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The Impact of Volume Fraction and Nanoparticles in the Flow of Nanofluids along the Boundary Layer Over a Flat Plate

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Abstract:
Introduction: The forced convective, steady, viscous, nonlinear boundary layer flow of a two-dimensional problem over a flat plate with heat transfer nanofluids. The flow is laminar. Two different types of nanoparticles, namely, Titanium dioxide (TiO_2) and Silver (Ag) are used to prepare nanofluids with base fluid is chosen as water. Nanofluids has a large number of applications in the area of biomedical, cooling and tribological applications.

Objectives: To study about the effect different nanoparticles and their volume fraction in the flow of nanofluids along the boundary layer over a flat plate. Analyse the changes in the viscosity, density, specific heat capacity, thermal conductivity, Prandtl number, skin friction coefficient and Nusselt number.

Methods: The flow of the fluid that are governed by the PDEs that are nonlinear are transformed into the ODEs that are nonlinear by employing similarity transformations, which then solved by the use of MATLAB bvp5c solution procedure.

Results: Both thermal conductivity and heat transfer coefficient increases with the increase in the volume fraction of the corresponding nanoparticle. Also, Silver-water nanofluid has the high thermal conductivity and heat transfer rate when compared with Titanium dioxide-water nanofluid.

Conclusions: Silver water nanofluid surpasses titanium dioxide water nanofluid in terms of thermal conductivity and heat transfer rate. Moreover, the physical attributes like viscosity, density, and thermal conductivity of nanofluids escalate proportionally with the volume fraction of nanoparticles. Conversely, specific heat capacity and Prandtl number exhibit an inverse relationship with the volume fraction of nanoparticles, decreasing as it increases.

Keywords: Nanofluids; flat plate; thermal conductivity; skin friction coefficient; Nusselt number.

Nomenclature

ϕ	Volume fraction of nanoparticle	k_f	Thermal conductivity of base liquid
$(Re_x)_{nf}$	Local Reynolds number	$(C_p)_{nf}$	Specific heat capacity of nanofluid

ρ_{nf}	Density of nanofluid	P_r	Prandtl number
μ_f	Dynamic viscosity of base liquid	C_f	Skin friction coefficient
μ_{nf}	Dynamic viscosity of nanofluid	Nu	Nusselt number
$(C_p)_f$	Specific heat capacity of base liquid	ρ_f	Density of base liquid
$(C_p)_s$	Specific heat capacity of nanoparticle	v_f	Kinematic viscosity of base liquid
k_s	Thermal conductivity of nanoparticle	ρ_s	Density of nanoparticle
k_{nf}	Thermal conductivity of nanofluid	α_f	Thermal diffusivity of base liquid

1. Introduction

In today's modern world, ultra-high-performance cooling is one of the most vital needs of many industrial technologies. Indeed, the concept of dispersing solid particles in liquids to enhance their thermal conductivity has been known for a while. This is because, solid particles generally have higher thermal conductivities than liquids. One of the significant challenges with dispersing solid particles in liquids to enhance thermal conductivity is sedimentation. In order to overcome this limitation, Choi and Eastman introduced the nano-suspensions [1]. Nanofluids were prepared by the stable suspension of nanometre sized particles (usually size less than 100nm) in a base fluid. This type of fluid has enhancive thermal properties than the conventional fluids.

Chein R and Huang G, presented the thermoelectric cooler applications in electronic cooling [2]. Xuan and Roetzel [3] and Xuan and Li [4] experimentally show that, adding nanoparticles to a fluid, like water and oil, will augment the thermal conductivity of base liquid.

In this work, forced convective, viscous, nonsequential boundary layer flow of a two-dimensional problem over a flat surface with heat transfer nanofluids is explored. Titanium dioxide (TiO_2) and Silver (Ag) are the nanoparticles used to prepare nanofluids and water is considered as the base fluid. The Pr value of water is taken as 7.02 with base temperature $293K$. The flow is modeled using PDEs which are nonlinear. By employing the similarity transformations, these heading equations are converted into ODEs and are then solved using the MATLAB bvp5c solution procedure.

