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On Soft Strongly $ƅ^* -$ Closed Via Soft Ideal

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Abstract:

In this paper, we introduce the soft strongly $ƅ^* -$closed via soft ideal and study the behavior of intersection and union of this level. Also, we define the soft strongly $ƅ^*\breve{I} -$continuous, irresolute, soft strongly $ƅ^*\breve{I} -$open map and soft strongly $ƅ^*\breve{I} -$closed map with some properties. Moreover, the relationship between another closed sets and soft strongly $ƅ^*\breve{I} -$closed with counterexamples are discuss.

Keywords: soft ideal, $sSƅ^* -$closed set, $sSƅ^*\breve{I} -$closed, $sSƅ^*\breve{I} -$continuous, $sSƅ^*\breve{I} -$irresolute.

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1. Introduction and Preliminaries

Molodtsov [1], instigated the concept of soft set as a new mathematical tool to deal with uncertainties problems in different fields of science. I. Arockiarani and A. Arokialancy [2] studied the soft $\beta -$open sets and continuous. Akdag and Ozkan [3, 4] introduced the soft α-open and define soft b-open and continuous. Hameed, S. Z., Hussein, A. K [5] defined the soft $ƅc -$open set. The soft $ƅ^* -$ closed, $sƅ^* -$continuous, $sSƅ^* -$closed sets and $sSƅ^* -$continuous functions are studied by Saif at el. in [6], [7] and [8]. Kandil et al. [9] define soft ideal and introduced the soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called $\mathcal{S}$TSs with soft ideal $(\mathcal{Z}, \mathfrak{W}, \Delta, \breve{I})$. Mustafa and Sleim [10] studied the notion of a soft ideal and they introduced the soft generalized closed sets with respect to a soft ideal and studied their properties in detail, which is extension of the concept of soft generalized closed sets. Later, K. Kannan [11] introduced the soft g-closed soft sets in a $\mathcal{S}$TS. In this work, we study the concept of $sSƅ^* -$closed set via soft ideal, Also, we study the relationship between $sSƅ^*\breve{I} -$closed sets and other existing soft sets have been investigated. Moreover, the $sSƅ^*\breve{I} -$continuous, irresolute, $sSƅ^*\breve{I} -$open map and $sSƅ^*\breve{I} -$closed map with counterexamples are discuss.

Definition 1.1: [1] Let $\mathcal{Z}$ be an initial universe set and E be a set of parameters. Let $P(\mathcal{Z})$ denote the power set of $\mathcal{Z}$, and $\Delta \subset E$. A pair (γ, Δ) is called a soft set over $\mathcal{Z}$. Where γ is a mapping given by $\gamma: \Delta \to P(\mathcal{Z})$. The family of all soft sets over $\mathcal{Z}$ denote by $SS(\mathcal{Z}, \Delta)$

Definition 1.2: [12] The soft set $(\delta, \Delta) \in SS(\mathcal{Z}, \Delta)$, where $\delta(c) = \emptyset$, for every $c \in \Delta$ is called A-null soft set of $SS(\mathcal{Z}, \Delta)$ and denoted by $\widetilde{\emptyset}$. The soft set $(\delta, \Delta) \in SS(\mathcal{Z}, \Delta)$, where $\delta(c) = \mathcal{Z}$, for every $c \in \Delta$ is called the A-absolute soft set of $SS(\mathcal{Z}, \Delta)$ and denoted by $\tilde{\mathcal{Z}}$.

Definition 1.3: [12] For two sets $(\gamma, \Delta), (\delta, B) \in SS(\mathcal{Z}, \Delta)$, we say that (γ, Δ) is a soft subset of (δ, B) denoted by $(\gamma, \Delta) \subseteq (\delta, B)$, if

(1) $\Delta \subseteq B$.

(2) $\gamma(\nabla) \subseteq \delta(\nabla), \forall \nabla \in \Delta$.

In this case, (γ, Δ) is said to be a soft superset of (δ, B), if (δ, B) is a soft subset of (γ, Δ), $(\gamma, \Delta) \supseteq (\delta, B)$.

Definition 1.4: [13] Let (γ, Δ) be a soft set over $\mathcal{Z}$ and $z \in \mathcal{Z}$. We say that $z \in (\gamma, \Delta)$ read as z belongs to the soft set (γ, Δ) whenever $z \in \gamma(\nabla)$ for all $\nabla \in \Delta$. The soft set (γ, Δ) over $\mathcal{Z}$ such that $\gamma(\nabla) = \{z\} \forall \nabla \in \Delta$ is called singleton soft point and denoted by z_Δ or (z, Δ).

Definition 1.5: [13] Let $\mathfrak{W}$ be a collection of soft sets over $\mathcal{Z}$, then $\mathfrak{W}$ is said to be $\mathcal{S}$TS on $\mathcal{Z}$ if

(1) $\widetilde{\emptyset}$ and $\tilde{\mathcal{Z}}$ belong to $\mathfrak{W}$.
(2) The union of any subcollection of soft sets of $\mathfrak{W}$ belongs to $\mathfrak{W}$.
(3) The intersection of any two soft sets in $\mathfrak{W}$ belongs to $\mathfrak{W}$.

It is denoted by $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \Delta)$ and briefly $\mathcal{Z}$.

Definition 1.6: [13] Let $(\mathcal{Z}, \mathfrak{W}, \Delta)$ be a soft space over $\mathcal{Z}$, then the members of $\mathfrak{W}$ are said to be soft open sets in $\mathfrak{W}$.

Definition 1.7: [13] Let $(\mathcal{Z}, \mathfrak{W}, \Delta)$ be a soft space over $\mathcal{Z}$. A soft set (P, Δ) over $\mathcal{Z}$ is said to be a soft closed set in $\mathcal{Z}$, if its relative complement $(\gamma, \Delta)'$ belongs to $\mathfrak{W}$.

