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On G$\mathcal{G}$-Closed Sets in Grill Topological Spaces

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Abstract:

In this article, we define a new class of gG-closed sets in a grill topological space and discuss the characterizations of gG-closed sets and gG-open sets by using the map s. Also analyze relationship between the gG-closed sets and some of the generalized closed sets.

Keywords: gG-closed, gG-open, s-semiclosed, s-semi-dense.

Mathematical Classification: 54A05, 54A10, 54D10.

1.Introduction

Levine[14, 15] introduced the concepts of semiopen sets and generalized closed sets in topological spaces. Crossly et al.[9, 10] described the concepts of semi-closure and analysed the semi-topological properties. Chattopadhyay et al.[6, 7] described the metropic spaces and created the extensions of closure spaces. In [2, 5, 13, 24, 25, 26], studied the concepts of generalized closed sets through semiclosed sets. Choquet[8] defined the grill structure in topological spaces. Roy et al.[20, 21] developed the grill concepts and induced $\tau_{\mathcal{G}}$ topological space. In [1, 11, 16, 27], initiated different types of grill sets such as $\mathcal{G}$-semiopen sets, $\mathcal{G}$-α-open sets and studied the decomposition of continuity via grill. Nasef[18], introduced φ^s operator in grill topological space via semiopen sets and analyzed the essential topological characterizations. Mandal[17], generalized the closed sets in grill topological space and Saravanakumar et al.[22, 23] defined $\mathcal{G}_{s_p}$-open sets and $\mathcal{G}_{s_\alpha}$-open sets through semiopen sets and characterized some topological structure. Anbarasan et al.[3, 4] studied generalized closed sets concepts via grill in generalized topological spaces.

In this paper, we introduced new grill closed sets namely, g$\mathcal{G}$-closed in grill topological spaces. We characterized g$\mathcal{G}$-closed sets and g$\mathcal{G}$-open sets in grill topological spaces by use the mapping φ^s and investigated some of their properties. We noticed that the idea of g$\mathcal{G}$-closed sets is a new generalization of $\mathcal{G}$g-closed sets. Also, we analyzed relationship between this g$\mathcal{G}$-closed sets with existence generalized closed sets such as g-closed, s*g-closed, gs-closed, $\tau_{\mathcal{G}}$-closed, $\mathcal{G}$g-closed etc.

2. Preliminaries

In a topological space X, a subset A of X is said to be semiopen[14] (resp. α-open[19], regular open[12]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(int(A)))$, $A = int(cl(A))$). The complement $X - A$ is called semiclosed (resp. α-closed, regular closed). For a subset A of X, semiclosure of A defined as scl(A)[9] = $\cap\{F \subseteq X : X - F$ is semiopen and $A \subseteq F\}$; semiinterior of A defined as sint(A)[9] = $\cup\{U \subseteq X : U$ is semiopen and $A \subseteq U\}$. A subset A of X is said to be g-closed[15] (resp. s*g-closed[13], gs-closed[2]) if $cl(A) \subseteq U$ (resp. $cl(A) \subseteq U$, $scl(A) \subseteq U$) whenever $A \subseteq U$ and U is open (resp. U is semi-open, U is open) in X. A nonempty collection $\mathcal{G}$ of subsets of a topological space (X, τ) is called a grill[8] on X if (i) $\emptyset \notin \mathcal{G}$, (ii) $A \in \mathcal{G}$ and $A \subseteq B$ implies that $B \in \mathcal{G}$, (iii) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$. A triple $(X\ \tau, \mathcal{G})$ is called a grill topological space[20]. Let Y be a subset of X. Then $\mathcal{G}_Y = \{\mathcal{G}_0 \cap Y : \mathcal{G}_0 \subseteq \mathcal{G}\}$ is a grill on Y and grill topological subspace denoted by $\{Y, \tau_Y, \mathcal{G}_Y\}$. A mapping φ[20] (resp. φ^s[18]) : $P(X) \to P(X)$ is defined by $\varphi(A)$[20] (resp. $\varphi^s(A)$[18]) $= \{x \in X: A \cap U \in \mathcal{G}$ for all $U \in \tau(x)$ (resp. $SO(X, x)\}$ for all $A \in P(X)$, where $\tau(x)$ (resp. $SO(X, x)$) denotes the collection of all open (resp. semiopen) neighbourhoods of x. A mapping ψ[20 (resp. ψ^s[18]) : $P(X) \to P(X)$ is defined by $\psi(A)$[20] (resp. $\psi^s(A)$[18]) $= A \cup \varphi(A)$ (resp. $A \cup \varphi^s(A)$) for all $A \in P(X)$. Also ψ (resp. ψ^s) satisfies the Kuratowski closure axioms. Corresponding to a grill $\mathcal{G}$ on a topological space (X, τ), there exists a unique topology $\tau_{\mathcal{G}}$[20] (resp. $\tau^s_{\mathcal{G}}$[18]) $= \{U \subseteq X : \psi(X - U)$ (resp. $\psi^s(X - U)) = X - U\}$, where for any $A \subseteq X$, $\psi(A)$ (resp. $\psi^s(A)) = A \cup \varphi(A)$ (resp. $A \cup \varphi^s(A)) = \tau_{\mathcal{G}}cl(A)$ (resp. $\tau^s_{\mathcal{G}}cl(A)) \subseteq cl(A)$ (resp. $scl(A)$) and $\tau \subseteq \tau_{\mathcal{G}}$ (resp. $SO(X) \subseteq \tau^s_{\mathcal{G}}$). A subset A of X is called (i) $\tau_{\mathcal{G}}$-closed[20] if

