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# A Comparison Study on Replacing Machines in Uncertain Environments whose Maintenance Costs Increase with Different Scrap Values Over Time

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**Abstract:**

Replacement is a type of maintenance performed on a system to ensure that it operates efficiently and effectively. Replacement theory is commonly used to describe the replacement of machines or equipment due to deterioration, decreased efficiency, failure, or breakdown. In applications dealing with uncertainty, an intuitionistic fuzzy set has been used. The purpose of this study is to investigate the replacement problem with uncertainty. This problem involves imprecise capital cost, imprecise scrap value or salvage value, and an imprecise maintenance cost. The imprecise values are assumed to be positive intuitionistic fuzzy numbers in this case. Scrap fuzzy value is the value of a physical item's separate parts when the asset is regarded to be no longer useful. After a long-term asset, such as machinery, a vehicle, or furniture, has served its purpose, it may be disposed of. Based on the age of the machine or equipment, this article examined three different replacement models. The first is based on rising imprecise maintenance costs and no scrap value. When the price of fuzzy Maintenance costs is rising. Because it is assumed that the asset will be obsolete at the end of the longer depreciation period. The second is based on increasing imprecise maintenance costs in conjunction with a scrap fuzzy value. In some circumstances, a piece of business machinery's scrap fuzzy value may remain constant for a number of years after it has reached the end of its useful life.. The third is based on rising imprecise maintenance fuzzy costs and falling scrap fuzzy value. When imprecise maintenance costs increase, most of the time, the scrap value falls. The soundness of the proposed method is to determine the age of a machine or piece of equipment without converting fuzzy values to crisp values.

**Keywords**: Replacement model, Fuzzy optimization, Intuitionistic Triangular fuzzy number.

## 1. Introduction

The Replacement Theory describes the process of replacing old equipment with newer, more efficient equipment. In the classical replacement model, M Abdel-Hameed [3] discussed the block replacement policy with classical parameters problem in 1987. It replaces devices when they fail and once every T units of time, with each failed device being replaced by a new device. Y. Lam[4] proposed two prospective replacement techniques based on working duration and failure rate later that year, in 1988, and the geometric method was proposed to describe operating and maintenance periods. Following that, Henning Oeltjenbruns, et al., [5] applied AHP strategic planning to an equipment replacement decision in manufacturing systems in 1995. Mr. Richard M. Feldman, et al., [6] introduced a minimal repair/replacement problem in 1996 to demonstrate the use of post-optimal analyses during policy implementation. In 1998,Karsak, E, et al.,[7] proposed an overhaul-replacement approach for machinery subject to technological change in an inflationary environment. X. In 2016, Zhao, et al.,[10] proposed policies to replace a parallel system with shortages and excess costs. In 2021, Bhattacharyya D, [14] investigated a two-sample nonparametric test for comparing mean time to failure functions in age replacement. In 2022, Vijay C. Makwana proposed a new hypothesis and solution for fuzzy equations. According to reports, many real-world problems cannot be solved using classical set theory, necessitating its extension. As a result, the illustrious scientist Zadeh [1] proposed Fuzzy Sets, an extension of classical set theory, in the 1960s. The fuzzy set theory and the classical set theory approach ambiguity differently. In 1986, Atanassov [2] proposed and demonstrated intuitionistic Fuzzy Sets as a fuzzy set generalization. In this study, we compared the annual mean time before sudden failure of three different replacement models with uncertain parameters to see if they could mitigate risks such as machine or equipment failure in an uncertain situation. The soundness of the proposed method in terms of age or replacement time is determined using intuitionistic fuzzy parameters without converting fuzzy values to crisp values.

## 2. Preliminaries

**Definition 2.1** A fuzzy number $\tilde{A}^{IFN}$ on R is said to be a triangular intuitionistic fuzzy number if its membership function $\mu_{\tilde{A}^{IFN}} : R \to [0,1]$ and non- membership function $\gamma_{\tilde{A}^{IFN}} : R \to [0,1]$ satisfies the following conditions,

$$\mu_{\tilde{A}^{IFN}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \le x \le a_2 \\ = 1, \text{ for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{for } a_2 \le x \le a_3 \\ = 0, \text{otherwise} \end{cases} \qquad \gamma_{\tilde{A}^{IFN}}(x) = \begin{cases} \dfrac{x - d_1}{d_2 - d_1}, \text{for } d_1 \le x \le d_2 \\ = 1, \text{ for } x = d_2 \\ \dfrac{d_3 - x}{d_3 - d_2}, \text{for } d_2 \le x \le d_3 \\ = 0, \text{otherwise} \end{cases}$$

$\tilde{A}^{IFN} = (a_1, a_2, a_3; d_1, d_2, d_3)$ where

$d_1 \le a_1 \le (d_2 = a_2) \le a_3 \le d_3$.

Let $\tilde{A}^{IFN} = (a_1, a_2, a_3; d_1, d_2, d_3)$ be a triangular intuitionistic fuzzy number then the following cases arise.