Definition 1.8: [14] Let $(\mathcal{Z}, \mathfrak{W}, \Delta)$ be a $\mathcal{S}$TS and $(\gamma, \Delta) \in SS(\mathcal{Z}, \Delta)$. Then

(1) The soft closure of (γ, Δ) is the soft set

$$cl(\gamma, \Delta) = \cap \{(L, \Delta) : (L, \Delta) \in \mathfrak{W}^c, (\gamma, \Delta) \subseteq (L, \Delta)\}.$$

(2) The soft interior of (γ, Δ) is the soft set

$$int(\gamma, \Delta) = \cup \{(H, \Delta) : (H, \Delta) \in \mathfrak{W}, (H, \Delta) \subseteq (\gamma, \Delta)\}.$$

Definition 1.9: [4, 5, 7, 19] A soft set (δ, Δ) of a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \Delta)$ is said to be

(1) soft α- open if $(\delta, \Delta) \subset int(cl(int((\delta, \Delta))))$.

(2) soft preopen if $(\delta, \Delta) \subset int(cl((\delta, \Delta)))$.

(3) soft semi - open if $(\delta, \Delta) \subset cl(int((\delta, \Delta)))$.

(4) soft β-open if $(\delta, \Delta) \subset cl(int(cl((\delta, \Delta))))$.

(5) soft ƅ $-$open if $(\delta, \Delta) \subset int(cl((\delta, \Delta))) \cup cl(int((\delta, \Delta))))$.

Definition 1.15: [8] A soft set (γ, Δ) of a $\mathcal{S}$TS$(\mathcal{Z}, \mathfrak{W}, \Delta)$ is called a soft strongly ƅ* $-$closed (briefly s$\mathcal{S}$ƅ* $-$closed) if $cl(int(\gamma, \Delta)) \subseteq (\delta, \Delta)$, whenever $(\gamma, \Delta) \subset (\delta, \Delta)$ and (δ, Δ) is soft ƅ $-$open. The complement of a ƅ* ƅ* $-$closed set is called ƅ* ƅ* $-$open set. The family of all ƅ* ƅ* $-$open sets denoted by $s\mathcal{S}$ƅ$^*OS(\mathcal{Z})$.

Theorem 1.16: [8] The following statements are true.

(i) Every soft open is $s\mathcal{S}$ƅ* $-$open.
(ii) Every $s\alpha$ $-$open is $s\mathcal{S}$ƅ* $-$open.
(iii) Every $s\mathcal{S}$ƅ* $-$open set is sƅ $-$open.
(iv) Every $s\omega$ $-$open is $s\mathcal{S}$ƅ* $-$open.

Definition 1.17: [9] Let Ĭ be a non-null collection of soft sets over a universe $\mathcal{Z}$ with the same set of parameters Δ. Then, Ĭ $\in SS(\mathcal{Z}, \Delta)$ is called a soft ideal on $\mathcal{Z}$ with the same set Δ if

(1) $(\gamma, \Delta) \in$ Ĭ and $(\delta, \Delta) \in$ Ĭ $\Rightarrow (\gamma, \Delta) \cup (\delta, \Delta) \in$ Ĭ,
(2) $(\gamma, \Delta) \in$ Ĭ and $(\delta, \Delta) \subseteq (\gamma, \Delta) \Rightarrow (\delta, \Delta) \in$ Ĭ.

i.e., Ĭ is closed under finite soft unions and soft subsets.

Definition 1.18: [10] A soft set $(\gamma, \Delta) \in SS(\mathcal{Z}, \Delta)$ is called soft generalized closed set with respect to soft ideal Ĭ (soft Ĭg$-$closed set) in $\mathcal{S}$TS$(\mathcal{Z}, \mathfrak{W}, \Delta)$ if $cl(\gamma, \Delta) \backslash (\delta, \Delta) \in$ Ĭ whenever $(\gamma, \Delta) \subset (\delta, \Delta)$ and $(\delta, \Delta) \in \mathfrak{W}$.

2. SS ƅ* $-$closed via soft ideal

In this section, we define $s\mathcal{S}$ƅ* $-$closed set via soft ideal and study some of their properties.

Definition 2.1: A soft set (γ, Δ) of a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \Delta)$ is called a soft strongly ƅ* $-$closed with respect to soft ideal Ĭ (briefly s$\mathcal{S}$ƅ*Ĭ $-$closed) if $cl(int(\gamma, \Delta)) \setminus (\delta, \Delta) \in$ Ĭ, whenever $(\gamma, \Delta) \subset (\delta, \Delta)$ and (δ, Δ) is $s\mathcal{S}$ƅ* $-$open.

Example 2.2. Let $\mathcal{Z} = \{\varepsilon, \mu\}$ and $\Delta = \{\nabla_1, \nabla_2\}$. Let $(\gamma_1, \Delta), (\gamma_2, \Delta)$ and (γ_3, Δ) be three soft sets, where
$(\gamma_1, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\varepsilon\})\}$,
$(\gamma_2, \Delta) = \{(\nabla_1, \{\mu\}), (\nabla_2, \emptyset)\}$ and
$(\gamma_3, \Delta) = \{(\nabla_1, \{\mu\}), (\nabla_2, \{\varepsilon\})\}$.
Then $(\gamma_1, \Delta), (\gamma_2, \Delta)$ and (γ_3, Δ) are soft sets over $\mathcal{Z}$ and
$\mathfrak{W} = \{\tilde{\mathcal{Z}}, \tilde{\emptyset}, (\gamma_1, \Delta), (\gamma_2, \Delta), (\gamma_3, \Delta)\}$ is the soft topology over $\mathcal{Z}$.
Let Ĭ $= \{\tilde{\emptyset}, (\delta_1, \Delta), (\delta_2, \Delta), (\delta_3, \Delta)\}$ be a soft ideal on $\mathcal{Z}$, where
$(\delta_1, \Delta) = \{(\nabla_1, \{\mu\}), (\nabla_2, \emptyset)\}$,

$(\delta_2, \Delta) = \{(\nabla_1, \{\mu\}), (\nabla_2, \{\varepsilon\})\}$ and
$(\delta_3, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\varepsilon\})\}$.
The soft sets $(\vartheta_1, \Delta), (\vartheta_2, \Delta), (\vartheta_3, \Delta)$ are $sSƅ^*\breve{I}$ −closed, where
$(\vartheta_1, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \mathcal{Z})\}$,
$(\vartheta_2, \Delta) = \{(\nabla_1, \mathcal{Z}), (\nabla_2, \{\varepsilon\})\}$ and
$(\vartheta_3, \Delta) = \{(\nabla_1, \{\mu\}), (\nabla_2, \{\mu\})\}$.
And we see the soft set (ξ, Δ) is not $sSƅ^*\breve{I}$ −closed, where $(\xi, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\varepsilon\})\}$.

