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A Note on Hybrid Coset of a Nearring

P. Narasimha Swamy [1a], Bhurgula Harika [1b], K. Vijay Kumar[2c], B. Jyothi[3d] and K. Rajani[1e]

[1]Department of Mathematics, School of Science, GITAM Deemed to be University, Hyderabad Campus-502329, India

[2]Department of Mathematics, KU College of Engineering and Technology, Kakatiya University, Warangal-506009, India

[3]Department of Mathematics, Telangana Mahila Vishwavidyalayam, Koti, Hyderabad-500095, India

Email: [a] swamy.pasham@gmail.com
[b] harika.burgula84@gmail.com
[c] vijay.kntm@gmail.com
[d] jyothireddydumbala@gmail.com
[e] rajanireddy2u@gmail.com

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Abstract:

The present study intends to learn hybrid coset of a nearring. It is explained with the adequate definitions and theorems of the hybrid coset of a nearing and near ring homomorphism. It has been demonstrated that hybrid coset of a nearring is a nearring epimorphism with kernel. Further, we established some important fundamental results in terms of hybrid structure corresponding to the nearrings.

Keywords: Hybrid structure, Near ring, coset, Hybrid coset

1. Introduction

A nearring is an algebraic system connected to two binary operations, which gratifies all of a ring's axioms, with the conceivable exception of one distributive law. The very idea of nearring was first adapted by G. Pilz [1]. Nearrings have been the subject of investigation by Nobusawa[2], Bh Satyanarayana [3] and T. Srinivas [4]. In 1965, L.A. Zadeh [5] introduced new concept about fuzzy set. This concept highlighting the membership status of an indeterminate or fuzzy set. In this concept, membership status is defined as a function whose value is in the interval [0, 1]. Fuzzy set theory is established in many directions by many scholars and has got evoked great interest in the minds of many researchers who are working in different fields of mathematics. The theory of fuzzy sets has found many applications, including engineering, robotics design, computer modelling, and water resource planning. A hesitant fuzzy set is an extraordinary tool for disclosing people's hesitancy in every life and for handling with uncertainty, which could be suitably and matching way labelled in terms of the decision makers' opinions. An extensive range of existing theories, like the probability theory, theory of fuzzy set, vague sets, interval mathematics theory, theory of rough set, etc., are observed to deal a variety of problems in many domains that which require data with uncertainties. V. Torra [6] established the perception of hesitant fuzzy sets. All these theories have their own limitations and difficulties which are elevated already [7]. To be free from these difficulties, D. Molodtsov [7] familiarized the soft set theory as a new mathematical tool for handling with uncertainties that is free from the difficulties. Molodtsov effectively applied the theory of soft set in many directions, such as functions' smoothness, theory of game, Riemann integration, Perron integration operations research, probability, measurement theory and many other.

As a parallel circuit of fuzzy sets and soft sets (or, hesitant fuzzy sets), Jun, Song and Muhiuddin [8] proposed the idea of hybrid structures in a set of parameters over an initial universe set, and illustrating numerous properties. Using this idea, they initiated the idea of a hybrid gamma near ring, hybrid ideal of a gamma near ring. B. Elavarasan [9] deliberated hybrid structures applied to ideals in near-rings. Saima Anis [10] has explored hybrid ideals in semigroups. M. Himaya Jaleela Begum [11] explored hybrid fuzzy bi-ideals in near-rings. S. Abou-zaid [12] and S.D. Kim [13] were developed fuzzy ideals of near rings. P. Narasimha swamy [14] has developed sim of fuzzy ideals of Γ-near-rings. K. Vijay Kumar [15] has proposed the idea on bipolar fuzzy quasi ideals and bipolar N-subgroups of Near rings. Satyanarayana Bhavanari [16] has explored on fuzzy cosets of gamma near rings. T. Srinivas [17], Harika Bhurgula [18] and B. Jyothi [19,20] have established the concepts on near algebra. In which I have inspired to study on near ring concepts. In the current study, we acquaint with the conception of hybrid coset of a near ring and hybrid structure is used to analyze the structural statements of near rings. All over this paper N means a (right) near ring.

2. Preliminaries

Definition 2.1: [8] Let U be a universal set, $P(U)$ be the power set, L be the set of parameters and I be the unit interval. A mapping $\tilde{\xi}_\lambda \coloneqq (\tilde{\xi}, \lambda): L \to P(U)\ X\ I,\ q \to \left(\tilde{\xi}(q), \lambda(q)\right)$ i.e., the image of q is chosen by $\left(\tilde{\xi}(q), \lambda(q)\right)$ is entitled a hybrid structure (HS) in L over U, where $\tilde{\xi}: L \to P(U)$ and $\lambda : L \to I$ are the mappings.