$\tau_{\mathcal{G}}cl(A) = \psi(A) = A \cup \varphi(A)$; φ-dense[20] if $\varphi(A) \subseteq A$; $\mathcal{G}$g-closed[17] if $\varphi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Theorem 2.1.[18] Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then, for every $A, B \subseteq X$, the following conditions are satisfied:

(i) if $A \subseteq B$, then $\varphi^s(A) \subseteq \varphi^s(B)$;

(ii) $\varphi^s(A) = scl(\varphi^s(A)) \subseteq scl(A)$ and $\varphi^s(A)$ is semiclosed in X;

(iii) $\varphi^s(\varphi^s(A)) \subseteq \varphi^s(A)$;

(iv) $\varphi^s(A \cup B) = \varphi^s(A) \cup \varphi^s(B)$;

(v) if $A \notin \mathcal{G}$, then $\varphi^s(A) = \emptyset$.

3. g$\mathcal{G}$-Closed sets

Definition 3.1. Let $(X, \tau, \mathcal{G})$ be a grill topological space and A be a subset of X. Then A is said to be g$\mathcal{G}$-closed if $\varphi^s(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of a g$\mathcal{G}$-closed set is called a g$\mathcal{G}$-open set.

Theorem 3.2. Let $(X, \tau, \mathcal{G})$ be a topological space and A be a subset of X. Then

(i) If A is a $\tau_{\mathcal{G}}$-closed set, then A is $\mathcal{G}$g-closed;

(ii) If if A is a g-closed set, then A is $\mathcal{G}$g-closed;

(iii) If A is a $\mathcal{G}$g-closed set, then A is g$\mathcal{G}$-closed;

(iv) If A is a s^*g-closed set, then A is g-closed;

(v) If A is a g-closed set, then A is gs-closed;

(vi) If A is a gs-closed set, then A is g$\mathcal{G}$-closed.

Proof. (i) and (ii) Follows from the Remark 2.2.(b) and (d)[17].

(iii) Let A be a $\mathcal{G}$g-closed set such that $A \subseteq U$ and $U \in \tau$. Then by assumption, $\varphi(A) \subseteq U$. Since every open set is semiopen, we have that $\varphi^s(A) \subseteq \varphi(A)$. Therefore $\varphi^s(A) \subseteq U$. Hence A is g$\mathcal{G}$-closed.

(iv) Let A be a s^*g-closed set such that $A \subseteq U$ and $U \in \tau$. Since every open set is semiopen, $A \subseteq U$ and U is semiopen. By assumption, $cl(A) \subseteq U$. Then by definition of g-closed, we have that A is g-closed.

