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Solving Fuzzy Assignment Problem of Pythagorean Fuzzy Numbers Exploiting a Software Tool

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Abstract:
A fresh approach to computing assignment issues in the pythagorean fuzzy domain is the focus of this body of work. Pythagorean fuzzy numbers are utilised in this work for the sake of analysing working hours and working costs. For converting fuzzy sets integers into a crisp number, a new ranking process is used. Optimal solutions have to be found by using software techniques for minimization, maximization and unbalanced assignment problems. In the end, numerical examples are being solved by using the proposed technique.

Keywords: Maximization pythagorean fuzzy assignment problem , Fuzzy sets, Triangular pythagorean fuzzy numbers, Ranking function, Software tool

1. Introduction

Operations research is an important field to finding new variety of solution in many real life problems. In OR, transportation problems are used to compute optimal value for goods shipping. The problem formulation is a subclass of the transportation problem. It refers to assign a work to the labour with the purpose of either minimizing the cost or reducing the number of hours into a single to one correspondence. Nowadays fuzzy sets are mainly used to all fields. Since many of the real life problems having only vagueness or imprecision values. An improvement on intuitionistic fuzzy sets, Pythagorean fuzzy sets take the precondition and wrapping legion regions and unwrap them. It is utilised for determining the best course of action in decision making, transportation, and business endeavours, among other applications. Fuzzy sets (FSs) have been crucial in various areas of contemporary culture. It is proposed to enhance the functionality of FSs and better express uncertain information. Information may be described in these sets and methods more thoroughly and precisely. Pythagorean fuzzy set (PFS), a new nonstandard fuzzy set, has just been developed. The PFS was created because we frequently come across these unique situations in practical decision-making processes, where an alternative may satisfy a decision-maker-provided criterion to a support (membership) degree and a nonmembership degree.

For simplicity, the Pythagorean fuzzy numbers (PFNs) will be the PFS's constituent parts—PFNs' fundamental operations, such as intersection, union, addition, and multiplication. The Pythagorean complement was the main focus of the study, and it was utilised to create PFS. Then, using these PFSs, he applied several aggregation techniques to situations involving multi-criteria decision-making (MCDM). They specifically demonstrated that the Pythagorean membership grades belong to the problematic number category known as -i numbers. Later, a straightforward and efficient Pythagorean fuzzy TOPSIS approach was presented for quickly

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addressing several MCDM issues. The research listed above, however, only considers how to handle PFSs by incorporating certain operations and aggregating techniques for discrete information.

Different fuzzy sets can be used to communicate ambiguous and uncertain information in actual decision-making processes. However, we may run into instances where an option fulfils a condition supplied by the decision maker. As the basic building blocks of PFSs, Pythagorean fuzzy numbers (PFNs) are initially described in this study as change values when viewed as variables.

Lotfi Zadeh (1965) pioneered the idea of fuzzy thinking in order to address issues relating to imprecision and vagueness. Atanassov (1986,1989) extended the concept of fuzzy sets with the fuzzy sets and discussed their many aspects. This was the first time the concept had been presented. Angelov made a suggestion for the optimization of an intuitively fuzzy environment in his paper. PP (1997). After that, he expanded the intuitive fuzzy set by adding new items and graphical representation (1999). The Hungarian technique, which had been suggested by Kuhn (1955), was utilised in order to resolve the assignment problem. Mahapatra and Roy (2009) conducted research on the reliability of fuzzy systems. The sorting function of triangular intuitionistic fuzzy numbers was analysed and explained by Nagoorgani and Ponnalagu (2012Yager (2013, 2014) provided an explanation of the concept of Pythagorean fuzzy sets, which are responsible for the complicated ambiguity and unreliability in a variety of domains. The distinguished attribute of the Pythagorean fuzzy method is to loosen up the restriction and which was more accessible than intuitionistic fuzzy sets. Yager (2014) presented a number of distinct kinds of aggregation methods for Pythagorean fuzzy subsets. Subsequently, in the instance of a multicriteria decision making problem, they described the problem in terms of Pythagorean degree of membership. Zhang and Xu (2014) presented a number of novel operational concepts of Pythagorean fuzzy sets and outlined their properties. The authors also discussed potential applications. In addition to this, they discussed a more comprehensive approach to the problems of decision-making based on many criteria, which can be resolved by using Pythagorean fuzzy sets. Luqman etal (2020) explained Pythagorean fuzzy risk matrices and calculate the risk priority idexes. An explanation of how to tackle the transportation problem in a pythagorean fuzzy environment was provided by Kumar etal (2019). To address the pythagorean fuzzy linear programming problem, Muhammad Akram, Inayat Ullah, and others (2021) came up with the idea of using non-negative and unconstrained triangular pythagorean fuzzy numbers in conjunction with equality constraints. Pythagorean fuzzy modified distribution approach is designed to identify optimality. This method was developed by Priyanka Nagar, Pankaj Kumar Srivasta, and their colleagues (2021) in order to cope with optimisation of fuzzy species pythagorean transportation problem with conserved uncertainties. Using the application of the signature approach, Nagoor Gani and V.N. Mohamed Mohamed (2019) were able to solve the intuitionistic fuzzy assignment problem. In 2016, Ronald R. Yager provided an explanation of the content of pythagorean fuzzy subsets as well as the relationship between those and intuitionistic fuzzy subsets. Using Fermat's fuzzy numbers, Laxminarayan Sahoo (2021) was able to find a solution to the transportation problem. Ting and Yu Chen (2017) centred their attention on the multiple criterion decision analysis by making use of the IVPF details. After that, they went into specifics on the outshining decision making strategy for the proximity-based assignment model.