Theorem 2.3:

(1) Every soft g −closed set is $sSƅ^*\breve{I}$ −closed.
(2) Every closed soft set is $sSƅ^*\breve{I}$ −closed.
(3) Every soft Ĭg −closed set is $sSƅ^*\breve{I}$ −closed.

Proof.

(1) Let $(\gamma, \Delta) \subseteq (\delta, \Delta)$ and (δ, Δ) is $sSƅ^*$ −open. Since (γ, Δ) is soft g −closed ⇒ $cl(\gamma, \Delta) \subseteq (\delta, \Delta)$ and $cl\big(int(\gamma, \Delta)\big) \subseteq cl(\gamma, \Delta)$. So, $cl\big(int(\gamma, \Delta)\big) \setminus (\delta, \Delta) = \emptyset \in \breve{I}$. Therefore, (γ, Δ) is $sSƅ^*\breve{I}$ −closed.
(2) Let $(\gamma, \Delta) \subseteq (\delta, \Delta)$ and (δ, Δ) is $sSƅ^*$ −open. Since (γ, Δ) is soft closed, then $cl\big(int(\gamma, \Delta)\big) \subseteq cl(\gamma, \Delta) = (\gamma, \Delta) \subseteq (\delta, \Delta)$. Hence, $cl\big(int(\gamma, \Delta)\big) \setminus (\delta, \Delta) = \emptyset \in \breve{I}$. Therefore, (γ, Δ) is $sSƅ^*\breve{I}$ −closed.
(3) Let $(\gamma, \Delta) \subseteq (\xi, \Delta)$ and (ξ, Δ) is $sSƅ^*$ −open. Then $cl\big(int(\gamma, \Delta)\big) \setminus (\xi, \Delta) \subseteq cl(\gamma, \Delta) \setminus (\xi, \Delta) \in \breve{I}$, $(\xi, \Delta) \in \breve{I}$ Hence, (γ, Δ) is $sSƅ^*\breve{I}$ −closed.

The converse of the above theorem is not true in general. The following examples support our claim.

Example 2.4: Let $\mathcal{Z} = \{\varepsilon, \mu\}$. Let $\Delta = \{\nabla_1, \nabla_2\}$. Let $(\gamma_1, \Delta), (\gamma_2, \Delta), (\gamma_3, \Delta)$ and (γ_4, Δ) be four soft sets, where
$(\gamma_1, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \mathcal{Z})\}$,
$(\gamma_2, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \emptyset)\}$,
$(\gamma_3, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \{\mu\})\}$ and
$(\gamma_4, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \{\varepsilon\})\}$.
Then, $\mathfrak{W} = \{\tilde{\mathcal{Z}}, \tilde{\emptyset}, (\gamma_1, \Delta), (\gamma_2, \Delta), (\gamma_3, \Delta), (\gamma_4, \Delta)\}$ is the soft topology over $\mathcal{Z}$.
Let $\breve{I} = \{\tilde{\emptyset}, (\delta_1, \Delta), (\delta_2, \Delta), (\delta_3, \Delta)\}$ be a soft ideal on $\mathcal{Z}$, where
$(\delta_1, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \emptyset)\}$,
$(\delta_2, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \{\varepsilon\})\}$ and
$(\delta_3, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\varepsilon\})\}$.
The soft sets (ϑ, Δ) is $sSƅ^*\breve{I}$ −closed but not soft g −closed, where
$(\vartheta, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\mu\})\}$.

Example 2.5: In Example 2.2, the soft set $(\Gamma, \Delta) = \{(\nabla_1, \mathcal{Z}), (\nabla_2, \{\varepsilon\})\}$ is $s\mathcal{S}ƅ^*\check{I}$ −closed but not closed soft se.

Example 2.6: In Example 2.4, the soft set (ζ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed but not soft $\check{I}g$ −closed set, where $(\zeta, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\mu\})\}$.

Remark 2.7: If a soft subset (γ, Δ) of a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \varDelta)$ is soft open, then it is $s\mathcal{S}ƅ^*\check{I}$ −closed if and only if it is soft $\check{I}g$ −closed.

Theorem 2.8: A soft set (ϑ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed in a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \varDelta)$ if and only if $(\gamma, \Delta) \subseteq cl(int(\vartheta, \Delta))\backslash(\vartheta, \Delta)$ and (γ, Δ) is soft closed implies $(\gamma, \Delta) \in \check{I}$.

Proof. (⇒) Let $(\gamma, \Delta) \subseteq cl(int(\vartheta, \Delta))\backslash(\vartheta, \Delta)$ and (γ, Δ) is soft closed. Then, $(\vartheta, \Delta) \subseteq (\gamma, \Delta)^c$. By hypothesis, $cl(int(\vartheta, \Delta))\backslash(\gamma, \Delta)^c \in \check{I}$. But $(\gamma, \Delta) \subseteq cl(int(\vartheta, \Delta)) \cap (\gamma, \Delta) = cl(int(\vartheta, \Delta))\backslash(\gamma, \Delta)^c$. Thus, $(\gamma, \Delta) \in \check{I}$ from Definition 1.17.
(⇐) Assume that $(\vartheta, \Delta) \subseteq (\delta, \Delta)$ and $(\delta, \varDelta)$ is $s\mathcal{S}ƅ^*$ −open. Then, $cl(int(\vartheta, \Delta))\backslash(\delta, \Delta) = cl(int(\vartheta, \Delta)) \cap (\delta, \Delta)^c$ is a $s\mathcal{S}ƅ^*$ −closed set and $cl(int(\vartheta, \Delta))\backslash(\delta, \Delta) \subseteq cl(int(\vartheta, \Delta))\backslash(\delta, \Delta)$. By assumption, $cl(int(\vartheta, \Delta))\backslash(\delta, \Delta) \in \check{I}$. So, (ϑ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed.