2. Objectives

In this work, forced convective, viscous, nonsequential boundary layer flow of a two-dimensional problem over a flat surface with heat transfer nanofluids is explored. Titanium dioxide (TiO_2) and Silver (Ag) are the nanoparticles used to prepare nanofluids and water is considered as the base fluid. To investigate the effect different nanoparticles and their volume fraction in the flow of nanofluids along the boundary layer over a flat plate. Also, analyse the changes in the viscosity, density, specific heat capacity, thermal conductivity, Prandtl number, skin friction coefficient and Nusselt number.

3. Methods

The infinite flat plate oriented parallel to the uniform velocity U_∞ of the liquid flow. The temperature and velocity which are away from the plate are represented as T_∞ and U_∞, respectively. Consider the origin of the coordinates be at the fore edge of the plate, the x- and y- axes are taken aligned with the uniform flow and perpendicular to the plate respectively. A pair of nanofluids, namely Titana-Water nanofluid and Silver-Water nanofluid, are considered in this study. Both the nanoparticles are off the spherical shape with diameter $100nm$. The measurements of viscosity, thermal conductivity specific heat capacity, Prandtl number and density for different volume fraction of different nanofluids with nanoparticle are numerically found out at the first stage. Then changes occurred in velocity field, Nusselt number and skin friction coefficient are analyzed.

The effective thermo-physical properties of nanofluids are given by Table 1.1 [5], [6] and the given physical properties of the base liquid(water) and nanoparticles are exhibited in Table 1.2 [7], [8].

Table 1.1

Effective density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$
Specific heat capacity	$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$
Dynamic viscosity	$\mu_{nf} = \mu_f(1-\emptyset)^{-2.5}$, where $\mu_f = 0.001002$
Thermal conductivity	$k_{nf} = k_f\left(\frac{\left({k_s}/{k_f}\right)+(n-1)-(n-1)\phi\left(1-\left({k_s}/{k_f}\right)\right)}{\left({k_s}/{k_f}\right)+(n-1)+\phi\left(1-\left({k_s}/{k_f}\right)\right)}\right)$, where $n = \frac{3}{\Psi}, \Psi$ is sphericity, $\Psi = 1$ for sphere
Prandtl number	$Pr_{nf} = \frac{\mu_{nf}(C_p)_{nf}}{k_{nf}}$

Table 1.2: Physical properties of water, TiO_2, and Ag

	ρ (Kg/m^3)	C_p $(J/Kg.K)$	k $(W/m.K)$
Water	997.1	4179	0.613
TiO_2	4250	686.2	8.9538
Ag	10500	235	429

The governing equations of the flow are [9]:

Continuity equation:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon_{nf}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Energy equation:
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\rho_{nf}(C_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

where u and v are the velocity components in the $x\ and\ y$ directions, T- temperature. The contribution of viscous dissipation is disregarded form the energy equation. The boundary constraints are given by [9]:

$$At\ y=0, u=v=0,\ T=T_w$$

$$As\ y\to\infty, u=U_\infty, T=T_\infty\ . \tag{4}$$

To solve the problem, the following dimensionless variables are implemented [10]:

$$\Psi(x,y)=\left(U_\infty v_f x\right)^{\frac{1}{2}}f(\eta),\qquad \eta=y\left(\frac{U_\infty}{v_f x}\right)^{\frac{1}{2}},\qquad \theta=\frac{T-T_\infty}{T_W-T_\infty}, \tag{5}$$

here $\Psi(x,y)$ is the stream function which satisfies the equation for continuity with $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$; θ is the dimensionless temperature.

By employing these variables, the components of velocity can be described in terms of the newly defined variables

$$u=U_\infty f'(\eta) \tag{6}$$

$$v=\frac{1}{2}\left(\frac{U_\infty v_f}{x}\right)^{\frac{1}{2}}(\eta f'(\eta)-f(\eta)) \tag{7}$$

Through the application of these transformations, the energy and momentum equations can be reduced to the ordinary differential equations which are nonlinear as:

$$f'''+\frac{1}{2}\left((1-\phi)^{2.5}\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\right)ff''=0 \tag{8}$$

$$\theta''+\frac{1}{2}\frac{\left(\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[(1-\phi)+\phi\frac{(C_p)_s}{(C_p)_f}\right]\right)(Pr)_f f\theta'}{\left(k_{nf}/k_f\right)}=0 \tag{9}$$

where $(Pr)_f=\frac{v_f}{\alpha_f}$, $v_f=\frac{\mu_f}{\rho_f}$ and $\alpha_f=\frac{k_f}{\rho_f(C_p)_f}$

