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Analysis of Flow of Blood Under the Influence of a Magnetic Field Using a Stenotic Multiple Artery Inclined with Suspended Silver Nanoparticles

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Abstract: A mathematical model is designed to study the continuous and incompressible flow of silver blood through an inclined artery with non uniform cross-section and multiple stenoses. The analysis considers the influence of an external magnetic field. The model studies how the stenosis height, Grashof number, heat source/sink parameter, magnetic field and inclination angle influence the resistance of flow and wall shear stress. The results have been displayed graphically and the impact of these parameters on the characteristics of arterial blood flow is explained and analyzed.

Key words: Multiple stenoses, silver nanoparticles, wall Shear Stress, Resistance to the flow.

Introduction:

Every year, heart disease becomes a major global health problem, ranking among the most serious medical problems on the planet. Understanding the dynamics of blood flow in different arterial geometries is essential to detecting and treating cardiovascular disease. One of the most common cardiovascular disorders is stenosis, which is a pathological narrowing of the arteries caused by abnormal growths along the arterial wall that can arise in various parts of the circulatory system. These constrictions dramatically alter the nature of blood flow compared to unobstructed arteries, compromising the regular function of the cardiovascular system.

Numerous theoretical and experimental studies have been conducted to study blood flow through stenotic arteries. These studies demonstrate that important hemodynamic variables, like velocity profiles, pressure distribution, and shear stress contribute crucially role in progression of stenotic disease. Understanding blood flow behavior in these conditions is essential for early diagnosis, treatment, and prevention of cardiovascular disease

Over the years, a vast literature has emerged addressing blood flow in stenotic arteries, modeling blood as a Newtonian or non-Newtonian fluid in various physiological situations. Significant contributions include Young (1968), Shukla et al., (1980), Rekha et al., (2012), and Arun Kumar (2015).

Nanotechnology's outstanding thermal and physical properties have had a substantial impact on fluid dynamics research in recent years. These fluids, which contain nanoparticles floating in a base fluid such as water, oil, or blood, have high thermal conductivity, viscosity, and heat transfer characteristics. Choi et al., (1995) pioneered the notion of nanofluids, and several researchers have since investigated the impact of nanoparticles in non-Newtonian fluid flows under different of circumstances R.Ellahi et al., (2014). Mekheimer et al., (2016) investigated the impact of metallic nanoparticles on blood flow in stenosed arteries. Mansi Tyagi and Atul Kumar Rai investigated the impact made metals on blood flow in a stenosed artery (2024). Gopinath Mandal and Dulal Pal (2024) study the heat transfer and entropy formation of blood as a hybrid nanofluid in stenotic arteries when magnetic field is present. Umadevi et al., (2021) have investigated the effects of magnetic field on blood containing copper nanoparticles passing through an overlapping stenosed artery. Maruthi Prasad et al., (2024) used SWCNT to analyze blood from an angled multiple stenosed artery with varying nano fluid viscosity.

It is important to note that many physiological channels, including arteries, are inclined rather than horizontal. Several studies have incorporated this bias into their models. Maruthi Prasad et al., (2008) studied Herschel- Bulkley fluid flow through inclined, non uniform tubes with multiple stenoses. Other notable studies include Maruthi Prasad et al.,(2015) on peristaltic transport of nanofluids in inclined tubes and Raja Agarwal et al., (2016) have studied on pulsatile Herschel- Bulkley fluid flow in inclined arteries with multiple stenoses and periodic acceleration of the body.
Taking the above into account, this work aims to analyse the effects of a magnetic field on flow of blood containing silver nanoparticles through a non-uniform inclined tube with multiple stenoses. The stenoses are assumed to be mild and analytical solutions were obtained. Expressions for temperature, velocity, pressure drop, flow resistance, and wall shear stress were calculated, and the impact of several important parameters on these flow variables were explored graphically

MATHEMATICAL FORMULATION:

Consider a continuous, incompressible, axisymmetric blood flow infused with silver nanoparticles through an inclined arterial segment with variable cross-sectional area and two stenotic constrictions. The flow is characterized using a cylindrical polar coordinate system (r, θ, z).The z-axis is aligned with the central axis of the tube. Figure 1 shows an artery with two stenoses and inclined at an angle α to the horizontal plane.