In this paper, we proposed an assignment problem technique with triangular pythagorean fuzzy numbers. Then we explained a ranking function, arithmetic operations and expand a current method for computing optimum result of the pythagorean fuzzy assignment model for non-negative constraints in both balanced and unbalanced problems. Then apply the suggested techniques for computing real world problems.

Pythagorean fuzzy numbers are a type of fuzzy numbers that extend the concept of fuzzy sets. They were introduced by Yager in 1996 as a way to represent uncertainty or vagueness in a more intuitive and expressive manner.

In traditional fuzzy sets, membership degrees are assigned to elements of a set using a membership function. Pythagorean fuzzy numbers take a different approach by assigning a membership degree to each value within a range or interval. These membership degrees are determined based on the Pythagorean fuzzy membership function.

A Pythagorean fuzzy number (PFN) is represented by a triple (L, M, R), where L, M, and R represent the left, middle, and right membership degrees, respectively. These membership degrees are assigned to the values within a specific interval, indicating the degree of membership of each value.

The membership function for Pythagorean fuzzy numbers is defined as follows:
- For values within the interval [L, M], the membership degree linearly increases from 0 to 1.
- For values within the interval [M, R], the membership degree linearly decreases from 1 to 0.
- Values outside the interval [L, R] have a membership degree of 0.

The shape of the membership function resembles a triangular shape, where the middle point M is the peak of the membership function, and the left and right sides determine the linear increase and decrease of the membership degree.

Pythagorean fuzzy numbers provide a more intuitive way to represent uncertainty, as they allow for the expression of both possibility and necessity. The middle value M represents the most typical or representative value, while the left and right values indicate the range of uncertainty or fuzziness around M.

Pythagorean fuzzy numbers have found applications in decision-making, expert systems, pattern recognition, and other areas where uncertainty needs to be captured and analyzed. Their representation allows for a more detailed and nuanced representation of uncertainty compared to traditional fuzzy sets.

Here we ordered as, in part 2 give out some primary definitions. Part 3 explain a proposed method to solve triangular pythagorean fuzzy assignment problem with non negative constraints. Part 4 gives some numerical examples for both balanced and unbalanced assignment problems. Part 5 devoted the conclusion.

Fuzzy Assignment Problem

The fuzzy assignment problem is an extension of the classical assignment problem that incorporates uncertainty or fuzziness in the assignment process. In the classical assignment problem, a set of agents or workers needs to be assigned to a set of tasks, each with a certain cost or benefit associated with it. The goal is to find the assignment that minimizes the total cost or maximizes the total benefit.

In the fuzzy assignment problem, instead of having crisp costs or benefits associated with each assignment, there are fuzzy numbers or fuzzy costs. Fuzzy numbers represent uncertainty or vagueness in the values, allowing for a more flexible representation of preferences or criteria.

The fuzzy assignment problem involves the following steps:

Formulating the fuzzy assignment matrix: Create a matrix where the rows represent agents and the columns represent tasks. Each element of the matrix contains a fuzzy number representing the cost or benefit of assigning a particular agent to a particular task. The fuzzy numbers can be represented using membership functions or other fuzzy representation methods.

Defining fuzzy assignment rules: Specify the rules or criteria for making the assignments based on the fuzzy numbers. These rules can be based on fuzzy logic, fuzzy reasoning, or other decision-making techniques. The rules consider the fuzzy numbers and their uncertainties to determine the best assignments.

Solving the fuzzy assignment problem: Use fuzzy optimization techniques or fuzzy decision-making approaches to find the optimal assignments that minimize the overall fuzzy cost or maximize the fuzzy benefit. This may involve aggregating the fuzzy numbers, performing fuzzy arithmetic operations, or applying fuzzy optimization algorithms to handle the uncertainties.