Definition 2.2: [8] Let $\tilde{\xi}_\lambda$ be a HS in L over U. Then the sets

$$\tilde{\xi}_\lambda[\alpha, t] = \{q \in L / \tilde{\xi}(q) \supseteq \alpha, \lambda(q) \leq t\},$$

$$\tilde{\xi}_\lambda(\alpha, t] = \{q \in L / \tilde{\xi}(q) \supsetneq \alpha, \lambda(q) \leq t\},$$

$$\tilde{\xi}_\lambda[\alpha, t) = \{q \in L / \tilde{\xi}(q) \supseteq \alpha, \lambda(q) < t\},$$

$$\tilde{\xi}_\lambda(\alpha, t) = \{q \in L / \tilde{\xi}(q) \supsetneq \alpha, \lambda(q) < t\}$$

are entitled the $[\alpha, t]$ – hybrid cut (HC), $(\alpha, t]$ – HC, $[\alpha, t)$ – HC, and (α, t) – HC of $\tilde{\xi}_\lambda$ respectively, where $\alpha \in P(U)$, $t \in I$. Obviously, $\tilde{\xi}_\lambda(\alpha, t) \subseteq \tilde{\xi}_\lambda(\alpha, t] \subseteq \tilde{\xi}_\lambda[\alpha, t]$ and $\tilde{\xi}_\lambda(\alpha, t) \subseteq \tilde{\xi}_\lambda[\alpha, t) \subseteq \tilde{\xi}_\lambda[\alpha, t]$.

Definition 2.3: [9] Let $\tilde{\varrho}_\gamma$ be a HS of a nearring N, $\tilde{\varrho}_\gamma$ is called a hybrid nearring of N over U if the following conditions clutch:

$(i)\tilde{\varrho}(q-\varsigma) \supseteq \tilde{\varrho}(q) \cap \tilde{\varrho}(\varsigma), \gamma(q-\varsigma) \leq \vee\{\gamma(q), \gamma(\varsigma)\}\ \forall\ q, \varsigma \in N$

$(ii)\tilde{\varrho}(q\varsigma) \supseteq \tilde{\varrho}(q) \cap \tilde{\varrho}(\varsigma), \gamma(q\varsigma) \leq \vee\{\gamma(q), \gamma(\varsigma)\}\ \forall\ q, \varsigma \in N.$

3. Main Results

In this, we introduce hybrid coset of a nearring (HCNR) and attain some of the properties of hybrid coset of a nearring.

Definition 3.1: Let N be a NR. A HS $\tilde{\varrho}_\gamma$ in N over U is entitled a HINR if the following conditions clutch.

$(i)\ \tilde{\varrho}(q-\varsigma) \supseteq \tilde{\varrho}(q) \cap \tilde{\varrho}(\varsigma), \gamma(q-\varsigma) \leq \vee\{\gamma(q), \gamma(\varsigma)\}\ \forall\ q, \varsigma \in N$

$(ii)\ \tilde{\varrho}(q\varsigma) \supseteq \tilde{\varrho}(q) \cap \tilde{\varrho}(\varsigma), \gamma(q\varsigma) \leq \vee\{\gamma(q), \gamma(\varsigma)\}\ \forall\ q, \varsigma \in N$

$(iii)\tilde{\varrho}(q+\varsigma-q) \supseteq \tilde{\varrho}(\varsigma), \gamma(q+\varsigma-q) \leq \gamma(\varsigma)\ \forall\ q, \varsigma \in N$

$(iv)\tilde{\varrho}(q\varsigma) \supseteq \tilde{\varrho}(q), \gamma(q\varsigma) \leq \gamma(q)\ \forall\ q, \varsigma \in N$

$(v)\tilde{\varrho}(\varsigma(q+i) - \varsigma q) \supseteq \tilde{\varrho}(i), \gamma(\varsigma(q+i) - \varsigma q) \leq \gamma(i)\ \forall\ q, \varsigma, i \in N.$

If $\tilde{\varrho}_\gamma$ gratifies $(i), (ii), (iii)$ and (iv) then $\tilde{\varrho}_\gamma$ is entitled a right HINR of N.

If $\tilde{\varrho}_\gamma$ gratifies $(i), (ii), (iii)$ and (v) then $\tilde{\varrho}_\gamma$ is entitled a left HINR of N.

Example 3.2: Let $N = \{0, a_\tau, b_\tau, c_\tau\}$ be a set with two binary operations '+', '.' as follows

$+$	0	a_τ	b_τ	c_τ
0	0	a_τ	b_τ	c_τ
a_τ	a_τ	0	c_τ	b_τ
b_τ	b_τ	c_τ	0	a_τ
c_τ	c_τ	b_τ	a_τ	0

$.$	0	a_τ	b_τ	c_τ
0	0	0	0	0
a_τ	a_τ	a_τ	a_τ	a_τ
b_τ	b_τ	b_τ	b_τ	b_τ
c_τ	c_τ	c_τ	c_τ	c_τ

Then $(N, +, .)$ is a nearring. Then the hybrid structure $\tilde{\varrho}_\gamma$ in N over $U = \{u_1, u_2, u_3, u_4, u_5\}$ which is given below

N	$\tilde{\varrho}$	γ
0	$\{u_1, u_2, u_3, u_4, u_5\}$	0.5
a_τ	$\{u_1, u_5\}$	0.6
b_τ	$\{u_2, u_3, u_5\}$	0.7
c_τ	$\{u_2, u_4, u_5\}$	0.8

Therefore $(\tilde{\varrho}_\gamma, N)$ is a HINR.

