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Zero Truncated Premium Linear-Exponential Mixture of Poisson Distribution

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Abstract:

This distribution is a modified form of the Zero Truncated Poisson-Lindley distribution (ZTPLD), which has been shown to be better alternative of ZTPLD for statistical modeling of count data related to mortality and biological sciences. All the characteristics required for this distribution are presented and explained in very well manner.

Keywords: Zero-truncated Probability distribution, Premium Linear-Exponential Mixture of Poisson distribution (PLEMPD), Premium Linear-exponential distribution (PLED), Chi-square Goodness of Fit Test, Probability distribution, Distribution.

INTRODUCTION:

Research is a regular process that allow us to constantly expand our knowledge by discovering new and unfamiliar things. The zero-truncated probability distribution is generated from a standard distribution excluding mass of the probability at zero. The proposed distribution is named as "Zero Truncated Premium Linear-Exponential Mixture of Poisson Distribution". It is abbreviated as ZTPLEMPD. It is a compound discrete distribution. It may be called Conditional Premium Linear-exponential Mixture of Poisson Distribution (CPLEMPD) or Positive Premium Linear-exponential Mixture of Poisson Distribution (PPLEMPD) or Zero-truncated Poisson-Premium Linear-exponential Distribution (ZTPPLED), (see [1] to [4]). It is a left truncated probability distribution which is truncated at zero.

This distribution an extension of Zero-truncated Poisson-Lindley distribution (ZTPLD) [5], which was obtained by using the definition of zero-truncated probability distribution as well as originated by size-biased Poisson distribution (1) when its parameter τ follows a distribution having probability density function

$$g(w/\tau)=\frac{e^{-\tau}\tau^{w-1}}{(w-1)!};w=1,2,3,...;\tau>0 \tag{1}$$

given in the expression (2) as

$$h(\tau,\beta)=\frac{\beta^2}{(\beta^2+3\beta+1)}\left\{(\beta+1)\tau+(\tau+2)\tau^0\right\}e^{-\beta\tau} \tag{2}$$

Hence, probability mass function of ZTPLD was obtained as

$$P_1(w,\beta)=\frac{\beta^2}{(\beta^2+3\beta+1)}\frac{(2+\beta+w)}{(1+\beta)^w} \tag{3}$$

Where $\beta>0$ and $w=1,2,...,\infty$

For details account, see [6-15]. Works of this paper have been arranged under the following headings.

1.0 Introduction

2.0 Results

3.0 Applications

4.0 Conclusions

2.0 Results:

2.1 Probability Mass Function of ZTPLEMPD:

It can be obtained by using (A) Definition of zero-truncated probability distribution (B) Mixing the expression (1) with (8).

(A) ZTPLEMPD of variable w with parameter β is denoted by $P(w,\beta)$ and defined as

$$P(w,\beta)=\frac{P_2(w,\beta)}{1-P_2(w=0,\beta)} \tag{4}$$

Where $P_2(w,\beta)$ is the Probability mass function of Premium Linear-exponential Mixture of Poisson Distribution (PLEMPD) [6] given by the equation (7).

$$P_2(w,\beta)=\frac{\beta^2}{(1+\pi\beta^2)}\left[\frac{\left\{1+\pi\beta(1+\beta)+w\right\}}{(1+\beta)^{w+2}}\right] \tag{5}$$

Where $\beta>0$ and $w=0,1,2,3,...,\infty$, and

$$P_2(w=0,\beta)=\frac{\beta^2}{(1+\pi\beta^2)}\left[\frac{\left\{1+\pi\beta(1+\beta)\right\}}{(1+\beta)^2}\right] \tag{6}$$

Putting the value of $P_2(w,\beta)$ and $P_2(w=0,\beta)$ in the expression (4) we get

$$P(w,\beta)=\frac{\beta^2}{(\pi\beta^3+\pi\beta^2+2\beta+1)}\frac{(1+\pi\beta+\pi\beta^2+w)}{(1+\beta)^w} \tag{7}$$

