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Necessity Modality and Basic Operators on Intuitionistic Fuzzy Matrices

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Abstract: In this paper, we have verified the necessity operator along with eight basic operations on intuitionistic fuzzy matrices, and their related properties have been proved.

Keywords: Necessity Modal Operator, Intuitionistic Fuzzy sets Intuitionistic Fuzzy matrices, Fuzzy Set, Fuzzy matrices, basic relations and operations on the Intuitionistic Fuzzy matrices.

AMS Classification: 03E99, 05C25, 08A72

1. Introduction: Afshar Alam and Sharfuddin Ahmad [1] discussed the Normalization of Intuitionistic Fuzzy Relations in the year 2004. Alpana Sharma [2] introduced by Fuzzy Logic and its Application in Real Life in the year 2020. Amrita Sarkar and U. C. Sahoo [3] are studying Application of Fuzzy Logic in Transport Planning this year 2012. I. M. Adamu [4] presented the Application of Intuitionistic Fuzzy Sets to Environmental Management in the year 2021. M. Asghari-Larimi [5] studied upper and Lower (α, β) - Intuitionistic Fuzzy set in the year 2012. Cengiz Kahraman, Murat Gulbay and Ozgur Kabak [6] are proposed by Applications of Fuzzy Sets in Industrial Engineering: A Topical Classification was this year 2006. Christophe Marsala [7] showed by Building Intuitionistic Fuzzy Sets in Machine Learning this year 2021. N. Deva and A. Felix [8] are writing on Bipolar Intuitionistic Fuzzy Matrices and Its Determinants in the year 2024. A. De Luca and S. Termini [9] were given Algebraic Properties of Fuzzy Sets in that year, 1972. P. A. Ejegwa and S.O. Akowe [10] are carrying out An Overview on Intuitionistic Fuzzy Sets this year 2014. P. A. Ejegwa, A. J. Akubo and O. M. Joshua [11] were proposed by Intuitionistic Fuzzy Set and Its Application this year 2014. P. A. Ejegwa [12] introduced the Intuitionistic Fuzzy Sets Approach to Appointment of Positions in an Organization via the Max-Min-Max Rule 2015 this year. Florentin Smarandache [13] proposed the Neutrosophic Set – A Generalization of the Intuitionistic Fuzzy Set 2010 this year. Harpreet Singh, Madan M. Gupta and Thomas Meitzler [14] introduced the Real-Life Applications of Fuzzy Logic 2013 this year. Hemlata Aggarwal and H.D. Arora [15] developed A Decision-making Problem as an Application of Intuitionistic Fuzzy Set Application this year. P. Jenita and E. Karuppusamy [16] are proposing Fuzzy Relational Equations of k - regular Intuitionistic Fuzzy and Block Fuzzy Matrices this year 2017. Jeevaraj Selvaraj and Abhijit Majumdar [17] proposed A New Ranking Method for Interval-Valued Intuitionistic Fuzzy Numbers and Its Application in Multi-Criteria Decision-Making this year 2021. Javaid Ahmad Shah [18] presented in Fuzzy Matrix Theory based Decision-Making for Machine Learning this year 2022. Krassimir T. Atanassov [19] gave an Intuitionistic Fuzzy Set that year, 1986. Kwang H. Lee [20] introduced the Fuzzy Theory and Applications that year, 2005. Krassimir T. Atanassov [21] was defined by Review and New Results on Intuitionistic Fuzzy Sets this year 2016. Krassimir T. Atanassov [22] proposed Intuitionistic Fuzzy Sets Theory this year 2012. Lilija Atanassova and Piotr Dworniczak [23] were introduced by On the Operation over Intuitionistic Fuzzy

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2. Objectives: Enhancing Representation of Uncertainty: The primary objective is to model and represent uncertainty more effectively in complex systems. By using the necessity modal operator, it is possible to quantify the degree of necessity of a relationship or condition in the context of both truth and falsehood in intuitionistic fuzzy systems. This allows for a richer interpretation of uncertain data, especially when dealing with incomplete or imprecise information. Formalizing Necessity and Possibility: The necessity modal operator helps formalize the concepts of necessity and possibility within intuitionistic fuzzy logic. It adds a layer of sophistication in how necessity is treated in fuzzy systems by distinguishing between necessary, possible, and impossible events or relationships. In the context of fuzzy matrices, this formalization provides a structured way to measure how "necessary" a given condition or relationship is within a set of fuzzy data. Improving Decision-Making in Uncertain Environments: One of the main goals of applying the necessity modal operator to intuitionistic fuzzy matrices is to support decision-making in environments with uncertainty. The necessity operator helps in making decisions where certain criteria or conditions need to be satisfied to a certain degree of necessity, despite the presence of fuzzy or uncertain information. It facilitates better evaluation of situations where outcomes must adhere to stricter constraints (necessity) or can be more relaxed (possibility). Strengthening Logical Reasoning and Inference: The necessity modal operator provides a formal method to express the logical necessity of propositions in fuzzy systems, enabling more robust logical reasoning. It allows the extraction of logical implications from intuitionistic fuzzy matrices, offering tools for inferring relationships between elements based on the degree of necessity assigned to each condition. Improving the Precision of Intuitionistic Fuzzy Systems: By incorporating the necessity operator, the system's precision in dealing with fuzzy relations is enhanced. This enables a more accurate representation of how necessary or impossible certain conditions are, thereby refining the Modeling of uncertainty in systems. It also contributes to better alignment of the fuzzy matrix's structure with real-world scenarios where certainty and necessity are key factors.

3. Preliminary

Fuzzy Set matrices

3.1 Definition: Fuzzy Set: A Fuzzy set is a set whose elements have degrees of membership. Fuzzy sets are an extension of the classical notion of set (known as a Crisp Set). More mathematically, a fuzzy set is a pair (A, μ_A) where A is a set and $\mu_A: A \rightarrow [0, 1]$. For all $x \in A$, $\mu_A(x)$ is called the grade of membership of x.

3.2 Example of Fuzzy Set

Mathematics and Logic: Height classification: Consider a fuzzy set for classifying the height of people as "tall," "average," and "short." Instead of having a strict definition (e.g., greater than 6 feet is tall), fuzzy logic allows for degrees of membership. A person who is 5'10" could have a 0.8 membership in the "tall" set and a 0.2 membership in the "average height" set.

Artificial Intelligence and Machine Learning: Fuzzy Clustering: A fuzzy clustering algorithm assigns data points to multiple clusters with varying degrees of membership. For example, in customer segmentation, a customer might have a 0.7 membership in the "frequent shopper" cluster and a 0.3 membership in the "bargain shopper" cluster.

Control Systems: Fuzzy Logic Temperature Control: A fuzzy logic system could be used in an air conditioning system where the temperature is classified as "cold," "comfortable," and "hot." If the room temperature is 22°C, it might have a 0.7 membership in "comfortable" and a 0.3 membership in "cold," so the air conditioner will adjust to maintain the temperature within the optimal range.

Computer Science and Data Mining: Fuzzy Database Querying: In a product database, the price might be classified into fuzzy categories like "cheap," "moderate," and "expensive." For a product with a price of $30, it could have a 0.6 membership in "moderate" and a 0.4 membership in "cheap."

Medicine and Healthcare: Diagnosis System: A fuzzy expert system might help in diagnosing diseases. For example, a patient with a fever of 38°C might have a 0.6 membership in the "high fever" category and a 0.4 membership in the "moderate fever" category. The system uses these degrees of membership to recommend possible diagnoses.

3.3 Fuzzy Sets Applications

Air Conditioners and Refrigerators: Fuzzy logic is used to control temperature, adjusting it smoothly based on input, such as the room's temperature.

Washing Machines: Fuzzy logic can be used to optimize washing cycles by considering various factors like load size, fabric type, and dirtiness, adjusting the washing time accordingly.

Automated Steering Systems: Fuzzy logic helps cars make decisions about steering, braking, and accelerating under various driving conditions, such as wet roads or heavy traffic.

Decision Making: Fuzzy logic systems are employed to assist in decision-making processes where there is uncertainty. For example, in finance, a fuzzy system might be used to evaluate investment opportunities or credit risk by considering factors like market volatility, company health, and economic conditions.

Image Processing: Fuzzy logic helps in segmenting images, reducing noise, and improving image quality, especially when working with unclear or ambiguous data. It is often used in medical imaging or satellite image processing.

Expert Systems: In fields like medicine, fuzzy logic can help create systems that simulate human expert reasoning. For instance, diagnosing diseases based on vague and incomplete symptoms is an area where fuzzy logic has been applied.

Robotics: Fuzzy control is used in robotics to help machines make more human-like decisions. This can include tasks like navigating unpredictable environments, grasping objects, or interacting with people.

Consumer Electronics: Products like cameras, smartphones, and smart home devices use fuzzy logic to adjust settings dynamically based on various factors, such as lighting conditions, motion detection, or noise levels.

Definition: Fuzzy Matrix Fuzzy matrices play a vital role in scientific development. A Fuzzy matrix may be matrix that has its parts from [0, 1]. Consider a matrix A $=[a_{ij}]_{3\times 3}$. Where $a_{ij} \in [0,1], 1 \leq j \leq n$.Then A is a Fuzzy Matrix.

Example: Suppose we have a fuzzy matrix A that represents the degree of membership of various objects to a set of fuzzy relations. $A = \begin{bmatrix} 0.8 & 0.5 & 0.3 \\ 0.6 & 0.9 & 0.7 \\ 0.4 & 0.6 & 0.9 \end{bmatrix}$

In this matrix,

$a_{11} = 0.8$: The degree of membership of the first object to the first relation is 0.8.

$a_{12} = 0.5$: The degree of membership of the first object to the second relation is 0.5.

$a_{13} = 0.3$: The degree of membership of the first object to the third relation is 0.3.

Similarly, the other entries represent degrees of membership for other objects and relations.

Fuzzy Matrix Applications

Decision Making: Fuzzy matrices are often used in decision-making problems, where the relationships between options or criteria are uncertain or vague. For example, in multi-criteria decision analysis, fuzzy matrices can represent the relative importance of different criteria.

Control Systems: In systems where control strategies are not precisely defined, fuzzy matrices can represent varying degrees of control actions that depend on multiple inputs (e.g., temperature, pressure, and speed).

Pattern Recognition: In pattern recognition and machine learning, fuzzy matrices can be used to model relationships between features or between clusters in a way that accommodates uncertainty in the data.

Fuzzy Relation Equations: Fuzzy matrices are used to solve fuzzy relation equations, where the relationships between entities are uncertain or imprecise.

4. Intuitionistic Fuzzy Matrix

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership function. An Intuitionistic fuzzy set A is a non-empty set X is an object having the form A = { x, μ_A (x), v_A (x)| x∈E } and satisfies $0 \leq \mu_A(x) + v_A(x) \leq 1$. An intuitionistic fuzzy matrix (IFM) A of order $m \times n$ is defined as A =$\left[x_{ij}, < a_{ij\mu}, a_{ijv} >\right]_{m\times n}$, where $a_{ij\mu}$ and a_{ijv} are called membership and non-membership values of x_{ij}in A, which maintaining the condition $0 \leq a_{ij\mu} + a_{ijv} \leq 1$. For simplicity, we write A = $[x_{ij}, a_{ij}]_{m\times n}$ or simply$[a_{ij}]_{m\times n}$.