Definition 3.3: Let $\tilde{\varrho}_\gamma$ be a HINR of N over U and $\varsigma \in N$. Then the hybrid coset (or coset) of $\tilde{\varrho}_\gamma$ is denoted by $\varsigma + \tilde{\varrho}_\gamma$ and is demarcated by $(\varsigma + \tilde{\varrho})(q) = \tilde{\varrho}(q - \varsigma)$ and $(\varsigma + \gamma)(q) = \gamma(q - \varsigma)\ \forall\ q \in N$.

Theorem 3.4: Let $\tilde{\varrho}_\gamma$ be a HINR of N over U and $q, \varsigma \in N$. Then $q + \tilde{\varrho}_\gamma = \varsigma + \tilde{\varrho}_\gamma$ if and only if $\tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(0)$ and $\gamma(q - \varsigma) = \gamma(0)$.

Proof: Let $q, \varsigma \in N$. Suppose that $q + \tilde{\varrho}_\gamma = \varsigma + \tilde{\varrho}_\gamma$. Then $\tilde{\varrho}(q - \varsigma) = (\varsigma + \tilde{\varrho})(q) = (q + \tilde{\varrho})(q) = \tilde{\varrho}(q - q) = \tilde{\varrho}(0)$ and $\gamma(q - \varsigma) = (\varsigma + \gamma)(q) = (q + \gamma)(q) = \gamma(q - q) = \gamma(0)$.

Conversely, suppose that $\tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(0)$ and $\gamma(q - \varsigma) = \gamma(0)$. For every $\kappa \in N$, we have $(q + \tilde{\varrho})(\kappa) = \tilde{\varrho}(\kappa - q) = \tilde{\varrho}(\kappa - \varsigma + \varsigma - q) = \tilde{\varrho}[(\kappa - \varsigma) + (\varsigma - q)] \supseteq \tilde{\varrho}(\kappa - \varsigma) \cap \tilde{\varrho}(\varsigma - q) = \tilde{\varrho}(\kappa - \varsigma) \cap \tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(\kappa - \varsigma) \cap \tilde{\varrho}(0) = \tilde{\varrho}(\kappa - \varsigma) = (\varsigma + \tilde{\varrho})(\kappa)$ and

$$\begin{aligned}(q + \gamma)(\kappa) &= \gamma(\kappa - q) = \gamma(\kappa - \varsigma + \varsigma - q) = \gamma[(\kappa - \varsigma) + (\varsigma - q)] \\ &\leq \vee \{\gamma(\kappa - \varsigma), \gamma(\varsigma - q)\} = \vee \{\gamma(\kappa - \varsigma), \gamma(q - \varsigma)\} = \vee \{\gamma(\kappa - \varsigma), \gamma(0)\} \\ &= \gamma(\kappa - \varsigma) = (\varsigma + \gamma)(\kappa).\end{aligned}$$

Thus $q + \tilde{\varrho} \supseteq \varsigma + \tilde{\varrho}$ and $(q + \gamma) \leq (\varsigma + \gamma)$. Now, $(\varsigma + \tilde{\varrho})(\kappa) = \tilde{\varrho}(\kappa - \varsigma) = \tilde{\varrho}(\kappa - q + q - \varsigma) = \tilde{\varrho}[(\kappa - q) + (q - \varsigma)] \supseteq \tilde{\varrho}(\kappa - q) \cap \tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(\kappa - q) \cap \tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(\kappa - q) \cap \tilde{\varrho}(0) = \tilde{\varrho}(\kappa - q) = (q + \tilde{\varrho})(\kappa)$ and $(\varsigma + \gamma)(\kappa) = \gamma(\kappa - \varsigma) = \gamma(\kappa - q + q - \varsigma) = \gamma[(\kappa - q) + (q - \varsigma)] \leq \vee \{\gamma(\kappa - q), \gamma(q - \varsigma)\} = \vee \{\gamma(\kappa - q), \gamma(0)\} = \gamma(\kappa - q) = (q + \gamma)(\kappa)$.

Thus $\varsigma + \tilde{\varrho} \supseteq q + \tilde{\varrho}$ and $(\varsigma + \gamma) \leq (q + \gamma)$. Hence $q + \tilde{\varrho} = \varsigma + \tilde{\varrho}$ and $q + \gamma = \varsigma + \gamma$.