Case 1:If $a_1 = d_1, a_2 = d_2, a_3 = d_3$, then

$\tilde{A}^{IFN}$ represent a triangular fuzzy number.

Case2: If $a_1 = d_1 = a_2 = d_2 = a_3 = d_3 = R$, then $\tilde{A}^{IFN}$ represent a real number R. The parametric form of triangular intuitionistic fuzzy number $\tilde{A}^{IFN}$ is represented as $\tilde{A}^{IFN} = (\alpha_{\mu_a}, a_\mu, \beta_{\mu_a}; \alpha_{\gamma_a}, a_\gamma, \beta_{\gamma_a})$ where $a_\mu, a_\gamma$ are the mid value of membership functions and non-membership functions and $\alpha_{\mu_a}, \alpha_{\gamma_a}$ & $\beta_{\mu_a}, \beta_{\gamma_a}$ represents the left spread and right spread membership and non – membership functions respectively.

**2.2 Ranking of triangular intuitionistic fuzzy number:**

$$R(\tilde{A}^{IFN}) = \sqrt{\frac{1}{2}\left(\left[\tilde{z}_\mu(\tilde{A}) - \tilde{w}_\mu(\tilde{A})\right]^2 + \left[\tilde{z}_\gamma(\tilde{A}) - \tilde{w}_\gamma(\tilde{A})\right]^2\right)}$$

here $\tilde{z}_\mu(\tilde{A}), \tilde{w}_\mu(\tilde{A}), \tilde{z}_\gamma(\tilde{A}), \tilde{w}_\gamma(\tilde{A})$ are centroid point of the membership and non-membership functions and it can be define by

$$\tilde{z}_\mu = \left[\frac{(a_3 + a_1 + a_2)}{3}\right]$$

,

$$\tilde{z}_\gamma = \left[\frac{(d_1 - (d_2 - d_1) + 2d_3)}{3}\right]$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{(a_1 - a_3)}{(a_1 - a_3)}\right] = \frac{1}{3}$$

,

$$\tilde{w}_\gamma = \frac{1}{3}\left[\frac{2(d_3 - d_1)}{(d_3 - d_1)}\right] = \frac{2}{3}$$

For the intuitionistic Fuzzy Number $\tilde{A}^{IFN}$

**3. Model Specifications:**

Replacement concepts based on age policy with uncertain parameters are proposed in three different situations, with two cases arising in each situation. In the first case, time is measured continuously, while in the second case, time is measured in discrete units.

**Case (i)**If time is measured continuously, replacing the machine or equipment when the year average cost equals the current running or maintenance fuzzy cost will reduce the annual average fuzzy costs.

**Case (ii)**If time is measured in discrete units, the machine or equipment must be replaced when the maintenance fuzzy costs for the next period exceed the current period's average fuzzy costs.

Let $\tilde{\varsigma}$ = Capital or purchase fuzzy cost of the machine or equipment, explanations as follows,

**Case (i)** The time 't' is assumed to be a continuous variable in this situation. Let $\tilde{k}(t)$ represent the current running or maintenance costs. The cumulative running fuzzy cost incurred over the course of the period "x" if the machine or equipment is utilized in the system for a period of time "x" will be:

$$\tilde{R}(x) = \int_0^x \tilde{k}(t)\,dt \quad (1)$$

The total fuzzy cost incurred on the machine or equipment during the period x= Capital fuzzy cost +total maintenance fuzzy cost in the period ‘x’ – Scrap fuzzy value= $\tilde{\varsigma} + \tilde{R}(x) - \tilde{S}$ .

Hence average fuzzy cost per unit of time incurred during the period ‘x’ on the machine or equipment is given by

$$\tilde{H}(x) = \frac{\tilde{\varsigma} + \tilde{R}(x) - \tilde{S}}{x} \quad (2)$$

To find the value of ‘x’ for which $\tilde{H}(x)$ is minimum .The 1$^{st}$derivative of $\tilde{H}(x)$ with respect to ‘x’ is equated to zero.

$$d\tilde{H} / dx = \left\{\frac{(\tilde{\varsigma}-\tilde{S})}{x^2}\right\} - \left\{\frac{\tilde{k}(x)}{x}\right\} + \left(\frac{1}{x^2}\right)\int_0^x \tilde{k}(t)dt = 0,$$

from (1) We get, $\left\{\frac{(\tilde{\varsigma}-\tilde{S})}{x^2}\right\} - \left\{\frac{\tilde{k}(x)}{x}\right\} + \left(\frac{1}{x^2}\right)\tilde{R}(x) = 0$ Or $\tilde{k}(x) = \left\{\frac{(\tilde{\varsigma}-\tilde{S})+\tilde{R}(x)}{x}\right\} = \tilde{H}(x)$ (3)

**Case (ii)** Time 't' is regarded as a discrete variable in this context. In this case, the time period is one year, and 't' can have values of 1,2,3,...etc., then,

$\tilde{R}(x) = \sum_{t=0}^{x} \tilde{k}(t)$ = Total running fuzzy cost of ‘x’ years.