The radius of the artery is calculated from the axial coordinates using the expression published by Maruthi Prasad et al., (2008).

$$h = R(z) = \begin{cases} R_0 & : \quad 0 \le z \le d_1 , \\ R_0 - \frac{\delta_1}{2}\left(1 + \operatorname{Cos} \frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) & : \quad d_1 \le z \le d_1 + L_1 , \\ R_0 & : \quad d_1 + L_1 \le z \le B_1 - \frac{L_2}{2} , \\ R_0 - \frac{\delta_2}{2}\left(1 + \operatorname{Cos} \frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 - \frac{L_2}{2} \le z \le B_1 , \\ R^*(z) - \frac{\delta_2}{2}\left(1 + \operatorname{Cos} \frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 \le z \le B_1 + \frac{L_2}{2} , \\ R^*(z) & : \quad B_1 + \frac{L_2}{2} \le z \le B . \end{cases} \tag{1}$$

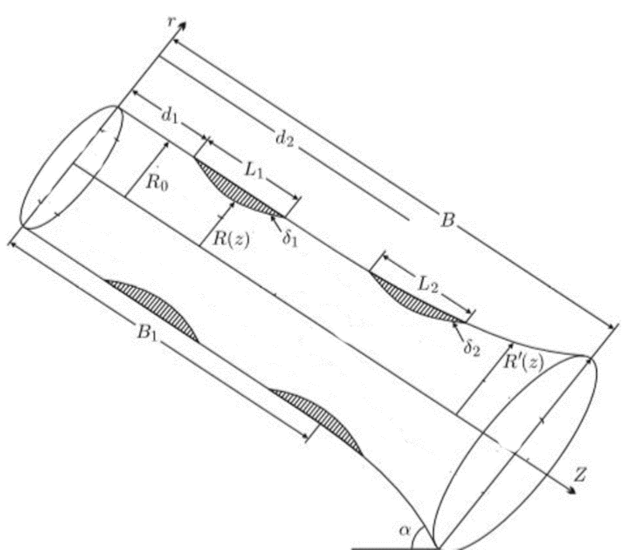

Figure 1: Design of an inclined tube with multiple stenoses.

The following conditions for mild stenosis (Maruthi Prasad et al., [2008]) are supposed to satisfy:

$$\delta_i \ll \min(R_0, R_{out}),$$
$$\delta_i \ll L_i \quad where\ R_{out} = R(z) at\ z = B.$$

Here L_i *and* δ_i $(i = 1,2)$ are the lengths and maximum heights of two stenoses (the suffixes 1 and 2 refer to the first and second stenoses respectively).

The governing equations for conservation of mass, momentum and temperature for a nano fluid is not compressible can be taken as (Akbar, N. S.,Wahid Butt.,(2015)).

$$\frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial z} = 0 \tag{2}$$

$$\rho_{nf}\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu_{nf}\frac{\partial}{\partial r}\left(2\frac{\partial v}{\partial r}\right) + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right) - \frac{\operatorname{Cos}\alpha}{F} \tag{3}$$

$$\rho_{nf}\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial u}{\partial z}\right) + \frac{\mu_{nf}}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right)\right] - g\rho_{nf}\alpha(T - T_0) - \sigma B_0^2 u + \frac{\operatorname{Sin}\alpha}{F} \tag{4}$$

$$\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = \frac{K_{\eta f}}{(\rho c_p)_{\eta f}}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{Q_0}{(\rho c_p)_{\eta f}} \tag{5}$$

With the conditions

$$\frac{\partial u}{\partial r} = 0, \frac{\partial T}{\partial r} = 0 \text{ at } r = 0 \tag{6}$$

$$u = 0, T = 0 \text{ at } r = h \tag{7}$$

Thermo physical features of blood and Silver nano particles (Ag) (Fang fang et al., [17]).