The boundary conditions are then reduced to

$$\eta=0, f=0, f'=0, \theta=1$$

$$\eta\to\infty, f'=1, \theta=0 \tag{10}$$

The dimensionless quantities like Nusselt number Nu and skin friction coefficient C_f are considered in this study. These quantities are defined as [11]

$$C_f = \frac{\tau_w}{\rho_f {U_\infty}^2}, \text{ where } \tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{11}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \text{ where } q_w = -k_{nf} \left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

By equation (5), we get

$$C_f (Re_x)^{\frac{1}{2}} = \frac{1}{(1-\phi)^{2.5}} f''(0) \tag{13}$$

$$Nu(Re_x)^{-\frac{1}{2}} = -\frac{k_{nf}}{k_f} \theta'(0) \tag{14}$$

where $Re_x = \frac{U_\infty x}{\upsilon_f}$, is the local Reynolds number.

The values that obtained for specific heat capacity, density, viscosity, thermal conductivity by the numerical method of both the nanofluids are found using MATLAB. The set of differential equations which are nonlinear $(8), (9)$, could be derived through similarity transformations, subject to the boundary conditions (10) are solved using MATLAB bvp5c solution procedure.

4. Results

Analysing the forced convective boundary layer flow of various nanofluids across a flat plate in the two-dimensional problem is analysed. The numerical values of thermo-physical characteristics of different nanofluids for different volume fraction of nanoparticle are noted. Also, the variations observed in the velocity field are examined through the Nusselt number and skin friction coefficient.

An alternative expression for the impact of volume fraction on the density of TiO_2-H_2O and Ag-H_2O nanofluids are exhibited in Figure 1.1. The figure illustrates a clear trend: as the volume fraction increases, so the density of the nanofluid. Also, the increase is more in the case of Ag--water nanofluid. This is because, Ag particles are denser than TiO_2 nanoparticles. Figure 1.2 shows that the viscosity is the same for both the nanofluid for a fixed volume fraction. The viscosity of the nanofluid is solely depended on the viscosity of base liquid. Further, the nanofluid’s viscosity is directly proportional to volume fraction.

