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Spin Dependent Steady State and Transient Gain Characteristics of Stimulated Brillouin Scattering in Magnetized Quantum Plasma

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ABSTRACT

The steady state stability and the transient gain features of stimulated Brillouin gain in the semiconductor plasmas are examined whenever an expanded quantum magnetohydrodynamic formula is considered. The model includes primeval quantum corrections including the Bohm potential, and also spin-generated effects of magnetization to better explain the electronic fluid behavior under excitation of the electromagnetic field. This is because the third order nonlinear susceptibility is responsible in the amplification mechanism known as the Brillouin amplification mechanism and therefore this is as a result of the nonlinear current density and electrostrictive coupling of the plasma medium. Our analysis demonstrates that both spin polarization and quantum corrections significantly alter the SBS gain dynamics. Notably, the spin effect enhances the Brillouin gain profile and leads to a substantial reduction in the threshold pump intensity, thereby improving the efficiency of SBS generation. These results underscore the critical role of spin dynamics in tailoring nonlinear optical responses in semiconductor plasmas and offer valuable insights for the development of spin-dependent photonic systems, plasma-based amplifiers, and quantum sensing technologies.

1. INTRODUCTION

The interaction of intense electromagnetic fields with dense media has led to the emergence of nonlinear optical phenomena such as Stimulated Brillouin Scattering (SBS), a process wherein a monochromatic optical wave couples with an induced acoustic wave through the mechanism of optical electrostriction. This interaction facilitates the generation and amplification of coherent optical radiation with fine spectral tunability and has been extensively studied in plasmas and condensed matter systems [1-3]. SBS has become a vital technique in photonics, offering applications ranging from laser frequency stabilization and signal processing to high-resolution spectroscopy and optical phase conjugation [4-5].

In recent years, the integration of quantum mechanical effects into classical plasma models has opened new avenues for understanding plasma dynamics at nanoscales. Notably, the role of the electron's intrinsic spin [6], once considered negligible in many plasma systems, has gained attention for its significant impact on wave dispersion, stability, and transport properties. Electron spin effects have not only enriched the theoretical framework of plasma physics but also inspired emerging applications in spintronics, quantum computation, and spin-based diagnostic techniques [7,8]. The spin of charged particles introduces additional magnetic moment interactions, leading to spin-current coupling and spin-induced forces, which are particularly relevant in magnetized environments such as astrophysical plasmas and semiconductor-based quantum plasmas.

To rigorously account for these quantum spin effects, the Quantum Magnetohydrodynamic model has been developed [9]. This model extends the classical Magnetohydrodynamic (MHD) framework by incorporating quantum statistical pressure or Fermi pressure, Bohm quantum potential (accounting for quantum diffraction), and spin-induced magnetization effects via the Pauli equation formalism.

The QMHD model thus provides a robust platform to study collective plasma behaviors influenced by both quantum mechanics and magnetism [10].

In semiconductor plasmas, particularly in piezoelectric crystals, quantum mechanical effects become increasingly significant due to the reduced carrier density, small effective masses, and the ability to confine carriers in nanostructured geometries [11]. These factors enhance the de Broglie wavelength of carriers, making quantum tunneling, nonlocality, and spin interactions essential to consider [12-13]. Under high magnetic fields or near cyclotron resonance, these systems exhibit enhanced nonlinearities, enabling experimental access to regimes where classical models fall short.

Despite the extensive literature on SBS in classical and quantum plasmas, the combined influence of quantum spin dynamics on steady-state and transient Brillouin gain characteristics has remained relatively unexplored. In response to this shortcoming, the proposed work examines, to the best of our knowledge, the spin induced quantum corrections on SBS in magnetized semiconductor plasmas using the QMHD model. We analytically derive the modified Brillouin gain profiles by incorporating spin magnetization forces and quantum potentials. Our results reveal that spin effects not only enhance Brillouin gain constants but also lower the threshold pump intensity for SBS excitation.

This manuscript is structured as follows: Section 2 outlines the theoretical foundation based on the QMHD model and derives the governing equations relevant to stimulated Brillouin scattering (SBS) in spin-influenced semiconductor plasmas. The subsections 2.1 and 2.2 detail the steady-state and transient gain characteristics, respectively, while Section 2.3 elaborates on the nonlinear structure of the model and its broader implications. Section 3 presents numerical simulations carried out for an n-InSb semiconductor crystal subjected to pulsed CO_2 laser excitation, highlighting the influence of spin polarization and quantum effects. Finally, Section 4 summarizes the principal findings and discusses their relevance to future developments in quantum plasma technologies and spin-sensitive optical applications.

2. THEORETICAL FORMULATIONS

Here in this section, we have talked about the field theoretical expression of the third order nonlinear optical susceptibility $\chi_B^{(3)}$of the Stokes part of the scattered electromagnetic wave of the doped semiconductor QMHD model. We have chosen a model which is the magneto hydrodynamical model of a homogenous one component (electron) plasma in thermodynamic equilibrium and that satisfies the condition $k_a l << 1$ (k_a is acoustic wave vector and l is the average distance which electrons move between collisions). The implication that follows this assumption is that the sound wavelength is very long relative to the mean free path of the electrons in the structure the carrier motion dictated with the external fields will smooth out. It also empowers us to disregard high frequency electric field non uniformity in dipole approximation [14]. To obtain third order Brillouin susceptibility induced by the induced polarization and electrostriction, the incident pump radiation $E_0(x,t) = \hat{z}E_0\exp[i(k_0 x - \omega_0 t)]$ is expected to be travelling along x direction and is polarized along z direction. The longitudinal polarized acoustic wave $\vec{u}(x,t) = \vec{u}_0\exp[i(k_a x - \omega_a t)]$ is also assumed to be travelling along x direction. The Brillouin back scattered Stoke wave $E_1(x,t) = \hat{z}E_1\exp[i(-k_1 x - \omega_1 t)]$ is traveling in -x direction and is polarized along z direction. So that SBS can be investigated within a medium, the phase matching criteria that must be achieved in the present case are: $\hbar\omega_0 = \hbar\omega_1 + \hbar\omega_a$ and $\hbar k_0 = \hbar k_1 + \hbar k_a$. These conditions provide $\omega_1 = \omega_0 - \omega_a$ and $k_a = 2k_s$ (since $|k_0| \approx |k_1|$). Since the crystal is supposed to be centro-symmetric, the impact of any pseudo-potential can be ignored in order to simplify the analysis.