Physical properties	Blood	Ag (Φ)
Cp ($J\ kg^{-1}K^{-1}$)	3594	235
ρ ($kg\ m^{-3}$)	1063	10500
κ ($W\ m^{-1}\ K^{-1}$)	0.492	385
σ (Ω /m)	6.67×10^{-1}	6.3×10^{7}

In the above equations $u$and v are components of velocity in the directions of r and z , T is the fluid temperature, Q_0 is the heat absorption or heat generation constant, $\mu_{\eta f}$ is the dynamic viscosity, ρ_{nf} is the density, $k_{\eta f}$ is the thermal conductivity, $\alpha_{\eta f}$ is the thermal diffusivity and $(\rho c_p)_{nf}$ is the heat capacitance of the nanofluid given as ,

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} . k_{nf} = k_f \left\{ \frac{k_s + 2k_f - 2\,\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)} \right\}$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_f, (\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_f$$

Introducing the following non dimensional variables

$$\bar{r} = \frac{r}{R_0}, \quad \bar{z} = \frac{z}{L_0}, \quad \bar{\upsilon} = \frac{L_0}{\delta U}\upsilon, \quad \bar{u} = \frac{u}{U}, \quad \bar{d} = \frac{d}{L_0}, \quad \bar{R} = \frac{R}{R_0}$$

$$M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f}, G_r = \frac{g\alpha R_0^2 T_0 \rho_{nf}}{U\mu_f}, \bar{\delta} = \frac{\delta}{R_0}, \theta = \frac{T - T_0}{T_0}$$

$$\bar{p} = \frac{UL_0\mu}{R_0^2}p, \quad \beta = \frac{Q_0 R_0^2}{k_f T_0},$$

Where U is the velocity that is averaged over the section of the tube exhibits radious R_0.

After using the above non dimensional variables in equations (2) - (5) and also making use of the mild stenoses conditions $\epsilon = \frac{R_0}{L_0} = o(1), \frac{\delta}{R_0} \ll 1$, the reduced equations along with the boundary conditions are (after dropping the bars)

$$\frac{\partial p}{\partial r} = -\frac{\cos\alpha}{F} \tag{8}$$

$$\frac{dp}{dz} = \frac{1}{(1-\varphi)^{2.5}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - M^2 u + G_r\theta + \frac{\sin\alpha}{F}, \tag{9}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) + \beta\left(\frac{k_{nf}}{k_f}\right) = 0 \tag{10}$$

Where M, β and G_r are the number proposed by Hartmann, parameter of heat absorption and number proposed by Grashof respectively.

The boundary conditions are

$$\frac{\partial u}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0 \quad at\; r = 0 \tag{11}$$

$$u = 0, \theta = 0 \quad at\; r = h \tag{12}$$

Solution

The solutions of equations (9) and (10) are obtained as

$$\text{Velocity, } u = \left\{ \frac{\frac{1}{M^2}\frac{dp}{dz} - \frac{G_r\beta}{M^2}\left(\frac{k_f}{k_{nf}}\right) - \frac{1}{M^2}\frac{Sin\,\alpha}{F}}{I_0\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)} \right\} I_0\left(Mr\sqrt{(1-\varphi)^{2.5}}\right)$$

$$-\frac{G_r\beta}{4M^2}\left(\frac{k_f}{k_{nf}}\right)(r^2 - h^2) - \frac{1}{M^2}\frac{dp}{dz} + \frac{G_r\beta}{M^2}\left(\frac{k_f}{k_{nf}}\right) - \frac{1}{M^2}\frac{Sin\,\alpha}{F} \tag{13}$$

$$\text{Temperature, } \theta = \frac{-\beta}{4}\left(\frac{k_f}{k_{nf}}\right)(r^2 - h^2) \tag{14}$$

The dimension less flux Q is

$Q = 2\int_0^h rudr.$

It implies,

$$\frac{dp}{dz} = \frac{Q - \frac{Gr\beta h^4}{16M^2}\left(\frac{k_f}{k_{nf}}\right) - \frac{Gr\beta h^2}{2M^2}\left(\frac{k_f}{k_{nf}}\right) + \frac{Gr\beta h}{M^2}\left(\frac{k_f}{k_{nf}}\right)\left(\frac{I_1\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)}{I_0\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)\sqrt{(1-\varphi)^{2.5}}}\right) + \frac{h}{M^2}\frac{Sin\,\alpha}{F}\left(\frac{I_1\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)}{I_0\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)\sqrt{(1-\varphi)^{2.5}}}\right) + \frac{h^2}{M^2}\frac{Sin\,\alpha}{F}}{\frac{-h^2}{2M^2} + \frac{h}{M^3}\left(\frac{I_1\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)}{I_0\left(Mh\sqrt{(1-\varphi)^{2.5}}\right)\sqrt{(1-\varphi)^{2.5}}}\right)} \tag{15}$$