Theorem 3.5: Let $\tilde{\varrho}_\gamma$ be a HINR of N over U. Then the following two statements hold:

If $q + \tilde{\varrho} = n + \tilde{\varrho},\ \varsigma + \tilde{\varrho} = v + \tilde{\varrho}$ then $(q + \varsigma) + \tilde{\varrho} = (n + v) + \tilde{\varrho}$, $q\varsigma + \tilde{\varrho} = nv + \tilde{\varrho}$ and

if $q + \gamma = n + \gamma,\ \varsigma + \gamma = v + \gamma$ then $(q + \varsigma) + \gamma = (n + v) + \gamma$,

$q\varsigma + \gamma = nv + \gamma\ \forall\ q, \varsigma, n, v \in N$.

Proof: Suppose that $q + \tilde{\varrho} = n + \tilde{\varrho},\ \varsigma + \tilde{\varrho} = v + \tilde{\varrho}$ and $q + \gamma = n + \gamma,\ \varsigma + \gamma = v + \gamma$.

Then $\tilde{\varrho}(q - n) = \tilde{\varrho}(0)$, $\tilde{\varrho}(\varsigma - v) = \tilde{\varrho}(0)$ and $\gamma(q - n) = \gamma(0)$, $\gamma(\varsigma - v) = \gamma(0)$. Consider $\tilde{\varrho}[(q + \varsigma) - (n + v)] = \tilde{\varrho}[(q - n) + (\varsigma - v)] \supseteq \tilde{\varrho}(q - n) \cap \tilde{\varrho}(\varsigma - v) = \tilde{\varrho}(0) \cap \tilde{\varrho}(0) = \tilde{\varrho}(0)$

and $\gamma[(q + \varsigma) - (n + v)] = \gamma[(q - n) + (\varsigma - v)] \leq \vee \{\gamma(q - n), \gamma(\varsigma - v)\} = \vee \{\gamma(0), \gamma(0)\} = \gamma(0)$. But $\tilde{\varrho}(0) \supseteq \tilde{\varrho}[(q + \varsigma) - (n + v)]$ and $\gamma(0) \leq \gamma[(q + \varsigma) - (n + v)]$. Therefore $\tilde{\varrho}[(q + \varsigma) - (n + v)] = \tilde{\varrho}(0)$ and $\gamma[(q + \varsigma) - (n + v)] = \gamma(0)$.

Thus $(q + \varsigma) + \tilde{\varrho} = (n + v) + \tilde{\varrho}$, $q\varsigma + \tilde{\varrho} = nv + \tilde{\varrho}$ and $(q + \varsigma) + \gamma = (n + v) + \gamma$.

Again $\tilde{\varrho}[q\varsigma - nv] = \tilde{\varrho}[nv - q\varsigma] = \tilde{\varrho}r(nv - qv + qv - q\varsigma)$

$$\begin{aligned}&= \tilde{\varrho}\big[(n - q)v + q\big(\varsigma + (-\varsigma + v)\big) - q\varsigma\big] \\ &\supseteq \tilde{\varrho}\big((n - q)v\big) \cap \tilde{\varrho}\big(q\big(\varsigma + (-\varsigma + v)\big) - q\varsigma\big)\end{aligned}$$

$$\supseteq \tilde{\varrho}(n-q) \cap \tilde{\varrho}(-\varsigma+v)$$
$$= \tilde{\varrho}(n-q) \cap \tilde{\varrho}(\varsigma-v)$$
$$= \tilde{\varrho}(0) \cap \tilde{\varrho}(0) = \tilde{\varrho}(0)$$

and $\gamma[q\varsigma - nv] = \gamma[nv - q\varsigma] = \gamma(nv - qv + qv - q\varsigma)$

$$= \gamma\big[(n-q)v + q\big(\varsigma + (-\varsigma+v)\big) - q\varsigma\big]$$
$$\leq \vee\big\{\gamma\big((n-q)v\big), \gamma\big(q\big(\varsigma+(-\varsigma+v)\big) - q\varsigma\big)\big\}$$
$$\leq \vee\{\gamma(n-q), \gamma(-\varsigma+v)\}$$
$$= \vee\{\gamma(u-q), \gamma(\varsigma-v)\}$$
$$= \vee\{\gamma(0), \gamma(0)\} = \gamma(0).$$

But $\tilde{\varrho}(0) \supseteq \tilde{\varrho}(q\varsigma - nv)$ and $\gamma(0) \leq \gamma(q\varsigma - nv)$. Therefore $\tilde{\varrho}(q\varsigma - nv) = \tilde{\varrho}(0)$ and $\gamma(q\varsigma - nv) = \gamma(0)$. Thus $q\varsigma + \tilde{\varrho} = nv + \tilde{\varrho}$ and $q\varsigma + \gamma = nv + \gamma$.

