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Bianchi Type V Universe with QVDP in f (R, T) Theory of Gravity

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Abstract:
Throughout the report, The Bianchi cosmological model is contemplating within a context of f (R, T) Gravity. Under the recently proposed the deceleration parameter in quadratic form, the solutions to the reconditioned field equations have been found by Bakery and Shafeek (2019). Big explosion uniqueness at infinite time t = 0 and shred with respect to t = 1 are expose to view by the model. Studies have been accompanied on the development of the universe. Physical and dynamical properties and a graphical depiction has also been provided. Typical parameter of field equations is determined Also we discussed the jerk parameter of the proposed model.

Key words: f (R, T) theory of gravity, B-V galactic model, Deceleration Parameter, State finder parameter.

I. INTRODUCTION

According to the discussion on many observations by many researchers, the cause of the accelerated universe is dark energy is most responsible candidate. sinister energy to be contemplate as a substance with under pressure which influence the current cosmological energy movement. By various observations, it becomes necessary to envision and concede the nature of force field [1-4]. Among the most broadly considered claimant of dark energy is the empty space of the cosmological constant Λ. But in cosmology, it brawls with remarkable problems among them like cosmic coincidence and fine tuning [5-7]. Some other features of dark energy are Chaplygin gas (a dark fluid which might describe the past decelerating matter dominated era and at present time it provides an accelerating expansion of the universe.) [8], tachyons (a hypothetical particle that travels faster than the speed of light.) [9], Quintessence (bottom distillation) [10], phantom (wraith) [11], k-essence [12].

By notify the angular part of the EH action, make alteration to gravity theories can be acquired. These theories have recently leap up to the top of the list of options for clarification the unsolved problem of the accelerating phase of the universe enlargement. The simplest and most common converting of GR in this situation is f(R) gravity. The f(R) gravity theory was first dispensed in Buchdah,1970 [13], and it was afterwards utilized to discover an isotropic, Non-Singular de-Sitter type Cosmological Solutions [14]. The masses authors have inquired various aspects of $f(R)$ gravity for different type cosmological models, [15-18]. Moreover, it is signified that the f(R) gravity is congenial with recent observations [19-22].

Nevertheless, modern cosmology now addresses the anisotropy problem in inclusion to the Dark Energy problem. Span the latest factual data supports the prospect of an anisotropic aspects in previous periods of the cosmos, the FRW models deduce the nature of the modern universe as spatially uniform and isotropic [23]. This observation is understood to mean that the universe's initial anisotropy is currently changing to an isotropy [24-25]. Therefore, the homogeneous and anisotropic Bianchi type models can be quite useful in examining the universe's early time anisotropic behaviour.

In this context, several theories of modified gravity were later developed. Otherwise, f (R, T) theory is a further imprecise one than the f (R). Geometry and matter are integrated in the gravitational action to enlarge f (R, T) theory [26]. Gravitational Lagrangian in this alter gravity theory requires an unknown function of the scalar curvature and the stress energy-momentum tensor trace.

Astronomers who have researched contemporary cosmological issues like as dark energy and anisotropy have a tendency to look at modified theories involving Bianchi type models. Formerly ten years f (R, T) gravity assumption increased attention, among other things. In this context, a large number of studies exist.

In f (R, T) gravity, Adhav stumbled on errorless solutions to the field equations with view to LRS Bianchi type-I space [27]. By embracing a linearly substituting deceleration parameter and taking into account numerous functional configurations of f (R, T) Sahoo and Sivakumar obtained the require solutions for LRS Bianchi type-I cosmological models in f (R, T) theory of gravity [28]. The wordsmith concluded that there is no way to avoid the Big Rip scenario. Using the presumption of the constant deceleration parameter for Bianchi type-V space-time in the f(R,T) theory of gravity and the variant law of the Hubble parameter, Shamir set up two exact solutions that correlate with singular and non-singular models [29].Using a linear deceleration parameter with a negative slope, Chaubey et al. analysed the broad class of Bianchi cosmological models in f(R, T) gravity with dark energy as standard and modified Chaplygin gas and bulk viscosity for three types of average scale factor [30].