Where $\beta>0$ and $w=1,2,...,\infty$

(B) ZTPLEMPD can also be obtained by mixing (1) with (8) when its parameter τ follows a distribution having probability density function given in the expression (8)

$$h(\tau,\beta)=\frac{\beta^2}{(\pi\beta^3+\pi\beta^2+2\beta+1)}\left\{(\beta+1)\tau+(\pi\beta^2+\pi\beta+1)\tau^0\right\}e^{-\beta\tau} \qquad (8)$$

Ans it can be derived as

$$P(w,\beta)=\int_0^\theta\left[\left\{(1+\beta)\tau^w+(\pi\beta^2+\pi\beta+1)\tau^{w-1}\right\}e^{-(1+\beta)\tau}\right]d\tau$$

Or, $$P(w,\beta)=\frac{\beta^2}{(\pi\beta^3+\pi\beta^2+2\beta+1)}\frac{1}{(w-1)!}\left[\frac{(1+\beta)\Gamma(w+1)}{(1+\beta)^{w+1}}+\frac{(1+\pi\beta+\pi\beta^2)\Gamma(w)}{(1+\beta)^w}\right]$$

Or, $$P(w,\beta)=\frac{\beta^2}{(\pi\beta^3+\pi\beta^2+2\beta+1)}\left[\frac{(1+w+\pi\beta+\pi\beta^2)}{(1+\beta)^w}\right] \qquad (9)$$

Where $\beta>0$ and $w=1,2,...,\infty$.

The expression (9) is the pmf of ZTPLEMPD.