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(v) Let A be a g-closed set such that $A \subseteq U$ and $U \in \tau$. Then by assumption, $cl(A) \subseteq U$. Since every closed set is semiclosed, we have that $scl(A) \subseteq cl(A)$. Therefore $scl(A) \subseteq U$. Hence A is gs-closed.

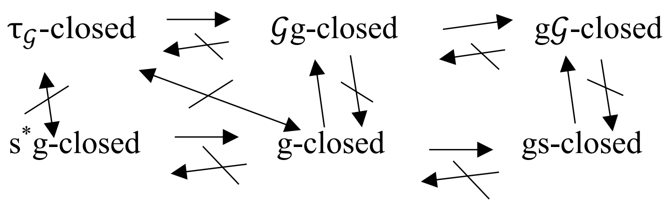
(vi) Let A be a gs-closed set such that $A \subseteq U$ and $U \in \tau$. Then by assumption, $scl(A) \subseteq U$. Since $\varphi^s(A) \subseteq scl(A)$. Therefore $\varphi^s(A) \subseteq U$. Hence A is g$\mathcal{G}$-closed.

Remark 3.3. The following examples shows that the reverse implication of above theorem is not true and the concepts of some generalizations of closed sets are independent.

(i) Let X = {a, b, c, d}, τ = {∅, X, {a, b}, {a, b, d}} and $\mathcal{G}$ = {X, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Then the set {a, c} is $\mathcal{G}$g-closed (resp. g-closed), but not $\tau_{\mathcal{G}}$-closed (resp. s^*g-closed). Also the set {d} is $\mathcal{G}$g-closed (resp. gs-closed) but not g-closed. Moreover, the set {b, d} is g$\mathcal{G}$-closed but it is not $\mathcal{G}$g-closed (resp. gs-closed). Here the set {b} is $\tau_{\mathcal{G}}$-closed, but not g-closed (resp. s^*g-closed). As well as the set {a, c, d} is g-closed (resp. s^*g-closed), but not $\tau_{\mathcal{G}}$-closed.

(ii) Let X = {a, b, c, d}, τ = {∅, X, {a}, {b}, {a, b}} and $\mathcal{G}$ = {X, {a}, {d}, {a, b}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Then the set {a} is gs-closed but not $\mathcal{G}$g-closed.

The following diagram shows the relationship among various generalizations of closed sets.



Remark 3.4. In a grill topological space (X, τ, $\mathcal{G}$),

(i) every non-member of $\mathcal{G}$ is g$\mathcal{G}$-closed;

(ii) φ^s is g$\mathcal{G}$-closed for every subset A of X;

(iii) if $\mathcal{G} = P(X) - \{\emptyset\}$, then $\varphi^s(A) = scl(A)$ and hence g$\mathcal{G}$-closed sets coincide with gs-closed sets.

Theorem 3.5. Let $(X, \tau, \mathcal{G})$ be a grill topological space and A be a subset of X. Then the following statements are equivalent:

(i) A is $g\mathcal{G}$-closed;

(ii) $scl(\varphi^s(A)) \subseteq U$ for every open set U containing A;

(iii) for all $x \in scl(\varphi^s(A))$, $cl(\{x\}) \cap A \neq \emptyset$;

(iv) $scl(\varphi^s(A)) - A$ contains no non empty closed set;

(v) $\varphi^s(A) - A$ contains no non empty closed set.

Proof. (i) ⇒ (ii). Let A be a $g\mathcal{G}$-closed set. Then clearly $\varphi^s(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X and so by Theorem 2.1, $scl(\varphi^s(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X. This proves (ii).

(ii) ⇒ (iii). Suppose $x \in scl(\varphi^s(A))$. If $cl(\{x\}) \cap A = \emptyset$, then $A \subseteq X - cl(\{x\})$. By (ii), $scl(\varphi^s(A)) \subseteq X - cl(\{x\})$. This contradicts the fact that $x \in scl(\varphi^s(A))$. Hence $cl(\{x\}) \cap A \neq \emptyset$. This proves (iii).