Theorem 2.9: If (γ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed in a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \varDelta)$ and $(\gamma, \Delta) \subseteq (\delta, \Delta) \subseteq cl(int(\gamma, \Delta))$, then (δ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed.

Proof. Let $(\delta, \Delta) \subseteq (\xi, \Delta)$ and (ξ, Δ) is $s\mathcal{S}ƅ^*$ −open. Then, $(\gamma, \Delta) \subseteq (\xi, \Delta)$. Since (γ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed, then $cl(int(\gamma, \Delta))\backslash(\xi, \Delta) \in \check{I}$. Now, $(\delta, \Delta) \subseteq cl(int(\gamma, \Delta))$ implies that $cl(\delta, \Delta) \subseteq cl(int(\gamma, \Delta))$. Thus, $cl(int(\delta, \Delta))\backslash(\xi, \Delta) \subseteq cl(\delta, \Delta)\backslash(\xi, \Delta) \subseteq cl(int(\gamma, \Delta))\backslash(\xi, \Delta)$. So, $cl(int(\delta, \Delta))\backslash(\xi, \Delta) \in \check{I}$ from Definition 1.17. Thus, (δ, Δ) is $s\mathcal{S}ƅ^*\check{I}$ −closed.

The intersection of two $s\mathcal{S}ƅ^*\check{I}$ −closed sets need not be a $s\mathcal{S}ƅ^*\check{I}$ −closed as shown by the following example.

Example 2.10: In Example 2.2, the soft sets (ϑ, Δ), (δ, Δ) are sSƅ$^*\check{I}$ −closed. But $(\xi, \Delta) = (\vartheta, \Delta) \cap (\delta, \Delta)$ is not sSƅ$^*\check{I}$ −closed, where $(\xi, \Delta) = \{(\nabla_1, \emptyset), (\nabla_2, \{\varepsilon\})\}$.

Theorem 2.11: If (ϑ, Δ) is sSƅ$^*\check{I}$ −closed and (γ, Δ) is soft closed in a $\mathcal{S}$TS $(\mathcal{Z}, \mathfrak{W}, \varDelta)$. Then, $(\vartheta, \Delta) \cap (\gamma, \Delta)$ is sSƅ$^*\check{I}$ −closed.

Proof. Let $(\vartheta, \Delta) \cap (\gamma, \Delta) \subseteq (\delta, \Delta)$ and $(\delta, \varDelta)$ is a sSƅ* −open. Then $(\vartheta, \Delta) \subseteq (\delta, \Delta) \cup (\gamma, \Delta)^c$. Since (ϑ, Δ) is sSƅ$^*\check{I}$ −closed, so $cl(int(\vartheta, \Delta))\backslash((\delta, \Delta) \cup (\gamma, \Delta)^c) \in \check{I}$. Now, $cl\left(int((\vartheta, \Delta) \cap (\gamma, \Delta))\right) \subseteq cl(int(\vartheta, \Delta)) \cap cl(int(\gamma, \Delta)) \subseteq cl(int(\vartheta, \Delta)) \cap cl(\gamma, \Delta) = cl(int(\vartheta, \Delta)) \cap (\gamma, \Delta) = [cl(int(\vartheta, \Delta)) \cap (\gamma, \Delta)]\backslash(\gamma, \Delta)^c$. Thus, $cl\left(int((\vartheta, \Delta) \cap (\gamma, \Delta))\right)\backslash(\delta, \Delta) \subseteq [cl(int(\vartheta, \Delta)) \cap (\gamma, \Delta)]\backslash((\delta, \Delta) \cup (\gamma, \Delta)^c)$

$$\subseteq cl\big(int(\vartheta,\Delta)\big)\backslash\big((\delta,\Delta)\cup(\gamma,\Delta)^c\big)\in \check{\mathrm{I}}$$

So, $(\vartheta,\Delta)\cap(\gamma,\Delta)$ is sS ƅ*Ǐ −closed.

Theorem 2.12: Let $(\mathcal{Z},\mathfrak{W},\Delta)$ is $\mathcal{S}$TS and $(\gamma,\Delta),(\delta,\Delta)$ are sSƅ*Ǐ −closed. Then $(\gamma,\Delta)\cup(\delta,\Delta)$ are sSƅ*Ǐ −closed.

Proof. Let (γ,Δ) and (δ,Δ) are sSƅ*Ǐ −closed. Suppose that $(\gamma,\Delta)\cup(\delta,\Delta)\subseteq(\vartheta,\Delta)$ and (ϑ,Δ) is sSƅ* −open. Then $(\gamma,\Delta)\subseteq(\vartheta,\Delta)$ and $(\delta,\Delta)\subseteq(\vartheta,\Delta)$. Since (γ,Δ) and (δ,Δ) are sSƅ*Ǐ −closed sets, then $cl\big(int(\gamma,\Delta)\big)\backslash(\vartheta,\Delta)\in\check{\mathrm{I}}$ and $cl\big(int(\delta,\Delta)\big)\backslash(\vartheta,\Delta)\in\check{\mathrm{I}}$. Therefore, $cl\Big(int\big((\gamma,\Delta)\cup(\delta,\Delta)\big)\Big)\setminus(\vartheta,\Delta)=\big[cl\big(int(\gamma,\Delta)\big)\backslash(\vartheta,\Delta)\big]\cup\big[cl\big(int(\delta,\Delta)\big)\backslash(\vartheta,\Delta)\big]\in\check{\mathrm{I}}$.
Hence, we obtain that $(\gamma,\Delta)\cup(\delta,\Delta)$ are sSƅ*Ǐ −closed.