Where $a_{ij} = < a_{ij\mu}, a_{ijv} >$

Example $A = \begin{bmatrix} (0.7, 0.2) & (0.5, 0.3) & (0.4, 0.5) \\ (0.6, 0.3) & (0.8, 0.1) & (0.7, 0.2) \\ (0.5, 0.4) & (0.6, 0.3) & (0.9, 0.05) \end{bmatrix}$

In this matrix:

The first entry (0.7, 0.2) means the degree of membership for the first element in the first relation is 0.7, and the degree of non-membership is 0.2.

The second entry (0.5, 0.3) means the degree of membership for the first element in the second relation is 0.5, and the degree of non-membership is 0.3.

The sum of membership and non-membership for each pair is less than or equal to 1.

Applications of Intuitionistic Fuzzy Matrices

Decision Making: In decision-making problems, especially multi-criteria decision

Making (MCDM), where decisions are based on uncertain or incomplete information. Intuitionistic fuzzy matrices are useful in representing the degrees of uncertainty between criteria and alternatives.

Control Systems: Intuitionistic fuzzy matrices can be used in fuzzy control systems where both the membership and non-membership information is needed for making real-time control decisions.

Data Mining: In clustering and classification tasks, intuitionistic fuzzy matrices can represent the uncertain membership of data points in multiple clusters.

Pattern Recognition: Intuitionistic fuzzy matrices are useful in representing the relationships between different features, especially when dealing with vague or incomplete data.

Expert Systems: In expert systems, where knowledge is often vague, intuitionistic fuzzy logic can be used to model the expert's uncertainty.

5. Necessity Fuzzy Operators

Definition: Let X be nonempty. If A is an IFS drawn from X, then $\Box A = \{x, \mu_A(x): x \in X\} = \{x, \mu_A(x), 1 - \mu_A(x): x \in X\}$

Example $A = \begin{bmatrix} (.2,.5) & (.3,.8) \\ (.4,.6) & (.5,.7) \end{bmatrix}$

$\Box A = \{x, \mu_A(x): x \in X\} = \{x, \mu_A(x), 1 - \mu_A(x): x \in X$, $\Box A = \begin{bmatrix} (.2,.8) & (.3,.7) \\ (.4,.6) & (.5,.5) \end{bmatrix}$

Applications of Necessity Fuzzy Operators

Fuzzy Decision Making: In situations where decisions are based on imprecise, uncertain, or fuzzy information, necessity fuzzy operators can be used to model the necessity of certain conditions being met. They are helpful.

Multi-criteria decision analysis (MCDA): These operators are used to assess the necessity of each criterion in decision-making when multiple factors are involved.

Preference modelling: In scenarios where preferences are not clearly defined but can be expressed in fuzzy terms (e.g., "highly preferable" or "somewhat acceptable"), necessity fuzzy operators help in deriving the necessity of preferences.

Fuzzy Logic Systems: Necessity fuzzy operators are an extension of classical fuzzy logic, used to make decisions based on degrees of necessity rather than certainty. In applications like:

Fuzzy control systems: These operators can enhance the control process, ensuring that the necessary conditions for the system to perform correctly are met.

Rule-based inference: In fuzzy inference systems, necessity operators help model how much a rule should apply under uncertain conditions, adding robustness to the reasoning process.

Pattern Recognition and Classification: When classifying data based on fuzzy criteria, necessity fuzzy operators can be used to evaluate the necessity of a feature or condition belonging to a certain class, especially in cases with uncertainty or imprecision in the data. This is common in:

Image processing: For example, recognizing patterns or objects in images where some features are ambiguous or unclear.

Medical diagnostics: In medical systems that diagnose diseases based on fuzzy symptoms, necessity fuzzy operators can determine the importance of certain diagnostic factors.

Expert Systems and Knowledge Representation: In expert systems where knowledge is represented using fuzzy rules, necessity fuzzy operators can model how necessary specific pieces of knowledge or rules are under given circumstances. This helps in:

Decision support systems: To provide guidance when experts have to make decisions based on vague or incomplete knowledge.

Uncertainty management: Assessing how necessary certain conditions are for drawing conclusions or taking actions in an uncertain environment.

Risk Analysis and Management: Necessity fuzzy operators are useful in scenarios where risks are uncertain or not precisely measurable. They are applied in:

Risk assessment: Evaluating the necessity of mitigating certain risks based on fuzzy criteria.

Reliability engineering: Determining the necessity of different components working correctly in systems with uncertainty in their performance.

Control and Automation Systems

In automated systems or robotics, where actions often depend on a combination of necessary conditions being met, necessity fuzzy operators can help evaluate and prioritize actions based on fuzzy input data. For example:

Robotic navigation: Robots make decisions based on a range of fuzzy conditions (e.g., obstacles, speed, distance), and necessity fuzzy operators help them decide what actions are most necessary to continue moving forward.

Manufacturing systems: In automated production lines, these operators can help optimize processes where the conditions for success are fuzzy and not perfectly defined.

Natural Language Processing (NLP)

Necessity fuzzy operators can be used to process and interpret natural language, where meanings are often vague or context-dependent. Applications include:

Sentiment analysis: Understanding the necessity of certain sentiments (e.g., "very happy" vs. "slightly happy") in a text or conversation.

Machine translation: In translating text with uncertain or context-dependent meanings, these operators help determine the most necessary translation options.

Artificial Intelligence and Machine Learning

AI and ML systems often deal with imprecise data or incomplete information, making necessity fuzzy operators valuable for:

Data mining: Extracting patterns or rules from fuzzy data and determining the necessity of various features or patterns in classification or clustering tasks.

Knowledge discovery: Identifying essential relationships or dependencies between variables in large datasets

Game Theory and Strategic Decision Making: In game theory, where players often act under uncertainty and imperfect information, necessity fuzzy operators can help evaluate the necessity of certain strategies or moves, especially when the players face uncertain payoffs or incomplete knowledge about other players' actions

Environmental Modelling and Simulation: In environmental modelling, where conditions can be highly uncertain (e.g., predicting weather patterns, climate change impacts), necessity fuzzy operators help assess the necessity of certain outcomes or actions under uncertain environmental conditions.

6. Some Modal Operators on Intuitionistic Fuzzy Sets

We define over the set of IFS, two modal operators which transform every IFS into fuzzy set. These operators are similar to the operator's '**necessity**' and '**possibility**' defined in some modal logics. This idea is drawn from the modal operators on IFS proposed by [3].

6.1 Definition: [Modal operators' necessity]: Let X be nonempty. If A is an IFS drawn from X, then □A = {x, μ_A(x): x∈X} = {x, μ_A(x), $1 - \mu_A$(x): x∈X}

6.2 Definition: [Modal operators' possibility]: Let X be nonempty. If A is an IFS drawn from X, then ◊A = {x, $1 - \vartheta_A$(x): x∈X} = {x, $1 - \vartheta_A$(x), ϑ_A(x): x∈X}

6.3 Definition: If A_E and B_F are two IFSs over different universes E and F gives as A_E = {x, μ_A(x), ϑ_A(x): x∈E} and B_F= {y, μ_B(y), ϑ_B(y): y∈F}

6.4 Definition: If $\overline{A_E}$ and$\overline{B_F}$ are two IFSs over different universes E and F gives as $\overline{A_E}$= {x, ϑ_A(x), μ_A(x): x∈E} and $\overline{B_F}$= {y, ϑ_B(y), μ_B(y): y∈F}

6.5 Definition: If A_E and B_F are two IFSs over different universes E and F gives as A_E = {x, μ_A(x), ϑ_A(x): x∈E} and B_F= {y, μ_B(y), ϑ_B(y): y∈F}

6.6 Definition: If $\overline{A_E}$ and$\overline{B_F}$ are two IFSs over different universes E and F gives as $\overline{A_E}$= {x, ϑ_A(x), μ_A(x): x∈E} and $\overline{B_F}$= {y, ϑ_B(y), μ_B(y): y∈F}

6.7 Definition: If $\overline{A_E}$ and$\overline{B_F}$ are two IFSs over different universes E and F gives as $\overline{A_E}$= {x, ϑ_A(x), μ_A(x): x∈E} and $\overline{B_F}$= {y, ϑ_B(y), μ_B(y): y∈F}

1. $\overline{A_E} \cap \overline{B_F}$ = {⟨x, y⟩, min ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), max ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y)): x∈E, y∈F}

2. $\overline{A_E} \cup \overline{B_F}$ = {⟨x, y⟩, max ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), min ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y)): x∈E, y∈F }

3. $\overline{A_E} + \overline{B_F}$ = {⟨x, y⟩, $\overline{\mu_A}$ (x) + $\overline{\mu_B}$ (y) - $\overline{\mu_A}$ (x) . $\overline{\mu_B}$ (y), $\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F }

4. $\overline{A_E}$. $\overline{B_F}$ = {⟨x, y⟩, $\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), ($\overline{\vartheta_A}$(x) +$\overline{\vartheta_A}$ (y) - ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F }

5. $\overline{A_E}$ @ $\overline{B_F}$ = {⟨x, y⟩,$\frac{\overline{\mu_A}(x) \cdot \overline{\mu_B}(y)}{2}$,$\frac{\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}{2}$: x∈E, y∈$F$ }

6. $\overline{A_E}$ \$ $\overline{B_F}$ = {⟨x, y⟩, $\sqrt{\overline{\mu_A}(x) \cdot \overline{\mu_B}(y)}$, $\sqrt{(\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y)}$: x∈E, y∈$F$ }

7. $\overline{A_E}$ # $\overline{B_F}$ = {⟨x, y⟩,$\frac{2\,\overline{\mu_A}(x) \cdot \overline{\mu_B}(y)}{\overline{\mu_A}(x) + \overline{\mu_B}(y)}$,$\frac{2\,(\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y)}{(\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}$: x∈E, y∈$F$ }

8. $\overline{A_E} * \overline{B_F}$ = {⟨x, y⟩,$\frac{\overline{\mu_A}(x) + \overline{\mu_B}(y)}{2(\overline{\mu_A}(x) \cdot \overline{\mu_B}(y) + 1)}$,$\frac{\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}{2(\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y) + 1)}$: x∈E, y∈$F$ }

9. $\overline{A_E} \Delta \overline{B_F}$ = {(x, y),$\frac{\bar{\mu}_A(x) + \bar{\mu}_B(y)}{\bar{\mu}_A(x) + \bar{\mu}_B(y) + \bar{\vartheta}_A(x) + \bar{\vartheta}_B(y)}$,$\frac{\bar{\vartheta}_A(x) + \bar{\vartheta}_B(y)}{\bar{\mu}_A(x) + \bar{\mu}_B(y) + \bar{\vartheta}_A(x) + \bar{\vartheta}_B(y)}$: x∈E, y∈F }