The drop of pressure per wave length $\Delta p = p(0) - p(\lambda)$ is $\Delta p = -\int_0^1 \frac{dp}{dz}\, dz$

The resistance to the flow λ in the stenosed artery is defined as $\lambda = \frac{\Delta p}{Q} = -\int_0^1 \frac{dp}{dz}\, dz$ (16)

The drop in pressure without stenosis $(h = 1)$ is defined as $\Delta p_n = \left[-\int_0^1 \frac{dp}{dz}\, dz\right]_{h=1}$

The resistance which is normalized in artery is defined as $\lambda_n = \frac{\Delta p_n}{Q}$

The normalized resistance defined as $\bar{\lambda} = \frac{\lambda}{\lambda_n}$ (17)

And the wall shear stressτ_h is defined as $\tau_h = -\frac{h}{2}\frac{dp}{dz}$ (18)

RESULTS AND ANALYSIS

After the analysis, the solutions for velocity (u), temperature (θ), flow resistance $(\bar{\lambda})$ and wall shear stress (τ_h) are provided by equations (13,14,17 and 18) respectively. The impact of various parameters on the flow resistance $(\bar{\lambda})$ and wall shear stress (τ_h) have been calculated numerically by taking

$\frac{R^*(z)}{R_0} = \exp\,[\beta_{sp}B^2(z - B_1)^2]$

Where $d_1 = 0.2, L_1 = L_2 = 0.2, B_1 = 0.7, B = 1\ and\ \beta_{sp} = 0.01.$

Figures 2-8 represent the change in flow resistance with respect to stenosis height $(\delta_1\ and\ \delta_2)$, angle (α), magnetic field (M), Grashof number (G_r), flow rate (Q), and thermal absorption constant (β). It has been observed that flow resistance $(\bar{\lambda})$ increases with an increase in stenosis height $(\delta_1\ and\ \delta_2)$, magnetic field restriction (M), Grashof number (G_r), thermal absorption constant (β), and decreases with angle (α) and flow rate (Q). The reduction in flow resistance is also observed when silver nanoparticles are added to blood compared to pure blood, since suspended silver nanoparticles improve heat transfer, reducing the viscosity of the fluid due to the increase in temperature.

The wall shear stress (τ_h) as an aspect of function of the stenosis height for varying values of tilt angle (α), magnetic field restriction (M), Grashof number (G_r) and thermal absorption constant (β) is shown in Figures (9-13). It has been observed that the wall shear stress (τ_h) increases with an increasing height of stenoses $(\delta_1 \; and \; \delta_2)$, and tilt angle (α), but decreases with an increasing magnetic field restriction (M), Grashof number(G_r) and thermal absorption constant (β). It is also observed that the wall shear stress increases when silver nanoparticles are added.

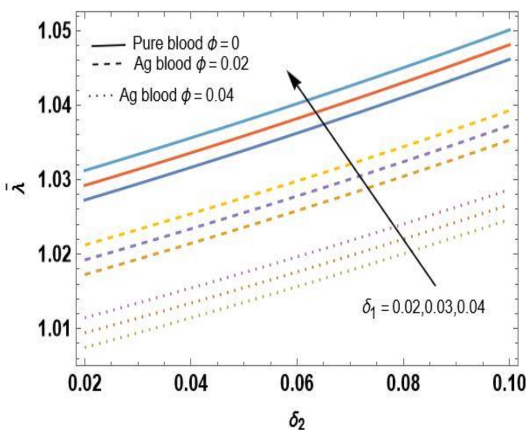

Figure 2: change of flow resistance $\bar{\lambda}$ with δ_2 for different δ_1 $(Q = 0.01, M = 1, F = 0.8, G_r = 0.2, \beta = 0.01, \alpha = \frac{\pi}{6})$