In these conditions when the density is considerably high and the plasma cooled into the quite low level of temperature, then the ultracold plasma will serve as the degenerate fermion gas and the quantum effects will be rather significant in dynamics of the charged particles [15]. For the appreciation of the above fact for this problem let us calculate the T_F in terms of carrier density ($n_0 =$

$3 \times 10^{24} m^{-3}$) through standard formula. It comes out to for InSb. We have considered the lattice temperature as 77 K, hence here T<<T$_F$ $(6.27 \times 10^3 K)$ which defines a fully degenerate quantum regime.

To make the discussions definite we suppose that the pressure law in one dimensional Fermi gas is satisfied by the plasma particles [16]

$$\boldsymbol{P} = \frac{m{V_{F\alpha}}^2 {n_{1\alpha}}^{5/3}}{5{n_{0\alpha}}^{2/3}} \qquad (1)$$

Where P stands for Fermi pressure with $V_{F\alpha j} = \sqrt{\xi_{3D}\hbar\left({n_{0\alpha}}^{1/3}\right)/m}$ is the Fermi velocity. ξ_{3D} is degree of spin polarization given by $\xi_{3D} = \left[(1-\eta)^{5/3} + (1+\eta)^{5/3}\right]/2$ with spin polarization (η) defined by $\eta = |n_{\uparrow} - n_{\downarrow}|/n_{\uparrow} + n_{\downarrow}$, ↑ and ↓ denotes the electrons in the spin up and spin down states correspondingly [17]. The quantum force F_Q on an electron in Eq. (1) includes two terms. The first term is the Bohm potential of quantum diffusion of electrons, and the other one is the energy of spin magnetization due to spin interaction with magnetic field. Here S_α is the spin of species α, $\alpha = \uparrow$ and $\downarrow$ denotes the electrons in the spin up and spin down states correspondingly. $n_{1\alpha}$ is the perturb number density of species α. It is an established fact that the quantum effects have no impacts on the transverse electromagnetic wave which is linearly polarized.

According to Guha et al. [18] and Manfredi [19], the other fundamental equations that are used are

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho}\frac{\partial^2 u}{\partial x^2} + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{\gamma}{2\rho}\frac{\partial}{\partial x}\left(E_0 E_{1x}^*\right) \qquad (2)$$

$$\frac{\partial v_0}{\partial t} + \nu v_0 = \frac{e}{m} E_0 \qquad (3)$$

$$\frac{\partial v_1}{\partial t} + \nu v_1 + v_1\left(v_0 \frac{\partial}{\partial x}\right) = -\frac{e}{m}\left(E_1 + v_1 \times B_1\right) - \frac{1}{m n_0}\frac{\partial P}{\partial x} + F_Q \qquad (4)$$

Where, $F_Q = \frac{2\mu_B}{\hbar m}\nabla(B_1 . S_\alpha) + \frac{\hbar^2}{4m^2 n}\frac{\partial^3 n_1}{\partial x^3}$

$$v_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial v_1}{\partial x} + \frac{\partial n_1}{\partial t} = 0 \qquad (5)$$

$$P_{es} = -\gamma E_0 \frac{\partial u^*}{\partial x} \qquad (6)$$

$$\frac{\partial E_1}{\partial x} = \frac{n_1 e}{\varepsilon} + \frac{\gamma}{\varepsilon} E_0 \frac{\partial u^*}{\partial x} \qquad (7)$$

The quantum force F_Q of an electron in Eq. (4) includes two terms: The former one is the Bohm potential as the energy that causes the quantum diffusion of the electrons and the energy of the spin magnetization as the outcome of the energy of spin magnetization due to spin interaction with magnetic field. Here S_α is the spin of species α, $\alpha = \uparrow$ and $\downarrow$ denotes the spin up and the spin down electrons respectively with $\mu = -\frac{g\mu_B}{2}, \mu_B = \frac{e\hbar}{2m}, g = 2.0023192$ is the electron g factor, $n_{1\alpha}$ is the perturb number density of species α. The quantum effects are unsurprisingly known to possess no effects on the transverse electromagnetic linearly polarised wave. The term proportional to $\hbar^2$ in Eq. (4), accounts the quantum diffraction represented by the Planck's constant where $\hbar$ is the Planck's constant divided by the 2π. The role of this term can also be perceived in the other way round; that they can also be described as the quantum pressure term or as the quantum Bohm potential [20]. In the other applications to semiconductor physics, the Bohm potential leads to tunnelling and contributions to differential resistances [21]. The model has incorporated quantum statistics, and it is incorporated by the equation of state (Eq. (1)) that takes into consideration the Fermionic nature of the electrons. Eq. (2) represents the dynamic of the lattice in the crystal, where ρ is the mass density of the crystal, u is the displacement of the lattice, Γ_a is the phenomenological damping factor of acoustic mode, C the elastic constant, γ the electrostriction coefficient of the crystal respectively. The conservative term at the right-hand side of Eq. (2) is proportional to the electrostrictive force, which is generated by the pump electric field through electrostriction. Eqs. (3) and (4) are the zeroth and first order oscillatory velocities of the fluid of an electron of constant mass m and charge e. ν is the phenomenological frequency of an electronic collision. In Eq. (3) we have neglected the pump magnetic field by assuming $\omega_p \approx \omega_c \approx \omega_0$. The quantum correction on the Eq. (4) manifests itself in terms of Fermi temperature and third term on the right hand-side. The conservation of charge is embodied by the continuity Eq. (5). Eq. (6) show that as a direct consequence of varying electronic strain there is a corresponding change in the dielectric constant of a material because the modulated or varying electrostrictive strain then leads to an electrostrictive induced polarizationP_{es}. When the frequency of the field is very high and therefore very high in comparison with the frequencies with which we have become familiar as related to the movement of electrons in the medium, we may say that the polarizability of the medium is negligible interaction of the electrons with each other and the nuclei of the atoms of the medium. The space charge field $E_1$is determined by Poisson Eq. (7) where ε is dielectric constant of semiconductor.