Total fuzzy cost incurred on the machine or equipment ‘x’

Years is $\tilde{T}(x) = \tilde{\varsigma} + \tilde{R}(x) - \tilde{S} = \tilde{\varsigma} - \tilde{S} + \tilde{k}(1) + \tilde{k}(2) + \tilde{k}(3) + .... + \tilde{k}(x)$

Annual average fuzzy cost incurred during ‘x’ years is

$\tilde{H}(x)=\left\{\frac{\tilde{T}(x)}{x}\right\}=\frac{\left\{\tilde{\xi}+\tilde{R}(x)-\tilde{S}\right\}}{x}$, $\tilde{H}(x)$ will be minimum for the value of ‘x’, for which $\tilde{H}(x+1)>\tilde{H}(x)$ &

$\tilde{H}(x-1)>\tilde{H}(x)$ or say that $\tilde{H}(x+1)-\tilde{H}(x)>0$ and $\tilde{H}(x-1)-\tilde{H}(x)>0$

$\tilde{H}(x+1)-\tilde{H}(x)>0$ if $\tilde{K}(x+1)>\tilde{H}(x)$ and $\tilde{H}(x-1)-\tilde{H}(x)>0$ if $\tilde{K}(x)>\tilde{H}(x-1)$.

**Model -1: Increasing maintenance fuzzy cost with zero scarp value:**

If time 't' is assumed to be a continuous variable then

$$\tilde{k}(x)=\left\{\frac{\tilde{\xi}+\tilde{R}(x)}{x}\right\}=\tilde{H}(x) \qquad (4)$$

So, replace the machine when the sum of the capital and total maintenance fuzzy costs in period 'x' equals the annual average fuzzy cost $\tilde{H}(x)$ after dividing it by the period 'x'.

If time’t’ is considered as a discrete variable then

$$\tilde{H}(x)=\left\{\frac{\tilde{T}(x)}{x}\right\}=\frac{\left\{\tilde{\xi}+\tilde{R}(x)\right\}}{x} \qquad (5)$$

It demonstrates that if the next year's running or maintenance fuzzy cost is less(<) than the previous year's average fuzzy cost, do not replace, but replace at the end of 'x' years if the next year's running or maintenance fuzzy cost is greater(>) than the 'x' th year’s average fuzzy cost.(Figure1 shows the graphical representation of model 1)

**Model -2: Increasing maintenance fuzzy cost with a scarp fuzzy value:**

If time 't' is assumed to be a continuous variable then

$$\tilde{k}(x)=\left\{\frac{(\tilde{\xi}-\tilde{S})+\tilde{R}(x)}{x}\right\}=\tilde{H}(x) \qquad (6)$$

So, replace the machine or equipment when the annual average fuzzy cost $\tilde{H}(x)$ reaches the minimum, which will always occur when the taverage fuzzy cost equals the current maintenance or running fuzzy cost.

If time’t’ is considered as a discrete variable then

$$\tilde{H}(x)=\left\{\frac{\tilde{T}(x)}{x}\right\}=\frac{\left\{\tilde{\xi}+\tilde{R}(x)-\tilde{S}\right\}}{x} \qquad (7)$$

It illustrates that if the next year's running or maintenance fuzzy cost is less(<) than the previous year's average fuzzy cost, do not replace, but replace at the end of 'x' year if the next year's running or maintenance fuzzy cost is greater(>) than the 'x' th year's eaverage fuzzy cost.(Figure2 shows the graphical representation of Model 2)

**Model -3: Increasing maintenance fuzzy cost with decreasing scarp fuzzy value:**

If time 't' is assumed to be a continuous variable then

$$\tilde{k}(x) = \left\{ \frac{(\tilde{\varsigma}-\tilde{S}_x)+\tilde{R}(x)}{x} \right\} = \tilde{H}(x) \tag{8}$$

Here $\tilde{s}_x$ is the scarp fuzzy value of $x^{th}$ year where x = 1,2,3,….n. As a result, replace the machine or equipment when the annual average fuzzy cost $\tilde{H}(x)$ is at its minimum, which occurs always when the annual average fuzzy cost equals the current running or maintenance fuzzy cost.

If time't' is considered as a discrete variable then

$$\tilde{H}(x) = \left\{ \frac{\tilde{T}(x)}{x} \right\} = \frac{\left\{\tilde{\varsigma}+\tilde{R}(x)-\tilde{S}_x\right\}}{x} \tag{9}$$

It clearly illustrates that if the next year's running or maintenance fuzzy cost is less(<) than the previous year's average fuzzy cost, do not replace; however, if the next year's running or maintenance fuzzy cost is greater(>) than the previous year's average fuzzy cost, replace at the end of 'x' years.(Figure 3 shows the graphical representation of model 3)