Interpreting the results: Analyze and interpret the obtained assignments, considering the uncertainties and fuzziness in the problem. The assignments may be evaluated based on their overall satisfaction of the fuzzy assignment rules and the associated fuzzy costs or benefits.

The fuzzy assignment problem allows for more realistic modeling of real-world situations where the costs or benefits of assignments are uncertain or imprecise. By incorporating fuzziness, it provides a more flexible and robust approach to decision-making and assignment optimization.

The solution for a fuzzy assignment problem involves finding the optimal assignments that minimize the overall fuzzy cost or maximize the fuzzy benefit, taking into account the uncertainties and fuzziness in the problem. Here is a general approach for solving a fuzzy assignment problem:

Formulate the fuzzy assignment problem: Define the problem by specifying the agents, tasks, and the fuzzy assignment matrix that represents the fuzzy costs or benefits associated with each assignment.

Define the fuzzy assignment rules: Determine the criteria or rules for making the assignments based on the fuzzy numbers. These rules can be based on fuzzy logic, fuzzy reasoning, or other decision-making techniques. Consider the preferences, constraints, and objectives of the problem.

Fuzzify the assignment matrix: Convert the fuzzy numbers in the assignment matrix into fuzzy sets using appropriate membership functions. This step involves assigning membership degrees to each fuzzy number based on the assigned fuzzy set.

Apply fuzzy optimization techniques: Use fuzzy optimization approaches to find the optimal assignments that satisfy the fuzzy assignment rules. This may involve aggregating the fuzzy numbers, performing fuzzy arithmetic operations, or applying fuzzy optimization algorithms.

Defuzzify the solutions: Convert the fuzzy solutions into crisp assignments by applying a defuzzification process. This process involves summarizing the fuzzy information and obtaining a crisp result.
Evaluate the solutions: Assess the quality of the obtained assignments based on the overall satisfaction of the fuzzy assignment rules and the associated fuzzy costs or benefits. Analyze the trade-offs between different assignments and consider the uncertainties and fuzziness in the problem.
It's important to note that the specific solution approach may vary depending on the problem context, the formulation of the fuzzy assignment rules, and the available optimization techniques. Various fuzzy optimization methods, such as fuzzy linear programming, fuzzy goal programming, or evolutionary algorithms, can be employed to solve fuzzy assignment problems.

Preliminaries

Definition 1: [18]

Consider $\mathcal{X}$ as a universal discourse and a pythagorean fuzzy set $\mathfrak{V} \in \mathcal{X}$ is defined as,

$$\mathfrak{V} = \{(\rho_{\mathfrak{V}}(\mathfrak{x}), \phi_{\mathfrak{V}}(\mathfrak{x}) / \mathfrak{x} \in \mathcal{X})\}$$

Here membership function is denoted as $\rho_{\mathfrak{V}}$ and a non-membership function is denoted as $\phi_{\mathfrak{V}}$ respectively, where

$$\rho_{\mathfrak{V}}: \mathcal{X} \rightarrow [0,1],\ \mathfrak{x} \in \mathcal{X} \rightarrow \rho_{\mathfrak{V}}(\mathfrak{x}) \in [0,1]$$

$$\phi_{\mathfrak{V}}: \mathcal{X} \rightarrow [0,1],\ \mathfrak{x} \in \mathcal{X} \rightarrow \phi_{\mathfrak{V}}(\mathfrak{x}) \in [0,1]$$

Such that $0 \leq \rho_{\mathfrak{V}}{}^2(\mathfrak{x}) + \phi_{\mathfrak{V}}{}^2(\mathfrak{x}) \leq 1\ \forall\ \mathfrak{x} \in \mathcal{X}$. Moreover, $\forall\ \mathfrak{x} \in \mathcal{X}, \pi_{\mathfrak{V}}(\mathfrak{x}) = \sqrt{1 - \rho_{\mathfrak{V}}{}^2(\mathfrak{x}) - \phi_{\mathfrak{V}}{}^2(\mathfrak{x})}$ is known as a Pythagorean fuzzy index or degree of hesitancy of $\mathfrak{x}$ in $\mathfrak{V}$.

In calculation purpose we consider, $\beta = (\rho_{\beta,}\ \phi_\beta)$ is called a pythagoean fuzzy numbers (Zhang and Xu 2014), where $\rho_{\beta,}\ \phi_\beta \in [0,1]$, $\rho_\beta{}^2 + \phi_\beta{}^2 \leq 1$ and $\pi_\beta(\mathfrak{x}) = \sqrt{1 - \rho_\beta{}^2 - \phi_\beta{}^2}$

Definition 2 [8]