Notation 3.6: Let $\tilde{\varrho}_\gamma$ be a HINR of N over U. Then the set of all cosets of $\tilde{\varrho}_\gamma$ is $N/_{\tilde{\varrho}_\gamma} = \{\varsigma + \tilde{\varrho}_\gamma : \varsigma \in N\}$, where $N/_{\tilde{\varrho}} = \{\varsigma + \tilde{\varrho} : \varsigma \in N\}$ and $N/_{\gamma} = \{\varsigma + \gamma : \varsigma \in N\}$.

Theorem 3.7: Let $\tilde{\varrho}_\gamma$ be a HINR of N over U. Then $N/_{\tilde{\varrho}_\gamma}$ is a near ring with respect to the operations defined by

$(q + \tilde{\varrho}) + (\varsigma + \tilde{\varrho}) = (q + \varsigma) + \tilde{\varrho}$, $(q + \tilde{\varrho})(\varsigma + \tilde{\varrho}) = (q\varsigma) + \tilde{\varrho}$ and

$(q + \gamma) + (\varsigma + \gamma) = (q + \varsigma) + \gamma$, $(q + \gamma)(\varsigma + \gamma) = (q\varsigma) + \gamma \;\forall\, q, \varsigma \in N$.

Proof: A direct verification shows that $\left(N/_{\tilde{\varrho}_\gamma}, +\right)$ is a group. Let $q + \tilde{\varrho}, \varsigma + \tilde{\varrho}, j + \tilde{\varrho} \in N/_{\tilde{\varrho}_\gamma}$ and $+\gamma,\ \varsigma + \gamma,\ j + \gamma \in N/_{\tilde{\varrho}_\gamma}$, where $q, \varsigma, j \in N$. Then $[(q + \tilde{\varrho})(\varsigma + \tilde{\varrho})](j + \tilde{\varrho}) = (q\varsigma + \tilde{\varrho})(j + \tilde{\varrho}) = (q\varsigma)j + \tilde{\varrho} = q(\varsigma j) + \tilde{\varrho} = (q + \tilde{\varrho})[(\varsigma + \tilde{\varrho})(j + \tilde{\varrho})]$ and $[(q + \gamma)(\varsigma + \gamma)](j + \gamma) = (q\varsigma + \gamma)(j + \gamma) = (q\varsigma)j + \gamma = q(\varsigma j) + \gamma = (q + \gamma)[(\varsigma + \gamma)(j + \gamma)]$. This shows that $N/_{\tilde{\varrho}_\gamma}$ is a semi group under multiplication. Consider

$$[(q + \tilde{\varrho}) + (\varsigma + \tilde{\varrho})](j + \tilde{\varrho}) = \big((q + \varsigma) + \tilde{\varrho}\big)(j + \tilde{\varrho}) = (q + \varsigma)j + \tilde{\varrho}$$
$$= (qj + \varsigma j) + \tilde{\varrho} = (qj + \tilde{\varrho}) + (\varsigma j + \tilde{\varrho})$$
$$= (q + \tilde{\varrho})(j + \tilde{\varrho}) + (\varsigma + \tilde{\varrho})(j + \tilde{\varrho}),$$
$$[(q + \gamma) + (\varsigma + \gamma)](j + \gamma) = \big((q + \varsigma) + \gamma\big)(j + \gamma) = (q + \varsigma)j + \gamma$$
$$= (qj + \varsigma j) + \gamma = (qj + \gamma) + (\varsigma j + \gamma)$$
$$= (q + \gamma)(j + \gamma) + (\varsigma + \gamma)(j + \gamma).$$

Hence $N/_{\tilde{\varrho}_\gamma}$ is a nearring.

Definition 3.8: Let $\tilde{\varrho}_\gamma$ be a HINR of N. Then $N/_{\tilde{\varrho}_\gamma}$, the set of all cosets of $\tilde{\varrho}_\gamma$ is called a hybrid quotient nearring of N by $\tilde{\varrho}_\gamma$ with respect to the following operations:

$(q+\tilde{\varrho})+(\varsigma+\tilde{\varrho})=(q+\varsigma)+\tilde{\varrho}$ and $(q+\gamma)+(\varsigma+\gamma)=(q+\varsigma)+\gamma$,

$(q+\tilde{\varrho})(\varsigma+\tilde{\varrho})=(q\varsigma)+\tilde{\varrho}$ and $(q+\gamma)(\varsigma+\gamma)=(q\varsigma)+\gamma$

for every $q,\varsigma\in N$.

Theorem 3.9: Let $\tilde{\varrho}_\gamma$ be a HINR of N. Define $\emptyset: N/_{\tilde{\varrho}_\gamma}\to P(U)XI$ by $\emptyset\big(q+\tilde{\varrho}_\gamma\big)=\tilde{\varrho}_\gamma(q)$

i.e., $\emptyset(q+\tilde{\varrho})=\tilde{\varrho}(q)$ and $\emptyset(q+\gamma)=\gamma(q)\ \forall\ q\in N$. Then $\emptyset$ is a hybrid ideal of $N/_{\tilde{\varrho}_\gamma}$.