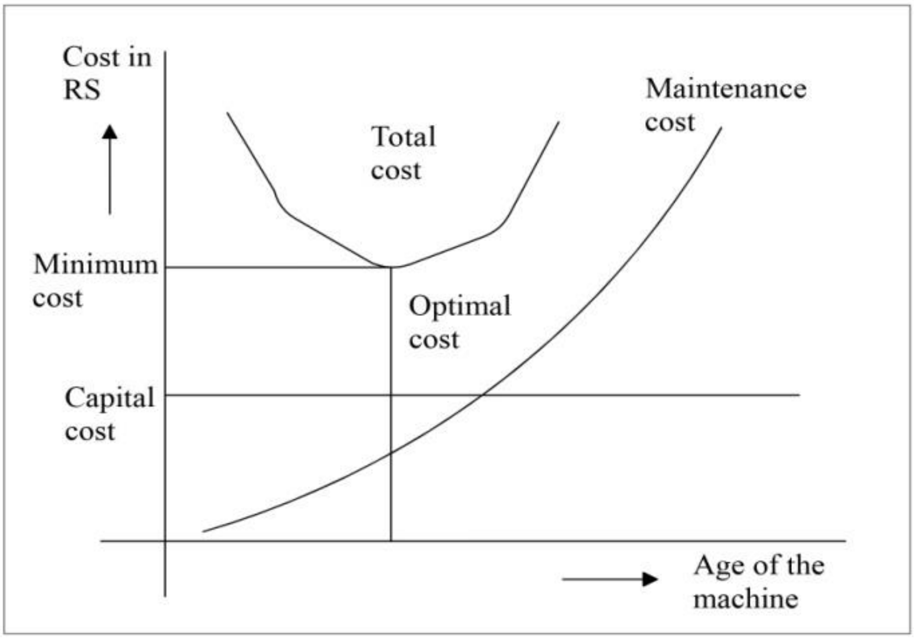


**Figure 1:** Graphical representation of increasing maintenance fuzzy cost with zero scarp fuzzy value

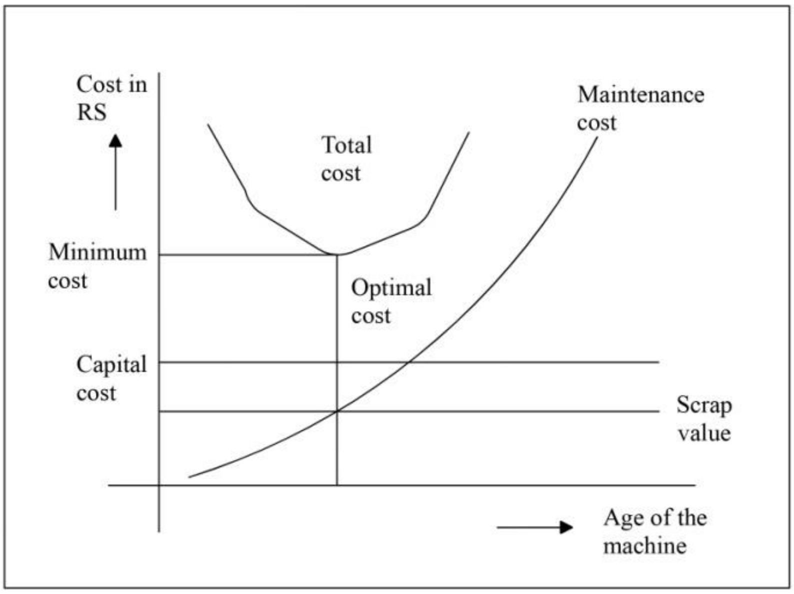


**Figure 2:** Graphical representation ofincreasing maintenance fuzzy cost with ascarp fuzzy value

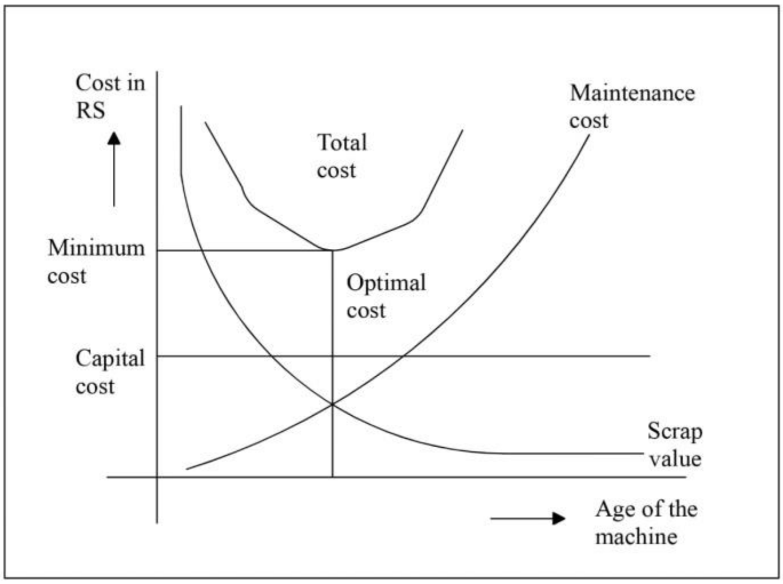


**Figure 3:** Graphical representation of increasing maintenance fuzzy cost with decreasing scarp fuzzy value

## 4. Computational descriptions:

### 4.1Model-1

The purchase fuzzy cost of Machine A is (80000, 90000, 100000; 70000, 90000, 110000). Determination of the machine's optimal age, in this case, machine A has a capital fuzzy cost and no scrap fuzzy value but is made in uncertainty. For Machine A, the rising maintenance costs are as follows: in the first year (18000,19000,20000; 17,000,19000,21000); in the second year (20,000,21000,22000; 19,000,21000,23000); in the third year (40000,41000,42000; 39,000,41000,43000); in the fourth year (45000,46000,47000; 44,000,46000,48000); in the fifth year (51000,52000,53000;50000,52000,54000) and in sixth year (73000,74000,75000;72000,74000,76000)