Figure-1: Showing graph of pmf of ZTPLEMPD

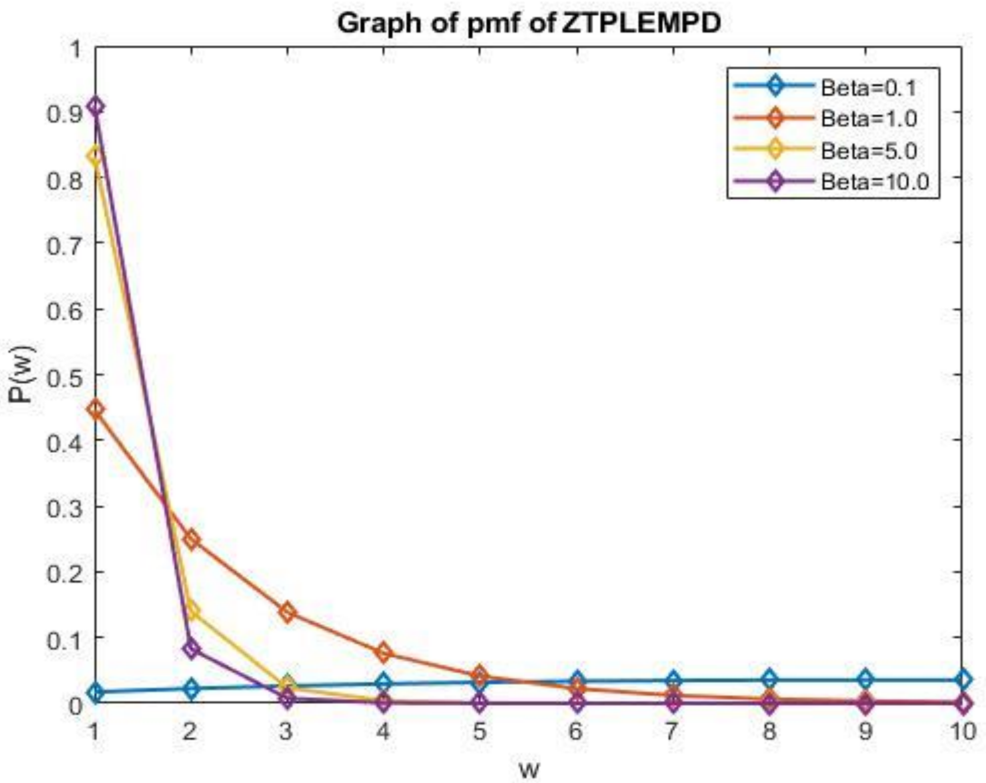


Figure-2: Showing graph of pmf of ZTPLEMPD

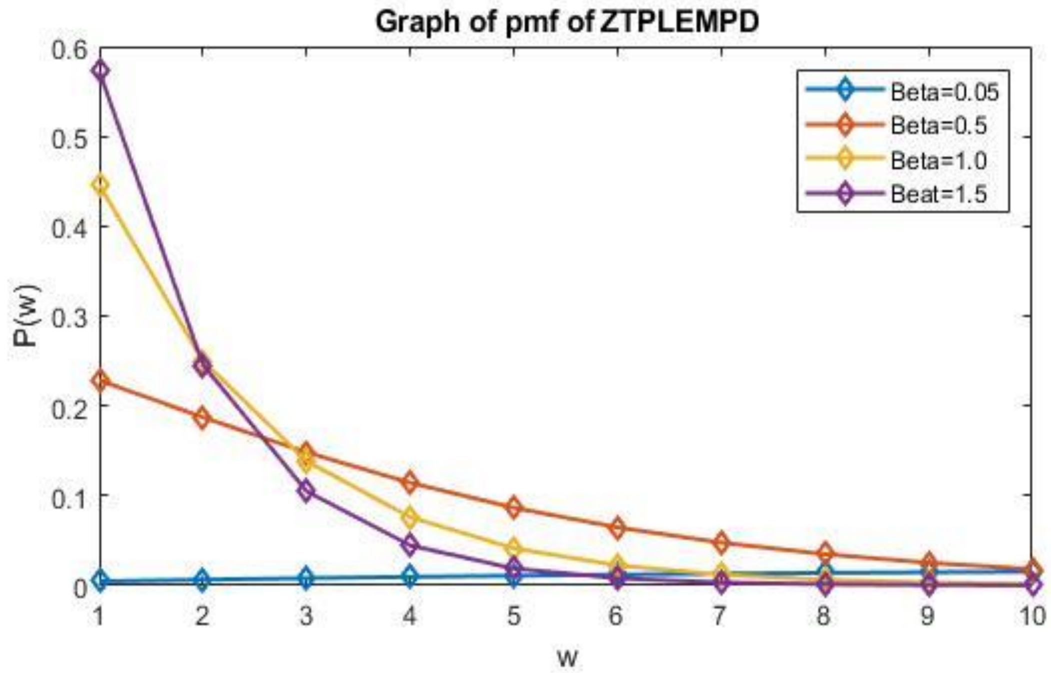
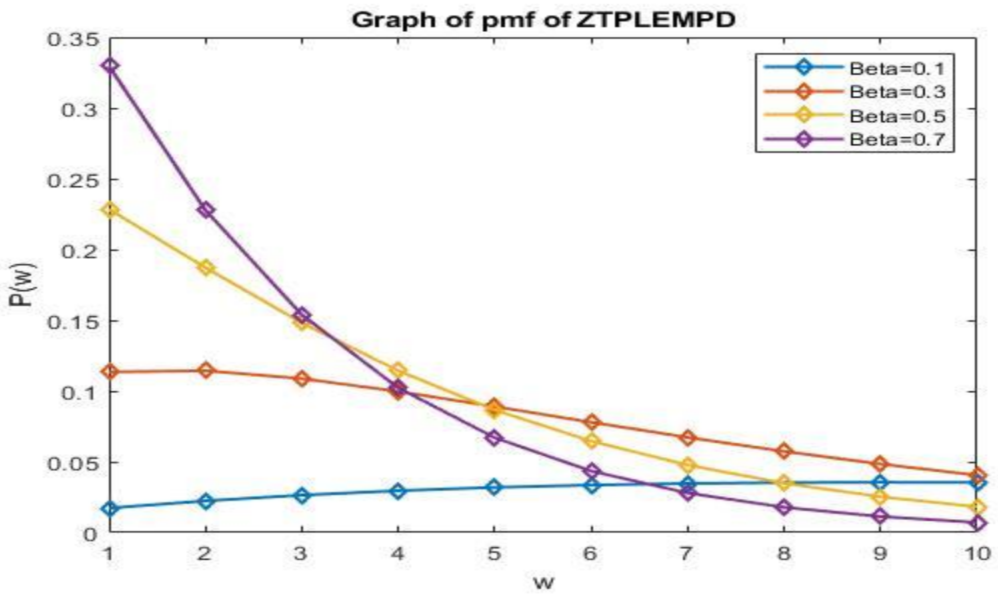


