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On Determining Shortest Path Problem under Pentagonal Neutrosophic Fuzzy Number

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Abstract:

Neutrosophic sets is the generalization of fuzzy set theory. In this discipline the crucial situation are widely explained in three components appear in it in distinct fields. The study of Neutrosophy deals with the interactions of different ideational spectra. And also the shortest path method is one of the important tool in network analysis. In this approach the pentagonal neutrosophic fuzzy number is applied to determine the shortest path. A score function is used to defuzzify the number into crisp number. The new algorithm evaluates the shortest path for pentagonal neutrosophic fuzzy environment and it is compared with a real life situation.

Keywords: Neutrosophic number, Pentagonal Neutrosophic Number (PNN), Shortest path.

Introduction:

In present situation finding a shortest route between two nodes with imprecise data is a great task. This type of uncertainties cannot be solved by classical theory. To overcome this Zadeh introduced fuzzy set theory in 1965 which involves membership function. Later many researchers worked in this area and determines that the available information is not enough to the level of accuracy. So, Atanassov introduced the concept of intuitionistic fuzzy set theory in 1986 with membership and non-membership function. It is used to adapt with imprecise information and it is applied in many real life problems.

Then the neutrosophic fuzzy number was introduced by Smarandache in 1998 to face the problem of indeterminate or inconsistent information. The concept of. neutrosophic set consists of three components namely 1.Truthfulness 2. Indeterminancy 3. Falsity. The shortest path problem is one of the important tool in network analysis. Many researchers worked on shortest path problem in various fuzzy domain. Shortest path problem has many applications in various disciplines.

Here the shortest path was first analyzed by Dubois and Prade. A.Praveenprakash, N. Jose Parvin Praveena, A. Rajkumar[6] characterized an innovative method for a New Intuitionistic Decagonal Fuzzy Number and its applications. A. Nagoorgani and A. Mumtaj Begum[5] has analyzed A new approach on shortest path in fuzzy environment.

Said Broumi, Assia Bakali, Mohammed Talea, Florentin Smarandache[7] they applied Dijkstra algorithm for solving Neutrosophic Shortest Path Problem. Avishek Chakraborty[1] contributed time dependency and neutrosophic cost function related work in shortest path problem in PNN area. N. Jose Parvin praveena, S. Ghousia Begum, Ganesh Kumar Thaker, Bandana Priya, Chirag Goyal[3] developed neutrosophic numbers in finding Shortest Path using Dynamic Programming.

In this proposed paper Pentagonal Neutrosophic fuzzy number is applied to calculate the shortest path. A score function is used to defuzzify into crisp number and a new algorithm is demonstrated to calculate the shortest path and the efficiency was discussed with a real life example.

Objectives:

In this paper the new algorithm evaluates the shortest path for pentagonal neutrosophic fuzzy number.

Methods:

Preliminaries:

1.Definition:

1.1 Neutrosophic set:

Let X be a non-empty set. Then the neutrosophic set $\tilde{A}$ of X is defined as $\tilde{A}$ = {(x, $T_{\tilde{A}}$(x), $I_{\tilde{A}}$(x), $F_{\tilde{A}}$(x)) , x∈X} where $T_{\tilde{A}}$(x), $I_{\tilde{A}}$(x), $F_{\tilde{A}}$(x) ∈ [0,1] and $T_{\tilde{A}}$(x), $I_{\tilde{A}}$(x), $F_{\tilde{A}}$(x) are defined as truth membership, indeterminacy membership and falsity membership function where