The electro strictive force causes the carrier density perturbation within the Brillouin active medium. This density perturbation is accessible in a doped semiconductor in the normal way [22] as

$$\frac{\partial^2 n_{1\alpha}}{\partial t^2} + \bar{\omega}^2{}_p n_{1\alpha} + n_0\left(\frac{2\mu_B}{\hbar m}\frac{ik^2\bar{E}}{\omega_0}S_{\alpha 0} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2}\right) + \nu\frac{\partial n_{1\alpha}}{\partial t} = \ \bar{E}\frac{\partial n_{1\alpha}}{\partial x} \tag{8}$$

Where $\bar{E} = \frac{e}{m}E_0(1 + \frac{\omega_c(\nu - i\omega_0)}{(\omega_c^2 - \omega_0^2 - 2i\omega_0\nu)})$, $\varpi_p^2 = \omega_p^2 + k^2 V_F'^2$, $V_F' = V_F\sqrt{1+\gamma_e}$, $\gamma_e = \frac{\hbar^2 k^2}{8mK_B T_F}$

In deriving of Eq. (8) we have neglected the Doppler shift under the assumption that $\omega_0 >> \nu > kv_0$; $\omega_p = \left(\frac{n_0 e^2}{m\varepsilon}\right)^{1/2}$ is the plasma frequency. It is obvious that the second term on left hand side of Eq. (8) has the composite effect of quantum correction and Fermi dispersion.

The perturbed concentration electron n_1 will consist of two parts which can be separated as slow and fast ($n_1 = n_{1s} + n_{1f}$). It is assumed that the slow part n_{1s} is related to the low frequency acoustic wave (ω_a), whereas the fast part n_{1f} oscillates only at the electromagnetic waves ($\omega_0 \pm \omega_a$). The higher order terms with frequencies $\omega_0 \pm p\omega_a (p = 2,3,4, \ldots)$ being off resonant are neglected. In this case we have taken the energy of the photons ($\hbar\omega_1$) slightly below the band gap energy ($\hbar\omega_g$); this approximation enables the optical energy to be treated in terms of a small perturbation. 7 characteristics of the sample to be altered significantly by the free charge carriers and not to be altered

The quantum magnetohydrodynamic is used to model the stimulated Brillouin scattering (SBS) process in this work, where quantum pressure, spin effects, and electrostrictive feedback are incorporated. The resultant system is a coupled non-linear system of governing equations. The present formulation is highly applicable in applied nonlinear analysis because of its highly structured nature and sensitivity to parameters.

2.1. Steady-state characteristics

On combining Eq. (8) we have the following coupled equations. Regarding rotating wave approximation

$$\frac{\partial^2 n_{1s}}{\partial t^2} + \nu\frac{\partial n_{1s}}{\partial t} + \varpi_p^2 n_{1s} = -\overline{E}\frac{\partial n_{1f}^*}{\partial x} \tag{9}$$

and

$$\frac{\partial^2 n_{1f}}{\partial t^2} + \varpi_p^2 n_{1f} + \nu\frac{\partial n_{1f}}{\partial t} - n_0\left(\frac{2\mu_B}{\hbar m}\frac{ik^2\bar{E}}{\omega_0}S_{\alpha 0} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2}\right) = -\overline{E}\frac{\partial n_{1s}^*}{\partial x} \tag{10}$$

subscripts s and f represents slow and fast components respectively. Asterisk (*) is used to indicate the complex conjugate of the quantities.

Based on Eqs. (9) and (10) that the slowly and rapidly varying components of the density perturbed are locked together by the pumping electric field. It thus becomes noticeable that the existence of the pump field forms the irreducible necessity of the SBS occurrence to take place.

Using the above equations, we obtain

$$n_{1s} = n_0 \left(\frac{2\mu_B}{\hbar m} \frac{ik^2 \bar{E}}{\omega_0} S_{\alpha 0} + \frac{e\beta}{m\varepsilon} \frac{\partial^2 u}{\partial x^2} \right) \left[1 - \frac{(\delta_1^2 - i\omega_a \nu)(\delta_2^2 + i\omega_1 \nu)}{k^2 |\bar{E}|^2} \right]^{-1} E_0^2 E_1(\omega_1) \qquad (11)$$

where $v_a = \sqrt{C/\rho}$ is the acoustic velocity in the medium. $\delta_1^2 = \varpi_p^2 - \omega_a^2$ and $\delta_2^2 = \varpi_p^2 - \omega_1^2$.