Figure-3: Showing graph of pmf of ZTPLEMPD



2.2 Factorial Moments ($\mu'_{(r)}$), Moments about Origin (μ'_r) and Central Moments (μ'_r) of ZTPLEMPD:

The factorial moment of order r and the first four factorial moments, the first four moments about the origin and the first four moments about the mean have been derived and given by the expression (10) to (22) in order respectively.

$\mu'_{(r)} = E\{w^r / \tau\}$

Or, $\mu'_{(r)} = \dfrac{\beta^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \int_0^\infty \left[\sum_{w=1}^{\infty} \dfrac{w^{(r)} e^{-\tau} \tau^{w-1}}{(w-1)!} \right] \left\{ (\beta+1)w + (\pi\beta^2+\pi\beta+1 \right\} e^{-\beta\tau} d\tau$

Or, $\mu'_{(r)} = \dfrac{\beta^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \int_0^\infty \left[\tau^{r-1}(\tau+r) \right] \left\{ (\beta+1)w + (\pi\beta^2+\pi\beta+1 \right\} e^{-\beta\tau} d\tau$

Or, $\mu'_{(r)} = \dfrac{(1+\beta)^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \dfrac{r!}{\beta^r}(1+r+\pi\beta^2)$ (10)

Therefore, from the expression (11) to (14) are first four factorial moments of ZTPLEMPD in order respectively.

$$\mu'_{(1)} = \frac{(1+\beta)^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \frac{1}{\beta}(2+\pi\beta^2) \tag{11}$$

$$\mu'_{(2)} = \frac{(1+\beta)^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \frac{2}{\beta^2}(3+\pi\beta^2) \tag{12}$$

$$\mu'_{(3)} = \frac{(1+\beta)^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \frac{6}{\beta^3}(4+\pi\beta^2) \tag{13}$$

$$\mu'_{(4)} = \frac{(1+\beta)^2}{(1+2\beta+\pi\beta^2+\pi\beta^3)} \frac{24}{\beta^4}(5+\pi\beta^2) \tag{14}$$

Conversion of Factorial Moments about Origin into Moments about the Origin:

$$\mu'_1 = \mu'_{(1)} = \frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)} \tag{15}$$

$$\mu'_2 = \mu'_{(2)} + \mu'_{(1)} = \frac{2(1+\beta)^2(3+\pi\beta^2)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)} + \frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}$$

Or, $\mu'_2 = \dfrac{(1+\beta)^2(6+2\beta+2\pi\beta^2+\pi\beta^3)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)}$ (16)

$$\mu'_3 = \mu'_{(3)} + 3\mu'_{(2)} + \mu'_{(1)} = \frac{6(1+\beta)^2(4+\pi\beta^2)}{\beta^3(1+2\beta+\pi\beta^2+\pi\beta^3)} + \frac{6(1+\beta)^2(3+\pi\beta^2)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)} + \frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}$$

$$\mu'_3 = \frac{(1+\beta)^2(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)} \tag{17}$$

$$\mu'_4 = \mu'_{(4)} + 6\mu'_{(3)} + 7\mu'_{(2)} + \mu'_{(1)} = \frac{24(1+\beta)^2(5+\pi\beta^2)}{\beta^4(1+2\beta+\pi\beta^2+\pi\beta^3)} + \frac{36(1+\beta)^2(4+\pi\beta^2)}{\beta^3(1+2\beta+\pi\beta^2+\pi\beta^3)}$$
$$+ \frac{14(1+\beta)^2(3+\pi\beta^2)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)} + \frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}$$