FLRW cosmological model with orthogonal deceleration parameter in f (R, T) gravity has been scrutinize by Bishi et al. [31]. Moreover, using a unique form of Hubble's parameter, Bishi, et al. range over the LRS Bianchi type-I cosmological model in f (R, T) gravity [32]. For LRS Bianchi type-I model, Tiwari *et al*. derived the suspension to the field equations of f (R, T) gravity by assuming that the deceleration parameter is a linear function of the Hubble parameter and indicated that the model starts with an initial singular phase and transitions from an early deceleration phase to a late-time acceleration phase [33]. Mamon and Das have examined the field equations of
f (R, T) gravity for the parametric remodelling model of the deceleration parameter [34].

Existed come across that the obtained model of the Universe restrain an initial singularity, a finite life full lenght, and a big rip at the end. Akarsu and Dereli explored cosmological models in which the cosmos evinces quintom like behavior and bring to a conclusion in a large rip with a linearly diverse deceleration value [35]. The big-rip theory advance that the universe expands before shrinking and moving back to its initial state at the time of the big bang. Bakry and Shafeek glanced into the universe model with the time-dependent deceleration parameter of the second degree and came to this inference [36]. Tiwari et al. inquire into the time-dependent deceleration parameter in the framework of f (R, T) gravity and concluded that the cosmos has become distended periodically. i.e., the universe expansion at embarks on with a slowing then facilitate super exponentially [37]. According to Tiwari and Sofuoglu's compulsion investigation of quadratic changing deceleration parameter in the context of f (R, T) gravity, universe set about with a big bang and ends with a enormous rip [38]. Kumar and Arora have studied Bi-Quadratic Varying Deceleration Parameter to Study the Cosmological Model [39]. Mishra et al. have provided a comprehensive analysis of the evolution of geometrical and physical parameters, complete with the numerically constrained values.[40].

In this paper, we have evolved anisotropic cosmological model in f(R, T) gravity theory to acquire the exact solution of the alter field equations in the activity of the newly offered condition of [36] quadratic form of deceleration parameter and requiring the functional form of f(R, T) = f_1(R) + f_2(T), where f_1(R) = λR and f_2(T) = λT. The tangible major of the considered deceleration parameter is presented as;

The cosmological model starts with a big bang at t = 0 and ends at a big rip at t=2β

i.e., The universe 'begins' with q = ($8\beta^2 - 1$) and ends with q = ($-4\beta^2 - 1$).

II. BASIC FORMALISM OF *F* (*R, T*) GRAVITY THEORY:

In f (R, T) gravity, the field equations are obtained from a dissimilarity, Hilbert-Einstein type, principle. The action for the specified f (R, T) gravity is,

$$S = \int \left(\frac{1}{16\pi} f(R,T) + s_m\right)\sqrt{-g}\, dx^4 \quad (1)$$

where f (R, T) is an arbitrary function of the Ricci tensor R and energy momentum tensor T.

Three obvious requirements of the operational from of f (R, T) are standardly considered,

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \quad (2)$$

Assuming as a choice of f (R, T) = f_1(R) + f_2(T), where f_1(R) = λR and f_2(T) = λT, where λ is free parameter.

Therefore, Equation (2) can be written as,

$$f_1'(R)R_{ij} - \frac{1}{2} f_1(R) g_{ij} + (g_{ij}\,\nabla^i\nabla_i - \nabla_i\nabla_j)\, f_1(R) = kT_{ij} - f_2'(T)\, T_{ij} - f_2'(T)\Theta_{ij} + \frac{1}{2} f_2(T) g_{ij} \quad (3)$$

Where the prime denotes differentiation with respect to the arguments. where Θ_{ij} is expressed as,

$$\Theta_{ij} = g^{lm}\frac{\delta T_{lm}}{\delta g^{ij}} \tag{4}$$

where Tij is energy momentum tensor with perfect fluid and defined as,

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} \tag{5}$$

where $u^i = (0,0,0,1)$ is four velocity vector satisfying $u^i u_j = 1$ (6)

p and ρ are pressure and energy density of the matter respectively.