As can be read in the above expression (11), n_{1s} is based on magnitude of the pump intensity (I_{in}), where $I_{in} = \frac{1}{2}\eta\varepsilon_0 c|E_0|^2$ with η and c being the refractive index of background of the crystal and velocity of light respectively. These induced density anguish affect the propagation aspect of the developed waves. The induced current density has a resonance on the Stoke component which is always given as

$$J_1(\omega_1) = n_0 e v_1 + n_{1s}^* e v_0 \qquad (12)$$

Which on using eq (11) may be obtained as

$$J_1(\omega_1) = \frac{i\varepsilon\varpi_p^2 \bar{E} + n_1 e k^2 V_F'^2}{\omega_1} - n_0 \left(\frac{2\mu_B}{\hbar m} \frac{ik^2 \bar{E}}{\omega_0} S_{\alpha 0} + \frac{e\beta}{m\varepsilon} \frac{\partial^2 u}{\partial x^2} \right) \left[1 - \frac{(\delta_1^2 + i\omega_a \nu)(\delta_2^2 - i\omega_a \nu)}{k^2 |\bar{E}|^2} \right]^{-1} E_0^2 E_1(\omega_1)$$
(13)

The first term at the right-hand side of above expression is linear term of produced current density. The second term holds the non-linear density current caused by the interaction of the three waves which interact with each other. In deriving Eq. (13) the contribution of the velocities of the oscillatory electron fluid in the presence of the pump and the disturbed fields can be obtained out of following Eqs. (3) and (4).

Now with the induced polarization P_{cd} as time integral of induced nonlinear current density $J_{nl}(\omega_1)$, using eq. (13) we may obtain the following relation

$$P_{cd}(\omega_1) = \left[\frac{\varepsilon\omega_P{}^2 (\nu - i\omega_0)}{(\omega_c{}^2 - \omega_0{}^2 - 2i\omega_0 \nu)} \right] \left(\frac{2\mu_B}{\hbar m} \frac{ik^2 \bar{E}}{\omega_0} S_{\alpha 0} + \frac{e\beta}{m\varepsilon} \frac{\partial^2 u}{\partial x^2} \right) \left[1 - \frac{(\delta_1^2 + i\omega_a \nu)(\delta_2^2 - i\omega_1 \nu)}{k^2 |\bar{E}|^2} \right]^{-1} E_0^2 E_1(\omega_1)$$
(14)

As we well know, the SBS process originates in that part of $P_{cd}(\omega_1)$ which is proportional to $E_0^2 E_1$, with that third-order susceptibility being the Brillouin susceptibility $\chi_B^{(3)}$.

Now the induced polarization at frequency ω_1 may also be defined as

$$P_{cd}(\omega_1) = \varepsilon_0 \left(\chi_B^{(3)} \right)_{cd} E_0^2 E_1(\omega_1) \qquad (15)$$

Using Eqs. (14) and (15) the Brillouin susceptibility with quantum correction becomes

$$\left(\chi_B^{(3)} \right)_{cd} = \left[\frac{\varepsilon\omega_P{}^2 (\nu - i\omega_0)}{(\omega_c{}^2 - \omega_0{}^2 - 2i\omega_0 \nu)} \right] \left(\frac{2\mu_B}{\hbar m} \frac{ik^2 E}{\omega_0} S_{\alpha 0} + \frac{e\beta}{m\varepsilon} \frac{\partial^2 u}{\partial x^2} \right) \left[1 - \frac{(\delta_1^2 + i\omega_a \nu)(\delta_2^2 - i\omega_1 \nu)}{k^2 |\bar{E}|^2} \right]^{-1} \qquad (16)$$

Equation (15) actually represents the intensity dependent Brillouin susceptibility of the medium. One may infer from it that $\left(\chi_B^{(3)} \right)_{cd}$ is dependent upon material parameters, including equilibrium carrier density n0 through the electron plasma frequency ω_p.

The Brillouin susceptibility in Equation (16) contains nonlinear dependencies on the spin polarization η, carrier density n, and field intensity E_0. These nonlinearities directly influence the threshold behavior and gain properties of the medium.

He-Liu [23] reported that the nonzero induced polarization directly gives the onset of SBS. Hence following He-Liu and others, from the Eq. (14) we can find out what the character of the threshold leading to the occurrence of SBS by setting$P_{cd}(\omega_1) = 0$as. This threshold corresponds to the vanishing nonlinear polarization condition defined in Eq. (17).

$$|E_{oth}| = \frac{m}{ek}\sqrt{(\delta_1^2 + i\omega_a \nu)(\delta_2^2 - i\omega_1 \nu)} \quad (17)$$

The threshold pump field as the SBS starts is highly perturbed by the quantum correction with δ_1^2 and δ_2^2. So the relations inside the pump and centrosymmetric crystal will be determined by the mechanisms of stimulated Brillouin scattering at the power level of the pump which is many times exceeds the power level of the threshold field E_{oth} .So the acoustical and the scattered optical beam are released in fixed directions, and can only be created above E_{oth} but in the common dielectric media, the effect of beam trapping causes the intensities to rise above the critical threshold of SBS at relatively modest input powers. Consequently, the threshold experimentally measured in majority of semiconductor materials encompasses beam trapping [24].

The electrostrictive strain interacts with the pump wave into the Brillouin active media generating an electrostrictive polarization $P_{es}(\omega_1)$. Therefore, in addition to the induced polarization that is not linear, because of the disturbed current density, the system ought to have electrostrictive polarization. This electrostrictive Polarization $P_{es}(\omega_1)$ is derived as one of the applications from the following equations (6) and (7) as

$$P_{es}(\omega_1) = \frac{-\gamma^2 k^2 E_0^2 E_1(\omega_1)}{2\rho(\omega_a^2 - k^2 v_a^2 - 2i\Gamma_a \omega_a)} = \varepsilon_0 \left(\chi_B^{(3)}\right)_{es} E_0^2 E_1(\omega_1) \quad (18)$$

An induced nonlinear polarization per unit volume is proportional to $E_0^2 E_1(\omega_1)$ n a centrosymmetric crystal doped, in which the electrostrictive terms contributing to the nonzero coupling to the square power are only of second order is:

$$P_{nl}(\omega_1) = P_{es}(\omega_1) + P_{cd}(\omega_1)$$

$$= E_0^2 E_1(\omega_1)\left[\frac{-\gamma^2 k^2}{2\rho(\omega_a^2 - k^2 v_a^2 - 2i\Gamma_a \omega_a)} + \frac{\varepsilon\omega_P{}^2(\nu - i\omega_0)}{(\omega_c{}^2 - \omega_0{}^2 - 2i\omega_0\nu)}\right]\left(\frac{2\mu_B}{\hbar m}\frac{ik^2 E}{\omega_0}S_{\alpha 0} + \frac{e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2}\right)\left[1 - \frac{(\delta_1^2 + i\omega_a\nu)(\delta_2^2 - i\omega_1\nu)}{k^2|\bar{E}|^2}\right]^{-1} = \varepsilon_0 \chi_B^{(3)} E_0^2 E_1(\omega_1) \quad (19)$$