Or, $$\mu_4' = \frac{(1+\beta)^2(120+144\beta+42\beta^2+24\pi\beta^2+2\beta^3+36\pi\beta^3+14\pi\beta^4+\pi\beta^5)}{\beta^4(1+2\beta+\pi\beta^2+\pi\beta^3)} \tag{18}$$

The first four moments about the mean has been obtained and expressed by the equations from (19) to (22) respectively.

$$\mu_1 = 0 \tag{19}$$

$$\mu_2 = \frac{(1+\beta)^2(\pi\beta^3+2\pi\beta^2+2\beta+6)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)} - \left[\frac{(1+\beta)^2(2\pi\beta^2+2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]^2$$

0r, $$\mu_2 = \frac{(1+\beta)^2(\pi^2\beta^5+\pi^2\beta^4+5\pi\beta^3+4\pi\beta^2+6\beta+2)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)^2} \tag{20}$$

$$\mu_3 = \frac{(1+\beta)^2(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)}{\beta^3(1+2\beta+\pi\beta^2+\pi\beta^3)} - 3\left[\frac{(1+\beta)^2(6+2\beta+2\pi\beta^2+\pi\beta^3)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]$$
$$\left[\frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right] + 2\left[\frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]^3$$

$$\mu_3 = \left[\frac{\begin{array}{l}(1+\beta)^2(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)\\(1+2\beta+\pi\beta^2+\pi\beta^3)^2-3(1+\beta)^4(2+\pi\beta^2)(6+2\beta+2\pi\beta^2+\pi\beta^3)\\(1+2\beta+\pi\beta^2+\pi\beta^3)+2(1+\beta)^6(2+\pi\beta^2)^3\end{array}}{\beta^3(1+2\beta+\pi\beta^2+\pi\beta^3)^3}\right] \tag{21}$$

$$\mu_4 = \frac{(1+\beta)^2(120+144\beta+42\beta^2+24\pi\beta^2+2\beta^3+36\pi\beta^3+14\pi\beta^4+\pi\beta^5)}{\beta^4(1+2\beta+\pi\beta^2+\pi\beta^3)}$$
$$-4\left[\frac{(1+\beta)^2(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)}{\beta^3(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]\left[\frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]$$
$$-6\left[\frac{(1+\beta)^2(6+2\beta+2\pi\beta^2+\pi\beta^3)}{\beta^2(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]\left[\frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]^2+3\left[\frac{(1+\beta)^2(2+\pi\beta^2)}{\beta(1+2\beta+\pi\beta^2+\pi\beta^3)}\right]^4$$

$$\mu_4 = \left[\frac{\begin{array}{l}(1+\beta)^2(120+144\beta+42\beta^2+24\pi\beta^2+2\beta^3+36\pi\beta^3+14\pi\beta^4+\pi\beta^5)\\(1+2\beta+\pi\beta^2+\pi\beta^3)^3-4(1+\beta)^4(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)\\(2+\pi\beta^2)(1+2\beta+\pi\beta^2+\pi\beta^3)^2+6(1+\beta)^6(6+2\beta+2\pi\beta^2+\pi\beta^3)\\(2+\pi\beta^2)^2(1+2\beta+\pi\beta^2+\pi\beta^3)-3(1+\beta)^8(2+\pi\beta^2)^4\end{array}}{\beta^4(1+2\beta+\pi\beta^2+\pi\beta^3)^4}\right] \tag{22}$$

