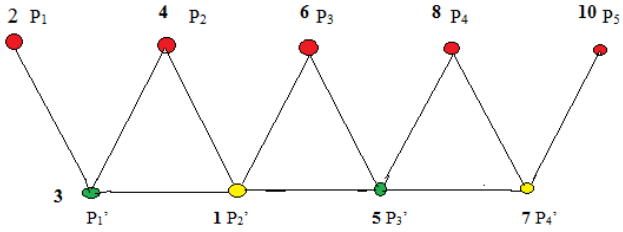


Figure 2.4 Colouring different colour for adjacent vertices

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