Proof: Suppose that $q+\tilde{\varrho}=\varsigma+\tilde{\varrho}$ and $q+\gamma=\varsigma+\gamma$. Then $\tilde{\varrho}(q-\varsigma)=\tilde{\varrho}(0)$ and $\gamma(q-\varsigma)=\gamma(0)$. This implies that $\tilde{\varrho}(q)=\tilde{\varrho}(\varsigma)$ and $\gamma(q)=\gamma(\varsigma)$ i.e., $\emptyset(q+\tilde{\varrho})=\emptyset(\varsigma+\tilde{\varrho})$ and $\emptyset(q+\gamma)=\emptyset(\varsigma+\gamma)$. Therefore $\emptyset$ is well defined.

We verify that $\emptyset$ is a hybrid ideal of $N/_{\tilde{\varrho}_\gamma}$.

Let $q+\tilde{\varrho}_\gamma,\varsigma+\tilde{\varrho}_\gamma,j+\tilde{\varrho}_\gamma\in N/_{\tilde{\varrho}_\gamma}$.

(i) $\emptyset\big((q+\tilde{\varrho})+(\varsigma+\tilde{\varrho})\big)=\emptyset\big((q+\varsigma)+\tilde{\varrho}\big)=\tilde{\varrho}(q+\varsigma)\supseteq\tilde{\varrho}(q)\cap\tilde{\varrho}(\varsigma)=\emptyset(q+\tilde{\varrho})\cap\emptyset(\varsigma+\tilde{\varrho})$, and $\emptyset\big((q+\gamma)+(\varsigma+\gamma)\big)=\emptyset\big((q+\varsigma)+\gamma\big)=\gamma(q+\varsigma)\le\vee\{\gamma(q),\gamma(\varsigma)\}=\vee\{\emptyset(q+\gamma),\emptyset(\varsigma+\gamma)\}$.

(ii) $\emptyset(q+\tilde{\varrho})=\tilde{\varrho}(q)=\tilde{\varrho}(-q)=\emptyset(-q+\tilde{\varrho})$ and $\emptyset(q+\gamma)=\gamma(q)=\gamma(-q)=\emptyset(-q+\gamma)$.

(iii) $\emptyset\big((\varsigma+\tilde{\varrho})+(q+\tilde{\varrho})-(\varsigma+\tilde{\varrho})\big)=\emptyset\big((\varsigma+q-\varsigma)+\tilde{\varrho}\big)=\tilde{\varrho}(\varsigma+q-\varsigma)=\tilde{\varrho}(q)=\emptyset(q+\tilde{\varrho})$ and $\emptyset\big((\varsigma+\gamma)+(q+\gamma)-(\varsigma+\gamma)\big)=\emptyset\big((\varsigma+q-\varsigma)+\gamma\big)=\gamma(\varsigma+q-\varsigma)=\gamma(q)=\emptyset(q+\gamma)$.

(iv) $\emptyset\big((\varsigma+\tilde{\varrho})((q+\tilde{\varrho})+(j+\tilde{\varrho})-(\varsigma+\tilde{\varrho})(q+\tilde{\varrho})\big)$

$$=\emptyset\Big((\varsigma+\tilde{\varrho})\big((q+j)+\tilde{\varrho}\big)-(\varsigma q+\tilde{\varrho})\Big)$$

$$=\emptyset\big((\varsigma(q+j)+\tilde{\varrho})-(\varsigma q+\tilde{\varrho})\big)$$

$$=\emptyset\big((\varsigma(q+j)-\varsigma q)+\tilde{\varrho}\big)=\tilde{\varrho}(\varsigma(q+j)-\varsigma q)=\tilde{\varrho}(j)=\emptyset(j+\tilde{\varrho})\text{ and}$$

$\emptyset\big((\varsigma+\gamma)((q+\gamma)+(j+\gamma)-(\varsigma+\gamma)(q+\gamma)\big)$

$$=\emptyset\Big((\varsigma+\gamma)\big((q+j)+\gamma\big)-(\varsigma q+\gamma)\Big)$$

$$=\emptyset\big((\varsigma(q+j)+\gamma)-(\varsigma q+\gamma)\big)$$

$$=\emptyset\big((\varsigma(q+j)-\varsigma q)+\gamma\big)=\gamma(\varsigma(q+j)-\varsigma q)=\gamma(j)=\emptyset(j+\gamma).$$

Hence $\emptyset$ is a hybrid ideal of $N/_{\tilde{\varrho}_\gamma}$.