Theorem 6.7.1: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E \cap \Box\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}$ *and* $\Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E=\begin{pmatrix}(\mu_{11},\pi_{11}) & (\mu_{12},\pi_{12}) & (\mu_{13},\pi_{13})\\ (\mu_{21},\pi_{21}) & (\mu_{22},\pi_{22}) & (\mu_{23},\pi_{23})\\ (\mu_{31},\pi_{31}) & (\mu_{32},\pi_{32}) & (\mu_{33},\pi_{33})\end{pmatrix} \text{ and } B_F=\begin{pmatrix}(\vartheta_{11},\theta_{11}) & (\vartheta_{12},\theta_{12}) & (\vartheta_{13},\theta_{13})\\ (\vartheta_{21},\theta_{21}) & (\vartheta_{22},\theta_{22}) & (\vartheta_{23},\theta_{23})\\ (\vartheta_{31},\theta_{31}) & (\vartheta_{32},\theta_{32}) & (\vartheta_{33},\theta_{33})\end{pmatrix}$$

intuitionistic fuzzy matrices and also

$$\overline{A_E}=\begin{pmatrix}(\pi_{11},\mu_{11}) & (\pi_{12},\mu_{12}) & (\pi_{13},\mu_{13})\\ (\pi_{21},\mu_{21}) & (\pi_{22},\mu_{22}) & (\pi_{23},\mu_{23})\\ (\pi_{31},\mu_{31}) & (\pi_{32},\mu_{32}) & (\pi_{33},\mu_{33})\end{pmatrix} \text{ and } \overline{B_F}=\begin{pmatrix}(\theta_{11},\vartheta_{11}) & (\theta_{12},\vartheta_{12}) & (\theta_{13},\vartheta_{13})\\ (\theta_{21},\vartheta_{21}) & (\theta_{22},\vartheta_{22}) & (\theta_{23},\vartheta_{23})\\ (\theta_{31},\vartheta_{31}) & (\theta_{32},\vartheta_{32}) & (\theta_{33},\vartheta_{33})\end{pmatrix}$$

intuitionistic fuzzy matrices. By the definition of $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two intuitionistic fuzzy matrices over different universes E and F.

Then the definition of $\Box\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\pi_A$(x): x∈X} and $\Box\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\theta_B$(y): y∈Y}

$$\Box\overline{A_E}=\begin{pmatrix}(\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13})\\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix} \text{ and}$$

$$\Box\overline{B_F}=\begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$ is also a intuitionistic fuzzy matrices

Then applying formula $\overline{A_E} \cap \overline{B_F}$ = {⟨x, y⟩, min ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), max ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_B}$ (y)): x∈E, y∈F}, we have

$$\Box\bar{A}_E \cap \Box\bar{B}_F=\begin{pmatrix}(\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13})\\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix} \cap \begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$

Where, $X_{11}=\big((\pi_{11},1-\pi_{11})\cap(\theta_{11},1-\theta_{11})\big)$, $X_{12}=\big((\pi_{12},1-\pi_{12})\cap(\theta_{21},1-\theta_{21})\big)$

$X_{13}=\big((\pi_{13},1-\pi_{13})\cap(\theta_{31},1-\theta_{31})\big)$, $X_{21}=\big((\pi_{21},1-\pi_{21})\cap(\theta_{12},1-\theta_{12})\big)$

$X_{22}=\big((\pi_{22},1-\pi_{22})\cap(\theta_{22},1-\theta_{22})\big)$, $X_{23}=\big((\pi_{23},1-\pi_{23})\cap(\theta_{32},1-\theta_{32})\big)$

$X_{31}=\big((\pi_{31},1-\pi_{31})\cap(\theta_{13},1-\theta_{13})\big)$, $X_{32}=\big((\pi_{32},1-\pi_{32})\cap(\theta_{23},1-\theta_{23})\big)$

$X_{33}=\big((\pi_{33},1-\pi_{33})\cap(\theta_{33},1-\theta_{33})\big)$

Now applying formula $\overline{A_E} \cap \overline{B_F}$ = {⟨x, y⟩, min ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), max ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_B}$ (y)): x∈E, y∈F}

X_{11}= {min (π_{11}(x) , θ_{11} (y)), max $(1-\pi_{11}$ (x) , $1-\ \theta_{11}$ (y))}

X_{12}= {min (π_{12} (x) , θ_{12} (y)), max $(1-\pi_{12}$ (x) , $1-\theta_{12}$(y))}

X_{13}= {min (π_{13} (x) , θ_{13} (y)), max $(1-\pi_{13}$ (x) , $1-\theta_{13}$ (y))}

X_{21}= {min (π_{21} (x) , θ_{21}(y)), max $(1-\pi_{21}$ (x) , $1-\theta_{21}$ (y))}

X_{22}= {min (π_{22} (x) , θ_{22}(y)), max $(1-\pi_{22}$ (x) , $1-\theta_{22}$(y))}

X_{23}= {min (π_{23} (x) , θ_{23} (y)), max $(1-\pi_{23}$ (x) ,$1-\theta_{23}$(y))}

X_{31}= {min (π_{31} (x) , θ_{31} (y)), max $(1-\pi_{31}$ (x) , $1-\theta_{31}$(y))}

X_{32}= {min (π_{32} (x) , θ_{32} (y)), max $(1-\pi_{32}$ (x) , $1-\theta_{32}$ (y))}

X_{33}= {min (π_{33} (x) , θ_{33} (y)), max $(1-\pi_{33}$ (x) , $1-\theta_{33}$ (y))}

We have, $\Box\overline{A_E} \cap \Box\overline{B_F} = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix set.

Theorem 6.7.2: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E \cup \Box\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}$ *and* $\Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11}, \pi_{11}) & (\mu_{12}, \pi_{12}) & (\mu_{13}, \pi_{13}) \\ (\mu_{21}, \pi_{21}) & (\mu_{22}, \pi_{22}) & (\mu_{23}, \pi_{23}) \\ (\mu_{31}, \pi_{31}) & (\mu_{32}, \pi_{32}) & (\mu_{33}, \pi_{33}) \end{pmatrix} \text{ and } B_F = \begin{pmatrix} (\vartheta_{11}, \theta_{11}) & (\vartheta_{12}, \theta_{12}) & (\vartheta_{13}, \theta_{13}) \\ (\vartheta_{21}, \theta_{21}) & (\vartheta_{22}, \theta_{22}) & (\vartheta_{23}, \theta_{23}) \\ (\vartheta_{31}, \theta_{31}) & (\vartheta_{32}, \theta_{32}) & (\vartheta_{33}, \theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11}, \mu_{11}) & (\pi_{12}, \mu_{12}) & (\pi_{13}, \mu_{13}) \\ (\pi_{21}, \mu_{21}) & (\pi_{22}, \mu_{22}) & (\pi_{23}, \mu_{23}) \\ (\pi_{31}, \mu_{31}) & (\pi_{32}, \mu_{32}) & (\pi_{33}, \mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11}, \vartheta_{11}) & (\theta_{12}, \vartheta_{12}) & (\theta_{13}, \vartheta_{13}) \\ (\theta_{21}, \vartheta_{21}) & (\theta_{22}, \vartheta_{22}) & (\theta_{23}, \vartheta_{23}) \\ (\theta_{31}, \vartheta_{31}) & (\theta_{32}, \vartheta_{32}) & (\theta_{33}, \vartheta_{33}) \end{pmatrix}$$

intuitionistic fuzzy matrices sets over different universes E and F. If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of $\Box\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\ \pi_A$(x): x∈X} and $\Box\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\ \theta_B$(y): y∈Y}

□$\overline{A_E}$= $\begin{pmatrix}(\pi_{11},1-\ \pi_{11}) & (\pi_{12},1-\ \pi_{12}) & (\pi_{13},1-\pi_{13})\\(\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\(\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix}$ and

□$\overline{B_F}$=$\begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\(\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\(\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$

Then applying formula $\overline{A_E} \cup \overline{B_F}$ = {⟨x, y⟩, max ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), min ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y)): x∈E, y∈F},

We have

$$□\bar{A}_E \cup □\bar{B}_F = \begin{pmatrix}(\pi_{11},1-\ \pi_{11}) & (\pi_{12},1-\ \pi_{12}) & (\pi_{13},1-\pi_{13})\\(\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\(\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix} \cup$$

$$\begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\(\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\(\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$

Where, X_{11} =$\big((\pi_{11},1-\ \pi_{11}) \cup (\theta_{11},1-\theta_{11})\big)$, X_{12}=$\big((\pi_{12},1-\ \pi_{12}) \cup (\theta_{21},1-\theta_{21})\big)$

X_{13}=$\big((\pi_{13},1-\pi_{13}) \cup (\theta_{31},1-\theta_{31})\big)$, X_{21}=$\big((\pi_{21},1-\pi_{21}) \cup (\theta_{12},1-\theta_{12})\big)$

X_{22}=$\big((\pi_{22},1-\pi_{22}) \cup (\theta_{22},1-\theta_{22})\big)$, X_{23}=$\big((\pi_{23},1-\pi_{23}) \cup (\theta_{32},1-\theta_{32})\big)$

X_{31}= $\big((\pi_{31},1-\pi_{31}) \cup (\theta_{13},1-\theta_{13})\big)$, X_{32}=$\big((\pi_{32},1-\pi_{32}) \cup (\theta_{23},1-\theta_{23})\big)$

X_{33}=$\big((\pi_{33},1-\pi_{33}) \cup (\theta_{33},1-\theta_{33})\big)$

Now applying the formula $\overline{A_E} \cup \overline{B_F}$ = {⟨x, y⟩, max ($\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), min ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y)): x∈E, y∈F},

X_{11}= {max (π_{11}(x) , θ_{11} (y)), min ($1-\pi_{11}$ (x) , $1-\ \theta_{11}$ (y))}

X_{12}= {max (π_{12} (x) , θ_{12} (y)), min ($1-\pi_{12}$ (x) , $1-\theta_{12}$(y))}

X_{13}= {max (π_{13} (x) , θ_{13} (y)), min ($1-\pi_{13}$ (x) , $1-\theta_{13}$ (y))}

X_{21}= {max (π_{21} (x) , θ_{21}(y)), min ($1-\pi_{21}$ (x) , $1-\theta_{21}$ (y))}

X_{22}= {max (π_{22} (x) , θ_{22}(y)), min ($1-\pi_{22}$ (x) , $1-\theta_{22}$(y))}

X_{23}= {max (π_{23} (x) , θ_{23} (y)), min ($1-\pi_{23}$ (x) ,$1-\theta_{23}$(y))}

X_{31}= {max (π_{31} (x) , θ_{31} (y)), min ($1-\pi_{31}$ (x) , $1-\theta_{31}$(y))}

X_{32}= {max (π_{32} (x) , θ_{32} (y)), min ($1-\pi_{32}$ (x) , $1-\theta_{32}$ (y))}

X_{33}= {max (π_{33} (x) , θ_{33} (y)), min ($1 - \pi_{33}$ (x) , $1 - \theta_{33}$ (y))}

We have, $\square\overline{A_E} \cup \square\overline{B_F} = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix set.