(iii) ⇒ (i). Suppose that A is not $g\mathcal{G}$-closed. There exists an open set U such that $A \subseteq U$ and $\varphi^s(A)$ is not contained in U. Then, there exists a point $x \in \varphi^s(A)$ such that $x \notin U$. Then we have $\{x\} \cap U = \emptyset$ and hence $cl(\{x\}) \cap U = \emptyset$. Since $A \subseteq U$, $cl(\{x\}) \cap A = \emptyset$. By Theorem 2.1, $scl(\varphi^s(A)) = \varphi^s(A)$ and it follows that (iii) does not hold. Therefore, the proof completes.

(iii) ⇒ (iv). Suppose F is a closed set of X contained in $scl(\varphi^s(A)) - A$ and $x \in F$. Since $F \cap A = \emptyset$, we have $cl(\{x\}) \cap A = \emptyset$. Again, since $x \in scl(\varphi^s(A))$, by (iii) we have $cl(\{x\}) \cap A \neq \emptyset$, a contradiction. This proves (iv). It follows from Theorem 2.1 that (iv) and (v) are equivalent.

Theorem 3.6. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $\{A_i: i \in J\}$ be a locally finite family of sets in X. Then $\cup_{i\in J}(\varphi^s(A_i)) = \varphi^s(\cup_{i\in J}A_i)$.

Proof. $A_i \subseteq \cup_{i\in J}A_i$ implies $\varphi^s(A_i) \subseteq \varphi^s(\cup_{i\in J}A_i)$ for every $i \in J$. This implies $\cup_{i\in J}(\varphi^s(A_i)) \subseteq \varphi^s(\cup_{i\in J}A_i)$. Conversely, let $x \in \varphi^s(\cup_{i\in J}A_i)$ and V be any semiopen set of X containing x. Since $\{A_i: i \in J\}$ is locally finite, there exists an open set U in X containing x that intersects only a finite number of members, says, $A_{i1}, A_{i2}, ..., A_{in}$ of $\{A_i: i \in J\}$. But x $\in \varphi^s(\cup_{i\in J}A_i)$ implies $(V \cap U) \cap (\cup_{i\in J}A_i) = \cup_{i\in J}((V \cap U) \cap A_i) \in \mathcal{G}$ for every V ∈ SO (X, x). This gives $\cup_{k=1}^{n}((V \cap U) \cap A_{ik}) \in \mathcal{G}$ for every V ∈ SO (X, x). Therefore, there exists at least one $A_{ij} \in \{A_{i1}, A_{i2}, ..., A_{in}\}$ such that $(V \cap U) \cap A_{ij} \in \mathcal{G}$ hence $V \cap A_{ij} \in \mathcal{G}$. This gives $x \in \varphi^s(A_{ij})$ which implies $x \in \cup_{k=1}^{n}(\varphi^s(A_{ik}))$ and hence $x \in \cup_{i\in J}(\varphi^s(A_i))$. This proves that $\varphi^s(\cup_{i\in J}A_i) \subseteq \cup_{i\in J}(\varphi^s(A_i))$. This completes the proof.

Theorem 3.7. Let $(X, \tau, \mathcal{G})$ be a grill topological space. If $\{A_i: i \in J\}$ is a locally finite family of sets and each A_i is g$\mathcal{G}$-closed, then $\cup_{i\in J}A_i$ is g$\mathcal{G}$-closed in X.

Proof. Let $\cup_{i\in J}A_i \subseteq U$, where U is open in X. Since A_i is g$\mathcal{G}$-closed for each i ∈ J, then $\varphi^s(A_i) \subseteq U$. Hence $\cup_{i\in J}(\varphi^s(A_i)) \subseteq U$. By Theorem 3.6, $\varphi^s(\cup_{i\in J}A_i) \subseteq U$. Hence $\cup_{i\in J}A_i$ is g$\mathcal{G}$-closed in X.

Remark 3.8. The following example shows that the intersection of two g$\mathcal{G}$-closed sets need not be g$\mathcal{G}$-closed.

Let X = {a, b, c, d}, τ = {∅, X, {a, b}} and $\mathcal{G}$ = {X, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Then A = {a, c} and B = {a, d} are g$\mathcal{G}$-closed sets, but A ∩ B = {a} is not g$\mathcal{G}$-closed.