Definition 3.10: Let N_1, N_2 be near rings. A mapping $\emptyset: N_1 \to N_2$ is called a nearing homomorphism if $\emptyset(q+\varsigma) = \emptyset(q) + \emptyset(\varsigma)$ and $\emptyset(q\varsigma) = \emptyset(q)\emptyset(\varsigma)$ $\forall\, q,\varsigma \in N_1$. Moreover if $\emptyset$ is one-one then $\emptyset$ is called as monomorphism, if $\emptyset$ is onto then $\emptyset$ is called epimorphism, if $\emptyset$ is bijective then $\emptyset$ is called an isomorphism.

Theorem 3.11: If $\tilde{\varrho}_\gamma$ is a HINR N then the mapping $\emptyset: N \to N/\tilde{\varrho}_\gamma$ defined by $\emptyset(q) = q + \tilde{\varrho}_\gamma$ $\forall\, q \in N$ where $\emptyset(q) = q + \tilde{\varrho}$ and $\emptyset(q) = q + \gamma$, is a near ring epimorphism with kernal $\tilde{\varrho}_{\gamma_*}$ where

$\tilde{\varrho}_{\gamma_*} = \{q \in N: \tilde{\varrho}_\gamma(q) = \tilde{\varrho}_\gamma(0)\}$ ie., $\tilde{\varrho}_{\gamma_*} = \{q \in N: \tilde{\varrho}(q) = \tilde{\varrho}(0) \ and \ \gamma(q) = \gamma(0)\}$.

Proof: Let $q, y \in N$. Suppose that $q = \varsigma$. Then $q + \tilde{\varrho} = \varsigma + \tilde{\varrho}$ and $q + \gamma = \varsigma + \gamma$. This implies that $\emptyset(q) = \emptyset(\varsigma)$. Therefore $\emptyset$ is well defined.

Now $\emptyset(q+\varsigma) = (q+\varsigma) + \tilde{\varrho} = (q+\tilde{\varrho}) + (\varsigma+\tilde{\varrho}) = \emptyset(q) + \emptyset(\varsigma)$ and

$\emptyset(q+\varsigma) = (q+\varsigma) + \gamma = (q+\gamma) + (\varsigma+\gamma) = \emptyset(q) + \emptyset(\varsigma)$,

$\emptyset(q\varsigma) = (q\varsigma) + \tilde{\varrho} = (q+\tilde{\varrho})(\varsigma+\tilde{\varrho}) = \emptyset(q)\emptyset(\varsigma)$ and $\emptyset(q\varsigma) = (q\varsigma) + \gamma = (q+\gamma)(\varsigma+\gamma) = \emptyset(q)\emptyset(\varsigma)$.

Therefore $\emptyset$ is a homomorphism.

Let $q + \tilde{\varrho}_\gamma \in N/_{\tilde{\varrho}_\gamma}$. Then $q \in N$. For this $q \in N$, we have $\emptyset(q) = q + \tilde{\varrho}_\gamma$. Therefore $\emptyset$ is a near ring epimorphism. And now $q \in \ker\emptyset \Leftrightarrow \emptyset(q) = 0 = 0 + \tilde{\varrho} = 0 + \gamma$

$\Leftrightarrow q + \tilde{\varrho} = 0 + \tilde{\varrho}$ and $q + \gamma = 0 + \gamma$

$\Leftrightarrow \tilde{\varrho}(q-0) = \tilde{\varrho}(0)$ and $\gamma(q-0) = \gamma(0) \Leftrightarrow \tilde{\varrho}(q) = \tilde{\varrho}(0)$ and $\gamma(q) = \gamma(0) \Leftrightarrow q \in \tilde{\varrho}_{\gamma_*}$.

This shows that kernal $\emptyset = \tilde{\varrho}_{\gamma_*}$.

Theorem 3.12: If $\tilde{\varrho}_\gamma$ is a HINR N. Then $N/_{\tilde{\varrho}_\gamma}$ is isomorphic to $N/_{\tilde{\varrho}_{\gamma_*}}$,

where $\tilde{\varrho}_{\gamma_*} = \{q \in N: \tilde{\varrho}(q) = \tilde{\varrho}(0) \ and \ \gamma(q) = \gamma(0)\}$.

Proof: We have that $\tilde{\varrho}_{\gamma_*}$ is an ideal(kernal) of N. We know that $N/_{\tilde{\varrho}_\gamma} = \{q + \tilde{\varrho}_\gamma : q \in N\}$ and $N/_{\tilde{\varrho}_{\gamma_*}} = \{q + \tilde{\varrho}_{\gamma_*} : q \in N\}$. Define a mapping $\emptyset$: $N/_{\tilde{\varrho}_\gamma} \to N/_{\tilde{\varrho}_{\gamma_*}}$ by $\emptyset(q+\tilde{\varrho}) = q + \tilde{\varrho}_*$ and $\emptyset(q+\gamma) = q + \gamma_*$ for every $q + \tilde{\varrho}, q + \gamma \in N/_{\tilde{\varrho}_\gamma}$. To prove that $N/_{\tilde{\varrho}_\gamma}$ is isomorphic to $N/_{\tilde{\varrho}_{\gamma_*}}$, it is sufficient to prove that $\emptyset$ is well defined, one-one, onto and homomorphism.