Theorem 6.7.3: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\square\bar{A}_E \oplus \square\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\square\overline{A_E}$ *and* $\square\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$A_E = \begin{pmatrix} (\mu_{11}, \pi_{11}) & (\mu_{12}, \pi_{12}) & (\mu_{13}, \pi_{13}) \\ (\mu_{21}, \pi_{21}) & (\mu_{22}, \pi_{22}) & (\mu_{23}, \pi_{23}) \\ (\mu_{31}, \pi_{31}) & (\mu_{32}, \pi_{32}) & (\mu_{33}, \pi_{33}) \end{pmatrix}$ and $B_F = \begin{pmatrix} (\vartheta_{11}, \theta_{11}) & (\vartheta_{12}, \theta_{12}) & (\vartheta_{13}, \theta_{13}) \\ (\vartheta_{21}, \theta_{21}) & (\vartheta_{22}, \theta_{22}) & (\vartheta_{23}, \theta_{23}) \\ (\vartheta_{31}, \theta_{31}) & (\vartheta_{32}, \theta_{32}) & (\vartheta_{33}, \theta_{33}) \end{pmatrix}$

$\overline{A_E} = \begin{pmatrix} (\pi_{11}, \mu_{11}) & (\pi_{12}, \mu_{12}) & (\pi_{13}, \mu_{13}) \\ (\pi_{21}, \mu_{21}) & (\pi_{22}, \mu_{22}) & (\pi_{23}, \mu_{23}) \\ (\pi_{31}, \mu_{31}) & (\pi_{32}, \mu_{32}) & (\pi_{33}, \mu_{33}) \end{pmatrix}$ and $\overline{B_F} = \begin{pmatrix} (\theta_{11}, \vartheta_{11}) & (\theta_{12}, \vartheta_{12}) & (\theta_{13}, \vartheta_{13}) \\ (\theta_{21}, \vartheta_{21}) & (\theta_{22}, \vartheta_{22}) & (\theta_{23}, \vartheta_{23}) \\ (\theta_{31}, \vartheta_{31}) & (\theta_{32}, \vartheta_{32}) & (\theta_{33}, \vartheta_{33}) \end{pmatrix}$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of $\square\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1 - \pi_A$(x): x∈X} and $\square\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1 - \theta_B$(y): y∈Y}

$\square\overline{A_E} = \begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix}$ and

$\square\overline{B_F} = \begin{pmatrix} (\theta_{11}, 1 - \theta_{11}) & (\theta_{12}, 1 - \theta_{12}) & (\theta_{13}, 1 - \theta_{13}) \\ (\theta_{21}, 1 - \theta_{21}) & (\theta_{22}, 1 - \theta_{22}) & (\theta_{23}, 1 - \theta_{23}) \\ (\theta_{31}, 1 - \theta_{31}) & (\theta_{32}, 1 - \theta_{32}) & (\theta_{33}, 1 - \theta_{33}) \end{pmatrix}$

Now applying formula $\overline{A_E} \oplus \overline{B_F}$ = {⟨x, y⟩, $\overline{\mu_A}$ (x) + $\overline{\mu_B}$ (y) - $\overline{\mu_A}$ (x) . $\overline{\mu_B}$ (y), $\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F}, We have,

$$\square\bar{A}_E + \square\bar{B}_F = \begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix} +$$

$$\begin{pmatrix} (\theta_{11}, 1-\theta_{11}) & (\theta_{12}, 1-\theta_{12}) & (\theta_{13}, 1-\theta_{13}) \\ (\theta_{21}, 1-\theta_{21}) & (\theta_{22}, 1-\theta_{22}) & (\theta_{23}, 1-\theta_{23}) \\ (\theta_{31}, 1-\theta_{31}) & (\theta_{32}, 1-\theta_{32}) & (\theta_{33}, 1-\theta_{33}) \end{pmatrix}$$

Where, $X_{11} = \big((\pi_{11}, 1-\ \pi_{11}) + (\theta_{11}, 1-\theta_{11})\big)$, $X_{12}= \big((\pi_{12}, 1-\ \pi_{12}) + (\theta_{21}, 1-\theta_{21})\big)$

$X_{13}= \big((\pi_{13}, 1-\pi_{13}) + (\theta_{31}, 1-\theta_{31})\big)$, $X_{21}= \big((\pi_{21}, 1-\pi_{21}) + (\theta_{12}, 1-\theta_{12})\big)$

$X_{22}= \big((\pi_{22}, 1-\pi_{22}) + (\theta_{22}, 1-\theta_{22})\big)$, $X_{23}= \big((\pi_{23}, 1-\pi_{23}) + (\theta_{32}, 1-\theta_{32})\big)$

$X_{31}= \big((\pi_{31}, 1-\pi_{31}) + (\theta_{13}, 1-\theta_{13})\big)$, $X_{32}= \big((\pi_{32}, 1-\pi_{32}) + (\theta_{23}, 1-\theta_{23})\big)$

$X_{33}= \big((\pi_{33}, 1-\pi_{33}) + (\theta_{33}, 1-\theta_{33})\big)$

Now applying the formula, $\overline{A_E} \oplus \overline{B_F}$ = {$\langle$x, y$\rangle$, $\overline{\mu_A}$ (x) + $\overline{\mu_B}$ (y) - $\overline{\mu_A}$ (x) . $\overline{\mu_B}$ (y), $\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F}

X_{11}= (π_{11}(x) + θ_{11}(y) - π_{11}(x). θ_{11}(y), $1-\ \pi_{11}$(x). $1-\theta_{11}$(y))

X_{12}= (π_{12}(x) + θ_{21}(y) - π_{12}(x). θ_{21}(y), $1-\ \pi_{12}$(x). $1-\theta_{21}$(y))

X_{13}= (π_{13}(x) + θ_{31}(y) - π_{13}(x). θ_{31}(y), $1-\ \pi_{13}$(x). $1-\theta_{31}$(y))

X_{21}= (π_{21}(x) + θ_{12}(y) - π_{21}(x). θ_{12}(y), $1-\ \pi_{21}$(x). $1-\theta_{12}$(y))

X_{22}= (π_{22}(x) + θ_{22}(y) - π_{22}(x). θ_{22}(y), $1-\ \pi_{22}$(x). $1-\theta_{22}$(y))

X_{23}= (π_{23}(x) + θ_{32}(y) - π_{23}(x). θ_{32}(y), $1-\ \pi_{23}$(x). $1-\theta_{32}$(y))

X_{31}= (π_{31}(x) + θ_{13}(y) - π_{31}(x). θ_{13}(y), $1-\ \pi_{31}$(x). $1-\theta_{13}$(y))

X_{32}= (π_{32}(x) + θ_{23}(y) - π_{32}(x). θ_{23}(y), $1-\ \pi_{32}$(x). $1-\theta_{23}$(y))

X_{33}= (π_{33}(x) + θ_{33}(y) - π_{33}(x). θ_{33}(y), $1-\ \pi_{33}$(x). $1-\theta_{33}$(y))

We have, $\Box\bar{A}_E \oplus \Box\overline{B}_F = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix set.

Theorem 6.7.4: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}\ and\ \overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E \otimes \Box\overline{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}\ and\ \Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11},\pi_{11}) & (\mu_{12},\pi_{12}) & (\mu_{13},\pi_{13}) \\ (\mu_{21},\pi_{21}) & (\mu_{22},\pi_{22}) & (\mu_{23},\pi_{23}) \\ (\mu_{31},\pi_{31}) & (\mu_{32},\pi_{32}) & (\mu_{33},\pi_{33}) \end{pmatrix} \text{and} B_F = \begin{pmatrix} (\vartheta_{11},\theta_{11}) & (\vartheta_{12},\theta_{12}) & (\vartheta_{13},\theta_{13}) \\ (\vartheta_{21},\theta_{21}) & (\vartheta_{22},\theta_{22}) & (\vartheta_{23},\theta_{23}) \\ (\vartheta_{31},\theta_{31}) & (\vartheta_{32},\theta_{32}) & (\vartheta_{33},\theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11},\mu_{11}) & (\pi_{12},\mu_{12}) & (\pi_{13},\mu_{13}) \\ (\pi_{21},\mu_{21}) & (\pi_{22},\mu_{22}) & (\pi_{23},\mu_{23}) \\ (\pi_{31},\mu_{31}) & (\pi_{32},\mu_{32}) & (\pi_{33},\mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11},\vartheta_{11}) & (\theta_{12},\vartheta_{12}) & (\theta_{13},\vartheta_{13}) \\ (\theta_{21},\vartheta_{21}) & (\theta_{22},\vartheta_{22}) & (\theta_{23},\vartheta_{23}) \\ (\theta_{31},\vartheta_{31}) & (\theta_{32},\vartheta_{32}) & (\theta_{33},\vartheta_{33}) \end{pmatrix}$$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of □$\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\pi_A$(x): x∈X} and □$\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\theta_B$(y): y∈Y}

$$\square\overline{A_E} = \begin{pmatrix} (\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13}) \\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23}) \\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33}) \end{pmatrix} \text{and}$$

$$\square\overline{B_F} = \begin{pmatrix} (\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13}) \\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23}) \\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33}) \end{pmatrix}$$

Then applying the formula, $\overline{A_E}\,.\overline{B_F} = \{\langle x,y\rangle, \overline{\mu_A}(x)\cdot\overline{\mu_B}(y), (\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)\ (\overline{\vartheta_A}(x)\,\overline{\vartheta_A}(y): x\in E, y\in F\}$,

We have, $\square\bar{A}_E \otimes \square\bar{B}_F =$

$$\begin{pmatrix} (\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13}) \\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23}) \\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33}) \end{pmatrix} \otimes \begin{pmatrix} (\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13}) \\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23}) \\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33}) \end{pmatrix}$$

Where, $X_{11} = \big((\pi_{11},1-\pi_{11})\,.\,(\theta_{11},1-\theta_{11})\big)$, $X_{12} = \big((\pi_{12},1-\pi_{12})\,.\,(\theta_{21},1-\theta_{21})\big)$

$X_{13} = \big((\pi_{13},1-\pi_{13})\,.\,(\theta_{31},1-\theta_{31})\big)$, $X_{21} = \big((\pi_{21},1-\pi_{21})\,.\,(\theta_{12},1-\theta_{12})\big)$

$X_{22} = \big((\pi_{22},1-\pi_{22})\,.\,(\theta_{22},1-\theta_{22})\big)$, $X_{23} = \big((\pi_{23},1-\pi_{23})\,.\,(\theta_{32},1-\theta_{32})\big)$

$X_{31} = \big((\pi_{31},1-\pi_{31})\,.\,(\theta_{13},1-\theta_{13})\big)$, $X_{32} = \big((\pi_{32},1-\pi_{32})\,.\,(\theta_{23},1-\theta_{23})\big)$

$X_{33} = \big((\pi_{33},1-\pi_{33})\,.\,(\theta_{33},1-\theta_{33})\big)$

Now applying formula $\overline{A_E}\,.\overline{B_F} = \{\langle x,y\rangle, \overline{\mu_A}(x)\cdot\overline{\mu_B}(y), (\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y) - (\overline{\vartheta_A}(x)\cdot\overline{\vartheta_A}(y): x\in E, y\in F\}$