Theorem 2.13: Let $(\mathcal{D},\mho,\Delta)$ be a soft subspace of a $\mathcal{S}$TS $(\mathcal{Z},\mathfrak{W},\Delta)$, and $(\mathfrak{R},\Delta)$ is a soft subset of $(\mathcal{D},\mho,\Delta)$ and $(\gamma,\Delta)\subseteq(\mathfrak{R},\Delta)$ and (γ,Δ) is a sSƅ*Ǐ −closed in $(\mathcal{Z},\mathfrak{W},\Delta)$. Then, (γ,Δ) is a sSƅ*Ǐ$_{\mathcal{D}}$ −closed in $(\mathcal{D},\mho,\Delta)$.

Proof. Assume that $(\gamma,\Delta)\subseteq(\mathcal{L},\Delta)\cap(\mathfrak{R},\Delta)$ and $(\mathcal{L},\Delta)\in\mathfrak{W}$. Then $(\mathcal{L},\Delta)\cap(\mathfrak{R},\Delta)\in\mho$ and $(\gamma,\Delta)\subseteq(\mathcal{L},\Delta)$. Since (γ,Δ) is a sSƅ*Ǐ −closed in $(\mathcal{Z},\mathfrak{W},\Delta)$, then $cl\big(int(\gamma,\Delta)\big)\backslash(\mathcal{L},\Delta)\in\check{\mathrm{I}}$. Now,

$$\big[cl\big(int(\gamma,\Delta)\big)\cap(\mathfrak{R},\Delta)\big]\setminus[(\mathcal{L},\Delta)\cap(\mathfrak{R},\Delta)]=\big[cl\big(int(\gamma,\Delta)\big)\backslash(\mathcal{L},\Delta)\big]\cap(\mathfrak{R},\Delta)\in\check{\mathrm{I}}_{\mathcal{D}}$$

Thus, (γ,Δ) is a sSƅ*Ǐ$_{\mathcal{D}}$ −closed in $(\mathcal{D},\mho,\Delta)$.

3. Soft Strongly ƅ* −open via soft ideal

In this section, we define sSƅ* −open set via soft ideal in $\mathcal{S}$TSs.

Definition 3.1: A soft set (γ,Δ) in $\mathcal{S}$TS $(\mathcal{Z},\mathfrak{W},\Delta)$, is called a soft stongly ƅ*Ǐ −open set with respect to soft ideal Ǐ (sSƅ*Ǐ −open) if and only if its relative complement $(\gamma,\Delta)^c$ is sSƅ*Ǐ −closed in $(\mathcal{Z},\mathfrak{W},\Delta)$.

Example 3.2: In Example 2.2. the soft sets $(\gamma_1,\Delta)^c,(\gamma_2,\Delta)^c$ and $(\gamma_3,\Delta)^c$ are sSƅ*Ǐ −open where $(\gamma_1,\Delta)^c,(\gamma_2,\Delta)^c$ and $(\gamma_3,\Delta)^c$ are given by
$(\gamma_1,\Delta)^c=\{(\nabla_1,\mathcal{Z}),(\nabla_2,\emptyset)\}$,
$(\gamma_1,\Delta)^c=\{(\nabla_1,\emptyset),(\nabla_2,\{\mu\})\}$ and
$(\gamma_3,\Delta)^c=\{(\nabla_1,\{\varepsilon\}),(\nabla_2,\{\mu\})\}$.

Theorem 3.3: A soft set (ϑ,Δ) is sSƅ*Ǐ −open in a $\mathcal{S}$TS $(\mathcal{Z},\mathfrak{W},\Delta)$ if and only if $(\gamma,\Delta)\setminus(\mathcal{L},\Delta)\subseteq cl\big(int(\vartheta,\Delta)\big)$ for some $(\mathcal{L},\Delta)\in\check{\mathrm{I}}$, whenever $(\gamma,\Delta)\subseteq(\vartheta,\Delta)$ and (γ,Δ) is soft closed in $(\mathcal{Z},\mathfrak{W},\Delta)$.

Proof. $(\Rightarrow)$ Let $(\gamma,\Delta)\subseteq(\vartheta,\Delta)$ and (γ,Δ) is soft closed. Then $(\vartheta,\Delta)^c\subseteq(\gamma,\Delta)^c$, $(\vartheta,\Delta)^c$ is a sSƅ*Ǐ −closed and $(\gamma,\Delta)^c\in\mathfrak{W}$. By assumption, $cl(int(\vartheta,\Delta)^c)\setminus(\gamma,\Delta)^c\in\check{\mathrm{I}}$. Then $cl(int(\vartheta,\Delta)^c)\setminus(\gamma,\Delta)^c=(\eta,\Delta)$ for some $(\eta,\Delta)\in\check{\mathrm{I}}$. Thus, $cl(int(\vartheta,\Delta)^c)\setminus(\gamma,\Delta)^c=cl(int(\vartheta,\Delta)^c)\cap(\gamma,\Delta)=(\eta,\Delta)\in\check{\mathrm{I}}$. So,

$[cl(int(\vartheta,\Delta)^c) \ \cap \ (\gamma,\Delta)] \cup (\gamma,\Delta)^c \underset{E}{=} (\eta,\Delta) \cup (\gamma,\Delta)^c$. This implies that, $cl(int(\vartheta,\Delta)^c) \subseteq cl(int(\vartheta,\Delta)^c) \cup (\gamma,\Delta)^c = (\mathcal{L},\Delta) \cup (\gamma,\Delta)^c$. Hence, $cl(int(\vartheta,\Delta)^c) \subseteq (\eta,\Delta) \cup (\gamma,\Delta)^c$ for some $(\eta,\Delta) \in \check{I}$. Furthermore, $(\eta,\Delta) \cup (\gamma,\Delta)^c \subseteq [cl(int(\vartheta,\Delta)^c)]^c = cl\big(int(\vartheta,\Delta)\big)$. Therefore, $(\gamma,\Delta)\backslash(\eta,\Delta) = (\gamma,\Delta) \cap (\eta,\Delta)^c \subseteq cl\big(int(\vartheta,\Delta)\big)$.
$(\Leftarrow)$ Let $(\gamma,\Delta)^c \subseteq (\delta,\Delta)$ such that (δ,Δ) is sSƅ* −open. Then, $(\delta,\Delta)^c \subseteq (\vartheta,\Delta)$. By assumption, $(\delta,\Delta)^c\backslash(\Gamma,\Delta) \subseteq cl\big(int(\vartheta,\Delta)\big) = [cl(cl(\vartheta,\Delta)^c)]^c$ for some $(\Gamma,\Delta) \in \check{I}$. Thus, $cl(int(\vartheta,\Delta)^c) = cl(cl(\vartheta,\Delta)^c) \subseteq [(\delta,\Delta)^c\backslash(\Gamma,\Delta)]^c = (\delta,\Delta) \cup (\Gamma,\Delta)$. So,