2.3 Nature of ZTPLEMPD Based on Variability, Shape and Size:

- *Based on Variability:*

$$I = \frac{(\pi^2\beta^5 + \pi^2\beta^4 + 5\pi\beta^3 + 4\pi\beta^2 + 6\beta + 2)}{(2+\pi\beta^2)(1+2\beta+\pi\beta^2+\pi\beta^3)} \quad (23)$$

It is index of dispersion. For over-dispersion

Or, $(\pi^2\beta^5 + 4\pi\beta^4 + 4\beta^2 + 4\pi\beta^2 - \pi^2\beta^4 - 2\pi\beta^3 - 4\pi\beta^2 - 4\beta - 2) < 0$ (24)

Table-1

Serial Number	Types of Variation	Condition
1	Over-dispersion	If $\beta < 1.14930004$
2	Equi-dispersed	If $\beta = 1.14930004$
3	Under-dispersed	If $\beta > 1.14930004$

For details (see, [15]).

- Coefficient of Skewness (γ_1):

$$\gamma_1 = \frac{\left\{\begin{array}{l}(1+\beta)^2(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)\\(1+2\beta+\pi\beta^2+\pi\beta^3)^2-3(1+\beta)^4(2+\pi\beta^2)(6+2\beta+2\pi\beta^2+\pi\beta^3)\\(1+2\beta+\pi\beta^2+\pi\beta^3)+2(1+\beta)^6(2+\pi\beta^2)^3\end{array}\right\}}{\left\{(1+\beta)^2(\pi^2\beta^5+\pi^2\beta^4+5\pi\beta^3+4\pi\beta^2+6\beta+2)\right\}^{3/2}} \quad (24)$$

- Coefficien of Kurtosis (β_2):

$$\beta_2 = \frac{\left\{\begin{array}{l}(1+\beta)^2(120+144\beta+42\beta^2+24\pi\beta^2+2\beta^3+36\pi\beta^3+14\pi\beta^4+\pi\beta^5)\\(1+2\beta+\pi\beta^2+\pi\beta^3)^3-4(1+\beta)^4(24+18\beta+2\beta^2+6\pi\beta^2+6\pi\beta^3+\pi\beta^4)\\(2+\pi\beta^2)(1+2\beta+\pi\beta^2+\pi\beta^3)^2+6(1+\beta)^6(6+2\beta+2\pi\beta^2+\pi\beta^3)\\(2+\pi\beta^2)^2(1+2\beta+\pi\beta^2+\pi\beta^3)-3(1+\beta)^8(2+\pi\beta^2)^4\end{array}\right\}}{\left\{(1+\beta)^2(\pi^2\beta^5+\pi^2\beta^4+5\pi\beta^3+4\pi\beta^2+6\beta+2)\right\}^2} \quad (25)$$

Remarks:

- $(\sqrt{2}) < \gamma_1 < \infty$
- $6 < \beta_2 < \infty$
- Hence the proposed distribution is positively skewed and leptokurtic in nature.

2.4 Estimation of Parameter: It can be obtained by using the following equation which has been derived by using μ_1' of this distribution.

$$k\pi\beta^4 + \pi(k-1)\beta^3 + (2k-\pi)\beta^2 + (k-3)\beta - 2 = 0 \quad (26)$$

Where $k = \bar{w} - 1$

3.0 Applications of ZTPLEMPD:

We may use this distribution for modelling of zero-truncated count data related to Mortality as well as Social Sciences (see, [16] to [21]). The example related to mortality are from (1) to (8) and the examples (9) and (10) are related to social sciences which are given in table from (2) to (11) respectively. Ultimately, we have used ten examples on which have applied chi-square goodness of fit test by using ZTPD as well as ZTPLEMPD which are placed in the table numbered from (12) to (21) in order respectively which indicate superiority of ZTPLEMPD over ZTPD.