Table 1: Estimation of Machine A's Cumulati.ve Running Fuzzy Cost and Averages Annual Fuzzy Cost.

| **Age Or Year** | **Cumulative Running fuzzy cost** $\tilde{R}(x)$ **of Machine A** | **Annual average fuzzy cost** $\tilde{H}(x) = \left\{\frac{\tilde{\xi}+\tilde{R}(x)}{x}\right\}$ **of Machine A** |
|---|---|---|
| 1 | (18000,19000,20000; 17000,19000,21000) | (99000,109000,119000; 89000,109000,129000) |
| 2 | (39000,40000,41000; 38000,40000,42000) | (55000,65000,75000; 45000,65000,85000) |
| 3 | (80000,81000,82000; 79000,81000,83000) | (47000,57000,67000; 37000,57000,77000) |
| 4 | (126000,127000,128000; 125000,127000,129000) | (44250,54250,64250; 34250,54250,74250) |
| 5 | (178000,179000,180000; 177000,179000,181000) | (43800,53800,63800; 33800,53800,73800) Minimum |
| 6 | (252000,253000,254000; 251000,253000,255000) | (47167,57167,67167; 37167,57167,77167) |

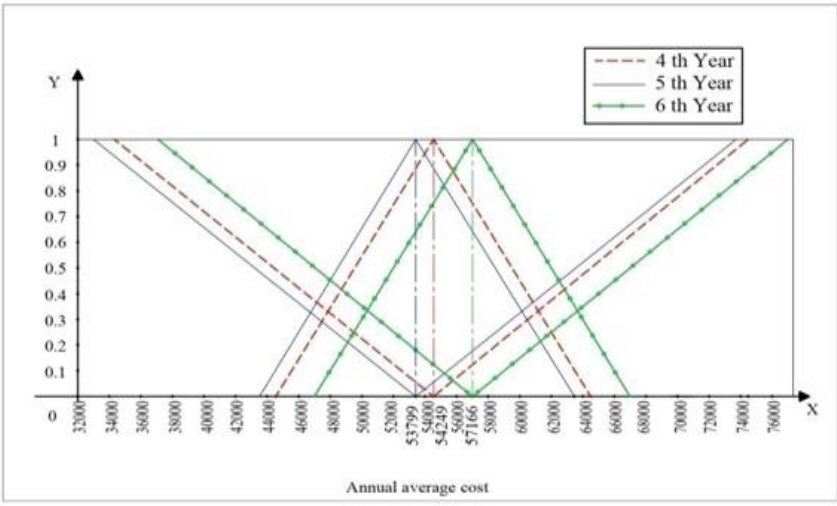


Figure 4: The graph depicts the annual average fuzzy cost of machine A for the fourth, fifth, and sixth years.

### 4.2Model-2

The price of a Machine B is Rs (90000, 100000, 110000; 98000, 100000, 120000), and its scrap value is (64000, 65000, 66000; 63000, 65000, 67000). It is established at what age a Machine B should be replaced under uncertain circumstances with constant capital costs and scrap values. In this case The costs of maintaining Machine B have increased over the years in the following order: in the first year (33500,34500,35500;32500,34500,36500); in the second year (34000,35000,36000;33000,35000,37000); in the third year (37000,38000,39000;36000,38000,40000); in the fourth year (40000,41000,42000;39000,41000,43000); in the fifth year (48000,49000,50000;47000,49000,51000),in the sixth year(50000;51000,52000;49000,51000,53000)

Table 2: Estimation of Machine B's Cumulati.ve Running Fuzzy Cost and Averages Annual Fuzzy Cost.

| **Age Or Year** | **Cumulative Running fuzzy cost** $\tilde{R}(x)$ **of Machine B** | **Annual average fuzzy cost** $\tilde{H}(x) = \left\{\frac{(\tilde{\varsigma}-\tilde{S})+\tilde{R}(x)}{x}\right\}$ **of Machine B** |
|---|---|---|
| 1 | (33500,34500,35500; 32500,34500,36500) | (59500,69500,79500; 49500,69500,89500) |
| 2 | (68500,69500,70500; 67500,69500,71500) | (42250,52250,62250; 32250,52250,72250) |
| 3 | (106500,107500,108500; 105500,107500,109500) | (37500,47500,57500; 27500,47500,67500) |
| 4 | (147500,148500,149500; 146500,148500,150500) | (35875,45875,55875; 25875,45875,65875)(Minimum) |
| 5 | (196500,197500,198500; 195500,197500,199500) | (36500,46500,56500; 26500,46500,66500) |
| 6 | (247500,248500,249500; 246500,248500,250500) | (37250,47250,57250; 27250,47250,67250) |