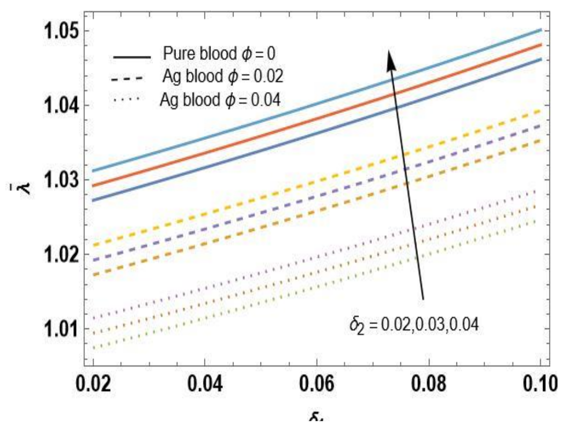

Figure 3: change of flow resistance $\bar{\lambda}$ with δ_1 for different δ_2 $(Q = 0.01, M = 1, F = 0.8, G_r = 0.2, \beta = 0.01, \alpha = \frac{\pi}{6})$

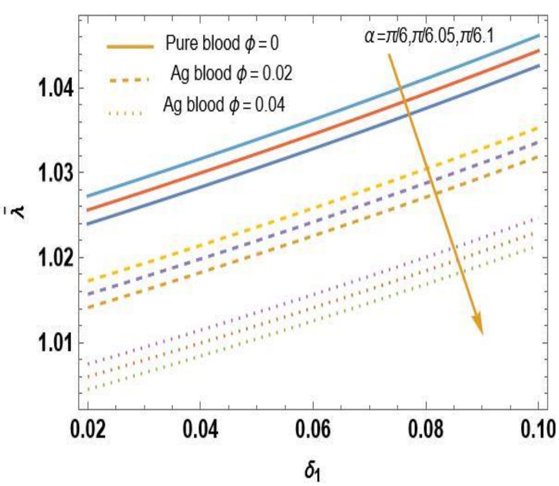

Figure 4: change of flow resistance $\bar{\lambda}$ with δ_1 for different α $(\delta_2 = 0.02, Q = 0.01, M = 1, F = 0.8, G_r = 0.2, \beta = 0.01)$

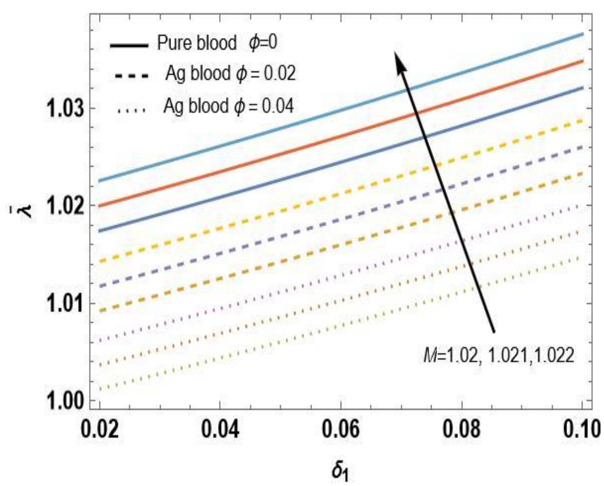

Figure 5:change of flow resistance $\bar{\lambda}$ with δ_1 for different M $(\delta_2 = 0.02, Q = 0.01, F = 0.8, G_r = 0.2, \beta = 0.01, \alpha = \frac{\pi}{6})$

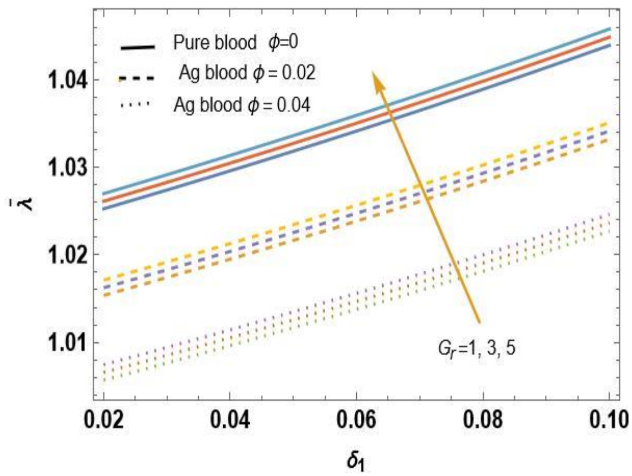

Figure 6: change of flow resistance $\bar{\lambda}$ with δ_1 for different G_r
($\delta_2 = 0.02, Q = 0.01, M = 1, F = 0.8, \beta = 0.01, \alpha = \frac{\pi}{6}$)