Example-1

Table-2

The table shows one live birth and at least one neonatal death of mothers of rural area, where W represents the number of neonatal death and observed number of mothers

W	1	2	3	4	5
O	409	88	19	5	1

Example-2

Table-3

The table shows the no. of mothers from estate area who have experienced one live birth and at least one neonatal death, where W represents the number of neonatal death and observed number of mothers.

W	1	2	3	4	5
O	71	32	7	5	3

Example-3

Table-4

The table shows the number of mothers from urban area who have experienced at least two live births by the numbers of infant and child deaths

W	1	2	3	4	5
O	176	44	16	6	2

Example-4

Table-5

The table shows the number of mothers from rural area who have experienced at least two live births by the numbers of infant and child deaths

W	1	2	3	4	5	6
O	745	212	50	21	7	2

Example-5

Table-6

The table shows the number of literate mothers who have experienced of at least one live birth and at least one death

W	1	2	3	4	5
O	683	145	29	11	5

Exaple-6

Table-7

The table shows the number of mothers who have completed fertility with at least one child death.

W	1	2	3	4	5	6
O	89	25	11	6	3	1

Exaple-7

Table-8

The table shows the number of mothers who have experienced at least one infant death

W	1	2	3	4	5
O	567	135	28	11	5

Exaple-8

Table-9

The table shows the no. of European res mites on apple leaves reported by Garman [19].

W	1	2	3	4	5	6	7	8
O	38	17	10	9	3	2	1	0

Where W and O represents the no. red mites and the no. of observed leaves respectively.

Exaple-9

Table-10

The table shows the count of yeast cell per mm square reported by Students [20].

W	1	2	3	4	5	6
O	128	37	18	3	1	0

Where W and O represents the no. of Yeast cell per mm square and the no. of observed leaves respectively.

Exaple-10

Table-11

The table shows the count of snows hares counts captured over 7 days reported by Keith and Meslow [21].

W	1	2	3	4	5
O	184	55	14	4	4

Where W and O represents the no. of snows hares caught and the no. of observed respectively.

Table-12
Fitting of ZTPLD and ZTPLEMPD to the example (1)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	409	408.1	408.5
2	88	89.4	88.8
3	19	19.3	19.3
4	5	4.1	4.2
5	1	1.1	1.2
Total	522	522.0	522.0
$\bar{w}=1.277777$	$m=\bar{w}-1=0.277777$		
$\hat{\beta}$		4.199697	3.68115
$d.f.$		2	2
χ^2		0.145	0.079
$P-value$		0.9301	0.9613

Table-13
Fitting of ZTPLD and ZTPLEMPD to the example (2)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	71	72.3	72.8
2	32	28.4	27.9
3	7	10.9	10.6
4	5	4.1	4.1
5	3	2.3	2.6
Total	118	118.0	118.0
$\bar{w}=1.618644068$	$m=\bar{w}-1=0.618644068$		
$\hat{\beta}$		2.0496094	1.75878
$d.f.$		2	2
χ^2		2.274	2.121
$P-value$		0.3204	0.3463

Table-14
Fitting of ZTPLD and ZTPLEMPD to the example (3)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD

1	176	171.6	172.0
2	44	53.3	50.8
3	16	15.0	15.0
4	6	4.3	4.4
5	2	1.7	1.8
Total	244	244.0	244.0
$\bar{w} = 1.418032787$	$m = \bar{w} - 1 = 0.418032787$		
$\hat{\beta}$		2.209411	2.50388
$d.f.$		2	2
χ^2		1.882	1.5915
$P-value$		0.3902	0.4516

Table-15
Fitting of ZTPLD and ZTPLEMPD to the example (4)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	745	738.1	739.9
2	212	214.8	212.6
3	50	61.3	61.0
4	21	17.2	17.5
5	7	4.8	5.0
6	3	1.8	2.0
Total	1038	1038.0	1038.0
$\bar{w} = 1.402697495$	$m = \bar{w} - 1 = 0.402697495$		
$\hat{\beta}$		3.007722	2.59203
$d.f.$		3	3
χ^2		4.773	4.0061
$P-value$		0.1892	0.2608