Theorem 3.9. If A and B are subsets of a grill topological space $(X, \tau, \mathcal{G})$, then $\varphi^s(A \cap B) \subseteq \varphi^s(A) \cap \varphi^s(B)$.

Theorem 3.10. Let $(X, \tau, \mathcal{G})$ be a grill topological space. If A is g$\mathcal{G}$-closed and B is closed in X, then A ∩ B is g$\mathcal{G}$-closed.

Proof. Let U be an open set in X containing A ∩ B. Then $A \subseteq U \cup (X - B)$. Since A is g$\mathcal{G}$-closed, we have $\varphi^s(A) \subseteq U \cup (X - B)$ and $B \cap \varphi^s(A) \subseteq U$. Using Theorem 3.9, $\varphi^s(A \cap B) \subseteq \varphi^s(A) \cap \varphi^s(B) \subseteq \varphi^s(A) \cap B \subseteq U$ because B is closed. This proves that A ∩ B is g$\mathcal{G}$-closed.

Definition 3.11. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is said to be φ^s-semiclosed if $\varphi^s(A) \subseteq A$.

Remark 3.12. Every $\tau_{\mathcal{G}}$-closed set is φ^s-semiclosed. The converse is not true. In example(ii) of Remark 3.3, A = {a} is φ^s-semiclosed but it is $\tau_{\mathcal{G}}$-closed.

Definition 3.13. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is said to be φ^s-semi-dense in-itself if $A \subseteq \varphi^s(A)$.

Remark 3.14. Every φ^s-semi dense in-itself set is φ-dense in-itself.

Theorem 3.15. In a grill topological space $(X, \tau, \mathcal{G})$, a g$\mathcal{G}$-closed and φ^s-semi-dense in-itself set is gs-closed.

Proof. Suppose A is φ^s-semi-dense in-itself and g$\mathcal{G}$-closed in X. Let U be any open set containing A. Since A is g$\mathcal{G}$-closed, $\varphi^s(A) \subseteq U$ and by Theorem 2.1, $scl(\varphi^s(A)) \subseteq U$. Since A is φ^s-semi-dense in-itself, $A \subseteq \varphi^s(A)$ and hence $scl(A) \subseteq U$ whenever $A \subseteq U$. This proves that A is gs-closed.

Theorem 3.16. Let $(X, \tau, \mathcal{G})$ be a grill topological space and A be a g$\mathcal{G}$-closed subset of X. If B is a subset of X such that $A \subseteq B \subseteq \varphi^s(A)$, then B is g$\mathcal{G}$-closed.

Proof. Let U be any open set of X such that $B \subseteq U$. Then $A \subseteq U$. Since A is g$\mathcal{G}$-closed, $\varphi^s(A) \subseteq U$. By Theorem 2.1, we have $\varphi^s(B) \subseteq \varphi^s(\varphi^s(A)) \subseteq \varphi^s(A) \subseteq U$ and hence B is g$\mathcal{G}$-closed.

Theorem 3.17. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq Y \subseteq X$, where Y is α-open in X. Then $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) = \varphi^s(A) \cap Y$.

Proof. Assume that $x \in X - (\varphi^s(A) \cap Y)$. Then either $x \in Y$ or $x \notin Y$

Case(i). $x \notin Y$: Since $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) \subseteq Y$, then $x \notin \varphi^s(A(\mathcal{G}_Y, \tau_Y))$.