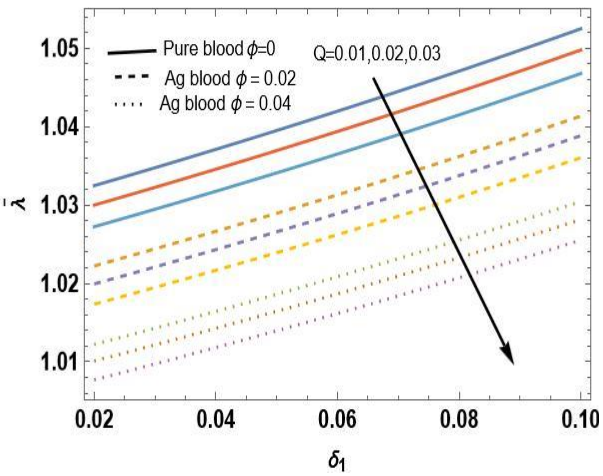

Figure 7: change of flow resistance $\bar{\lambda}$ with δ_1 for different Q
($\delta_2 = 0.02, M = 1, F = 0.8, G_r = 0.2, \beta = 0.01, \alpha = \frac{\pi}{6}$)

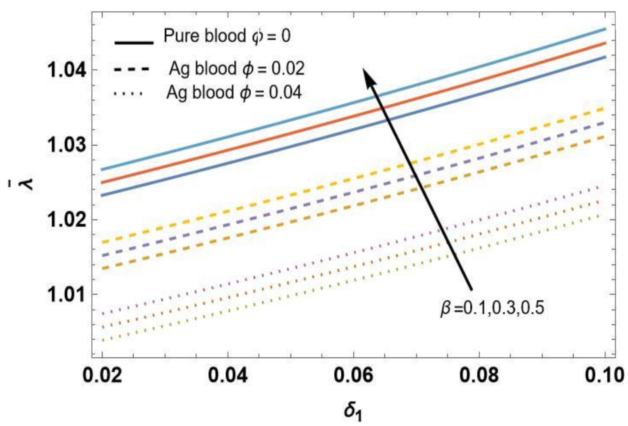

Figure 8: change of flow resistance $\bar{\lambda}$ with δ_1 for different β
($\delta_2 = 0.02, Q = 0.01, F = 0.8, M = 1, G_r = 0.2, \alpha = \frac{\pi}{6}$)

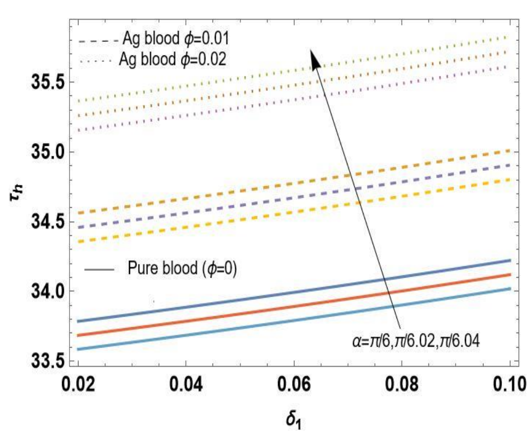

Figure 9: change of wall shear stress τ_h with δ_1 for different α
$Q = 0.01, F = 0.8, M = 1, G_r = 0.2, \delta_2 = 0.02, \beta = 0.1)$)

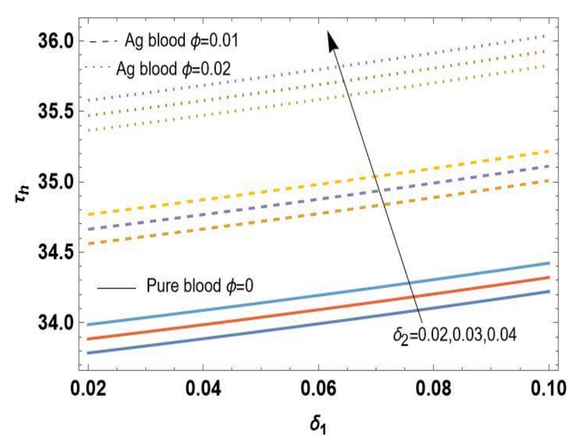

Figure 10: change of wall shear stress τ_h with δ_1 for different δ_2
($\alpha = \frac{\pi}{6}, F = 0.8. Q = 0.01, G_r = 0.2, M = 1, , \beta = 0.1$)