Assuming that the matter content consists of a perfect fluid and defined as,

$$\Theta_{ij} = -2\, T_{ij} - p g_{ij} \tag{7}$$

The field equation takes the form,

$$\lambda R_{ij} - \frac{1}{2}\lambda(R+T)g_{ij} + (g_{ij} \quad \nabla^i\nabla_i \; - \; \nabla_i\nabla_j)\,\lambda \quad = \; kT_{ij} - \quad \lambda\, T_{ij} \; + \lambda\left[\,2T_{ij} + \frac{1}{2}g_{ij}\,)\right] \tag{8}$$

Generally, the field equations also pivot on the whole time the tensor θ_{ij}, on the physical nature of the matter
field. Hence in the case of f (R, T) gravity depending on the nature of the matter source, we prevailed several theoretical models corresponding to each choice of f (R, T). Out of three explicit specifications
of the functional form of *f* (*R, T*).

Now, we have

$$R_{ij} - \frac{1}{2}R g_{ij} - (p + \frac{1}{2}\,T)g_{ij} \;\; = \left(\frac{k+\lambda}{\lambda}\right)\, T_{ij} \tag{9}$$

Recollecting Einstein equations inclusive of cosmological constant,

$$G_{ij} - \Lambda g_{ij} = -kT_{ij} \tag{10}$$

Pick out k=1.

We choose a negative small value for the arbitrary *λ,* so that we have the same sign of the RHS. of (9) and (10), we keep this choice of *λ* throughout. (p + $\frac{1}{2}$ T) can be regarded now as a cosmological constant term t. Hence, we can write,

$$\Lambda(T) = p + \frac{1}{2}T \tag{11}$$

Hence Eq. (11) reduces to ,

$\Lambda(T) = \frac{1}{2}(\rho - p)$

(12)

III. GRAVITATIONAL FIELD EQUATION

We consider a homogeneous Bianchi type–V metric, which is given by,

$$ds^2 = dt^2 - a_1^2 dx^2 - e^{2mx}[a_2^2 dy^2 + a_3^2 dz^2] \tag{13}$$

where, A, B and C are functions of cosmic time t only.

Now assuming commoving coordinate system, the field Equation (9) with the help of (5) and (6) for the metric (12) can be written as,

$$\frac{\dot{a_2}\dot{a_3}}{a_2 a_3} + \frac{\ddot{a_3}}{a_3} + \frac{\ddot{a_2}}{a_2} - \frac{m}{{a_1}^2} = \left(\frac{8\pi+\lambda}{\lambda}\right)\text{p - }\Lambda$$

(14)

$$\frac{\dot{a_1}\dot{a_3}}{a_1 a_3} + \frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_3}}{a_3} - \frac{m}{{a_1}^2} = \left(\frac{8\pi+\lambda}{\lambda}\right)\text{p - }\Lambda$$

(15)

$$\frac{\dot{a_1}\dot{a_2}}{a_1 a_2} + \frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} - \frac{m}{{a_1}^2} = \left(\frac{8\pi+\lambda}{\lambda}\right)\text{p} - \Lambda$$

(16)

$$\frac{\dot{a_1}}{a_1}\frac{\dot{a_2}}{a_2} + \frac{\dot{a_2}\dot{a_3}}{a_2 a_3} + \frac{\dot{a_1}\dot{a_3}}{a_1 a_3} - \frac{3m}{{a_1}^2} = -\left(\frac{8\pi+\lambda}{\lambda}\right)\rho - \Lambda$$

(17)

$$2\frac{\dot{a_1}}{a_1} - \frac{\dot{a_2}}{a_2} - \frac{\dot{a_3}}{a_3} = 0$$

(18)

From equations (14)-(16) together with equation (17), we get –

$$\frac{\dot{a_1}}{a_1} = \frac{\dot{a}}{a}$$

(19)

$$\frac{\dot{a_2}}{a_2} = \frac{\dot{a}}{a} - \frac{k}{a^3}$$

(20)

$$\frac{\dot{a_3}}{a_3} = \frac{\dot{a}}{a} + \frac{k}{a^3}$$

(21)

where k is constant.