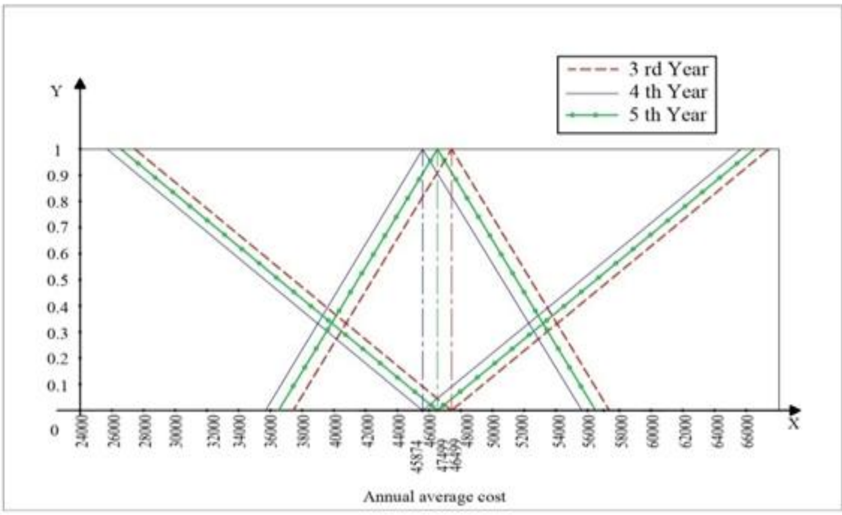


**Figure 5:**The graph depicts the annual average fuzzy cost of Machine B for the third, fourth, and fifth years.

### 4.3 Model-3

The price of a Machine C was Rs (590000, 600000, 610000;580000,600000,620000). In this case historical data of a Machine C with annual Running fuzzy cost and resale fuzzy value as follows. Under uncertain circumstances, it is decided when a Machine C needs to be replaced

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Table 3: Machine C's Running fuzzy cost and Resale fuzzy value:

| Age or (year) | Running fuzzy cost | Resale fuzzy value |
|---|---|---|
| 1 | (39000,40000,41000; 38000,40000,42000) | (290000,300000,310000; 280000,300000,320000) |
| 2 | (89000,90000,91000; 88000,90000,92000) | (280000,290000,300000; 270000,290000,310000) |
| 3 | (114000,115000,116000; 113000,115000,117000) | (240000,250000,260000; 230000,250000,270000) |
| 4 | (129000,130000,131000; 128000,130000,132000) | (210000,220000,230000; 200000,220000,240000) |
| 5 | (131000,132000,133000; 130000,132000,134000) | (200000,210000,220000; 190000,210000,230000) |
| 6 | (147000,148000,149000; 146000,148000,150000) | (190000,200000,210000; 180000,200000,220000) |
| 7 | (150000,151000,152000; 149000,151000,153000) | (180000,190000,200000; 170000,190000,210000) |
| 8 | (449000,450000,451000; 448000,450000,452000) | (90000,100000,110000; 80000,100000,120000) |

Capital fuzzy cost $(\tilde{\varsigma})$ = Rs.(590000,600000,610000;580000,600000,620000), Scarp fuzzy value $(\tilde{S}_x)$ changes yearly,

Table 4: Estimation of Machine C's Total fuzzy cost and Annual average Fuzzy Cost.

| Age Or Year | Total fuzzy cost $\tilde{T}(x) = \left\{\tilde{\varsigma}+\tilde{R}(x)-\tilde{S}_x\right\}$ of the Machine C | Annual Average fuzzy cost $\tilde{H}(x) = \left\{\frac{\tilde{T}(x)}{x}\right\} = \frac{\left\{\tilde{\varsigma}+\tilde{R}(x)-\tilde{S}_x\right\}}{x}$ of Machine C |
|---|---|---|
| 1 | (330000,340000,350000; 320000,340000,360000) | (330000,340000,350000; 320000,340000,360000) |
| 2 | (430000,440000,450000; 420000,440000,460000) | (210000,220000,230000; 200000,220000,240000) |
| 3 | (585000,595000,605000; 575000,595000,615000) | (188334,198334,208334; 178334,198334,218334) |
| 4 | (745000,755000,765000; 735000,755000,775000) | (178750,188750,198750; 168750,188750,208750) |
| 5 | (887000,897000,907000; 877000,897000,917000) | (169400,179400,189400; 159400,179400,199400) |
| 6 | (1045000,1055000,1065000; 1035000,1055000,1075000) | (165833,175833,185833; 155833,175833,195833) |
| 7 | (1206000,1216000,1226000; 1196000,1216000,1236000) | (163714,173714,183714; 153714, 173714,193714) Minimum |
| 8 | (1746000,1756000,1766000; 1736000,1756000,1776000) | (209500,219500,229500; 199500,219500,239500) |