A triangular pythagorean fuzzy number $\mathfrak{V}= \{(\mathbb{e}, \mathbb{f}, \mathbb{g}); \mathcal{u}, \mathcal{v}\}$ is a pythagorean fuzzy set on $\mathfrak{R}$, then their $(\rho_{\mathfrak{V}})$ and $(\phi_{\mathfrak{V}})$ it is defined as

$$\rho_{\mathfrak{V}}(\mathfrak{x}) = \begin{cases} \frac{(\mathfrak{x} - \mathbb{e})\mathcal{u}}{\mathbb{f} - \mathbb{e}}, \mathbb{e} \leq \mathfrak{x} \leq \mathbb{f} \\ \mathcal{u}, \mathfrak{x} = \mathbb{f} \\ \frac{(\mathbb{g} - \mathfrak{x})\mathcal{u}}{\mathbb{g} - \mathbb{f}}, \mathbb{f} \leq \mathfrak{x} \leq \mathbb{g} \\ 0, \mathfrak{x} < \mathbb{e}\quad or\quad \mathfrak{x} > \mathbb{g} \end{cases}$$

$$\phi_{\mathfrak{B}}(\mathfrak{x}) = \begin{cases} \frac{[\mathbb{f}-\mathfrak{x}+v(\mathfrak{x}-\mathbb{e})]}{\mathbb{f}-\mathbb{e}}, \mathbb{e} \le \mathfrak{x} \le \mathbb{f} \\ v, \mathfrak{x} = \mathbb{f}, \\ [\mathfrak{x} - \mathbb{f} + v(\mathbb{g} - \mathbb{f}), \mathbb{f} < x \le \mathbb{g}] \\ 1, \mathfrak{x} < \mathbb{e} \quad or \quad \mathfrak{x} > \mathbb{g} \end{cases}$$

Here, the merit of u & v defined the maximum and minimum degree of $\rho_{\mathfrak{B}}$ & $\phi_{\mathfrak{B}}$ properly, like $u \in [0,1]$, $v \in [0,1]$ and $0 \le u^2 + v^2 \le 1$.

Definition 3

Considering two triangular Pythagorean fuzzy numbers $\mathfrak{B} = \{(\mathbb{e}, \mathbb{f}, \mathbb{g});(e', \mathbb{f}, \mathbb{g}')\}$ $\mathfrak{A} = \{(\hbar, i, j)(\hbar', i, j')\}$ and δ be any real number . Then we defined,

$\mathfrak{B} \oplus \mathfrak{A} = \{(\mathbb{e} + \hbar, \mathbb{f} + i, \mathbb{g} + j);(\mathbb{e}' + \hbar', \mathbb{f} + i, \mathbb{g}' + j')\}$

$-\mathfrak{B} = \{(-\mathbb{g}, -\mathbb{f}, -\mathbb{e});(-\mathbb{g}', -\mathbb{f}, -\mathbb{e}')\}$

$\mathfrak{B} \ominus \mathfrak{A} = \{(\mathbb{e} - j, \mathbb{f} - i, \mathbb{g} - \hbar);(\mathbb{e}' - j', \mathbb{f} - i, \mathbb{g}' - \hbar')\}$

$\delta\,\mathfrak{B} = \{(\delta\,\mathbb{e}, \delta\mathbb{f}, \delta\mathbb{g});(\delta\,\mathbb{e}e', \delta\mathbb{f}, \delta\mathbb{g}')\}, \delta > 0$

$\{(\delta\mathbb{g}, \delta\mathbb{f}, \delta\,\mathbb{e});(\delta\mathbb{g}', \delta\mathbb{f}, \delta\,\mathbb{e}')\}, \delta < 0$

Definition 4

$\mathfrak{B} = \{(\mathbb{e}, \mathbb{f}, \mathbb{g});(e', \mathbb{f}, \mathbb{g}')\}$ be a triangular Pythagorean non-negative fuzzy number, $(\mathfrak{B} \ge 0)$ if $\mathbb{e}' \ge 0$.

Definition 5 [11]

Ranking for triangular Pythagorean fuzzy number $\mathfrak{B} = \{(\mathbb{e}, \mathbb{f}, \mathbb{g});(e', \mathbb{f}, \mathbb{g}')\}$ can be stated as,

$$\mathfrak{R}(\mathfrak{B}) = \frac{(\mathbb{e}+2\mathbb{f}+\mathbb{g})+(\mathbb{e}'+2\mathbb{f}+\mathbb{g}')}{8}$$

Definition 6

Consider an assignment problem of assigning ‘m’workers to ‘m’ work and each worker having the capacity of working any work at various money / time period. Let $C^{\sim}_{kl}$ be a cost coefficient of triangular pythagorean fuzzy number of allocating the l^{th} work to the k^{th} worker. Let the variable $\mathbb{X}^{\sim}_{kl}$serve as the decision maker for the allocation of the assignment. Our objective is to reduce the overall cost of the minimization problem by distributing all of the labour to the workers who are available while keeping the net cost as low as possible. In maximization problem, we have to increase the profit. In unbalanced assignment problem, this condition holds but, rows and columns are not equal.