The spatial volume and the scale factor of the Metric (13) are defined by, $V = a^3 = a_1 a_2 a_3$ (22)

The physical quantities of observational interest in cosmology are as;

The mean generalized Hubble parameter becomes,

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a_1}}{a_1} + \frac{\dot{a_2}}{a_2} + \frac{\dot{a_3}}{a_3}\right) \quad (23)$$

The energy density, pressure and cosmological term for this model are demonstrates as,

$$\rho = \frac{2\lambda^2}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\dot{H} - \frac{3(8\pi+\lambda)}{\lambda}H^2 + \frac{(8\pi+2\lambda)k^2}{\lambda a^6} + \frac{(24\pi+4\lambda)\lambda}{a^2}\right] \quad (24)$$

$$p = \frac{2\lambda^2}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{(16\pi+3\lambda)}{\lambda}\dot{H} + \frac{3(8\pi+\lambda)}{\lambda}H^2 + \frac{(8\pi+2\lambda)k^2}{\lambda a^6} - \frac{8\pi\lambda}{a^2}\right] \quad (25)$$

and

$$\Lambda = \frac{\lambda^2}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{(16\pi+2\lambda)}{\lambda}\dot{H} - \frac{6(8\pi+\lambda)}{\lambda}H^2 + \frac{(32\pi+4\lambda)}{a^2}\lambda\right] \quad (26)$$

In sequence to find the exact solution of field equations following [36], we assume the quadratic deceleration parameter q which is given by,

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = 3t^2 - 12\beta t + (8\beta^2 - 1) \quad (27)$$

where β is positive constant. At t=0, $q = (8\beta^2 - 1)$,which is always positive for all $\beta > 0$ i.e. Given form of the deceleration parameter in (27) concludes the constant form of deceleration parameter which is discussed by [41,42]. Remoter this was extended in the quadratic form of time. At the cyclic term, this model starts with the big-bang singularity and late time in a big rip.

The equation for the deceleration parameter is given by,

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1 \quad (28)$$

With the help of equation (27) and (28) Hubble parameter can be found as,

$$H = \frac{1}{t(t-2\beta)(t-4\beta)} \quad (29)$$

we can see from equation (29), that the Hubble parameter H is always positive in the range $0 \leq t \leq 4\beta$. Also we can observe that the Hubble parameter diverges at t=0, t = 2β and t =4β

Integrating equation (29), we obtain the scale factor as,

$$a = \frac{[t(4\beta-t)]^{\frac{1}{8\beta^2}}}{[(2\beta-t)]^{\frac{1}{4\beta^2}}} \tag{30}$$

which gives the declaration of the scale factor as function of cosmic time. It is remarked that the scale factor a= 0 for t=0 and at t =4β and scale factor diverges at t= 2β, which manifest that the universe starts with big-bang at t=0 and ends with big crunch at t=2 β.

From equations (19)-(21) together with equation (31) & (32), we get –

$$\frac{\dot{a_1}}{a_1} = \frac{1}{t(2\beta-t)(4\beta-t)} \tag{31}$$

$$\frac{\dot{a_2}}{a_2} = \frac{1}{t(2\beta-t)(4\beta-t)} - \frac{[(2\beta-t)]^{\frac{3}{4\beta^2}}k}{[t(4\beta-t)]^{\frac{3}{8\beta^2}}} \tag{32}$$

$$\frac{\dot{a_3}}{a_3} = \frac{1}{t(2\beta-t)(4\beta-t)} + \frac{[(2\beta-t)]^{\frac{3}{4\beta^2}}k}{[t(4\beta-t)]^{\frac{3}{8\beta^2}}} \tag{33}$$

which gives,

$$a_1 = \left[\frac{t(4\beta-t)}{(2\beta-t)^2}\right]^{\frac{1}{8\beta^2}} \tag{34}$$

$$a_2 = \left[\frac{t(4\beta-t)}{(2\beta-t)^2}\right]^{\frac{1}{8\beta^2}} - \int \frac{[(2\beta-t)]^{\frac{3}{4\beta^2}}k}{[t(4\beta-t)]^{\frac{3}{8\beta^2}}}\, dt \tag{35}$$

$$a_3 = \left[\frac{t(4\beta-t)}{(2\beta-t)^2}\right]^{\frac{1}{8\beta^2}} + \int \frac{[(2\beta-t)]^{\frac{3}{4\beta^2}}k}{[t(4\beta-t)]^{\frac{3}{8\beta^2}}}\, dt \tag{36}$$