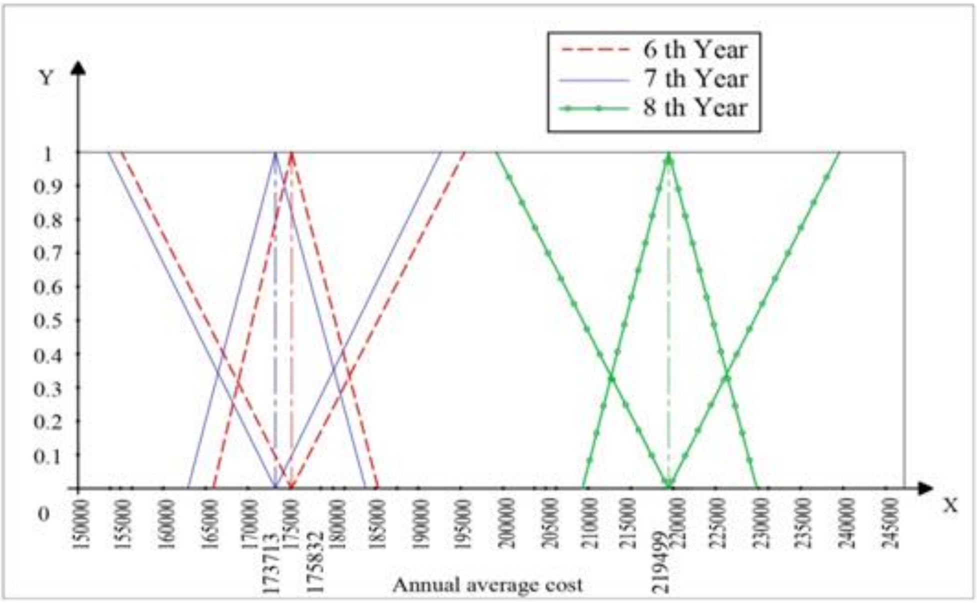


**Figure 6:** The graph depicts the annual average fuzzy cost of Machine C for the sixth, seventh, and eighth years.

## 5. Results and discussions:

### 5.1 Model -1

- (From table 1)The annual average fuzzy cost of the Machine A for first year is 108999.57, which is approximately 109000 where the left & right fuzziness of the (MF) membership function are 99000 and 119000, respectively, and where the left & right fuzziness of the (NMF) non membership function are 89000 and 129000, respectively. The annual average fuzzy cost of the Machine A for second year is 64999.61, which is approximately 65000 where the left & right fuzziness of the (MF) membership function are 55000 and 75000, respectively, and where the left & right fuzziness of the (NMF) non membership function are 45000 and 85000, respectively.
- The annual average fuzzy cost of the Machine A for third year is 56999.52, which is approximately 57000 where the left & right fuzziness of the (MF) membership function are 47000 and 67000, respectively, and where the left & right fuzziness of the (NMF) non membership function are 37000 and 77000, respectively. The annual average fuzzy cost of the Machine A for fourth year is 54249.91, which is approximately 54250 where the left & right fuzziness of the (MF) membership function are 44250 and 64250, respectively, and where the left & right fuzziness of the(NMF)non membership function are 34250 and 74250, respectively.
- The annual average fuzzy cost of the Machine A for fifth year is 53799.73, which is approximately 53800 where the left & right fuzziness of the (MF) membership function are 43800 and 63800, respectively, and where the left & right fuzziness of the (NMF)non membership function are 33800 and 73800, respectively. The annual average fuzzy cost of the Machine A for sixth year is 57166.85, which is approximately 57167 where the left & right fuzziness of the (MF) membership function are 47167 and 67167, respectively, and where the left & right fuzziness of the(NMF)non

membership function are 37167 and 77167, respectively. Based on the aforementioned findings, we could conclude that Machine A's minimum average fuzzy cost is 53799.73, which is approximately 53800, and that it occurred in the fifth year. Therefore, in accordance with the proposed1replacement policy, machine A needs to be replaced at theend of the fifth year. Otherwise, it can result in a loss due to rising maintenance costs. Figure 4 illustrates machine A's minimum annual average cost.