$X_{11} = \{(\pi_{11}\,.\,\theta_{11}), (1-\pi_{11}+1-\theta_{11}) - (1-\pi_{11}\,.\,1-\theta_{11})\}$

$X_{12}= \{(\pi_{12} \,.\, \theta_{21}), (1- \pi_{12} + 1 - \theta_{21}) - (1- \pi_{12} \,.\, 1 - \theta_{21})\}$

$X_{13}= \{(\pi_{13} \,.\, \theta_{31}), (1- \pi_{13} + 1 - \theta_{31}) - (1- \pi_{13} \,.\, 1 - \theta_{31})\}$

$X_{21}= \{(\pi_{21} \,.\, \theta_{12}), (1- \pi_{21} + 1 - \theta_{12}) - (1- \pi_{21} \,.\, 1 - \theta_{12})\}$

$X_{22}= \{(\pi_{22} \,.\, \theta_{22}), (1- \pi_{22} + 1 - \theta_{22}) - (1- \pi_{22} \,.\, 1 - \theta_{22})\}$

$X_{23}= \{(\pi_{23} \,.\, \theta_{32}), (1- \pi_{23} + 1 - \theta_{32}) - (1- \pi_{23} \,.\, 1 - \theta_{32})\}$

$X_{31}= \{(\pi_{31} \,.\, \theta_{13}), (1- \pi_{31} + 1 - \theta_{13}) - (1- \pi_{31} \,.\, 1 - \theta_{13})\}$

$X_{32}= \{(\pi_{32} \,.\, \theta_{23}), (1- \pi_{32} + 1 - \theta_{23}) - (1- \pi_{32} \,.\, 1 - \theta_{23})\}$

$X_{33}= \{(\pi_{33} \,.\, \theta_{33}), (1- \pi_{33} + 1 - \theta_{33}) - (1- \pi_{33} \,.\, 1 - \theta_{33})\}$

We have, $\Box\bar{A}_E \otimes \Box\bar{B}_F = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix.

Theorem 6.7.5: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E \,\#\, \Box\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}$ *and* $\Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11},\pi_{11}) & (\mu_{12},\pi_{12}) & (\mu_{13},\pi_{13}) \\ (\mu_{21},\pi_{21}) & (\mu_{22},\pi_{22}) & (\mu_{23},\pi_{23}) \\ (\mu_{31},\pi_{31}) & (\mu_{32},\pi_{32}) & (\mu_{33},\pi_{33}) \end{pmatrix} \text{ and } B_F = \begin{pmatrix} (\vartheta_{11},\theta_{11}) & (\vartheta_{12},\theta_{12}) & (\vartheta_{13},\theta_{13}) \\ (\vartheta_{21},\theta_{21}) & (\vartheta_{22},\theta_{22}) & (\vartheta_{23},\theta_{23}) \\ (\vartheta_{31},\theta_{31}) & (\vartheta_{32},\theta_{32}) & (\vartheta_{33},\theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11},\mu_{11}) & (\pi_{12},\mu_{12}) & (\pi_{13},\mu_{13}) \\ (\pi_{21},\mu_{21}) & (\pi_{22},\mu_{22}) & (\pi_{23},\mu_{23}) \\ (\pi_{31},\mu_{31}) & (\pi_{32},\mu_{32}) & (\pi_{33},\mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11},\vartheta_{11}) & (\theta_{12},\vartheta_{12}) & (\theta_{13},\vartheta_{13}) \\ (\theta_{21},\vartheta_{21}) & (\theta_{22},\vartheta_{22}) & (\theta_{23},\vartheta_{23}) \\ (\theta_{31},\vartheta_{31}) & (\theta_{32},\vartheta_{32}) & (\theta_{33},\vartheta_{33}) \end{pmatrix}$$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of $\Box\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\pi_A$(x): x∈X} and $\Box\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\theta_B$(y): y∈Y}

$$\Box\overline{A_E} = \begin{pmatrix} (\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13}) \\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23}) \\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33}) \end{pmatrix} \text{ and}$$

$$\Box\overline{B_F} = \begin{pmatrix} (\theta_{11}, 1-\theta_{11}) & (\theta_{12}, 1-\theta_{12}) & (\theta_{13}, 1-\theta_{13}) \\ (\theta_{21}, 1-\theta_{21}) & (\theta_{22}, 1-\theta_{22}) & (\theta_{23}, 1-\theta_{23}) \\ (\theta_{31}, 1-\theta_{31}) & (\theta_{32}, 1-\theta_{32}) & (\theta_{33}, 1-\theta_{33}) \end{pmatrix}$$

Then applying the formula $\overline{A_E}$ # $\overline{B_F}$ = $\{\langle x, y\rangle, \frac{2(\overline{\mu_A}(x) \cdot \overline{\mu_B}(y))}{\overline{\mu_A}(x) + \overline{\mu_B}(y)}, \frac{2(\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y))}{(\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}: x \in E, y \in F\}$,We have

$\Box\bar{A}_E$ # $\Box\bar{B}_F$ =

$$\begin{pmatrix} (\pi_{11}, 1-\pi_{11}) & (\pi_{12}, 1-\pi_{12}) & (\pi_{13}, 1-\pi_{13}) \\ (\pi_{21}, 1-\pi_{21}) & (\pi_{22}, 1-\pi_{22}) & (\pi_{23}, 1-\pi_{23}) \\ (\pi_{31}, 1-\pi_{31}) & (\pi_{32}, 1-\pi_{32}) & (\pi_{33}, 1-\pi_{33}) \end{pmatrix} \# \begin{pmatrix} (\theta_{11}, 1-\theta_{11}) & (\theta_{12}, 1-\theta_{12}) & (\theta_{13}, 1-\theta_{13}) \\ (\theta_{21}, 1-\theta_{21}) & (\theta_{22}, 1-\theta_{22}) & (\theta_{23}, 1-\theta_{23}) \\ (\theta_{31}, 1-\theta_{31}) & (\theta_{32}, 1-\theta_{32}) & (\theta_{33}, 1-\theta_{33}) \end{pmatrix}$$

Where, $X_{11} = \big((\pi_{11}, 1-\pi_{11}) \# (\theta_{11}, 1-\theta_{11})\big)$, $X_{12} = \big((\pi_{12}, 1-\pi_{12}) \# (\theta_{21}, 1-\theta_{21})\big)$

$X_{13} = \big((\pi_{13}, 1-\pi_{13}) \# (\theta_{31}, 1-\theta_{31})\big)$, $X_{21} = \big((\pi_{21}, 1-\pi_{21}) \# (\theta_{12}, 1-\theta_{12})\big)$

$X_{22} = \big((\pi_{22}, 1-\pi_{22}) \# (\theta_{22}, 1-\theta_{22})\big)$, $X_{23} = \big((\pi_{23}, 1-\pi_{23}) \# (\theta_{32}, 1-\theta_{32})\big)$

$X_{31} = \big((\pi_{31}, 1-\pi_{31}) \# (\theta_{13}, 1-\theta_{13})\big)$, $X_{32} = \big((\pi_{32}, 1-\pi_{32}) \# (\theta_{23}, 1-\theta_{23})\big)$

$X_{33} = \big((\pi_{33}, 1-\pi_{33}) \# (\theta_{33}, 1-\theta_{33})\big)$

Now applying the formula $\overline{A_E}$ # $\overline{B_F}$ = $\{\langle x, y\rangle, \frac{2(\overline{\mu_A}(x) \cdot \overline{\mu_B}(y))}{\overline{\mu_A}(x) + \overline{\mu_B}(y)}, \frac{2((\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y))}{(\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}: x \in E, y \in F\}$

$X_{11} = \left\{\frac{2(\pi_{11}(x) . \theta_{11}(y))}{\pi_{11}(x) + \theta_{11}(y)}, \frac{2(1-\pi_{11}(x) .\ 1-\theta_{11}(y))}{1-\pi_{11}(x) + 1-\theta_{11}(y)}\right\}$, $X_{12} = \left\{\frac{2(\pi_{12}(x) . \theta_{21}(y))}{\pi_{12}(x) + \theta_{21}(y)}, \frac{2(1-\pi_{12}(x) .\ 1-\theta_{21}(y))}{1-\pi_{12}(x) + 1-\theta_{21}(y)}\right\}$

$X_{13} = \left\{\frac{2(\pi_{13}(x) . \theta_{31}(y))}{\pi_{13}(x) + \theta_{31}(y)}, \frac{2(1-\pi_{13}(x) .\ 1-\theta_{31}(y))}{1-\pi_{13}(x) + 1-\theta_{31}(y)}\right\}$, $X_{21} = \left\{\frac{2(\pi_{21}(x) . \theta_{12}(y))}{\pi_{21}(x) + \theta_{12}(y)}, \frac{2(1-\pi_{21}(x) .\ 1-\theta_{12}(y))}{1-\pi_{21}(x) + 1-\theta_{12}(y)}\right\}$

$X_{22} = \left\{\frac{2(\pi_{22}(x) . \theta_{22}(y))}{\pi_{22}(x) + \theta_{22}(y)}, \frac{2(1-\pi_{22}(x) .\ 1-\theta_{22}(y))}{1-\pi_{22}(x) + 1-\theta_{22}(y)}\right\}$, $X_{23} = \left\{\frac{2(\pi_{23}(x) . \theta_{32}(y))}{\pi_{23}(x) + \theta_{32}(y)}, \frac{2(1-\pi_{23}(x) .\ 1-\theta_{32}(y))}{1-\pi_{23}(x) + 1-\theta_{32}(y)}\right\}$

$X_{31} = \left\{\frac{2(\pi_{31}(x) . \theta_{13}(y))}{\pi_{31}(x) + \theta_{13}(y)}, \frac{2(1-\pi_{31}(x) .\ 1-\theta_{13}(y))}{1-\pi_{31}(x) + 1-\theta_{13}(y)}\right\}$, $X_{32} = \left\{\frac{2(\pi_{32}(x) . \theta_{23}(y))}{\pi_{32}(x) + \theta_{23}(y)}, \frac{2(1-\pi_{32}(x) .\ 1-\theta_{23}(y))}{1-\pi_{32}(x) + 1-\theta_{23}(y)}\right\}$

$X_{33} = \left\{\frac{2(\pi_{33}(x) . \theta_{33}(y))}{\pi_{33}(x) + \theta_{33}(y)}, \frac{2(1-\pi_{33}(x) .\ 1-\theta_{33}(y))}{1-\pi_{33}(x) + 1-\theta_{33}(y)}\right\}$

We have, $\Box\overline{A_E}$ # $\Box\overline{B_F}$ = $\begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix.