Relation between scale factor and redshift is given as,

$$\frac{a}{a_0} = \frac{1}{1+z} \tag{37}$$

From equation (30) and (37) we have,

$$\left[\frac{t(4\beta-t)}{(2\beta-t)^2}\right]^{\frac{1}{8\beta^2}} = \frac{1}{1+z} \tag{38}$$

To find the value of time in the term of red shift, we have taken the choice of β .
So, we assume, $\frac{1}{4\beta^2} = 1 \Longrightarrow \ \beta = 0.5$, which can help to express t as,

$$t = 1 \pm \frac{1+Z}{\sqrt{Z(Z+2)+2}} \tag{39}$$

The Hubble Parameter in the terms of redshift (z) conceivably expressed as,

$$H = \frac{H_0}{\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^3 - 6\beta\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^2 + 8\beta^2\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]}$$

concerning red shift, Deceleration parameter can be stated as,

$$q\,(z) = \frac{z^2+2z-1}{z^2+2z+2} \tag{40}$$

as the consequence (26), (27), (28), (31) & (32), the energy density, pressure and cosmological term of model are express as,

$$\rho = \frac{2\lambda^2}{[(8\pi+3\lambda)^2-\lambda^2}\left[\frac{3\lambda t^2-12\beta\lambda t + 8\lambda\beta^2-(24\pi+3\lambda)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2} + \frac{(8\pi+2\lambda)k^2\,[(2\beta-t)]^{\frac{3}{2\beta^2}}}{\lambda t^{\frac{3}{4\beta^2}}(4\beta-t)]^{\frac{3}{4\beta^2}}} + \frac{(24\pi+4\lambda)[(2\beta-t)]^{\frac{1}{2\beta^2}}\lambda}{t^{\frac{1}{4\beta^2}}(4\beta-t)]^{\frac{1}{4\beta^2}}}\right] \tag{41}$$

$$p = \frac{2\lambda^2}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{(16\pi+3\lambda)3t^2-12\beta t+24\beta^2(8\pi+\lambda)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2} + \frac{(8\pi+2\lambda)k^2\,[(2\beta-t)]^{\frac{3}{2\beta^2}}}{\lambda t^{\frac{3}{4\beta^2}}(4\beta-t)^{\frac{3}{4\beta^2}}} - \frac{(8\pi\lambda)[(2\beta-t)]^{\frac{1}{2\beta^2}}}{t^{\frac{1}{4\beta^2}}(4\beta-t)^{\frac{1}{4\beta^2}}}\right]$$

(42)

$$\Lambda = \frac{\lambda^2(8\pi+\lambda)}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{(6t^2-24\beta t+16\beta^2+3)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2} + \frac{4[(2\beta-t)]^{\frac{1}{2\beta^2}}}{t(4\beta-t)]^{\frac{1}{4\beta^2}}}\right]$$

(43)

The Equalization of state parameter (p = $\omega\rho$)is given by,

$$\omega=\frac{\left[\frac{(16\pi+3\lambda)3t^2-12\beta t+24\beta^2(8\pi+\lambda)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2}+\frac{(8\pi+2\lambda)k^2[(2\beta-t)]^{\frac{3}{2\beta^2}}}{\lambda t^{\frac{3}{4\beta^2}}(4\beta-t)^{\frac{3}{4\beta^2}}}-\frac{(8\pi\lambda)[(2\beta-t)]^{\frac{1}{2\beta^2}}}{t^{\frac{1}{4\beta^2}}(4\beta-t)^{\frac{1}{4\beta^2}}}\right]}{\left[\frac{3\lambda t^2-12\beta\lambda t+8\lambda\beta^2-(24\pi+3\lambda)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2}+\frac{(8\pi+2\lambda)k^2[(2\beta-t)]^{\frac{3}{2\beta^2}}}{\lambda t^{\frac{3}{4\beta^2}}(4\beta-t)]^{\frac{3}{4\beta^2}}}+\frac{(24\pi+4\lambda)\lambda[(2\beta-t)]^{\frac{1}{2\beta^2}}}{t^{\frac{1}{4\beta^2}}(4\beta-t)]^{\frac{1}{4\beta^2}}}\right]}$$