Hence the total third order Brillouin susceptibility can be obtained from Eq. (19) as

$$\left(\chi_B^{(3)}\right) = \left[\frac{-\gamma^2 k^2}{2\rho(\omega_a^2 - k^2 v_a^2 - 2i\Gamma_a \omega_a)} + \frac{\varepsilon\omega_P{}^2(\nu - i\omega_0)}{(\omega_c{}^2 - \omega_0{}^2 - 2i\omega_0\nu)}\right]\left(\frac{2\mu_B}{\hbar m}\frac{ik^2 E}{\omega_0}S_{\alpha 0} + \frac{e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2}\right)\left[1 - \frac{(\delta_1^2 + i\omega_a\nu)(\delta_2^2 - i\omega_1\nu)}{k^2|\bar{E}|^2}\right]^{-1} \quad (20)$$

where $\chi_B^{(3)} = \chi_{Br}^{(3)} + \chi_{Bi}^{(3)}$, the quantities are written with subscripts r and i, hence signifying real and imaginary parts respectively.

The primary aim of the present article is to take into consideration the sensitivity of the threshold power SBS and the gain coefficient of the backward scattered mode Brillouin g_B In doing so, the following expression [25] is used

$$g_B = -\frac{k}{2\varepsilon_1}\left[\chi_{Bi}{}^{(3)}\right] \; |E_0|^2 \quad (21)$$

The expression of steady state Brillouin gain given above can further be written in terms of the input pump power when considering centrosymmetric semiconductor plasma as

$$g_B = 1.54 \times 10^{-7} I_{in} \tag{22}$$

The relevant physical parameters are given in section 3. Based on the above equations, one can see that quantum effect has significant effect on the third-order nonlinearity of the medium and subsequently the steady state properties of the Brillouin active medium.

Table 1 shows the list of key physical and mathematical symbols used throughout the manuscript, along with their corresponding descriptions.

Table 1. Definitions of key symbols used in the paper.

Symbol	Description
η	Spin polarization parameter
n_0, n_1	Equilibrium and perturbed carrier density, respectively
$\mathbf{v}_0$, $\mathbf{v}_1$	Equilibrium and oscillatory (perturbed) electron fluid velocities
E_0, E_1	Pump (incident) electric field and scattered (Stokes) electric field
ω_0, ω_1	Frequencies of the pump and scattered electromagnetic waves
$\mathbf{k}$	Wave vector associated with propagating fields
$\chi^{(3)}$	General third-order nonlinear susceptibility
$\chi_B^{(3)}$	Third-order Brillouin susceptibility
g_B	Brillouin gain coefficient
τ_{opt}	Optimum pulse duration for maximum transient gain
P_{cd}	Induced nonlinear polarization at the scattered frequency
ν	Electron collision frequency (damping rate)
ω_p	Electron plasma frequency
μ_B	Bohr magneton (magnetic moment of the electron)
δ_1, δ_2	Detuning parameters used in the gain model
u	Longitudinal displacement of the ion acoustic wave
β	Pressure-related coupling parameter in the quantum force model

2.2. Transient characteristics

This section gives the dynamics of SBS i.e. the time behavior of the Stoke wave intensity. In the macroscopic perspective, the transient coherent phenomena are caused by the ability of the material system in the capacity of the ability of retaining the certain phase of a coherent excitation over certain period of time. Such effects have transient features that determine their speed in which different types of optical functions are executed. From Eqs. (21) and (22), it can be assumed that only high-power laser source exists that will produce considerable gain of the SBS mode. Therefore, the laser pump source must be in the pulsed mode with a time range of the order of 10^{-12} s or in the pulse-train mode with a pulse duration of the order of 10^{-9} s in the case of Q Switched lasers and mode locked lasers, respectively. These durations of time are either comparable to or smaller than the phonon life times (for acoustic phonon $\geq 10^{-9}$ s); and thus, the transient effect study comes to the fore. Conversely the steady state formulations are quite handicapped both in their ability to correctly predict the threshold pump intensity (I_{th}) at which SBS will start to occur with positive gains as well as in their ability to correctly predict the optimum pulse duration over which such instabilities may be observed. These will indicate that SBS has to be studied with the inclusion of transient effect. Overall, according to [26] the transient gain factors are a linear combination of the steady-state gain coefficients with the following relation

$$g_{TB} = \left[2g_B x \Gamma_B \tau_p\right]^{1/2} - \Gamma_B \tau_p \tag{23}$$

where Γ_B is the acoustic phonon lifetime, and x is interaction length, τ_p is pulse duration. In case of an extremely short pulse duration ($\tau_p \leq 10^{-10} s$) the interaction length can be substituted by $(c_l \tau_p/2)$ where $c_l$is the velocity of light in crystal lattice.

2.2.1. Threshold pump intensity and optimum pulse duration

By making $g_B = 0$ in the Eq. (23) we are able to get the threshold pump intensity of the initiation of SBS as

$$I_{th} = \frac{\Gamma_B \tau_p}{2 G_B c_l} \tag{24}$$

with $G_B = \frac{g_B}{I_{in}}$, the gain per unit pump intensity.

The threshold condition under which SBS will occur is at the point corresponding to Brillouin gain coefficient g_B becomes zero. One of the key factors influencing this threshold is the spin polarization η. As shown in Figure 1, an increase in spin polarization results in a nonlinear decrease in the required threshold pump intensity. This suggests that spin-polarized systems allow for SBS to initiate at significantly lower energy inputs, which is beneficial for low-power device applications.