Theorem 6.7.6: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E$ @ $\Box\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and

□$\overline{A_E}$ and □$\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11}, \pi_{11}) & (\mu_{12}, \pi_{12}) & (\mu_{13}, \pi_{13}) \\ (\mu_{21}, \pi_{21}) & (\mu_{22}, \pi_{22}) & (\mu_{23}, \pi_{23}) \\ (\mu_{31}, \pi_{31}) & (\mu_{32}, \pi_{32}) & (\mu_{33}, \pi_{33}) \end{pmatrix} \text{ and } B_F = \begin{pmatrix} (\vartheta_{11}, \theta_{11}) & (\vartheta_{12}, \theta_{12}) & (\vartheta_{13}, \theta_{13}) \\ (\vartheta_{21}, \theta_{21}) & (\vartheta_{22}, \theta_{22}) & (\vartheta_{23}, \theta_{23}) \\ (\vartheta_{31}, \theta_{31}) & (\vartheta_{32}, \theta_{32}) & (\vartheta_{33}, \theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11}, \mu_{11}) & (\pi_{12}, \mu_{12}) & (\pi_{13}, \mu_{13}) \\ (\pi_{21}, \mu_{21}) & (\pi_{22}, \mu_{22}) & (\pi_{23}, \mu_{23}) \\ (\pi_{31}, \mu_{31}) & (\pi_{32}, \mu_{32}) & (\pi_{33}, \mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11}, \vartheta_{11}) & (\theta_{12}, \vartheta_{12}) & (\theta_{13}, \vartheta_{13}) \\ (\theta_{21}, \vartheta_{21}) & (\theta_{22}, \vartheta_{22}) & (\theta_{23}, \vartheta_{23}) \\ (\theta_{31}, \vartheta_{31}) & (\theta_{32}, \vartheta_{32}) & (\theta_{33}, \vartheta_{33}) \end{pmatrix}$$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of □$\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1 - \pi_A$(x): x∈X} and □$\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1 - \theta_B$(y): y∈Y}

$$\square\overline{A_E} = \begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix} \text{ and}$$

$$\square\overline{B_F} = \begin{pmatrix} (\theta_{11}, 1 - \theta_{11}) & (\theta_{12}, 1 - \theta_{12}) & (\theta_{13}, 1 - \theta_{13}) \\ (\theta_{21}, 1 - \theta_{21}) & (\theta_{22}, 1 - \theta_{22}) & (\theta_{23}, 1 - \theta_{23}) \\ (\theta_{31}, 1 - \theta_{31}) & (\theta_{32}, 1 - \theta_{32}) & (\theta_{33}, 1 - \theta_{33}) \end{pmatrix}$$

Then applying the formula $\overline{A_E}$ @ $\overline{B_F}$ = {⟨x, y⟩,$\frac{\overline{\mu_A}\,(x) \cdot \overline{\mu_B}\,(y)}{2}$,$\frac{\overline{\vartheta_A}(x) + \overline{\vartheta_A}\,(y)}{2}$: x∈E, y∈$F$}, We have

□$\bar{A}_E$ @ □$\bar{B}_F$ =

$$\begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix} @ \begin{pmatrix} (\theta_{11}, 1 - \theta_{11}) & (\theta_{12}, 1 - \theta_{12}) & (\theta_{13}, 1 - \theta_{13}) \\ (\theta_{21}, 1 - \theta_{21}) & (\theta_{22}, 1 - \theta_{22}) & (\theta_{23}, 1 - \theta_{23}) \\ (\theta_{31}, 1 - \theta_{31}) & (\theta_{32}, 1 - \theta_{32}) & (\theta_{33}, 1 - \theta_{33}) \end{pmatrix}$$

Where, $X_{11} = \big((\pi_{11}, 1 - \pi_{11})\ @\ (\theta_{11}, 1 - \theta_{11})\big)$, $X_{12} = \big((\pi_{12}, 1 - \pi_{12})\ @\ (\theta_{21}, 1 - \theta_{21})\big)$

$X_{13} = \big((\pi_{13}, 1 - \pi_{13})\ @\ (\theta_{31}, 1 - \theta_{31})\big)$, $X_{21} = \big((\pi_{21}, 1 - \pi_{21})\ @\ (\theta_{12}, 1 - \theta_{12})\big)$

$X_{22} = \big((\pi_{22}, 1 - \pi_{22})\ @\ (\theta_{22}, 1 - \theta_{22})\big)$, $X_{23} = \big((\pi_{23}, 1 - \pi_{23})\ @\ (\theta_{32}, 1 - \theta_{32})\big)$

$X_{31} = \big((\pi_{31}, 1 - \pi_{31})\ @\ (\theta_{13}, 1 - \theta_{13})\big)$, $X_{32} = \big((\pi_{32}, 1 - \pi_{32})\ @\ (\theta_{23}, 1 - \theta_{23})\big)$

$X_{33} = \big((\pi_{33}, 1 - \pi_{33})\ @\ (\theta_{33}, 1 - \theta_{33})\big)$

Now applying the formula $\overline{A_E}$ @ $\overline{B_F}$ = {⟨x, y⟩,$\frac{\overline{\mu_A}\,(x) \cdot \overline{\mu_B}\,(y)}{2}$,$\frac{\overline{\vartheta_A}(x) + \overline{\vartheta_A}\,(y)}{2}$: x∈E, y∈$F$}

$X_{11}=\left\{\frac{\pi_{11}(x).\ \theta_{11}(y)}{2}, \frac{1-\pi_{11}(x)+1-\theta_{11}(y)}{2}\right\}$, $X_{12}=\left\{\frac{\pi_{12}(x).\ \theta_{21}(y)}{2}, \frac{1-\pi_{12}(x)+1-\theta_{21}(y)}{2}\right\}$

$X_{13}=\left\{\frac{\pi_{13}(x).\ \theta_{31}(y)}{2}, \frac{1-\pi_{13}(x)+1-\theta_{31}(y)}{2}\right\}$, $X_{21}=\left\{\frac{\pi_{21}(x).\ \theta_{12}(y)}{2}, \frac{1-\pi_{21}(x)+1-\theta_{12}(y)}{2}\right\}$

$X_{22}=\left\{\frac{\pi_{22}(x).\ \theta_{22}(y)}{2}, \frac{1-\pi_{22}(x)+1-\theta_{22}(y)}{2}\right\}$, $X_{23}=\left\{\frac{\pi_{23}(x).\ \theta_{32}(y)}{2}, \frac{1-\pi_{23}(x)+1-\theta_{32}(y)}{2}\right\}$

$X_{31}=\left\{\frac{\pi_{31}(x).\ \theta_{13}(y)}{2}, \frac{1-\pi_{31}(x)+1-\theta_{13}(y)}{2}\right\}$, $X_{32}=\left\{\frac{\pi_{32}(x).\ \theta_{23}(y)}{2}, \frac{1-\pi_{32}(x)+1-\theta_{23}(y)}{2}\right\}$

$X_{33}=\left\{\frac{\pi_{33}(x).\ \theta_{33}(y)}{2}, \frac{1-\pi_{33}(x)+1-\theta_{33}(y)}{2}\right\}$

We have, $\Box\overline{A_E}$ @ $\Box\overline{B_F}$ = $\begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix.

Theorem 6.7.7: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\Box\bar{A}_E$ \$ $\Box\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}$ *and* $\Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$A_E=\begin{pmatrix} (\mu_{11},\pi_{11}) & (\mu_{12},\pi_{12}) & (\mu_{13},\pi_{13}) \\ (\mu_{21},\pi_{21}) & (\mu_{22},\pi_{22}) & (\mu_{23},\pi_{23}) \\ (\mu_{31},\pi_{31}) & (\mu_{32},\pi_{32}) & (\mu_{33},\pi_{33}) \end{pmatrix}$ and $B_F=\begin{pmatrix} (\vartheta_{11},\theta_{11}) & (\vartheta_{12},\theta_{12}) & (\vartheta_{13},\theta_{13}) \\ (\vartheta_{21},\theta_{21}) & (\vartheta_{22},\theta_{22}) & (\vartheta_{23},\theta_{23}) \\ (\vartheta_{31},\theta_{31}) & (\vartheta_{32},\theta_{32}) & (\vartheta_{33},\theta_{33}) \end{pmatrix}$

$\overline{A_E}=\begin{pmatrix} (\pi_{11},\mu_{11}) & (\pi_{12},\mu_{12}) & (\pi_{13},\mu_{13}) \\ (\pi_{21},\mu_{21}) & (\pi_{22},\mu_{22}) & (\pi_{23},\mu_{23}) \\ (\pi_{31},\mu_{31}) & (\pi_{32},\mu_{32}) & (\pi_{33},\mu_{33}) \end{pmatrix}$ and $\overline{B_F}=\begin{pmatrix} (\theta_{11},\vartheta_{11}) & (\theta_{12},\vartheta_{12}) & (\theta_{13},\vartheta_{13}) \\ (\theta_{21},\vartheta_{21}) & (\theta_{22},\vartheta_{22}) & (\theta_{23},\vartheta_{23}) \\ (\theta_{31},\vartheta_{31}) & (\theta_{32},\vartheta_{32}) & (\theta_{33},\vartheta_{33}) \end{pmatrix}$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of $\Box\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\pi_A$(x): x∈X} and $\Box\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\theta_B$(y): y∈Y}

$\Box\overline{A_E}=\begin{pmatrix} (\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13}) \\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23}) \\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33}) \end{pmatrix}$ and

$\Box\overline{B_F}=\begin{pmatrix} (\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13}) \\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23}) \\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33}) \end{pmatrix}$

Then applying the formula $\overline{A_E}$ \$ $\overline{B_F}$ = {⟨x, y⟩, √$\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), √ ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F},

We have, □$\bar{A}_E$ \$ □$\bar{B}_F$ =

$$\begin{pmatrix}(\pi_{11},1-\ \pi_{11}) & (\pi_{12},1-\ \pi_{12}) & (\pi_{13},1-\pi_{13}) \\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23}) \\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix} \$ \begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13}) \\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23}) \\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$

Where, X_{11} =$\left((\pi_{11},1-\ \pi_{11})\ \$\ (\theta_{11},1-\theta_{11})\right)$, X_{12}=$\left((\pi_{12},1-\ \pi_{12})\ \$\ (\theta_{21},1-\theta_{21})\right)$

X_{13}=$\left((\pi_{13},1-\pi_{13})\ \$\ (\theta_{31},1-\theta_{31})\right)$, X_{21}=$\left((\pi_{21},1-\pi_{21})\ \$\ (\theta_{12},1-\theta_{12})\right)$

X_{22}=$\left((\pi_{22},1-\pi_{22})\ \$\ (\theta_{22},1-\theta_{22})\right)$, X_{23}=$\left((\pi_{23},1-\pi_{23})\ \$\ (\theta_{32},1-\theta_{32})\right)$

X_{31}= $\left((\pi_{31},1-\pi_{31})\ \$\ (\theta_{13},1-\theta_{13})\right)$, X_{32}=$\left((\pi_{32},1-\pi_{32})\ \$\ (\theta_{23},1-\theta_{23})\right)$

X_{33}=$\left((\pi_{33},1-\pi_{33})\ \$\ (\theta_{33},1-\theta_{33})\right)$ .Now applying the formula

$\overline{A_E}$ \$ $\overline{B_F}$ = {⟨x, y⟩, √$\overline{\mu_A}$ (x) · $\overline{\mu_B}$ (y), √ ($\overline{\vartheta_A}$(x) · $\overline{\vartheta_A}$ (y): x∈E, y∈F}

X_{11}= $\left\{\sqrt{\pi_{11}(\mathrm{x}).\ \theta_{11}(\mathrm{y})}, \sqrt{1-\ \pi_{11}(\mathrm{x}).1-\theta_{11}(\mathrm{y})}\right\}$,