Case(ii). $x \in Y$: Since $x \notin \varphi^s(A)$, there exists a semiopen set V in X containing x such that $V \cap A \notin \mathcal{G}$. Since $x \in Y$ and Y is α-open in X, we have a set $Y \cap V \in SO(Y, \tau_Y)$ such that $x \in Y \cap V$ and $(Y \cap V) \cap A \notin \mathcal{G}$ and hence $(Y \cap V) \cap A \notin \mathcal{G}_Y$. Consequently, $x \notin \varphi^s(A(\mathcal{G}_Y, \tau_Y))$. Hence, we get $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) \subseteq \varphi^s(A) \cap Y$. To prove the reverse implication, consider $x \notin \varphi^s(A(\mathcal{G}_Y, \tau_Y))$. Then, for some semiopen set V in (Y, τ_Y) containing x there exists $U \in SO(X, x)$ such that $V = U \cap Y$ and we have $(U \cap Y) \cap A \notin \mathcal{G}_Y$. Since $A \subseteq Y$, then $U \cap A \notin \mathcal{G}_Y \subseteq \mathcal{G}$ gives $U \cap A \notin \mathcal{G}$ for some semiopen set U in (X, τ) containing x. This proves $x \notin \varphi^s(A)$. This completes the proof.

Theorem 3.18. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq Y \subseteq X$. If A is g$\mathcal{G}$-closed in $(Y, \tau_Y, \mathcal{G}_Y)$ and X is α-open and φ^s-semiclosed in X, then A is g$\mathcal{G}$-closed in X.

Proof. Let $A \subseteq U$ and U be open in X. Then $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) = \varphi^s(A) \cap Y \subseteq U \cap Y$. Then we have $Y \subseteq U \cup (X - \varphi^s(A))$. Since Y is φ^s-semiclosed, we have $\varphi^s(A) \subseteq \varphi^s(Y) \subseteq Y \subseteq U \cup (X - \varphi^s(A))$. This proves that $\varphi^s(A) \subseteq U$. This completes the proof.

Theorem 3.19. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq Y \subseteq X$. If A is g$\mathcal{G}$-closed in X and $Y \in \tau$, then A is g$\mathcal{G}$-closed in $(Y, \tau_Y, \mathcal{G}_Y)$.

Proof. Let U be an open subset of (Y, τ_Y) such that $A \subseteq U$. Since $Y \in \tau$, then $U \in \tau$. Thus $\varphi^s(A) \subseteq U$. By Theorem 3.17, $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) = \varphi^s(A) \cap Y \subseteq U \cap Y = U$ and we have $\varphi^s(A(\mathcal{G}_Y, \tau_Y)) \subseteq U$. Hence A is g$\mathcal{G}$-closed in $(Y, \tau_Y, \mathcal{G}_Y)$.

Corollary 3.19. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq Y \subseteq X$, where Y is a regular open subset of X. Then A is g$\mathcal{G}$-closed in $(Y, \tau_Y, \mathcal{G}_Y)$ if and only if A is g$\mathcal{G}$-closed in X.

Theorem 3.20. Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq X$. If A is g$\mathcal{G}$-closed, then $A \cup (X - \varphi^s(A))$ is g$\mathcal{G}$-closed.

Proof. Suppose A is g$\mathcal{G}$-closed. Let U be an open set such that $A \cup (X - \varphi^s(A)) \subseteq U$. Then $X - U \subseteq X - (A \cup (X - \varphi^s(A))) = \varphi^s(A) - A$. Since A is g$\mathcal{G}$-closed, by Theorem 3.5, it follows that $\varphi^s(A) - A$ contains no non-empty closed set. This implies $X - U = \varnothing$ or $X = U$. Thus, X is the only open set containing $A \cup (X - \varphi^s(A))$. This gives $\varphi^s(A \cup (X - \varphi^s(A))) \subseteq X$. This proves $A \cup (X - \varphi^s(A))$ is g$\mathcal{G}$-closed.

Theorem 3.21. Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then $A \cup (X - \varphi^s(A))$ is g$\mathcal{G}$-closed if and only if $\varphi^s(A) - A$ is g$\mathcal{G}$-open.

Proof. Since $X - (\varphi^s(A) - A) = A \cup (X - \varphi^s(A))$, the proof follows immediately.

Theorem 3.22. Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then every subset of X is g$\mathcal{G}$-closed if every open set is φ^s-semiclosed.

Proof. Suppose that every open set is φ^s-semiclosed. If $A \subseteq X$ and U is an open set such that $A \subseteq U$, then $\varphi^s(A) \subseteq \varphi^s(U) \subseteq U$ and $\varphi^s(A) \subseteq U$. This proves that A is g$\mathcal{G}$-closed.

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