**5.2 Model - 2**

- (From table 2) The annual average fuzzy cost of the Machine B for first year is 69499.62, which is approximately 69500 where the left & right fuzziness of the (MF) membership function are 59500 and 79500, respectively, and where the left & right fuzziness of the (NMF)non membership function are 49500 and 89500, respectively. The annual average fuzzy cost of the Machine B for second year is 52249.57, which is approximately 52250 where the left & right fuzziness of the (MF) membership function are 42250 and 62250, respectively, and where the left & right fuzziness of the (NMF) non membership function are 32250 and 72250, respectively.
- The annual average fuzzy cost of the Machine B for third year is 47499.84, which is approximately 47500 where the left & right fuzziness of the (MF) membership function are 37500 and 57500, respectively, and where the left & right fuzziness of the (NMF)non membership function are 27500 and 67500, respectively. The annual average fuzzy cost of the Machine B for fourth year is 45874.91, which is approximately 46500 where the left & right fuzziness of the (MF) membership function are 35875 and 55875, respectively, and where the left & right fuzziness of the (NMF)non membership function are 25875 and 65875 respectively.
- The annual average fuzzy cost of the Machine B for fifth year is 46499.63, which is approximately 46500 where the left & right fuzziness of the (MF) membership function are 36500 and 56500, respectively, and where the left & right fuzziness of the (NMF) non membership function are 26500 and 66500, respectively. The annual average fuzzy cost of the Machine B for the sixth year is 47249.61, which is approximately 47250 where the left & right fuzziness of the (MF) membership function are 37250 and 57250, respectively, and where the left & right fuzziness of the (NMF) non membership function are 27250 and 67250, respectively. Based on the aforementioned findings, we could conclude that Machine B's minimum average fuzzy cost is 45874.91, which is approximately 45875, and that it occurred in the fourth year. Therefore, in accordance with the proposedage-based replacement policy, machine B needs to be replaced at ttheend of the fourth year. Otherwise, it can result in a loss due to rising maintenance costs and Figure 5 illustrates machine B's minimum annual average cost

**5.3 Model – 3**

- (From table 4) The annual average fuzzy cost of the Machine C for first year is 339999.56, which is approximately 340000 where the left & right fuzziness of the (MF) membership function are

330000 and 350000, respectively, and where the left & right fuzziness of the (NMF)non membership function are 320000 and 360000, respectively. The annual average fuzzy cost of the Machine C for second year is 219999.73, which is approximately 220000 where the left & right fuzziness of the (MF)membership function are 210000 and 230000, respectively, and where the left &right fuzziness of the(NMF)non membership function are 200000 and 240000, respectively.The annual average fuzzy cost of the Machine C for third year is 198333.71, which is approximately 198334 where the left & right fuzziness of the (MF)membership function are188334 and 208334, respectively, and where the left & right fuzziness of the (NMF)non membership function are 178334 and 218334, respectively. The annual average fuzzy cost of the Machine C for fourth year is 188749.69, which is approximately 188750 where the left & right fuzziness of the (MF) membership function are 178750 and 198750, respectively, and where the left & right fuzziness of the (NMF)non membership function are 168750 and 208750, respectively.The annual average fuzzy cost of the Machine C for fifth year is 179399.83, which is approximately 179400 where the left & right fuzziness of the (MF)membership function are 169400 and 189400, respectively, and where the left & right fuzziness of the (NMF)non membership function are 159400 and 199400, respectively. The annual average fuzzy cost of the Machine C for sixth year is 175832.51, which is approximately 175833 where the left & right fuzziness of the (MF)membership function are 165833 and 185833, respectively, and where the left & right fuzziness of the (NMF)non membership function are 155833 and 195833, respectively.

- The annual average fuzzy cost of the Machine C for seventh year is 173713.62, which is approximately 173714 where the left & right fuzziness of the (MF)membership function are 163714 and 183714, respectively, and where the left & right fuzziness of the (NMF)non membership function are 153714 and 193714, respectively. The annual average fuzzy cost of the Machine C for eighth year is 219499.84, which is approximately 219500 where the left & right fuzziness of the (MF)membership function are 209500 and 229500, respectively, and where the left & right fuzziness of the (NMF)non membership function are 199500 and 239500, respectively. Based on the aforementioned findings, we could conclude that Machine C's minimum average fuzzy cost is 173713.62, which is approximately 173713, and that it occurred in the seventh year. Therefore, in accordance with the proposedsage-based replacement policy, machine C needs to be replacedat the end of the seventh year. Otherwise, it can result in a loss due to rising maintenance costs and Figure 6 illustrates machine C's minimum annual average cost.

**Conclusion**

The cognitive process known as decision-making is thought to be employed to address issues that arise in daily life. In situations when there is ambiguity, Decision Making becomes more difficult. Zadeh's fuzzy set is a useful tool for handling uncertainty, but as time goes on, it becomes apparent that it is insufficient. In order to solve the issues that arose as a result, intuitionistic fuzzy sets have proven to be successful. In an uncertain environment, this study focuses on replacing machines with rising imprecise maintenance costs. It engaged three distinct models: an increasing maintenance fuzzy cost with a zero scarp value, an increasing maintenance fuzzy cost with a stable scarp fuzzy value, and an increasing maintenance fuzzy cost with a decreasing scarp fuzzy value. All of the models are discussed along with two different cases. the first of which is If time 't' is assumed to be a continuous variable, then replace the machine when the sum of the imprecise capital and cumulative maintenance fuzzy costs in period 'x' equals to the a average fuzzy cost after dividing it by period 'x'. If time't' is regarded as a discrete variable, it demonstrates that If the running fuzzy cost for the next year is less than the previous year's average fuzzy cost, do not replace; however, if the running fuzzy cost for the next year is greater than the 'x' th year's average fuzzy cost, replace at the end of 'x' years. Exemplifications are provided for all three models. The proposed intuitionistic fuzzy replacement model's soundness is evaluated using numerical examples, and it has been established that the replacement time or age of the machine or equipment can be found using the proposed method without converting the fuzzy values into crisp values.

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