X_{12}= $\left\{\sqrt{\pi_{12}(\mathrm{x})\ .\ \theta_{21}(\mathrm{y})}, \sqrt{1-\ \pi_{12}(\mathrm{x})\ .1-\theta_{21}(\mathrm{y})}\right\}$

X_{13}= $\left\{\sqrt{\pi_{13}(\mathrm{x}).\ \theta_{31}(\mathrm{y})}, \sqrt{1-\ \pi_{13}(\mathrm{x}).1-\theta_{31}(\mathrm{y})}\right\}$,

X_{21}= $\left\{\sqrt{\pi_{21}(\mathrm{x})\ .\ \theta_{12}(\mathrm{y})}, \sqrt{1-\ \pi_{21}(\mathrm{x})\ .1-\theta_{12}(\mathrm{y})}\right\}$

X_{22}= $\left\{\sqrt{\pi_{22}(\mathrm{x})\ .\ \theta_{22}(\mathrm{y})}, \sqrt{1-\ \pi_{22}(\mathrm{x})\ .1-\theta_{22}(\mathrm{y})}\right\}$

X_{23}= $\left\{\sqrt{\pi_{23}(\mathrm{x})\ .\ \theta_{32}(\mathrm{y})}, \sqrt{1-\ \pi_{23}(\mathrm{x})\ .1-\theta_{32}(\mathrm{y})}\right\}$

X_{31}= $\left\{\sqrt{\pi_{31}(\mathrm{x})\ .\ \theta_{13}(\mathrm{y})}, \sqrt{1-\ \pi_{31}(\mathrm{x})\ .1-\theta_{13}(\mathrm{y})}\right\}$

X_{32}= $\left\{\sqrt{\pi_{32}(\mathrm{x})\ .\ \theta_{23}(\mathrm{y})}, \sqrt{1-\ \pi_{32}(\mathrm{x})\ .1-\theta_{23}(\mathrm{y})}\right\}$

X_{33}= $\left\{\sqrt{\pi_{33}(\mathrm{x})\ .\ \theta_{33}(\mathrm{y})}, \sqrt{1-\ \pi_{33}(\mathrm{x})\ .1-\theta_{33}(\mathrm{y})}\right\}$

We have, □$\overline{A_E}$ \$ □$\overline{B_F}$ = $\begin{pmatrix}X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33}\end{pmatrix}$ is also intuitionistic fuzzy matrix.

Theorem 6.7.8: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then □$\bar{A}_E$ ∗ □$\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\Box\overline{A_E}$ *and* $\Box\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11}, \pi_{11}) & (\mu_{12}, \pi_{12}) & (\mu_{13}, \pi_{13}) \\ (\mu_{21}, \pi_{21}) & (\mu_{22}, \pi_{22}) & (\mu_{23}, \pi_{23}) \\ (\mu_{31}, \pi_{31}) & (\mu_{32}, \pi_{32}) & (\mu_{33}, \pi_{33}) \end{pmatrix} \text{ and } B_F = \begin{pmatrix} (\vartheta_{11}, \theta_{11}) & (\vartheta_{12}, \theta_{12}) & (\vartheta_{13}, \theta_{13}) \\ (\vartheta_{21}, \theta_{21}) & (\vartheta_{22}, \theta_{22}) & (\vartheta_{23}, \theta_{23}) \\ (\vartheta_{31}, \theta_{31}) & (\vartheta_{32}, \theta_{32}) & (\vartheta_{33}, \theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11}, \mu_{11}) & (\pi_{12}, \mu_{12}) & (\pi_{13}, \mu_{13}) \\ (\pi_{21}, \mu_{21}) & (\pi_{22}, \mu_{22}) & (\pi_{23}, \mu_{23}) \\ (\pi_{31}, \mu_{31}) & (\pi_{32}, \mu_{32}) & (\pi_{33}, \mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11}, \vartheta_{11}) & (\theta_{12}, \vartheta_{12}) & (\theta_{13}, \vartheta_{13}) \\ (\theta_{21}, \vartheta_{21}) & (\theta_{22}, \vartheta_{22}) & (\theta_{23}, \vartheta_{23}) \\ (\theta_{31}, \vartheta_{31}) & (\theta_{32}, \vartheta_{32}) & (\theta_{33}, \vartheta_{33}) \end{pmatrix}$$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of $\Box\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1 - \pi_A$(x): x∈X} and $\Box\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1 - \theta_B$(y): y∈Y}

$$\Box\overline{A_E} = \begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix} \text{ and}$$

$$\Box\overline{B_F} = \begin{pmatrix} (\theta_{11}, 1 - \theta_{11}) & (\theta_{12}, 1 - \theta_{12}) & (\theta_{13}, 1 - \theta_{13}) \\ (\theta_{21}, 1 - \theta_{21}) & (\theta_{22}, 1 - \theta_{22}) & (\theta_{23}, 1 - \theta_{23}) \\ (\theta_{31}, 1 - \theta_{31}) & (\theta_{32}, 1 - \theta_{32}) & (\theta_{33}, 1 - \theta_{33}) \end{pmatrix}$$

Then applying the formula $\overline{A_E} * \overline{B_F} = \{\langle x, y\rangle, \frac{\overline{\mu_A}(x) + \overline{\mu_B}(y)}{2(\overline{\mu_A}(x) \cdot \overline{\mu_B}(y) + 1)}, \frac{\overline{\vartheta_A}(x) + \overline{\vartheta_A}(y)}{2(\overline{\vartheta_A}(x) \cdot \overline{\vartheta_A}(y) + 1)} : x \in E, y \in F\}$,

We have, $$\Box\bar{A}_E * \Box\bar{B}_F = \begin{pmatrix} (\pi_{11}, 1 - \pi_{11}) & (\pi_{12}, 1 - \pi_{12}) & (\pi_{13}, 1 - \pi_{13}) \\ (\pi_{21}, 1 - \pi_{21}) & (\pi_{22}, 1 - \pi_{22}) & (\pi_{23}, 1 - \pi_{23}) \\ (\pi_{31}, 1 - \pi_{31}) & (\pi_{32}, 1 - \pi_{32}) & (\pi_{33}, 1 - \pi_{33}) \end{pmatrix} * \begin{pmatrix} (\theta_{11}, 1 - \theta_{11}) & (\theta_{12}, 1 - \theta_{12}) & (\theta_{13}, 1 - \theta_{13}) \\ (\theta_{21}, 1 - \theta_{21}) & (\theta_{22}, 1 - \theta_{22}) & (\theta_{23}, 1 - \theta_{23}) \\ (\theta_{31}, 1 - \theta_{31}) & (\theta_{32}, 1 - \theta_{32}) & (\theta_{33}, 1 - \theta_{33}) \end{pmatrix}$$

Where, $X_{11} = \big((\pi_{11}, 1 - \pi_{11}) * (\theta_{11}, 1 - \theta_{11})\big)$, $X_{12} = \big((\pi_{12}, 1 - \pi_{12}) * (\theta_{21}, 1 - \theta_{21})\big)$

$X_{13} = \big((\pi_{13}, 1 - \pi_{13}) * (\theta_{31}, 1 - \theta_{31})\big)$, $X_{21} = \big((\pi_{21}, 1 - \pi_{21}) * (\theta_{12}, 1 - \theta_{12})\big)$

$X_{22} = \big((\pi_{22}, 1 - \pi_{22}) * (\theta_{22}, 1 - \theta_{22})\big)$, $X_{23} = \big((\pi_{23}, 1 - \pi_{23}) * (\theta_{32}, 1 - \theta_{32})\big)$

$X_{31} = \big((\pi_{31}, 1 - \pi_{31}) * (\theta_{13}, 1 - \theta_{13})\big)$, $X_{32} = \big((\pi_{32}, 1 - \pi_{32}) * (\theta_{23}, 1 - \theta_{23})\big)$

$X_{33}= \left((\pi_{33}, 1-\pi_{33}) \ * \ (\theta_{33}, 1-\theta_{33})\right)$

Then applying the formula $\overline{A_E} * \overline{B_F} = \{\langle x, y\rangle, \frac{\overline{\mu_A}(x)+\overline{\mu_B}(y)}{2(\overline{\mu_A}(x)\cdot\overline{\mu_B}(y)+1)}, \frac{\overline{\vartheta_A}(x)+\overline{\vartheta_A}(y)}{2(\overline{\vartheta_A}(x)\cdot\overline{\vartheta_A}(y)+1)}: x\in E,\ y\in F\}$

$$X_{11}= \left\{\frac{\pi_{11}(x)+\theta_{11}(y)}{2(\pi_{11}(x).\ \theta_{11}(y)+1)},\ \frac{1-\pi_{11}(x)+1-\theta_{11}(y)}{2(1-\pi_{11}(x).\ 1-\theta_{11}(y)+1)}\right\},$$

$$X_{12}= \left\{\frac{\pi_{12}(x)+\theta_{21}(y)}{2(\pi_{12}(x).\ \theta_{21}(y)+1)},\ \frac{1-\pi_{12}(x)+1-\theta_{21}(y)}{2(1-\pi_{12}(x).\ 1-\theta_{21}(y)+1)}\right\}$$

$$X_{13}= \left\{\frac{\pi_{13}(x)+\theta_{31}(y)}{2(\pi_{13}(x).\ \theta_{31}(y)+1)},\ \frac{1-\pi_{13}(x)+1-\theta_{31}(y)}{2(1-\pi_{13}(x).\ 1-\theta_{31}(y)+1)}\right\},$$

$$X_{21}= \left\{\frac{\pi_{21}(x)+\theta_{12}(y)}{2(\pi_{21}(x).\ \theta_{12}(y)+1)},\ \frac{1-\pi_{21}(x)+1-\theta_{12}(y)}{2(1-\pi_{21}(x).\ 1-\theta_{12}(y)+1)}\right\}$$

$$X_{22}= \left\{\frac{\pi_{22}(x)+\theta_{22}(y)}{2(\pi_{22}(x).\ \theta_{22}(y)+1)},\ \frac{1-\pi_{22}(x)+1-\theta_{22}(y)}{2(1-\pi_{22}(x).\ 1-\theta_{22}(y)+1)}\right\},$$

$$X_{23}= \left\{\frac{\pi_{23}(x)+\theta_{32}(y)}{2(\pi_{23}(x).\ \theta_{32}(y)+1)},\ \frac{1-\pi_{23}(x)+1-\theta_{32}(y)}{2(1-\pi_{23}(x).\ 1-\theta_{32}(y)+1)}\right\}$$

$$X_{31}= \left\{\frac{\pi_{31}(x)+\theta_{13}(y)}{2(\pi_{31}(x).\ \theta_{13}(y)+1)},\ \frac{1-\pi_{31}(x)+1-\theta_{13}(y)}{2(1-\pi_{31}(x).\ 1-\theta_{13}(y)+1)}\right\},$$

$$X_{32}= \left\{\frac{\pi_{32}(x)+\theta_{23}(y)}{2(\pi_{32}(x).\ \theta_{23}(y)+1)},\ \frac{1-\pi_{32}(x)+1-\theta_{23}(y)}{2(1-\pi_{32}(x).\ 1-\theta_{23}(y)+1)}\right\}$$

$$X_{33}= \left\{\frac{\pi_{33}(x)+\theta_{33}(y)}{2(\pi_{33}(x).\ \theta_{33}(y)+1)},\ \frac{1-\pi_{33}(x)+1-\theta_{33}(y)}{2(1-\pi_{33}(x).\ 1-\theta_{33}(y)+1)}\right\}$$

We have, $\square\overline{A_E} * \square\overline{B_F} = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$ is also intuitionistic fuzzy matrix.