Let the variable $\mathbb{X}^{\sim}_{kl}$serve as the decision maker for the allocation of the assignment. Our objective is to reduce the overall cost of the minimization problem by distributing all of the labour to the workers who are available while keeping the net cost as low as possible.

Mathematical formulation of Pythagorean fuzzy assignment problem as,

Maximize/Minimize $\mathcal{Z}^{\sim} = \sum_{k=1}^{m}\sum_{l=1}^{m}\mathbb{C}^{\sim}_{kl} \times \mathbb{X}^{\sim}_{kl}$ $\quad \rightarrow(1)$

Subject to $\sum_{l=1}^{m}\mathbb{X}^{\sim}_{kl}=1$ for k = 1, 2 ...m

$\sum_{k=1}^{m}\mathbb{X}^{\sim}_{kl}=1$ for l = 1, 2 ... m $\quad \rightarrow(2)$

$$\mathbb{X}^{\sim}{}_{kl} = 0 \text{ (or) } 1$$

3 Methodology

The proposed method is helped to compute the optimum result of the assignment problem under Pythagorean fuzzy environment.

1. Construct the table of costs associated with the assignment based on the supplied problem.
2. Determine whether the issue that has been provided is balanced or unbalanced.
3. If the situation that has been presented to you is imbalanced, then you should introduce a dummy row or column with entries that have no cost.
4. Consider $\mathbb{C}^{\sim}{}_{kl} = \{(\mathfrak{z}_{kl,}^{1\prime}\ \mathfrak{z}_{kl,}^{2\prime}\ \mathfrak{z}_{kl}^{3\prime})\ ;\ (\mathfrak{z}_{kl,}^{1\prime\prime}\ \mathfrak{z}_{kl,}^{2\prime}\ \mathfrak{z}_{kl,}^{3\prime\prime})\}$

   $\mathbb{X}^{\sim}{}_{kl} = \{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})\ (\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\}$ in (1).

   Where $\mathbb{C}^{\sim}{}_{kl}$, $\mathbb{X}^{\sim}{}_{kl}$ are all triangular Pythagorean fuzzy numbers .

   Then problem (1) becomes,

Maximize/Minimize $\mathcal{Z}^{\sim} = \sum_{k=1}^{m}\sum_{l=1}^{m}\{(\mathfrak{z}_{kl,}^{1\prime}\ \mathfrak{z}_{kl,}^{2\prime}\ \mathfrak{z}_{kl,}^{3\prime});\ (\mathfrak{z}_{kl,}^{1\prime\prime}\ \mathfrak{z}_{kl,}^{2\prime}\mathfrak{z}_{kl,}^{3\prime\prime})\}$
$\times \{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\}$

Subject to

$\sum_{l=1}^{m}\{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\} = 1$ for k = 1,2,...m

$\sum_{k=1}^{m}\{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\} = 1$ for l = 1,2,... m

5. By using ranking method, then the problem becomes,

Maximize/Minimize $\mathbb{R}\ (\mathcal{Z})^{\sim} = \mathbb{R}(\sum_{k=1}^{m}\sum_{l=1}^{m}\{(\mathfrak{z}_{kl,}^{1\prime}\ \mathfrak{z}_{kl,}^{2\prime}\ \mathfrak{z}_{kl,}^{3\prime});\ (\mathfrak{z}_{kl,}^{1\prime\prime}\ \mathfrak{z}_{kl,}^{2\prime}\mathfrak{z}_{kl,}^{3\prime\prime})\}$
$\times \{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\})$

$\sum_{l=1}^{m}\{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\} = 1$ for k = 1,2,...m

$\sum_{k=1}^{m}\{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\} = 1$ for l = 1,2,... m

5. With the help of lingo software technique, solve the crisp problem we get the values

   $\mathbb{X}^{\sim}{}_{kl} = \{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})\ (\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\}$, for k = 1,2,...m and l = 1,2,... m

6. Substitute the above values in

   $\sum_{k=1}^{m}\sum_{l=1}^{m}\{(\mathfrak{z}_{kl,}^{1\prime}\ \mathfrak{z}_{kl,}^{2\prime}\ \mathfrak{z}_{kl,}^{3\prime});\ (\mathfrak{z}_{kl,}^{1\prime\prime}\ \mathfrak{z}_{kl,}^{2\prime}\mathfrak{z}_{kl,}^{3\prime\prime})\}\ \times \{(\mathfrak{p}_{kl}^{'}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{'})(\mathfrak{p}_{kl}^{''}, \mathfrak{q}_{kl}^{'}, \mathfrak{r}_{kl}^{''})\}$,

   we can find the optimal solution and the allocation of the Pythagorean fuzzy assignment problem.