$$0 \leq T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 3$$

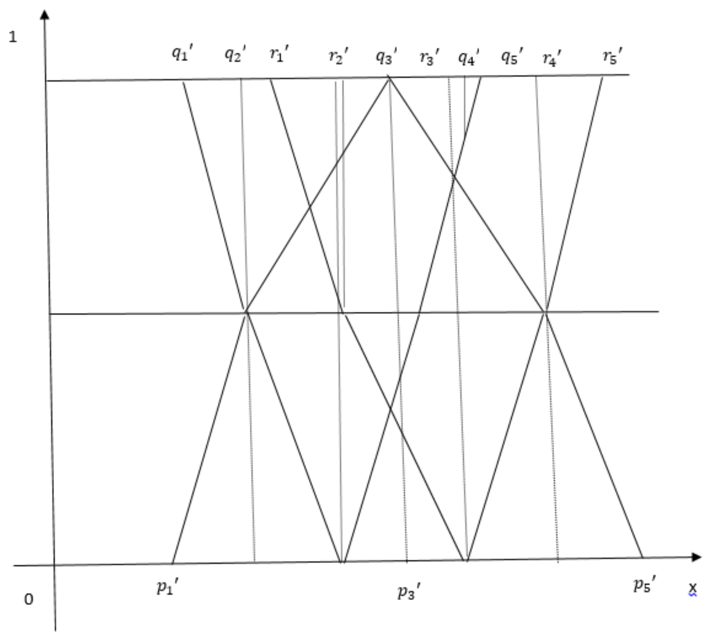
1.2 Pentagonal Neutrosophic Fuzzy Number:

A single valued PNN $\tilde{P} = (p_1', p_2', p_3', p_4', p_5')$ where $p_1', p_2', p_3', p_4', p_5' \in$ R such that $p_1' \leq p_2' \leq p_3' \leq p_4' \leq p_5'$ whose truth membership, indeterminacy membership and falsity membership is given as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \; for \; x < p_1' \\ \frac{x-p_1'}{p_2'-p_1'} \; for \; p_1' \leq x \leq p_2' \\ \frac{x-p_2'}{p_3'-p_2'} \; for \; p_2' \leq x \leq p_3' \\ 1 \; for \; x = p_3' \\ \frac{p_4'-x}{p_4'-p_3'} \; for \; p_3' \leq x \leq p_4' \\ \frac{p_5'-x}{p_5'-p_4'} \; for \; p_4' \leq x \leq p_5' \\ 0 \; for \; x > p_5' \end{cases}$$

$$\gamma_{\tilde{A}}(x) = \begin{cases} 0 \; for \; x < q_1' \\ \frac{x-q_1'}{q_2'-q_1'} \; for \; q_1' \le x \le q_2' \\ \frac{x-q_2'}{q_3'-q_2'} \; for \; q_2' \le x \le q_3' \\ 1 \; for \; x = q_3' \\ \frac{q_4'-x}{q_4'-q_3'} \; for \; q_3' \le x \le q_4' \\ \frac{q_5'-x}{q_5'-q_4'} \; for \; q_4' \le x \le q_5' \\ 0 \; for \; x > q_5' \end{cases}$$

$$\emptyset_{\tilde{A}}(x) = \begin{cases} 0 \; for \; x < r_1' \\ \frac{x-r_1'}{r_2'-r_1'} \; for \; r_1' \le x \le r_2' \\ \frac{x-r_2'}{r_3'-r_2'} \; for \; r_2' \le x \le r_3' \\ 1 \; for \; x = r_3' \\ \frac{r_4'-x}{r_4'-r_3'} \; for \; r_3' \le x \le r_4' \\ \frac{r_5'-x}{r_5'-r_4'} \; for \; r_4' \le x \le r_5' \\ 0 \; for \; x > r_5' \end{cases}$$



2. Algorithm for Fuzzy Smallest Path Problem using Pentagonal Neutrosophic Fuzzy Number:

Step 1: Assume a cyclic network N(V, E) where V is the vertex set and E is the edge set. $e^*_{ij} = \{ e^*_{ij(1)}, e^*_{ij(2)}, e^*_{ij(3)}, e^*_{ij(4)}, e^*_{ij(5)}, e^*_{ij(6)}, e^*_{ij(7)} \}$ represents the PNN where ij represents the edge.

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Step 2: In the fuzzy sense the shortest path problem is given by

f*(i) = min(e*$_{ij}$ – f*(j) , / i, j Є E)

Step 3: Assume f*(n) = 0 where f*(i) is the length of the shortest path from the vertex i to n.

Step 4: By using the score function

$$S(\tilde{A}) = \frac{1}{3}\left(2 + \left(\frac{p_1' + p_2' + p_3' + p_4' + p_5'}{5}\right) - \left(\frac{q_1' + q_2' + q_3' + q_4' + q_5'}{5}\right)\left(\frac{r_1' + r_2' + r_3' + r_4' + r_5'}{5}\right)\right)$$

the value is calculated.

Step 5: Then by comparing the score function the minimum of the path length is calculated for each nodes. Hence the shortest path is calculated.