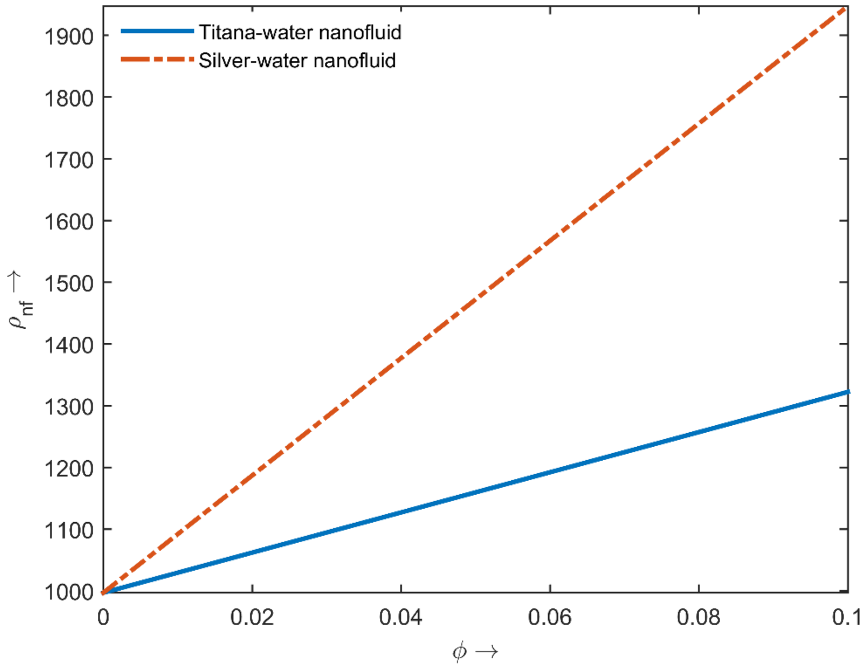

Figure 1.1: Density of TiO_2-H_2O and Ag- H_2O nanofluids vs ϕ

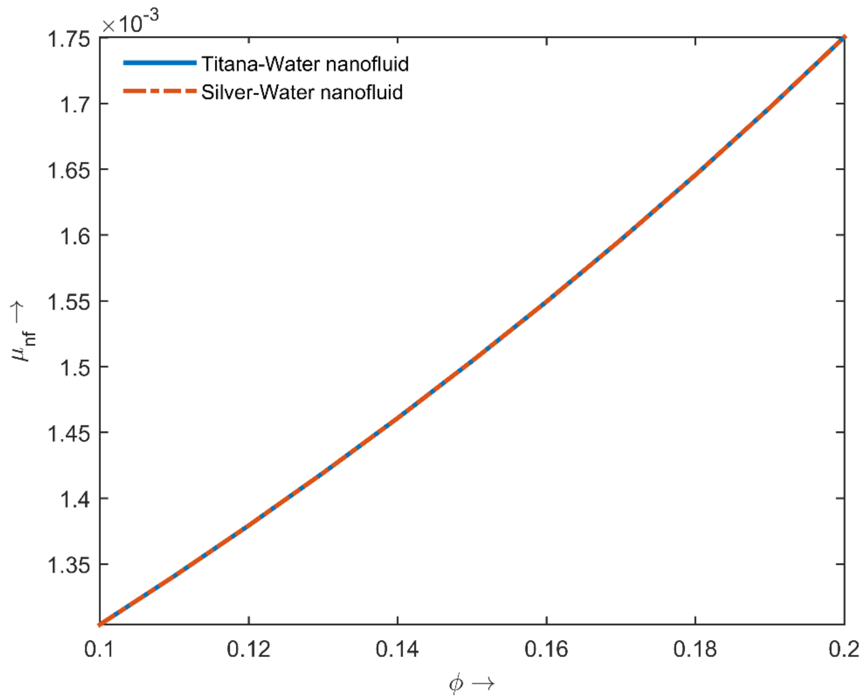

Figure 1.2: Viscosity of TiO_2- H_2O and Ag- H_2O nanofluids vs ϕ

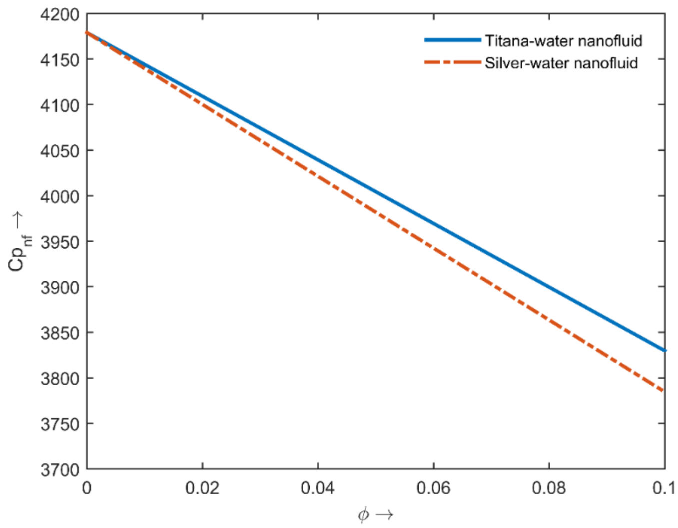

Figure 1.3: $(C_p)_{nf}$ of TiO_2- H_2O and Ag- H_2O nanofluids vs ϕ

The variation specific heat capacity $(C_p)_{nf}$ of nanofluid against the volume fraction is depicted in Figure 1.3. It is evident that as the volume fraction increases, $(C_p)_{nf}$ decreases. Also, the decrease is slightly more for Ag-water nanofluid. Figure 1.4 gives the variation in the thermal conductivity of nanofluids with respect to the volume fraction of nanoparticles. The data suggests that Ag-water nanofluid exhibits sufficiently greater thermal conductivity compared to TiO_2-water nanofluid. Further, k_{nf} increases with the increase in ϕ. The variation in the Prandtl number of different nanofluids against ϕ is depicted in Figure 1.5. It is clear that Pr decreases with the increase in ϕ. Ag-water nanofluid shows much more decrease in Pr with increasing ϕ, as compared to TiO_2-water nanofluid.