Numerical Examples

4.1. Minimization Pythagorean Fuzzy Assignment Problem

A two wheeler dealer wishes to put 2 machines into 2 works. The mechanics have truly exceptional types of competencies and they showcase one of kind degrees of affectivity from one job to another. The supplier has estimated the wide variety of man-hours that would be required for every job-man combination. From the following desk locate the greatest venture that will end result in minimal man-hours needed.

	$\mathcal{J}_1$	$\mathcal{J}_2$
$\mathcal{M}_1$	(3,5,9)(1,5,11)	(5,7,10)(3,7,12)
$\mathcal{M}_2$	(4,7,11)(2,7,13)	(5,8,10)(3,8,12)

Where the time entries are in triangular Pythagorean fuzzy number

Solution

1 Let $\mathcal{X}^{\sim}{}_{11} = \{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}$

$\mathcal{X}^{\sim}{}_{12} = \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\}$

$\mathcal{X}^{\sim}{}_{21} = \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} \qquad \rightarrow \quad (I)$

$\mathcal{X}^{\sim}{}_{22} = \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\}$

Then the above problem can be written as,

Minimize $\mathcal{Z}^{\sim} = \{(3, 5, 9)\ (1, 5, 11)\}\ \{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\} +$
$\{(5, 7, 10)\ (3, 7, 12)\}\ \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\} +$

$\{(4, 7, 11)\ (2, 7, 13)\}\ \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} +$
$\{(5, 8, 10)\ (3, 8, 12)\}\ \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\}$

Subject to

$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\} + \{(p'_{12}, q'_{12}, r'_{12})\ (p''_{12}, q'_{12}, r'_{12})\} = 1$
$\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} + \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\} = 1$
$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\} + \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} = 1$
$\{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\} + \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\} = 1$

Where

$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}, \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\}, \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}$, $\{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})$ are all ≥ 0.

2 Minimize $\mathcal{Z}^{\sim} = \{(3p'_{11}, 5q'_{11}, 9r'_{11})\ (1p''_{11}, 5q'_{11}, 11r''_{11})\} +$
$\{(5p'_{12}, 7q'_{12}, 10r'_{12})\ (3p''_{12}, 7q'_{12}, 12r'_{12})\} +$
$\{(4p'_{21}, 7q'_{21}, 11r'_{21})\ (2p''_{21}, 7q'_{21}, 13r''_{21})\} +$
$\{(5p'_{22}, 8q'_{22}, 10r'_{22})\ (3p''_{22}, 8q'_{22}, 12r''_{22})\}$

Subject to

$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\} + \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\} = 1$
$\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} + \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\} = 1$
$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\} + \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\} = 1$
$\{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'_{12})\} + \{(p'_{22}, q'_{22}, r'_{22})(p'_{22}, q'_{22}, r'_{22})\} = 1$

3 Applying the ranking function,

Minimize $\mathcal{Z}^{\sim} = \mathbb{R}\ \{(3p'_{11} + 5p'_{12} + 4p'_{21} + 5p'_{22}), (5q'_{11} + 7q'_{12} + 7q'_{21} + 8q'_{22}),$
$(9r'_{11} + 10r'_{12} + 11r'_{21} + 10r'_{22});$
$(1p''_{11} + 3p''_{12} + 2p''_{21} + 3p''_{22}), (5q'_{11} + 7q'_{12} + 7q'_{21} + 8q'_{22}),$

$$(11\mathfrak{r}''_{11}+12\mathfrak{r}''_{12}+13\mathfrak{r}''_{21}+12\mathfrak{r}''_{22})\}$$

Subject to

$$\{(\mathfrak{p}'_{11}+\mathfrak{p}'_{12},\mathfrak{q}'_{11}+\mathfrak{q}'_{12},\mathfrak{r}'_{11}+\mathfrak{r}'_{12});(\mathfrak{p}''_{11}+\mathfrak{p}''_{12},\mathfrak{q}'_{11}+\mathfrak{q}'_{12},\mathfrak{r}''_{11}+\mathfrak{r}''_{12})\}=1$$
$$\{(\mathfrak{p}'_{21}+\mathfrak{p}'_{22},\mathfrak{q}'_{21}+\mathfrak{q}'_{22},\mathfrak{r}'_{21}+\mathfrak{r}'_{22});(\mathfrak{p}''_{21}+\mathfrak{p}''_{22},\mathfrak{q}'_{21}+\mathfrak{q}'_{22},\mathfrak{r}''_{21}+\mathfrak{r}''_{22})\}=1$$
$$\{(\mathfrak{p}'_{11}+\mathfrak{p}'_{21},\mathfrak{q}'_{11}+\mathfrak{q}'_{21},\mathfrak{r}'_{11}+\mathfrak{r}'_{21});(\mathfrak{p}''_{11}+\mathfrak{p}''_{21},\ \mathfrak{q}'_{11}+\mathfrak{q}'_{21},\ \mathfrak{r}''_{11}+\mathfrak{r}''_{21})\}=1$$
$$\{(\mathfrak{p}'_{12}+\mathfrak{p}'_{22},\mathfrak{q}'_{12}+\mathfrak{q}'_{22},\mathfrak{r}'_{12}+\mathfrak{r}'_{22});(\mathfrak{p}''_{12}+\mathfrak{p}''_{22},\mathfrak{q}'_{12}+\mathfrak{q}'_{22},\ \mathfrak{r}''_{12}+\mathfrak{r}''_{22})\}=1$$