(44)

identification of restriction (ω), as regards red shift as,

$$\omega(z)=\frac{\left[\frac{(16\pi+3\lambda)3\left[1\pm\frac{1+Z}{\sqrt{Z(Z+2)+2}}\right]^2-12\beta\left[1\pm\frac{1+Z}{\sqrt{Z(Z+2)+2}}\right]+24\beta^2(8\pi+\lambda)}{\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^2\left(\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]-2\beta\right)^2\left(\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]-4\beta\right)^2}+\frac{(8\pi+2\lambda)k^2\left(2\beta-\left[1\pm\frac{1+Z}{\sqrt{Z(Z+2)+2}}\right]\right)^{\frac{3}{2\beta^2}}}{\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^{\frac{3}{4\beta^2}}\left(4\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{3}{4\beta^2}}}-\frac{8\pi\lambda\left(2\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{1}{2\beta^2}}}{\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^{\frac{1}{4\beta^2}}\left(4\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{1}{4\beta^2}}}\right]}{\left[\frac{3\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^2-12\beta\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]+8\lambda\beta^2-(24\pi+3\lambda)}{\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^2\left(\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]-2\beta\right)^2\left(\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]-4\beta\right)^2}+\frac{(8\pi+2\lambda)k^2\left(2\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{3}{2\beta^2}}}{\lambda\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^{\frac{3}{4\beta^2}}\left(4\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{3}{4\beta^2}}}+\frac{(24\pi+4\lambda)\lambda\left(2\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{1}{2\beta^2}}}{\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]^{\frac{1}{4\beta^2}}\left(4\beta-\left[1\pm\frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{1}{4\beta^2}}}\right]}$$

(45)

The Density parameter,

$$\Omega(t)=\frac{\rho}{3H^2}$$

$$\Omega(t)=\frac{2\lambda^2t^2(2\beta-t)^2(4\beta-t)^2}{3[(8\pi+3\lambda)^2-\lambda^2}\left[\frac{3\lambda t^2-12\beta\lambda t+8\lambda\beta^2-(24\pi+3\lambda)}{\lambda t^2(2\beta-t)^2(4\beta-t)^2}+\frac{[(2\beta-t)]^{\frac{1}{2\beta^2}}\{(8\pi+2\lambda)k^2[(2\beta-t)]^{\frac{1}{\beta^2}}-(24\pi+4\lambda)\}}{\lambda t^{\frac{3}{4\beta^2}}(4\beta-t)]^{\frac{3}{4\beta^2}}}\right]$$

(46)

Density parameter in the terms of red shift,

$$\Omega(\mathrm{z}) = \frac{2\lambda^2\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]^2\left(\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right] - 2\beta\right)^2\left(\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right] - 4\beta\right)^2}{3[(8\pi+3\lambda)^2 - \lambda^2}$$

$$\left[\frac{3\lambda\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]^2 - 12\beta\lambda\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right] + 8\,\lambda\beta^2 - (24\pi + 3\lambda\,)}{\lambda\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]^2\left(\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right] - 2\beta\right)^2\left(\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right] - 4\beta\right)^2}\right.$$

$$\left.+\frac{\left(2\beta - \left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{1}{2\beta^2}}\{(8\pi + 2\lambda\,)\mathrm{k}^2\left[\left(2\beta - \left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)\right]^{\frac{1}{\beta^2}} - (24\pi + 4\lambda\,)}{\lambda\left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]^{\frac{3}{4\beta^2}}\left(4\beta - \left[1 \pm \frac{1+z}{\sqrt{z(z+2)+2}}\right]\right)^{\frac{3}{4\beta^2}}}\right]$$