3. Arithmetic Operations under Pentagonal Neutrosophic Number:

Let $\tilde{P} = (p_1', p_2', p_3', p_4', p_5')$ and $\tilde{Q} = (q_1', q_2', q_3', q_4', q_5')$ are two functions then the arithmetic operations are given by

1. Addition of $\tilde{P}$ and $\tilde{Q}$ are defined as

   $\tilde{P} + \tilde{Q} = (p_1' + q_5', p_2' + q_4', p_3' + q_3', p_4' + q_2', p_5' + q_1')$
2. Subtraction of $\tilde{P}$ and $\tilde{Q}$ are defined as

   $\tilde{P} - \tilde{Q} = (p_1' - q_5', p_2' - q_4', p_3' - q_3', p_4' - q_2', p_5' - q_1')$
3. Multiplication of $\tilde{P}$ and $\tilde{Q}$ are defined as

   $\tilde{P}\,\tilde{Q} = (p_1' q_5', p_2' q_4', p_3' q_3', p_4' q_2', p_5' q_1')$

4. Score and Accuracy function for a Pentagonal Neutrosophic Number:

Score and accuracy function are used to compare the two single valued PNN. Let $\tilde{A} = (p_1', p_2', p_3', p_4', p_5')$ $(q_1', q_2', q_3', q_4', q_5')$ $(r_1', r_2', r_3', r_4', r_5')$ then

$$S(\tilde{A}) = \frac{1}{3}\left(2 + \left(\frac{p_1' + p_2' + p_3' + p_4' + p_5'}{5}\right) - \left(\frac{q_1' + q_2' + q_3' + q_4' + q_5'}{5}\right) - \left(\frac{r_1' + r_2' + r_3' + r_4' + r_5'}{5}\right)\right)$$

And the accuracy function is defined as

$$\left(\left(\frac{p_1' + p_2' + p_3' + p_4' + p_5'}{5}\right) - \left(\frac{r_1' + r_2' + r_3' + r_4' + r_5'}{5}\right)\right)$$

when comparing it should satisfy the condition

(i) $S(\tilde{A}) < S(\tilde{B})$ then a < b
(ii) $S(\tilde{A}) > S(\tilde{A})$ then a > b
(iii) $S(\tilde{A}) = S(\tilde{A})$ then a = b

5.1 Real Time example:

Data collected from the students those who are appearing for Engineering entrance exam. They must fulfil the criteria for engineering entrance exams. They must have scored passing marks in class 12th and must have studied Physics, Chemistry and Mathematics. Candidates must have secured a minimum of 75% aggregate in class 12th board examinations. The entrance exam has various scopes since it provides chance of forming well- built educational foundation in present competitive market. The main purpose of conducting entrance exam is to judge the students ability, sharpness, knowledge etc. The aptitude of students is tested in entrance exam. Definite pattern is used in entrance exam after the students get it done with written test, the shortlisted candidate in written test are followed with group discussion round and personal interview. If the majority of the respondents were uncertain about the entrance result and group discussion then the indeterminacy occurs.

Here the nodes are represented as

Node 1: Enrol top rank universities

Node 2: Major Engineering courses

Node 3: Record placements with salary packages

Node 4: Best hostel facility

Node 5: Good Infrastructure

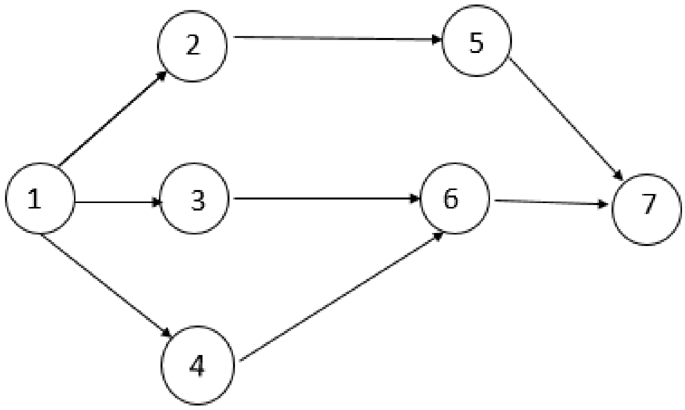
Node 6: International collaboration for Internships