4 By using arithmetic operations, then the problem converts into crisp assignment problem,
Minimize $\mathcal{Z}^{\sim}=$

$$\frac{3\mathfrak{p}'_{11}+5\mathfrak{p}'_{12}+4\mathfrak{p}'_{21}+5\mathfrak{p}'_{22}+1\mathfrak{p}''_{11}+3\mathfrak{p}''_{12}+2\mathfrak{p}''_{21}+3\mathfrak{p}''_{22}+20\mathfrak{q}'_{11}+28\mathfrak{q}'_{12}+28\mathfrak{q}'_{21}+32\mathfrak{q}'_{22}+9\mathfrak{r}'_{11}+10\mathfrak{r}'_{12}+11\mathfrak{r}'_{21}+10\mathfrak{r}'_{22}+11\mathfrak{r}''_{11}+12\mathfrak{r}''_{12}+13\mathfrak{r}''_{21}+12\mathfrak{r}''_{22}}{8}$$

Subject to

$$\mathfrak{p}'_{11}+\mathfrak{p}'_{12}=1 \quad \mathfrak{q}'_{11}+,\mathfrak{q}'_{12}=1 \quad \mathfrak{r}'_{11}+\mathfrak{r}'_{12}=1 \quad \mathfrak{p}''_{11}+\mathfrak{p}''_{12}=1 \quad \mathfrak{r}''_{11}+\mathfrak{r}''_{12}=1$$
$$\mathfrak{p}'_{21}+\mathfrak{p}'_{22}=1 \quad \mathfrak{q}'_{21}+\mathfrak{q}'_{22}=1 \quad \mathfrak{r}'_{11}+\mathfrak{r}'_{12}=1 \quad \mathfrak{p}''_{21}+\mathfrak{p}''_{22}=1 \quad \mathfrak{r}''_{21}+\mathfrak{r}''_{22}=1$$
$$\mathfrak{p}'_{11}+\mathfrak{p}'_{21}=1 \quad \mathfrak{q}'_{11}+\mathfrak{q}'_{21}=1 \quad \mathfrak{r}'_{11}+\mathfrak{r}'_{21}=1 \quad \mathfrak{p}''_{11}+\mathfrak{p}''_{21}=1 \quad \mathfrak{r}''_{11}+\mathfrak{r}''_{21}=1$$
$$\mathfrak{p}'_{12}+\mathfrak{p}'_{22}=1 \quad \mathfrak{q}'_{12}+\mathfrak{q}'_{22}=1 \quad \mathfrak{r}'_{12}+\mathfrak{r}'_{22}=1 \quad \mathfrak{p}''_{12}+\mathfrak{p}''_{22}=1 \quad \mathfrak{r}''_{12}+\mathfrak{r}''_{22}=1$$

5. By using software computations we get the following values,

$\mathfrak{p}'_{11}=1$	$\mathfrak{q}'_{11}=1$	$\mathfrak{r}'_{11}=1$	$\mathfrak{p}''_{11}=1$	$\mathfrak{r}''_{11}=1$
$\mathfrak{p}'_{12}=0$	$\mathfrak{q}'_{12}=0$	$\mathfrak{r}'_{12}=0$	$\mathfrak{p}''_{12}=0$	$\mathfrak{r}''_{12}=0$
$\mathfrak{p}'_{21}=0$	$\mathfrak{q}'_{21}=0$	$\mathfrak{r}'_{21}=0$	$\mathfrak{p}''_{21}=0$	$\mathfrak{r}''_{21}=0$
$\mathfrak{p}'_{22}=1$	$\mathfrak{q}'_{22}=1$	$\mathfrak{r}'_{22}=1$	$\mathfrak{p}''_{22}=1$	$\mathfrak{r}''_{22}=1$