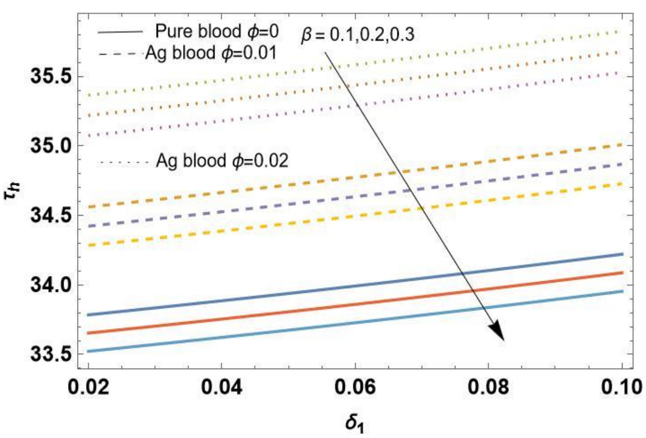

Figure 11: change of wall shear stress τ_h with δ_1 for different β $(\alpha = \frac{\pi}{6}, F = 0.8. Q = 0.01, F = 0.8, M = 1, \delta_2 = 0.02, G_r = 0.2)$

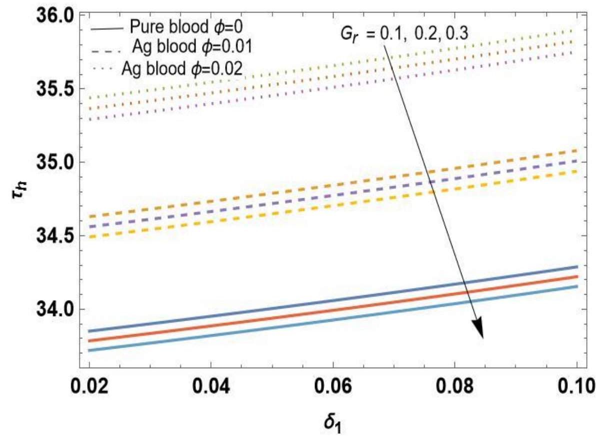

Figure 12: change of wall shear stress τ_h with δ_1 for different $\boldsymbol{G_r}$ $(\alpha = \frac{\pi}{6}, F = 0.8. Q = 0.01, F = 0.8, M = 1, \delta_2 = 0.02, \beta = 0.1)$

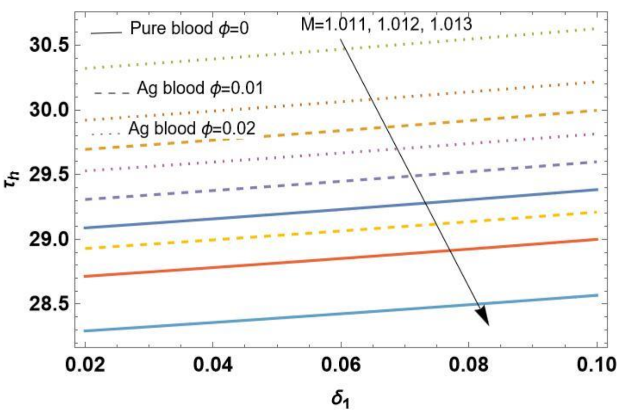

Figure 13: change of wall shear stress τ_h with δ_1 for different M $(\alpha = \frac{\pi}{6}, F = 0.8. Q = 0.01, F = 0.8, G_r = 0.2, \delta_2 = 0.02, \beta = 0.1)$

Conclusion:

This study focuses on examining the influence of magnetic field on silver nano blood with multiple stenoses. Solutions for the mild stenoses were obtained. The equations of flow were linearized and expressions for pressure drop, flow resistance and wall shear stress were derived. The results have been summarized below :

- Flow resistance increases with stenosis height, magnetic field restriction, Grashof number and absorption of heat constant and reduces with tilt angle and flow rate.
- Flow resistance is lower for silver nano blood than for pure blood due to higher thermal conductivity.
- Wall shear stress increases with stenosis height and tilt angle. However it decreases with magnetic field, Grashof number and heat absorption constant.
- By adding silver nanoparticles an increase in wall shear stress is observed.

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