Theorem 6.7.9: If E and F be two universal sets. For every necessity function of intuitionistic fuzzy matrix $\overline{A_E}$ *and* $\overline{B_F}$ are intuitionistic fuzzy matrix in E and F then $\square\bar{A}_E \ \Delta \ \square\bar{B}_F$ is also necessity function of intuitionistic fuzzy matrix.

Proof: If A_E and B_F are two intuitionistic fuzzy matrices sets over different universes E & F and $\square\overline{A_E}$ *and* $\square\overline{B_F}$ are also intuitionistic fuzzy matrices sets over different universes E and F.

If we consider to 3× 3 matrix.

$$A_E = \begin{pmatrix} (\mu_{11},\pi_{11}) & (\mu_{12},\pi_{12}) & (\mu_{13},\pi_{13}) \\ (\mu_{21},\pi_{21}) & (\mu_{22},\pi_{22}) & (\mu_{23},\pi_{23}) \\ (\mu_{31},\pi_{31}) & (\mu_{32},\pi_{32}) & (\mu_{33},\pi_{33}) \end{pmatrix} \text{ and } B_F= \begin{pmatrix} (\vartheta_{11},\theta_{11}) & (\vartheta_{12},\theta_{12}) & (\vartheta_{13},\theta_{13}) \\ (\vartheta_{21},\theta_{21}) & (\vartheta_{22},\theta_{22}) & (\vartheta_{23},\theta_{23}) \\ (\vartheta_{31},\theta_{31}) & (\vartheta_{32},\theta_{32}) & (\vartheta_{33},\theta_{33}) \end{pmatrix}$$

$$\overline{A_E} = \begin{pmatrix} (\pi_{11},\mu_{11}) & (\pi_{12},\mu_{12}) & (\pi_{13},\mu_{13}) \\ (\pi_{21},\mu_{21}) & (\pi_{22},\mu_{22}) & (\pi_{23},\mu_{23}) \\ (\pi_{31},\mu_{31}) & (\pi_{32},\mu_{32}) & (\pi_{33},\mu_{33}) \end{pmatrix} \text{ and } \overline{B_F} = \begin{pmatrix} (\theta_{11},\vartheta_{11}) & (\theta_{12},\vartheta_{12}) & (\theta_{13},\vartheta_{13}) \\ (\theta_{21},\vartheta_{21}) & (\theta_{22},\vartheta_{22}) & (\theta_{23},\vartheta_{23}) \\ (\theta_{31},\vartheta_{31}) & (\theta_{32},\vartheta_{32}) & (\theta_{33},\vartheta_{33}) \end{pmatrix}$$

If $\overline{A_E}$ = {x, π_A(x), μ(x): x∈X} and $\overline{B_F}$ = {y, θ_A(y), ϑ_A(y): y∈Y} are two also intuitionistic fuzzy matrices sets over different universes E and F.

By the definition of □$\overline{A_E}$ = {x, π_A(x): x∈X} = {x, π_A(x), $1-\pi_A$(x): x∈X} and □$\overline{B_F}$ = {y, θ_B(y): y∈Y} = {y, θ_B(y), $1-\theta_B$(y): y∈Y}

$$\square\overline{A_E}=\begin{pmatrix}(\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13})\\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix} \text{ and}$$

$$\square\overline{B_F}=\begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$

Then the applying formula

$\overline{A_E}\,\Delta\,\overline{B_F}$ = {(x, y),$\frac{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})}{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})+\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})},\frac{\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})}{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})+\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})}$: x∈E, y∈F},We have

$$\square\bar{A}_E\,\Delta\,\square\bar{B}_F=$$

$$\begin{pmatrix}(\pi_{11},1-\pi_{11}) & (\pi_{12},1-\pi_{12}) & (\pi_{13},1-\pi_{13})\\ (\pi_{21},1-\pi_{21}) & (\pi_{22},1-\pi_{22}) & (\pi_{23},1-\pi_{23})\\ (\pi_{31},1-\pi_{31}) & (\pi_{32},1-\pi_{32}) & (\pi_{33},1-\pi_{33})\end{pmatrix}\Delta\begin{pmatrix}(\theta_{11},1-\theta_{11}) & (\theta_{12},1-\theta_{12}) & (\theta_{13},1-\theta_{13})\\ (\theta_{21},1-\theta_{21}) & (\theta_{22},1-\theta_{22}) & (\theta_{23},1-\theta_{23})\\ (\theta_{31},1-\theta_{31}) & (\theta_{32},1-\theta_{32}) & (\theta_{33},1-\theta_{33})\end{pmatrix}$$

Where, $X_{11}=\left((\pi_{11},1-\pi_{11})\,\Delta\,(\theta_{11},1-\theta_{11})\right)$, $X_{12}=\left((\pi_{12},1-\pi_{12})\,\Delta\,(\theta_{21},1-\theta_{21})\right)$

$X_{13}=\left((\pi_{13},1-\pi_{13})\,\Delta\,(\theta_{31},1-\theta_{31})\right)$, $X_{21}=\left((\pi_{21},1-\pi_{21})\,\Delta\,(\theta_{12},1-\theta_{12})\right)$

$X_{22}=\left((\pi_{22},1-\pi_{22})\,\Delta\,(\theta_{22},1-\theta_{22})\right)$, $X_{23}=\left((\pi_{23},1-\pi_{23})\,\Delta\,(\theta_{32},1-\theta_{32})\right)$

$X_{31}=\left((\pi_{31},1-\pi_{31})\,\Delta\,(\theta_{13},1-\theta_{13})\right)$, $X_{32}=\left((\pi_{32},1-\pi_{32})\,\Delta\,(\theta_{23},1-\theta_{23})\right)$

$X_{33}=\left((\pi_{33},1-\pi_{33})\,\Delta\,(\theta_{33},1-\theta_{33})\right)$

Now applying the formula

$\overline{A_E}\,\Delta\,\overline{B_F}$ = {(x, y),$\frac{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})}{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})+\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})},\frac{\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})}{\bar{\mu}_A(\text{x})+\bar{\mu}_B(\text{y})+\bar{\vartheta}_A(\text{x})+\bar{\vartheta}_B(\text{y})}$: x∈E, y∈F}

$$X_{11}=\left\{\frac{\pi_{11}(\text{x})+\theta_{11}(\text{y})}{\pi_{11}(\text{x})+\theta_{11}(\text{y})+1-\pi_{11}(\text{x})+1-\theta_{11}(\text{y})},\frac{1-\pi_{11}(\text{x})+1-\theta_{11}(\text{y})}{\pi_{11}(\text{x})+\theta_{11}(\text{y})+1-\pi_{11}(\text{x})+1-\theta_{11}(\text{y})}\right\}$$

$$X_{12}=\left\{\frac{\pi_{12}(\text{x})+\theta_{21}(\text{y})}{\pi_{12}(\text{x})+\theta_{21}(\text{y})+1-\pi_{12}(\text{x})+1-\theta_{21}(\text{y})},\frac{1-\pi_{12}(\text{x})+1-\theta_{21}(\text{y})}{\pi_{12}(\text{x})+\theta_{21}(\text{y})+1-\pi_{12}(\text{x})+1-\theta_{21}(\text{y})}\right\}$$

$$X_{13}=\left\{\frac{\pi_{13}(\text{x})+\theta_{31}(\text{y})}{\pi_{13}(\text{x})+\theta_{31}(\text{y})+1-\pi_{13}(\text{x})+1-\theta_{31}(\text{y})},\frac{1-\pi_{13}(\text{x})+1-\theta_{31}(\text{y})}{\pi_{13}(\text{x})+\theta_{31}(\text{y})+1-\pi_{13}(\text{x})+1-\theta_{31}(\text{y})}\right\}$$

$$X_{21}=\left\{\frac{\pi_{21}(x)+\theta_{12}(y)}{\pi_{21}(x)+\theta_{12}(y)+1-\pi_{21}(x)+1-\theta_{12}(y)},\frac{1-\pi_{21}(x)+1-\theta_{12}(y)}{\pi_{21}(x)+\theta_{12}(y)+1-\pi_{21}(x)+1-\theta_{12}(y)}\right\}$$

$$X_{22}=\left\{\frac{\pi_{22}(x)+\theta_{22}(y)}{\pi_{22}(x)+\theta_{22}(y)+1-\pi_{22}(x)+1-\theta_{22}(y)},\frac{1-\pi_{22}(x)+1-\theta_{22}(y)}{\pi_{22}(x)+\theta_{22}(y)+1-\pi_{22}(x)+1-\theta_{22}(y)}\right\}$$

$$X_{23}=\left\{\frac{\pi_{23}(x)+\theta_{32}(y)}{\pi_{23}(x)+\theta_{32}(y)+1-\pi_{23}(x)+1-\theta_{32}(y)},\frac{1-\pi_{23}(x)+1-\theta_{32}(y)}{\pi_{23}(x)+\theta_{32}(y)+1-\pi_{23}(x)+1-\theta_{32}(y)}\right\}$$

$$X_{31}=\left\{\frac{\pi_{31}(x)+\theta_{13}(y)}{\pi_{31}(x)+\theta_{13}(y)+1-\pi_{31}(x)+1-\theta_{13}(y)},\frac{1-\pi_{31}(x)+1-\theta_{13}(y)}{\pi_{31}(x)+\theta_{13}(y)+1-\pi_{31}(x)+1-\theta_{13}(y)}\right\}$$

$$X_{32}=\left\{\frac{\pi_{32}(x)+\theta_{23}(y)}{\pi_{32}(x)+\theta_{23}(y)+1-\pi_{32}(x)+1-\theta_{23}(y)},\frac{1-\pi_{32}(x)+1-\theta_{23}(y)}{\pi_{32}(x)+\theta_{23}(y)+1-\pi_{32}(x)+1-\theta_{23}(y)}\right\}$$

$$X_{33}=\left\{\frac{\pi_{33}(x)+\theta_{33}(y)}{\pi_{33}(x)+\theta_{33}(y)+1-\pi_{33}(x)+1-\theta_{33}(y)},\frac{1-\pi_{33}(x)+1-\theta_{33}(y)}{\pi_{33}(x)+\theta_{33}(y)+1-\pi_{33}(x)+1-\theta_{33}(y)}\right\}$$

We have, $\Box\overline{A_E}\ \Delta\ \Box\overline{B_F}=\begin{pmatrix}X_{11} & X_{12} & X_{13}\\ X_{21} & X_{22} & X_{23}\\ X_{31} & X_{32} & X_{33}\end{pmatrix}$ is also intuitionistic fuzzy

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