Node 7: Skilled faculties

5.2 Numerical example:

Calculate the shortest path for the following PNN

Activity	Duration
1 - 2	(0.5, 0.6, 0.6 ,0.8, 0.9) (0.4, 0.5, 0.5, 0.7, 0.8) (0.3, 0.4, 0.5, 0.5, 0.7)
1 - 3	(0.4, 0.5, 0.7, 0.8, 0.8) (0.3, 0.4, 0.4, 0.6, 0.8) (0.3, 0.4, 0.5, 0.5, 0.7)
1 - 4	(0.6, 0.6, 0.7, 0.7, 0.9) (0.2, 0.4, 0.5, 0.6, 0.6) (0.3, 0.4, 0.6, 0.7, 0.9)
2 - 5	(0.5, 0.6, 0.7, 0.7, 0.8) (0.1, 0.3, 0.3, 0.4, 0.7) (0.4, 0.4, 0.6, 0.8, 0.8)
3 - 6	(0.4, 0.5, 0.6, 0.6, 0.7) (0.2, 0.2, 0.3, 0.5, 0.6) (0.3, 0.4, 0.5, 0.7, 0.8)
4 - 6	(0.5, 0.7, 0.7, 0.8, 0.9) (0.3, 0.4, 0.4, 0.6, 0.7) (0.2, 0.3, 0.3, 0.4, 0.6)
5 - 7	(0.6, 0.7, 0.8, 0.8, 0.9) (0.2, 0.5, 0.5, 0.6, 0.8) (0.1, 0.3, 0.4, 0.4, 0.7)
6 - 7	(0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8)



Assuming f(7) = 0

f(6) = e_{67} – f(7)

= ((0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8)) - 0

= (0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8)

S ((0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8))

= $\frac{1}{3}(2 + 0.64 - 0.48 - 0.4)$ = 0.5866

f(5) = e_{57} – f(7)

= (0.6, 0.7, 0.8, 0.8, 0.9) (0.2, 0.5, 0.5, 0.6, 0.8) (0.1, 0.3, 0.4, 0.4, 0.7)

S((0.6, 0.7, 0.8, 0.8, 0.9) (0.2, 0.5, 0.5, 0.6, 0.8) (0.1, 0.3, 0.4, 0.4, 0.7))

= $\frac{1}{3}(2 + 0.76 - 0.52 - 0.38)$ = 0.62

f(4) = e_{46} – f(6)

= (0.5, 0.7, 0.7, 0.8, 0.9) (0.3, 0.4, 0.4, 0.6, 0.7) (0.2, 0.3, 0.3, 0.4, 0.6)

- (0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8)

= (-0.4, 0, 0.1, 0.3, 0.4) (-0.3, -0.2, -0.1, 0.2, 0.4) (-0.6, -0.2, 0, 0.2, 0.4)

S((-0.4, 0, 0.1, 0.3, 0.4) (-0.3, -0.2, -0.1, 0.2, 0.4) (-0.6, -0.2, 0, 0.2, 0.4))

= $\frac{1}{3}\left(2 + 0.08 - 0 - (-0.04)\right)$ = 0.706

f(3) = e_{36} – f(6)

= (0.4, 0.5, 0.6, 0.6, 0.7) (0.2, 0.2, 0.3, 0.5, 0.6) (0.3, 0.4, 0.5, 0.7, 0.8)

- (0.5, 0.5, 0.6, 0.7, 0.9) (0.3, 0.4, 0.5, 0.6, 0.6) (0.2, 0.2, 0.3, 0.5, 0.8)

= (-0.5, -0.2, 0, 0.1, 0.2) (-0.4, -0.4, -0.2, 0.1, 0.3) (-0.5, -0.1, 0.2, 0.5, 0.6)

S((-0.5, -0.2, 0, 0.1, 0.2) (-0.4, -0.4, -0.2, 0.1, 0.3) (-0.5, -0.1, 0.2, 0.5, 0.6))

$= \frac{1}{3}\left(2 + (-0.08) - (0.12) - (0.14)\right) = 0.6333$

f(2) = e_{25} – f(5)

= (0.5, 0.6, 0.7, 0.7, 0.8) (0.1, 0.3, 0.3, 0.4, 0.7) (0.4, 0.4, 0.6, 0.8, 0.8)