Let $q + \tilde{\varrho}_\gamma, \varsigma + \tilde{\varrho}_\gamma \in N/_{\tilde{\varrho}_\gamma}$, where $q, \gamma \in N$.

Then $q + \tilde{\varrho} = \varsigma + \tilde{\varrho} \Leftrightarrow \tilde{\varrho}(q - \varsigma) = \tilde{\varrho}(0) \Leftrightarrow q - \varsigma \in \tilde{\varrho}_* \Leftrightarrow q + \tilde{\varrho}_* = \varsigma + \tilde{\varrho}_* \Leftrightarrow \emptyset(q + \tilde{\varrho}) = \emptyset(\varsigma + \tilde{\varrho})$ and $q + \gamma = \varsigma + \gamma \Leftrightarrow \gamma(q - \varsigma) = \gamma(0) \Leftrightarrow q - \varsigma \in \gamma_* \Leftrightarrow q + \gamma_* = \varsigma + \gamma_* \Leftrightarrow \emptyset(q + \gamma) = \emptyset(\varsigma + \gamma)$. Therefore $\emptyset$ is well defined and one-one.

Let $\varsigma + \tilde{\varrho}_{\gamma_*} \in N/_{\tilde{\varrho}_{\gamma_*}}$. Then $\varsigma \in N$. For this $\varsigma \in N$, we have $q + \tilde{\varrho}_\gamma \in N/_{\tilde{\varrho}_\gamma}$ and $\emptyset(\varsigma + \tilde{\varrho}) = \varsigma + \tilde{\varrho}_*$ and $\emptyset(\varsigma + \gamma) = \varsigma + \gamma_*$. That is for each $\varsigma + \tilde{\varrho}_{\gamma_*} \in N/_{\tilde{\varrho}_{\gamma_*}}$ there exists $\varsigma + \tilde{\varrho}_\gamma \in N/_{\tilde{\varrho}_\gamma}$ such that $\emptyset(\varsigma + \tilde{\varrho}) = \varsigma + \tilde{\varrho}_*$ and $\emptyset(\varsigma + \gamma) = \varsigma + \gamma_*$. Therefore $\emptyset$ is onto.

Let $q + \tilde{\varrho}_\gamma, \varsigma + \tilde{\varrho}_\gamma \in N/_{\tilde{\varrho}_\gamma}$, $q, \gamma \in N$. Then $\emptyset\big((q + \tilde{\varrho}) + (\varsigma + \tilde{\varrho})\big) = \emptyset\big((q + \varsigma) + \tilde{\varrho}\big) = (q + \varsigma) + \tilde{\varrho}_* = (q + \tilde{\varrho}_*) + (\varsigma + \tilde{\varrho}_*) = \emptyset(q + \tilde{\varrho}) + \emptyset(\varsigma + \tilde{\varrho})$ and $\emptyset\big((q + \gamma) + (\varsigma + \gamma)\big) = \emptyset\big((q + \varsigma) + \gamma\big) = (q + \varsigma) + \gamma_* = (q + \gamma_*) + (\varsigma + \gamma_*) = \emptyset(q + \gamma) + \emptyset(\varsigma + \gamma)$,

$\emptyset\big((q + \tilde{\varrho})(\varsigma + \tilde{\varrho})\big) = \emptyset\big((q\varsigma) + \tilde{\varrho}\big) = (q\varsigma) + \tilde{\varrho}_* = (q + \tilde{\varrho}_*)(\varsigma + \tilde{\varrho}_*) = \emptyset(q + \tilde{\varrho})\emptyset(\varsigma + \tilde{\varrho})$ and $\emptyset\big((q + \gamma)(\varsigma + \gamma)\big) = \emptyset\big((q\varsigma) + \gamma\big) = (q\varsigma) + \gamma_* = (q + \gamma_*)(\varsigma + \gamma_*) = \emptyset(q + \gamma)\emptyset(\varsigma + \gamma.)$

Therefore $\emptyset$ is a homomorphism. Hence $N/_{\tilde{\varrho}_\gamma}$ is isomorphic to $N/_{\tilde{\varrho}_{\gamma_*}}$.

4. Conclusion

In this study, we familiarized the idea of hybrid coset of a nearring and explored numerous properties. Using these ideas, we familiarized the ideas of nearring homomorphism and isomorphism. Research can be prolonged to the hybrid ideal of a gamma near algebra, sum of hybrid ideals of a near algebra and sum of hybrid ideals of a gamma near algebra.

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