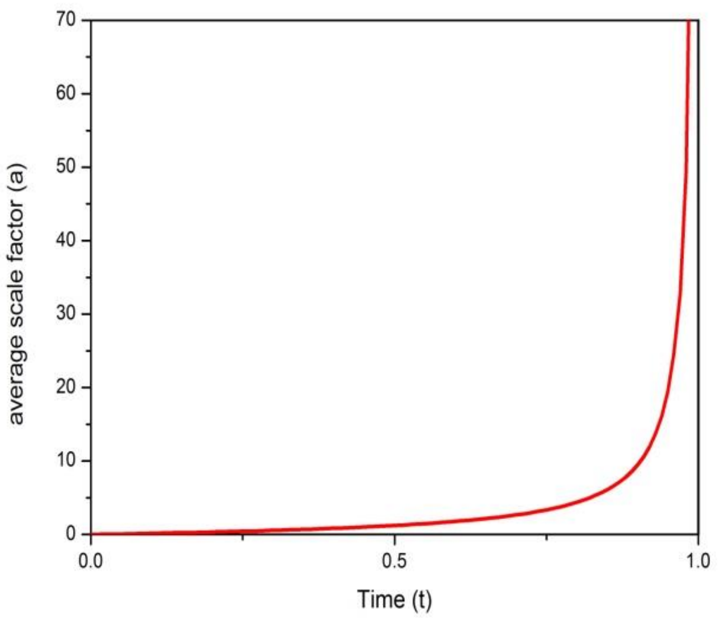

Figure.1 average scale factor (a) against cosmic time(t)

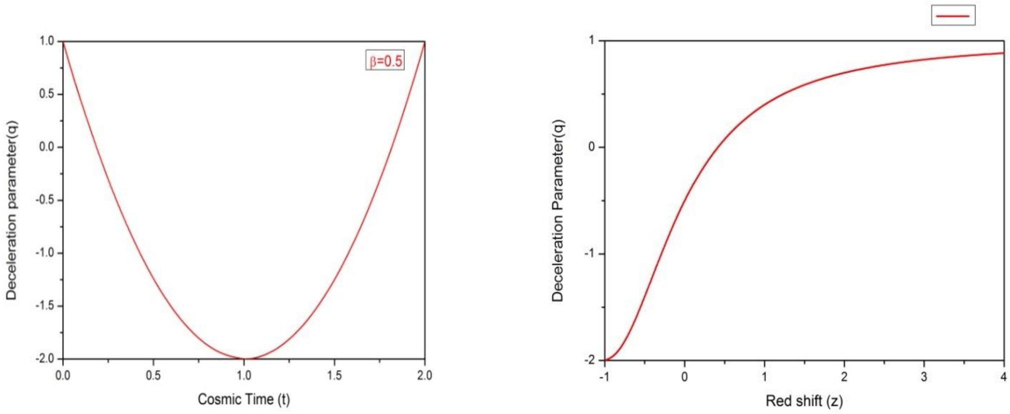

Figure.2 Deceleration parameter against cosmic time(t) and redshift (z)

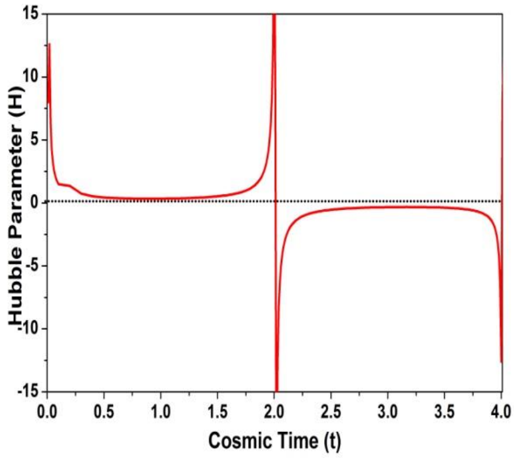

Figure 3 Hubble Parameter (H) averse to cosmic time(t)