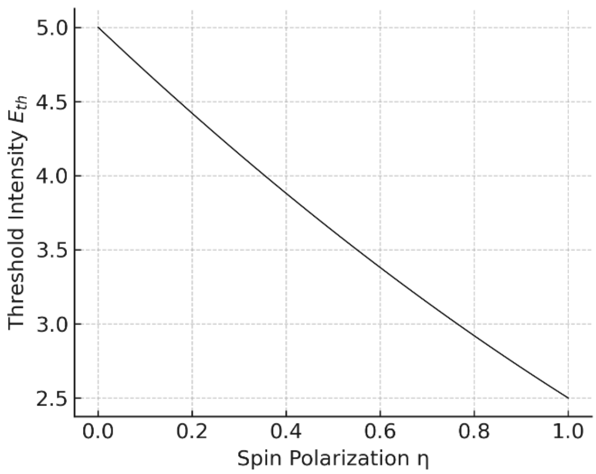

Figure 1: Threshold intensity vs spin polarization

But, in the case of the relatively long pulse duration ($\tau_p \geq 10^{-9} s$), the cell length can be considered equal to x and under such circumstances, one finds

$$g_{TB} = \left(\Gamma_B \tau_p\right)^{1/2} \left[-\left(\Gamma_B \tau_p\right)^{1/2} + \left[g_B \quad x \quad\right)^{1/2}\right] \qquad (25)$$

With the help of the above equation, we can get the impression of optimum pulse duration $\left(\tau_p\right)_{opt}$beyond which no gain can be attained by equating g_{TB} to zero as

$$\left(\tau_p\right)_{opt} \approx \left(\frac{g_B\, x}{\Gamma_B}\right) \qquad (26)$$

It can be noted that reversible gain characteristics of the Brillouin scattered mode are suggested to be modified by the quantum terms.

The optimum pulse duration τ opt is not fixed and varies with the pump intensity I. This relationship is nonlinear due to gain saturation and phonon response times in the plasma. As shown in Figure 2, τ opt τ opt increases slowly with I, suggesting a logarithmic-type behavior. This helps determine suitable pulse durations for transient SBS gain in spin-sensitive plasmas.

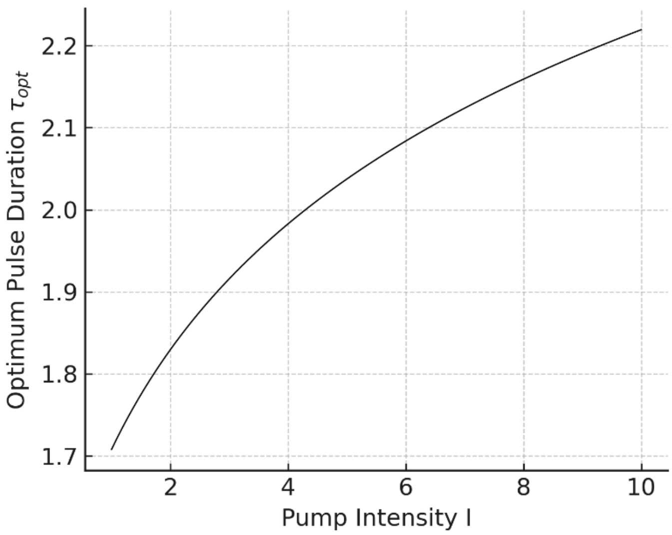

Figure 2: Optimum pulse duration vs pump intensity

2.3 Mathematical Structure and Nonlinear Properties

The QMHD model described above gives rise to set of coupled nonlinear partial differential equations characterizing the dynamics of the charge density perturbations, acoustic wave propagation and nonlinear induced polarization. The spin-polarized Fermi pressure adds cubic nonlinear terms, whereas the Bohm potential adds quantum corrections, which are functions of second derivatives of density. Such effects render the system analytically very rich and would be quite applicable to additional research in terms of bifurcation analysis, perturbation methods and theory of nonlinear stability.

3. RESULTS AND DISCUSSION

The given section is a study of the SBS gain behavior with the calculated QMHD model. The nonlinear dependence on wave vector, spin polarization, magnetic field, and carrier density is also illustrated in Figures 3 through 5 and the transient characteristics are illustrated in Figures 6 and 7. These findings show the efficacy of the model as a nonlinear dynamical system and indicates the contribution of quantum network interactions and of spins in changing the optical response.

To explain the validity of the model, an inward-looking numerical quest of the threshold condition and the gain behaviour of the SBS process with the addition of the quantum correction term and the use of the narrow-bandgap semiconductors has been presented in the present section. The above semiconductor bulk crystal is n-InSb that is a narrow direct gap semi-conductor which has gained zinc blende structure that is virtually a cube type of pattern. We have considered the irradiation of n-InSb medium by a pulsed 10.6 μm CO_2 laser at liquid nitrogen temperature (77 ^{0}K). Absorption coefficient of a sample under such a temperature is therefore very low and we can neglect the transition mechanism of band to band. Scattering of the electron in the acoustic phonon in InSb is the dominant mechanism of transferring the momentum and energy of the electron in the scattered state of the kind [27].

The representative values of the following material parameters have been taken into account to make up the theoretical formulation: $m = 0.015m_0$, $m_0$being the free electron mass, $\varepsilon_1 = 15.8$, $\gamma = 5 \times 10^{-10} Fm^{-1}$,$\rho = 5.8 \times 10^3 kgm^{-3}$, $\omega_1 = 2 \times 10^{11} s^{-1}$, $\omega_0 = 1.78 \times 10^{14} s^{-1}$, $\nu = 4 \times 10^{11} s^{-1}$.