$$cl(int(\vartheta,\Delta)^c)\backslash(\delta,\Delta) \subseteq [(\delta,\Delta) \cup (\Gamma,\Delta)] \cap (\delta,\Delta)^c = (\Gamma,\Delta) \cap (\delta,\Delta)^c \subseteq (\Gamma,\Delta) \in \check{I}$$

This shows that, $cl(int(\vartheta,\Delta)^c)\backslash(\delta,\Delta) \in \check{I}$. Therefore, $(\vartheta,\Delta)^c$is sSƅ$^*\check{I}$ −closed and hence (ϑ,Δ) is sSƅ$^*\check{I}$ −open.

Theorem 3.4:

(1) Every open soft set is sSƅ$^*\check{I}$ −open.

(2) Every soft $\check{I}$g −open set is sSƅ$^*\check{I}$ −open.

Proof. Immediate from Theorem 2.3.

The converse of the above theorem is not true in general as shall show in the following examples.

Example 3.5: In Example 2.2, the soft set (ψ,Δ) is sSƅ$^*\check{I}$ −open but not open soft set, where $(\psi,\Delta) = \{(\nabla_1,\emptyset),(\nabla_2,\{\mu\})\}$.

Example 3.6: In Example 2.5, the soft set (η,Δ) is sSƅ$^*\check{I}$ −open but not soft $\check{I}$g −open, where $(\eta,\Delta) = \{(\nabla_1,\mathcal{Z}),(\nabla_2,\{\varepsilon\})\}$.

The soft intersection (resp. union) of two sSƅ$^*\check{I}$ −open sets need not be a sSƅ$^*\check{I}$ −open as shown by the following example.

Example 3.7: In Example 2.2, the soft sets $(\gamma_1,\Delta)^c,(\gamma_2,\Delta)^c,(\gamma_3,\Delta)^c$ are sSƅ$^*\check{I}$ −open. But $(\mathcal{L},\Delta) = (\gamma_1,\Delta)^c \cup (\gamma_2,\Delta)^c$ is not sSƅ$^*\check{I}$ −open, where $(\mathcal{L},\Delta) = \{(\nabla_1,\mathcal{Z}),(\nabla_2,\{\mu\})\}$.

Theorem 3.8: If (γ,Δ) is sSƅ$^*\check{I}$ −open in a $\mathcal{S}$TS $(\mathcal{Z},\mathfrak{W},\Delta)$ and $cl\big(int(\gamma,\Delta)\big) \subseteq (\delta,\Delta) \subseteq (\gamma,\Delta)$, then (δ,Δ) is a sSƅ$^*\check{I}$ −open.

Proof. Let $(\eta,\Delta) \subseteq (\delta,\Delta)$ and (η,Δ) is a sSƅ$^*\check{I}$ −closed. Then, $(\eta,\Delta) \subseteq (\gamma,\Delta)$. Since (γ,Δ) is sS ƅ$^*\check{I}$ −open, then $(\delta,\Delta)\backslash cl\big(int(\eta,\Delta)\big) \subseteq (\gamma,\Delta)\backslash cl\big(int(\eta,\Delta)\big) \in \check{I}$. It follows that, $(\delta,\Delta)\backslash cl\big(int(\eta,\Delta)\big) \in \check{I}$. Thus, (δ,Δ) is sSƅ$^*\check{I}$ −open.

Theorem 3.9: A soft set (ϑ,Δ) is sSƅ$^*\check{I}$ −closed in a $\mathcal{S}$TS $(\mathcal{Z},\mathfrak{W},\Delta)$ if and only if $cl\big(int(\vartheta,\Delta)\big)\backslash(\vartheta,\Delta)$ is sSƅ$^*\check{I}$ −open.

Proof. (⇒) Let $(\gamma,\Delta) \subseteq cl\big(int(\vartheta,\Delta)\big)$ and (γ,Δ) be a soft closed set. Then, $(\gamma,\Delta) \in \check{I}$ from Theorem 2.9 so there exists $(\sigma,\Delta) \in \check{I}$ such that $(\gamma,\Delta)\backslash(\sigma,\Delta) = \widetilde{\emptyset}$. Thus, that $(\gamma,\Delta)\backslash(\sigma,\Delta) = \widetilde{\emptyset} \subseteq int(cl[cl(\vartheta,\Delta)\backslash(\vartheta,\Delta)])$. Hence, $cl\big(int(\vartheta,\Delta)\big)\backslash(\vartheta,\Delta)$ is a sSƅ*Ĭ −open from Theorem 3.3.

(⇐) Let $(\vartheta,\Delta) \subseteq (\delta,\Delta)$ such that (δ,Δ) is sSƅ*Ĭ −open. Then, $cl\big(int(\vartheta,\Delta)\big) \cap (\delta,\Delta)^c \subseteq cl\big(int(\vartheta,\Delta)\big) \cap (\vartheta,\Delta)^c = cl\big(int(\vartheta,\Delta)\big)\backslash(\vartheta,\Delta)$. By hypothesis, $\big[cl\big(int(\vartheta,\Delta)\big) \cap (\delta,\Delta)^c\big]\backslash (\sigma,\Delta) \subseteq int\big(cl\big[cl\big(int(\vartheta,\Delta)\big)\backslash(\vartheta,\Delta)\big]\big) = \widetilde{\emptyset}$, for some $(\sigma,\Delta) \in \check{I}$ from Theorem 3.3. So, $cl\big(int(\vartheta,\Delta)\big) \cap (\delta,\Delta)^c \subseteq (\sigma,\Delta) \in \check{I}$. Thus, $cl\big(int(\vartheta,\Delta)\big)\backslash(\delta,\Delta) \in \check{I}$. So, (ϑ,Δ) is a sS ƅ*Ĭ −closed.