- (0.6, 0.7, 0.8, 0.8, 0.9) (0.2, 0.5, 0.5, 0.6, 0.8) (0.1, 0.3, 0.4, 0.4, 0.7)

= (-0.4,- 0.2, -0.1, 0, 0.2) (-0.7, -0.3,- 0.2, -0.1, 0.5) (- 0.3, 0, 0.2, 0.5, 0.7)

S((-0.4,- 0.2, -0.1, 0, 0.2) (-0.7, -0.3,- 0.2, -0.1, 0.5) (- 0.3, 0, 0.2, 0.5, 0.7))

$= \frac{1}{3}\left(2 + (-0.1) - (0.1) - (0.22)\right) = 0.5933$

f(1) = min (e_{12} – f(2), e_{13} – f(3), e_{14} – f(4))

e_{12} – f(2) = (0.5, 0.6, 0.6 ,0.8, 0.9) (0.4, 0.5, 0.5, 0.7, 0.8) (0.3, 0.4, 0.5, 0.5, 0.7)

- (-0.4,- 0.2, -0.1, 0, 0.2) (-0.7, -0.3,- 0.2, -0.1, 0.5) (- 0.3, 0, 0.2, 0.5, 0.7)

= (0.3, 0.6, 0.7,1.0, 1.3) (-0.1, 0.6, 0.7, 1.0, 1.5) (-0.4, -0.1, 0.3, 0.5, 1.0)

S((0.3, 0.6, 0.7,1.0, 1.3) (-0.1, 0.6, 0.7, 1.0, 1.5) (-0.4, -0.1, 0.3, 0.5, 1.0))

$= \frac{1}{3}\left(2 + (0.78) - (0.74) - (0.26)\right) = 0.5933$

e_{13} – f(3) = (0.4, 0.5, 0.7, 0.8, 0.8) (0.3, 0.4, 0.4, 0.6, 0.8) (0.3, 0.4, 0.5, 0.5, 0.7)

-(-0.5, -0.2, 0, 0.1, 0.2) (-0.4, -0.4, -0.2, 0.1, 0.3) (-0.5, -0.1, 0.2, 0.5, 0.6)

= (0.2, 0.4, 0.7, 1.0, 1.3) (0, 0.3, 0.6,1.0, 1.2) (- 0.4, -0.2, 0.1, 0.5, 1.1)

S((0.2, 0.4, 0.7, 1.0, 1.3) (0, 0.3, 0.6,1.0, 1.2) (- 0.4, -0.2, 0.1, 0.5, 1.1))

$= \frac{1}{3}\left(2 + (0.72) - (0.62) - (0.22)\right) = 0.626$

e_{14} – f(4) = (0.6, 0.6, 0.7, 0.7, 0.9) (0.2, 0.4, 0.5, 0.6, 0.6) (0.3, 0.4, 0.6, 0.7, 0.9)

- (-0.4, 0, 0.1, 0.3, 0.4) (-0.3, -0.2, -0.1, 0.2, 0.4) (-0.6, -0.2, 0, 0.2, 0.4)

= (0.2, 0.3, 0.8, 0.7, 1.3) (-0.2, 0.2, 0.6, 0.8, 0.9) (- 0.1, 0.2, 0.6, 0.9, 1.5)

S((0.2, 0.3, 0.8, 0.7, 1.3) (-0.2, 0.2, 0.6, 0.8, 0.9) (- 0.1, 0.2, 0.6, 0.9, 1.5))

$= \frac{1}{3}\left(2 + (2.26) - (0.46) - (0.62)\right) = 1.06$

f(1) = min (0.5933, 0.626, 1.06)

= 0.5933

= e_{12} – f(2)

= e_{12} – e_{25} – e_{57}

Therefore the shortest path is 1→ 2 →5→ 7

Results:

From the above method the shortest path is calculated for the pentagonal neutrosophic fuzzy number. By using this new algorithm the shortest path remains to be the same when compared with the existing algorithm.

Conclusions:

In neutrosophic set the indeterminacy can be represented along with uncertainty since it is a generalization of intuitionistic set. Here using the score function the shortest path problem is constructed in PNN environment. An algorithm is demonstrated to calculate the shortest path and it was merged with real life problems. Further approach can be extended in neutrosophic sets in various domains such as medical diagnosis, and many indeterminacy conditions, etc.

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