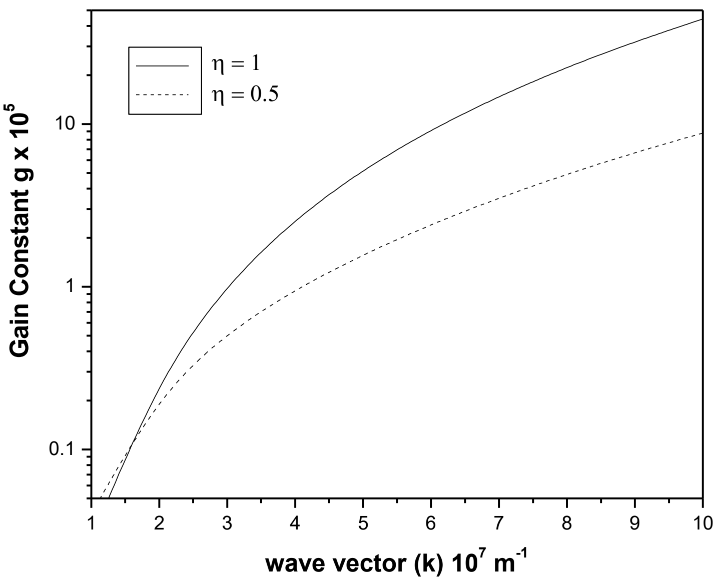

Figure 3: Variation of steady state gains with wave vector k at $n_0 = 3 \times 10^{24} m^{-3}$ and $E_0 = 8 \times 10^7 Vm^{-1}$.

In Figure 3, steady-state gain characteristics of the SBS are displayed including quantum spin effect. The solid, dashed line showed the variation for fully spin polarized η=1 and partially spin-polarized i.e η=0.5 respectively. In this case, gain increases with increasing spin-polarization. It is also observed that the nature of both the curve is same and increases with increasing value of wave vector k. This graph demonstrates how changes in wave vector k directly impact the gain constant in the system.

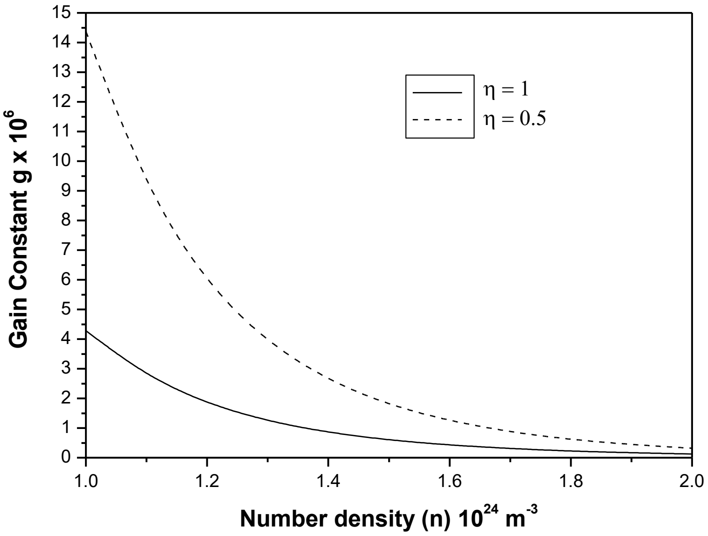

Figure 4: Variation of steady state gains with number density n_0 at $k = 3 \times 10^8 m^{-1}$ and $E_0 = 8 \times 10^7 V m^{-1}$.

Figure 4 shows how g_B varies as a function of free carrier density n_0. The steady state SBS gain properties of the medium are found sensitive to the concentration of doping. It can be concluded that gain decreases parabolically as the carrier density rises

The SBS in partially spin polarized decreases linearly whereas fully spin polarized curve increases parabolically with free carrier density. It is found that $\eta = 0.5$ favorable in achieving the larger gain constants and therefore is beneficial in for the construction of SBS cell.

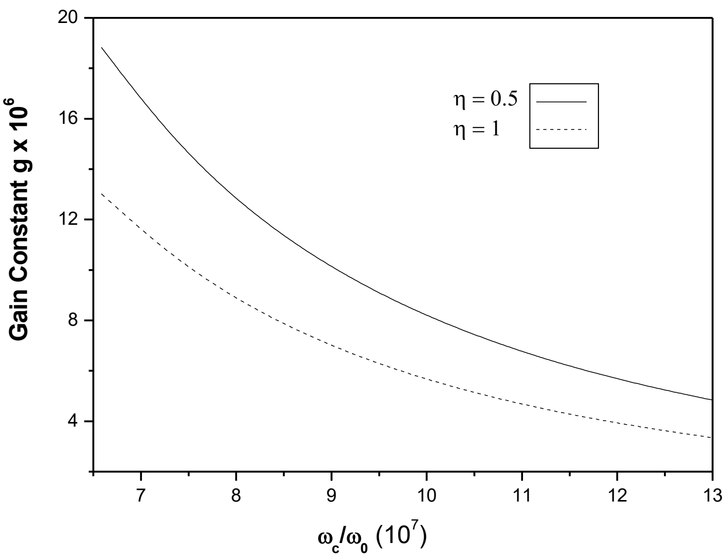

Figure 5: Variation of steady state gains with cyclotron frequency ω_c at $k = 3 \times 10^8 m^{-1}$ and $n_0 = 3 \times 10^{24} m^{-3}$.

Figure 5 shows how the Brillouin gain changes with cyclotron frequency ω_c that relies on the outer applied magnetic field. Spin is important in the exposure of the plasma to an external magnet field; this interact with the magnetization in the plasma because of the electron spin. The solid, dashed line showed the variation for fully polarized i.e η = 1 and partially polarized i.e η = 0.5 respectively. The nature of both curves is the same, but with the partially spin-polarized curve, medium reaching a higher susceptibility compared to the fully spin-polarized. The spin of the electrons is coupled with the direction of the external magnetic field in the magnetized plasmas in such a way that it increases the intensity of the external magnetic field. When this spin is aligned it results in enhancement of the overall magnetic characteristics of the plasma. Within the analysis the steady state SBS gain is determined to be of the order of 1.96×10^{-10} SI units while considering $s_{\uparrow 0} = -s_{\downarrow 0}$ and $n_\alpha = n_\uparrow - n_\downarrow = 3n\frac{\mu_B B_0}{2k_B T_F}$ with carrier density $n_0 = 10^{24} m^{-3}$.