4. SS ƅ* −continuous via soft ideal

In this section, we introduce a sSƅ* −continuous function with respect to a soft ideal in $\mathcal{S}$TSs.

Definition 4.1: Let $\Omega: (\mathcal{Z},\mathfrak{W},\Delta) \to (\mathcal{D},\mho,\Theta)$ be a soft mapping. If $\Omega^{-1}\big((\delta,\Delta)\big)$ is sSƅ*Ĭ −open in $(\mathcal{Z},\mathfrak{W},\Delta)$ for each soft open set (δ,Δ) of $(\mathcal{D},\mho,\Theta)$, then Ω is called soft strongly ƅ*Ĭ −continuous function.

Corollary 4.2: Let Ω: $(\mathcal{Z},\mathfrak{W},\Delta) \to (\mathcal{D},\mho,\Theta)$ be a soft function. Then:

1- Every soft continuous function is sSƅ*Ĭ −continuous functions.

2- Every soft Ĭg −continuous is sSƅ*Ĭ −continuous function.

Proof. Immediate from Theorem 3.4.

The converse of the above theorem is not true in general as shall show in the following example.

Example 4.3: Let $\mathcal{Z} = \{\varepsilon,\mu\}$ and $\Delta = \{\nabla_1,\nabla_2\}$. Let $(\gamma_1,\Delta),(\gamma_2,\Delta)$ be two soft sets where
$(\gamma_1,\Delta) = \{(\nabla_1,\{\varepsilon\})\}$, $\quad (\gamma_2,\Delta) = \{(\nabla_1,\{\varepsilon\}),(\nabla_2,\{\mu\})\}$.
$\mathfrak{W} = \{\tilde{\mathcal{Z}},\widetilde{\emptyset},(\gamma_1,\Delta),(\gamma_2,\Delta)\}$ is the soft topology over $\mathcal{Z}$.
Let $\check{I} = \{\widetilde{\emptyset}\}$ be a soft ideal on $\mathcal{Z}$.
Let $\mathcal{D} = \{\sigma,\rho\}$ and $\Theta = \{\varrho_1,\varrho_2\}$,
$\mho = \{\widetilde{\mathcal{D}},\widetilde{\emptyset},(\delta,\Theta)\}$ is soft topology on $\mathcal{D}$. Where $(\delta,\Theta) = \{(\varrho_1,\{\sigma\}),(\varrho_2,\{\sigma\})\}$. Then let $\Omega: (\mathcal{Z},\mathfrak{W},\Delta) \to (\mathcal{D},\mho,\Theta)$ be a soft function and $u: \mathcal{Z} \to \mathcal{D}$ and $p: \Delta \to \Theta$ denoted by $u(\varepsilon) = \sigma, \quad u(\mu) = \rho, \quad p(\nabla_1) = \varrho_1, \quad p(\nabla_2) = \varrho_2$.
Let take $(\vartheta,\Delta) = \{(\nabla_1,\{\varepsilon\}),(\nabla_2,\{\varepsilon\})\}$. Then Ω is sSƅ*Ĭ −continuous but not soft continuous. Since If $\Omega^{-1}\big((\delta,\Theta)\big) = (\vartheta,\Delta)$ is sSƅ*Ĭ −open set but not soft open set.

Example 4.4: Let $\mathcal{Z} = \{\varepsilon,\mu,\omega\}$ and $\Delta = \{\nabla_1,\nabla_2\}$. Let $(\gamma_1,\Delta),(\gamma_2,\Delta)$ and (γ_3,Δ) be soft sets where:
$(\gamma_1,\Delta) = \{(\nabla_1,\{\varepsilon\},(\nabla_2,\{\varepsilon\}))\}$,
$(\gamma_2,\Delta) = \{(\nabla_1,\{\mu\}),(\nabla_2,\emptyset)\}$ and

$(\gamma_3, \Delta) = \{(\nabla_1, \{\varepsilon, \omega\}), (\nabla_2, \{\varepsilon\})\}$ and
$\mathfrak{W} = \{\tilde{\mathcal{Z}}, \widetilde{\emptyset}, (\gamma_1, \Delta), (\gamma_2, \Delta), (\gamma_3, \Delta)\}$ is the soft topology over $\mathcal{Z}$.
Let $\breve{\mathrm{I}} = \{\widetilde{\emptyset}\}$ be a soft ideal on $\mathcal{Z}$.
Let $\mathcal{D} = \{\sigma, \rho, \pi\}$ and $\Theta = \{\varrho_1, \varrho_2\}$,
$\mho = \{\widetilde{\mathcal{D}}, \widetilde{\emptyset}, (\delta, \Theta)\}$ is soft topology on $\mathcal{D}$. Where $(\delta, \Theta) = \{(\varrho_1, \{\sigma\}), (\varrho_1, \emptyset)\}$. Then let $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ be a soft function and $u: \mathcal{Z} \to \mathcal{D}$ and $p: \Delta \to \Theta$ denoted by $u(\varepsilon) = \sigma$, $u(\mu) = \rho$, $u(\omega) = \pi$, $p(\nabla_1) = \varrho_1$, $p(\nabla_2) = \varrho_2$.
Let take $(\vartheta, \Delta) = \{(\nabla_1, \{\varepsilon\}), (\nabla_2, \emptyset)\}$. Then Ω is sSƅ*Ĭ −continuous but not soft Ĭg −continuous. Since If $\Omega^{-1}((\delta, \Theta)) = (\vartheta, \Delta)$ is sSƅ*Ĭ −closed set but not soft Ĭg −closed set.