Table-16
Fitting of ZTPLD and ZTPLEMPD to the example (5)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	683	674.4	675.0
2	145	154.1	153.1
3	29	34.6	34.7
4	11	7.7	7.9
5	5	2.2	2.3

Total	873	873.0	873.0
$\bar{w} = 1.293241695$	$m = \bar{w} - 1 = 0.293241695$		
$\hat{\beta}$		4.00231	3.49508
$d.f.$		2	2
χ^2		5.310	4.7577
$P - value$		0.0703	0.0927

Table-17

Fitting of ZTPLD and ZTPLEMPD to the example (6)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	89	83.4	83.9
2	25	32.3	31.8
3	11	12.2	12.0
4	6	4.5	4.5
5	3	1.6	1.7
6	1	0.9	1.1
Total	135	135.0	135.0
$\bar{w} = 1.607407407$	$m = \bar{w} - 1 = 0.607407407$		
$\hat{\beta}$		2.089084	1.78732
$d.f.$		2	2
χ^2		3.428	2.8460
$P - value$		0.1801	0.2410

Table-18

Fitting of ZTPLD and ZTPLEMPD to the example (7)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	567	561.4	562.1
2	135	139.7	138.6
3	28	34.2	34.2
4	11	8.2	8.4
5	5	2.6	2.7
Total	746	746.0	746.0
$\bar{w} = 1.327077748$	$m = \bar{w} - 1 = 0.327077748$		
$\hat{\beta}$		3.625737	3.15020
$d.f.$		2	2
χ^2		3.839	3.4232

$P-value$		0.1467	0.1808

Table-19
Fitting of ZTPLD and ZTPLEMPD to the example (8)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	38	36.1	36.9
2	17	20.5	20.1
3	10	11.5	10.8
4	9	5.6	5.8
5	3	3.1	3.1
6	2	1.6	1.6
7	1	0.8	0.9
8	0	0.8	0.8
Total	80	80.0	80.0
$\bar{w}=2.15$	$m=\bar{w}-1=1.15$		
$\hat{\beta}$		1.185582	1.04542665
$d.f.$		3	3
χ^2		2.467	2.3606
$P-value$		0.4813	0.5010

Table-20
Fitting of ZTPLD and ZTPLEMPD to the example (9)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	128	127.6	128.1
2	37	40.9	40.4
3	18	12.8	12.7
4	3	4.0	4.0
5	1	1.2	1.3
6	0	0.5	0.5
Total	187	187.0	187.0
$\bar{w}=1.459893048$	$m=\bar{w}-1=0.459893048$		
$\hat{\beta}$		2.667323	2.29372882
$d.f.$		1	1
χ^2		1.034	0.948
$P-value$		0.309	0.330

Table-21
Fitting of ZTPLD and ZTPLEMPD to the example (10)

W	O	Theoretical frequency due to	
		ZTPLD	ZTPLEMPD
1	184	182.6	183.1
2	55	55.3	54.7
3	14	16.4	16.3
4	4	4.8	4.9
5	4	1.9	2.0
Total	261	261.0	261.0
$\bar{w}=1.425287356$	$m=\bar{w}-1=0.425287356$		
$\hat{\beta}$		2.863957	2.46444202
$d.f.$		2	2
χ^2		0.61	0.5059
$P-value$		0.7371	0.7765

4.0 Conclusion:

- It is suggested to apply ZTPNLEMPD instead of ZTPLD for the zero-truncated count data related to mortality and social sciences because P-value obtained by using ZTPLEMPD is greater than those obtained by using ZTPLD for the table numbered from (12) to (21).
- This distribution is Positively skewed and Leptokurtic in nature.
- This distribution is over-dispersed if $\beta < 1.14930004$.

Conflict of Interest:
The sole purpose of this paper is to contribute to the field of zero-truncated Poisson-Continuous distribution. Our intention is not to hurt anyone's feelings.
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