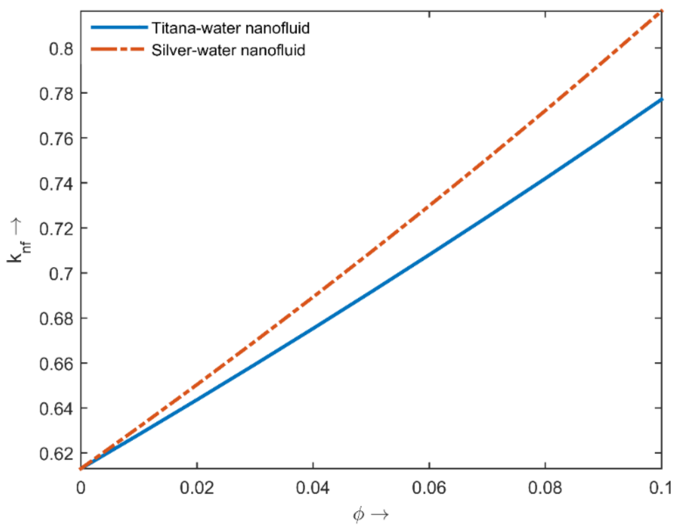

Figure 1.4: Thermal conductivity of TiO_2-H_2O and Ag- H_2O nanofluids vs ϕ

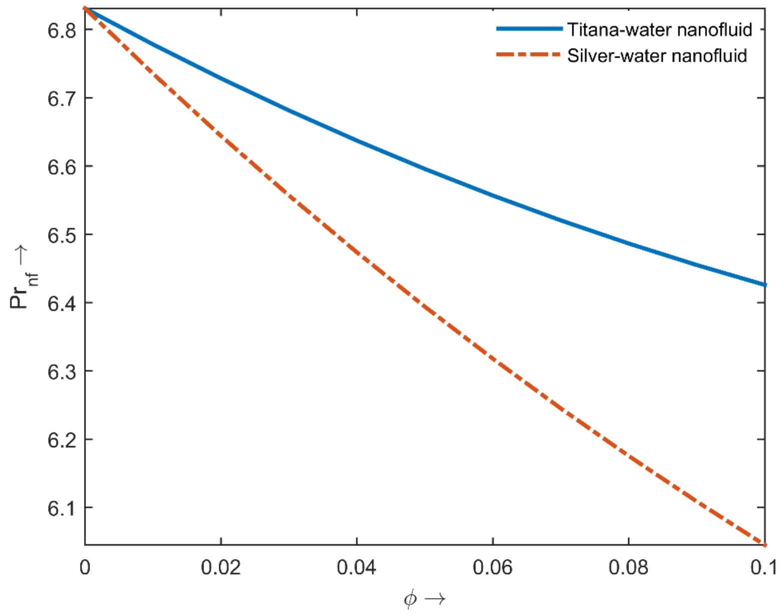

Figure 1.5: Prandtl number of TiO_2- H_2O and Ag- H_2O nanofluids vs ϕ

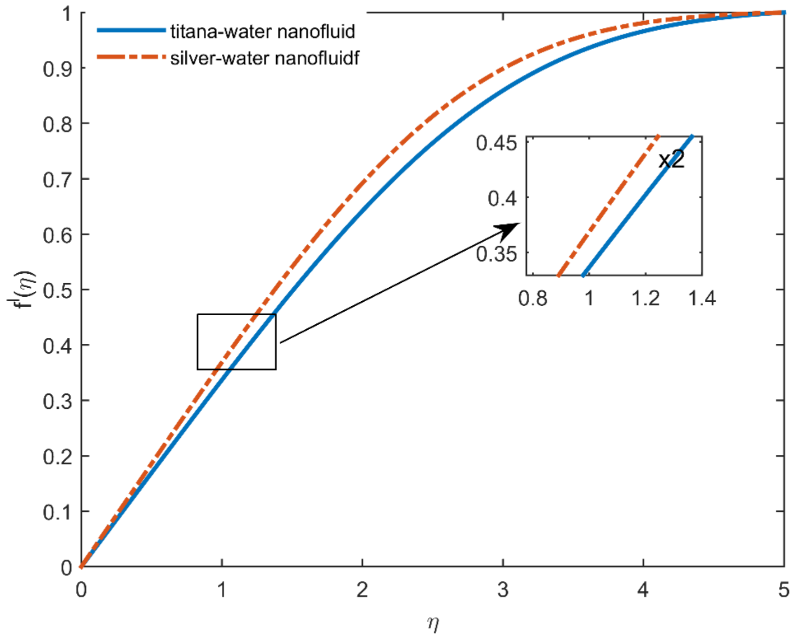

Figure 1.6(a): Change in velocity profile of two nanofluids for $\phi = 0.04$

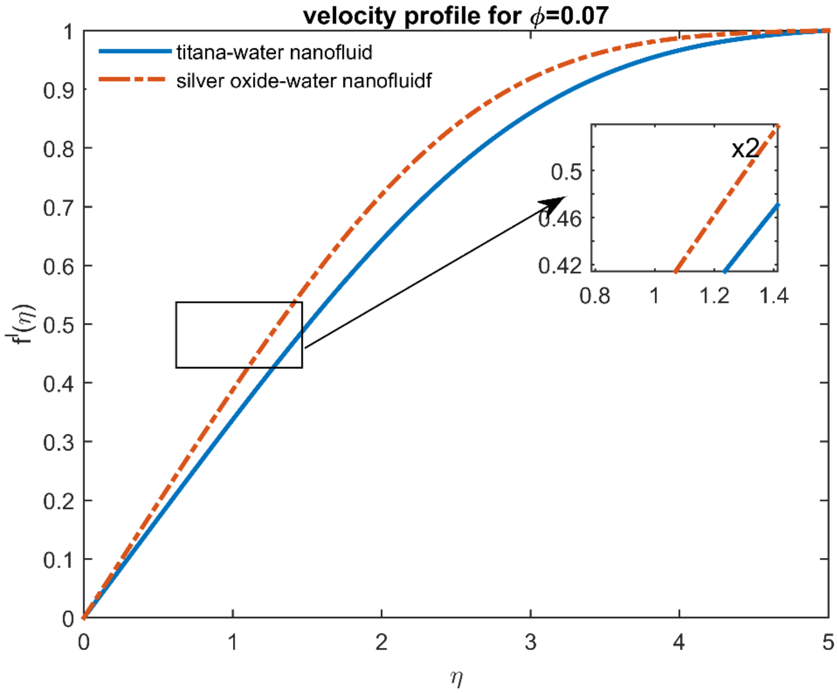

Figure 1.6(b): Change in velocity profile of two nanofluids for $\phi = 0.07$

The change in the velocity profile for both the nanofluids at fixed volume fraction values ($\phi = 0.04$ *and* $\phi = 0.07$) are displayed in Figure 1.6(a) and Figure 1.6(b), respectively. Figures shows that, the velocity profile is slightly more for the Ag-water nanofluid when compared with TiO_2-water nanofluid. The table 1.3 offers another perspective on how the non-dimensional skin friction coefficient changes for both nanofluids as the volume fraction varies. The trend shows that with higher volume fraction ϕ, there is a noticeable increase in the skin friction coefficient. Additionally, the skin friction coefficient is higher for Ag-water nanofluid as compared to TiO_2-water nanofluid.

Table 1.3: Skin friction coefficient

ϕ	TiO_2-water nanofluid	Ag-water nanofluid
0.01	0.34578	0.35508
0.02	0.35557	0.37407
0.03	0.36554	0.39313
0.04	0.3757	0.41227
0.05	0.38606	0.43151

The variation in the Nusselt number for the two nanofluids is displayed in Table 1.4. There is a noted increase in the Nusselt number as ϕ increases. The Nusselt number is more for Ag-water nanofluid for a fixed volume fraction of nanoparticle.

Table 1.4: Local Nusselt number

ϕ	TiO_2-water nanofluid	Ag-water nanofluid
0.01	0.66578	0.68716
0.02	0.68234	0.72464
0.03	0.69887	0.7617
0.04	0.71536	0.79839
0.05	0.73182	0.83474

5. Discussion

The effect of TiO_2-water and Ag-water nanofluids is investigated. This investigation focuses on a two-dimensional forced convective, laminar, nonlinear boundary layer flow over a flat plate. Observations reveal alterations in both the thermal and velocity fields. Additionally, variations are observed in the skin friction coefficient and Nusselt number across different nanoparticles. It's evident from the research that silver water nanofluid surpasses titanium dioxide water nanofluid in terms of thermal conductivity and heat transfer rate. Moreover, the physical attributes like viscosity, density, and thermal conductivity of nanofluids escalate proportionally with the volume fraction of nanoparticles. Conversely, specific heat capacity and Prandtl number exhibit an inverse relationship with the volume fraction of nanoparticles, decreasing as it increases.

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