Definition 4.5: Let $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ be a soft mapping. If $\Omega^{-1}((\delta, \Delta))$ is sSƅ*Ĭ −closed in $(\mathcal{Z}, \mathfrak{W}, \Delta)$ for each sSƅ* −closed set (δ, Δ) of $(\mathcal{D}, \mho, \Theta)$, then Ω is said to be soft strongly ƅ*Ĭ −irresolute function.

Theorem 4.6: A map $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ is sSƅ* $-$ *irresolute* if and only if the inverse image of every soft strongly ƅ*Ĭ $-$ *open* set in $\mathcal{D}$ is soft strongly ƅ* $-$ *open* in $\mathcal{Z}$.
Proof. Clearly.

Theorem 4.8: Every sSƅ*Ĭ −irresolute mapping is sSƅ*Ĭ −continuous functions.
Proof. Let $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ be a sSƅ*Ĭ −irresolute mapping. Let (δ, Δ) be a soft closed set in $\mathcal{D}$. Then (δ, Δ) is sSƅ*Ĭ −closed set in $\mathcal{D}$. Since Ω is sSƅ*Ĭ −irresolute mapping, $\Omega^{-1}((\delta, \Delta))$ is sSƅ*Ĭ −closed set in $\mathcal{Z}$. Hence, Ω is sSƅ*Ĭ −ccontinuous function.

Definition 4.9: A soft mapping $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ is said to be soft strongly ƅ*Ĭ −open (soft strongly ƅ*Ĭ −closed) map if the image of every soft *open* (soft *closed*) set in $\mathcal{Z}$ is sSƅ*Ĭ −open (sSƅ*Ĭ −closed) set in $\mathcal{D}$.

Remark 4.10: (1) Every soft open map is sSƅ*Ĭ −open.
(2) Every sSƅ*Ĭ −open map is sSƅ* −open.

In the following examples as observed the converses are not true.
Example 4.11: Let $\mathcal{Z} = \{\varepsilon, \mu\}$ and $\Delta = \{\nabla_1, \nabla_2\}$. $\mathfrak{W} = \{\tilde{\mathcal{Z}}, \widetilde{\emptyset}, (\gamma, \Delta)\}$ is the soft topology over $\mathcal{Z}$ where $(\gamma, \Delta) = \{(\nabla_1, \{\varepsilon\})\}$. Let $\breve{J} = \{\widetilde{\emptyset}\}$ be a soft ideal on $\mathcal{Z}$.
Also, let $\mathcal{D} = \{\sigma, \rho\}$ and $\Theta = \{\varrho_1, \varrho_2\}$,
$\mho = \{\widetilde{\mathcal{D}}, \widetilde{\emptyset}, (\delta_1, \Theta), (\delta_2, \Theta)\}$ is soft topology on $\mathcal{D}$. Where $(\delta_1, \Theta) = \{(\varrho_1, \{\sigma\}), (\varrho_2, \{\rho\})\}$ and $(\delta_2, \Theta) = \{(\varrho_1, \{\rho\}), (\varrho_2, \{\sigma\})\}$. Then the soft function $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \to (\mathcal{D}, \mho, \Theta)$ where $u: \mathcal{Z} \to \mathcal{D}$ and $p: \Delta \to \Theta$ denoted by
$u(\varepsilon) = \sigma$, $u(\mu) = \rho$, $u(\omega) = \pi, p(\nabla_1) = \varrho_1$, $p(\nabla_2) = \varrho_2$ is sSƅ*Ĭ −open but not soft open. Since for each soft open (γ, Δ) in $\mathcal{Z}$, $\Omega((\gamma, \Delta)) = (\vartheta, \Theta) = \{(\varrho_1, \{\sigma\})\}$ is sSƅ*Ĭ −open set but not soft open set.

Example 4.12: Let $\mathcal{Z} = \{\varepsilon, \mu\}$ and $\Delta = \{\nabla_1, \nabla_2\}$. $\mathfrak{W} = \{\tilde{\mathcal{Z}}, \tilde{\emptyset}, (\gamma, \Delta)\}$ is the soft topology over $\mathcal{Z}$ where $(\gamma, \Delta) = \{(\nabla_1, \mathcal{Z}), (\nabla_1, \{\varepsilon\})\}$. Also, let $\mathcal{D} = \{\sigma, \rho\}$ and $\Theta = \{\varrho_1, \varrho_2\}$, $\mho = \{\tilde{\mathcal{D}}, \tilde{\emptyset}, (\delta_1, \Theta), (\delta_2, \Theta)\}$ is soft topology on $\mathcal{D}$, where $(\delta_1, \Theta) = \{(\varrho_1, \{\sigma\})\}$ and $(\delta_2, \Theta) = \{(\varrho_1, \{\sigma\}), (\varrho_2, \{\rho\})\}$. Let $\check{J} = P(\mathcal{D})$ be a soft ideal on $\mathcal{D}$. Then the soft function $\Omega: (\mathcal{Z}, \mathfrak{W}, \Delta) \rightarrow (\mathcal{D}, \mho, \Theta)$ where $u: \mathcal{Z} \rightarrow \mathcal{D}$ and $p: \Delta \rightarrow \Theta$ denoted by
$u(\varepsilon) = \sigma, \quad u(\mu) = \rho, \quad u(\omega) = \pi, p(\nabla_1) = \varrho_1, \quad p(\nabla_2) = \varrho_2$ is sSƅ*Ĭ −open but not sSƅ* −open. Since for each soft open (γ, Δ) in $\mathcal{Z}$, $\Omega((\gamma, \Delta)) = (\vartheta, \Theta) = \{(\varrho_1, \mathcal{D}), (\varrho_1, \{\sigma\})\}$ is sSƅ*Ĭ −open set but not sSƅ* −open set.

5. Conclusions

In this work, we study the sSƅ*Ĭ −closed sets and sSƅ*Ĭ −open sets and some of their properties and investigated. Also, we define the sSƅ*Ĭ −continuous and sSƅ*Ĭ −irresolute. In future, more general types of sSƅ*Ĭ −closed sets may be defined and using of them characterizations related with soft separation axioms and soft continuity may be studied.

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