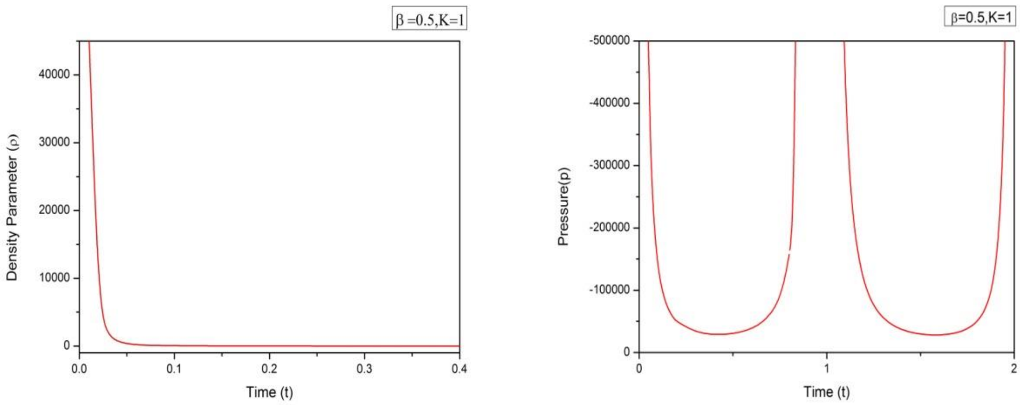

Figure 4. Density parameter and pressure against cosmic time(t)

STATE- FINDER PARAMETER

The state finder parameter is decisive in order to study the cosmological models. The pair $\{r, s\}$ is useful for distinguishing the properties of dark Energy. The $r - s$ plane advisable the different qualitative behaviour for different cosmological models. The state appropriator diagnostic along with SNAP (Super Novae Acceleration Probe) inspection may distinguish different dark energy models. The state finder pair $\{r, s\}$ is defined as,

$$r = \frac{\ddot{a}}{aH^3} \text{ together with } s = \frac{r-1}{3(q-0.5)}$$

Here, the associated with parameters have the accustomed denotation talk throughout the paper. The state finder parameters for the discussed model are obtained as,

$$r = \frac{\ddot{a}}{aH^3} = \frac{z^2+2z-1}{z^2+2z+2}\left[2\left(\frac{z^2+2z-1}{z^2+2z+2}\right)+1\right]+\frac{6(1+z)^2}{(z^2+2z+2)^2}$$

$$s = \frac{-18\,(1+z)^2\,(2z^2+4z+1)}{(z^2+2z+2)^2} + 3z\,\frac{(z^2+2z-1)\,(z+2)\,(5z^2+10z+1)}{(z^2+2z+2)^3}$$

IV. RESULT AND DISCUSSION

In The figure1, opposed to time we have taken the interval [0, 2] for cosmic time, while the roughly is notable in interval [0,1] only. Noteworthy this, the creativity from the value of scale factor i.e., scale factor is negative in the domain [1,2]. So that this region is insignificant from the observed value.

Figure 2, represents behaviour of the deceleration parameter hostile to cosmic time and red shift separately from the figures for equation (29), it can be observed that at $\beta = 1/2$, the deceleration parameter starts at q = 1 at initial time t = 0 and ends at q= -2 at t =1. So, observations show that deceleration parameter be sited in the interval [-2,1] for the time interval [0,1]. In the course of the time interval of t ∈ [0,0.18], q is positive., q lies between [1, 0.0172] which advisable the decelerating phase. i.e. (q > 0). At t = 0.19, q is negative, which indicates the accelerating phase. i.e. (q < 0). In the interval t ∈ [1, 1.57] present model forecasting the super exponential phase i.e., (q < -1). The qualitative behaviour of q(z) is consistent with deceleration parameter discussed by Farook [43].

Figure 3 represents The Hubble parameter against cosmic time which shows that, H diverges at t = 0 and $t = 2\beta$, i.e., t = 1. Further, one can say that the big rip moment occurs at t = 1.

Figure 4, characterized the variation of the energy density and pressure of model for the flat, open and closed universe [44]. It is strong matches out that in both the models, for flat, open and closed universes, the energy density has singularities at t = 0 and t = 1, is non-negative ($\rho \geq 0$). The energy density in all flat, open and closed universes has a big rip moment at t = 1 due to the divergent essence of the scale factor and energy density of the fluid at that time.

At the initial point t = 0, the pressure has the maximum value. The pressure is positive valued in the interval [0, 0.08] and negative valued in the interval [0.09, 1]. for the closed universe of the model.

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