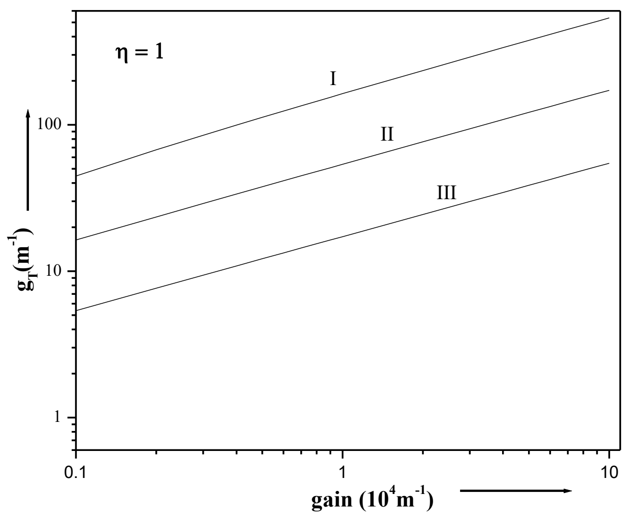

Figure 6: Variation of g_{TB} with $g_B$in SBS process at different pump pulse durations.

Figure 6 displays the rivalry between transient Brillouin gain and steady state Brillouin gain across various pulse durations. It can be seen from the graph that g_{TB} increases linearly with g_B at one specific pulse length of three different values for the pump. On the contrary, as the pump pulse is increased in duration, g_{TB} increases at a particular value of g_B. At $\tau_p \approx 10^{-2}/\Gamma_B$ and smaller values of g_B, the transient gain g_{TB} is found less than 1. But on increasing the steady state Brillouin gain, transient Brillouin gain becomes larger than 1 For any longer pulse duration g_{TB} is always larger than 1.

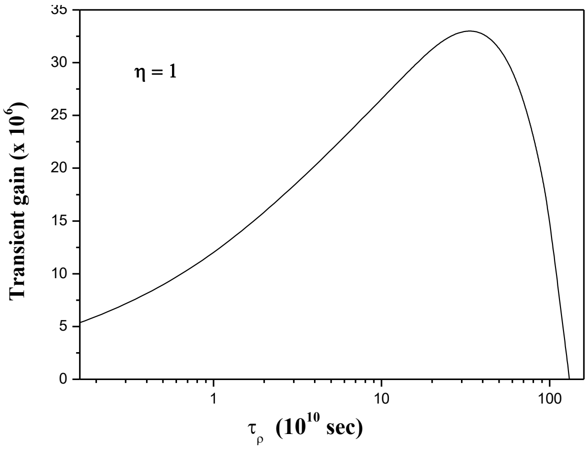

Figure 7: Variation of transient gain coefficients with pump pulse durationτ_p.

Figure 7 depicts the dynamical behaviour of transient gain factor with the pump pulse duration τ_p. The acoustic phonon lifetime $\approx 10^{-9}s$, hence, to draw this behavior, we have considered pulse duration in the range $10^{-12} \leq \tau_p \geq 10^{-9}$ s. For backward Brillouin mode the interaction length (that is the cell length) is $c_l\tau_p/2$ or x, whichever is shorter. For fixed I_{in}, $(g_{TB})_{QE}$ increases with rise in pulse duration and at a particular value of τ_p, $(g_{TB})_{QE}$ reaches an optimum. The maximum value stays almost similar to some range of τ_p. These zones may be considered both as quasi steady state or quasi saturation zones. When τ_p is further raised outside of quasi saturation regime g_{TB} decreases extremely rapidly and eventually falls to zero. This figure also indicates that the consideration of quantum effects causes the maximum gain point to shift to the larger value of τ_p and expands the τ_p range over which transient phenomena may be measured. Therefore, when examining the transient behaviour of Brillouin mode, it would be desirable to include terms which correct the quantum condition

From Eq. (26), one can get the numerical approximation of optimum pulse duration $(\tau_p)_{\text{opt}}$ for nearly centrosymmetric crystal ($\text{n} - \text{InSb}$) (using the values of g_{TB} and g_B obtained earlier and $x = 3.5 \times 10^{-5}m$) above which no gain is possible, as

$(\tau_p)_{opt} = 1.2 \times 10^{-10}s$ with spin effect

Resting on these values an inference can be drawn that the rise in magnitude of optimum pulse duration can be attributed to both quantum correction term and such that the optimum pulse duration can also be increased by increasing the pump intensity.

4. CONCLUSION

The implication of spin on the steady state and transient characteristics of a stimulated Brillouin scattering light on the semi-conductor plasma medium has been discussed in the present paper based on quantum magneto-hydrodynamic model which considers Fermi pressure, Bohm potential and spin terms. The analysis demonstrates that electron spin significantly influences plasma's response to an external magnetic field, altering its gain characteristics, thereby impacting electron transport and enhancing magnetic properties. The role of spin polarization, which reflects the distribution of unpaired electrons, is found to be critical in governing the magnetic behavior of the semiconductor plasma under such conditions. Furthermore, the transient Brillouin gain profile reveals that SBS develops efficiently for pulse durations shorter than the phonon lifetime, while it is suppressed for longer pulses. Notably, the quantum effects reduce the SBS threshold, allowing for substantial Brillouin gain at lower laser powers, which has practical implications in minimizing power requirements and reducing the cost of SBS-based device fabrication.

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