6 Substitute the above values in (I)
we get $\mathcal{X}^{\sim}{}_{11}=\{(1,1,1)(1,1,1)\}$, $\mathcal{X}^{\sim}{}_{12}=\{(0,0,0)(0,0,0)\}$, $\mathcal{X}^{\sim}{}_{21}=\{(0,0,0)(0,0,0)\}$,
$\mathcal{X}^{\sim}{}_{22}=\{(1,1,1)(1,1,1)\} \quad \rightarrow \quad (II)$

7 Substitute (II) in the objective function, we get the optimum solution of the Pythagorean fuzzy assignment problem is {(8,13,19);(4,13,23)}
and the job allocations are, $\mathcal{M}_1 \rightarrow \mathcal{J}_1,\ \mathcal{M}_2 \rightarrow \mathcal{J}_2$

4.2. Maximization Pythagorean Fuzzy Assignment Problem

A software technology consultancy wants to assign the project work to two team leader. They are receiving the quotation from the team leader for various working cost. Company wants to increase their revenue. How can they assign the work to the team for increasing their profit? The working costs are given in the following table.

	Team 1	Team 2
Project 1	(4,6,8) (1,6,11)	(4,7,9) (2,7,14)
Project 2	(3,4,6) (1,4,8)	(5,8,9) (3,8,13)

Where the cost entries are in triangular Pythagorean fuzzy number.

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 2 (2024)

Solution

1 Let $\mathcal{X}^{\sim}{}_{11} = \{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}$

$\mathcal{X}^{\sim}{}_{12} = \{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}$

$\mathcal{X}^{\sim}{}_{21} = \{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}$ $\rightarrow$ (I)

$\mathcal{X}^{\sim}{}_{22} = \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\}$

Then the above problem can be written as,

Maximize $\mathcal{Z}^{\sim} =$ $\{(4, 6, 8)\ (1, 6, 11)\}\ \{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}+$
$\{(4, 7, 9)\ (2, 7, 14)\}\ \{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}+$
$\{(3,4, 6)\ (1,4,8)\}\ \{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}+$
$\{(5, 8, 9)\ (3, 8,13)\}\ \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\}$

Subject to

$\{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}+ \{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\} = 1$
$\{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}+ \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\}= 1$
$\{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}+ \{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}= 1$
$\{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}+ \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\} = 1$

Where
$\{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}, \{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}, \{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\},$
$\{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})$ are all ≥ 0.

2 Maximize $\mathcal{Z}^{\sim} = \{(4\mathfrak{p}'_{11}, 6\mathfrak{q}'_{11}, 8\mathfrak{r}'_{11})\ (1\mathfrak{p}''_{11}, 6\mathfrak{q}'_{11}, 11\mathfrak{r}''_{11})\}$ +
$\{(4\mathfrak{p}'_{12}, 7\mathfrak{q}'_{12}, 9\mathfrak{r}'_{12})\ (2\mathfrak{p}''_{12}, 7\mathfrak{q}'_{12}, 14\mathfrak{r}'_{12})\}+$
$\{(3\mathfrak{p}'_{21}, 4\mathfrak{q}'_{21}, 6\mathfrak{r}'_{21})\ (1\mathfrak{p}''_{21}, 4\mathfrak{q}'_{21}, 8\mathfrak{r}''_{21})\}+$
$\{(5\mathfrak{p}'_{22}, 8\mathfrak{q}'_{22}, 9\mathfrak{r}'_{22})\ (3\mathfrak{p}''_{22}, 8\mathfrak{q}'_{22}, 13\mathfrak{r}''_{22})\}$

Subject to

$\{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}+ \{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}=1$
$\{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}+ \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\}= 1$
$\{(\mathfrak{p}'_{11}, \mathfrak{q}'_{11}, \mathfrak{r}'_{11})(\mathfrak{p}''_{11}, \mathfrak{q}'_{11}, \mathfrak{r}''_{11})\}+\{(\mathfrak{p}'_{21}, \mathfrak{q}'_{21}, \mathfrak{r}'_{21})(\mathfrak{p}''_{21}, \mathfrak{q}'_{21}, \mathfrak{r}''_{21})\}= 1$
$\{(\mathfrak{p}'_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})(\mathfrak{p}''_{12}, \mathfrak{q}'_{12}, \mathfrak{r}'_{12})\}+ \{(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})(\mathfrak{p}'_{22}, \mathfrak{q}'_{22}, \mathfrak{r}'_{22})\} = 1$

3 Applying the ranking function,

Maximize $\mathcal{Z}^{\sim} = \mathbb{R}\ \{(4\mathfrak{p}'_{11}+4\mathfrak{p}'_{12} + 3\mathfrak{p}'_{21} + 5\mathfrak{p}'_{22}), (6\mathfrak{q}'_{11} +7\mathfrak{q}'_{12} + 4\mathfrak{q}'_{21} + 8\mathfrak{q}'_{22}),$
$(8\mathfrak{r}'_{11}+9\mathfrak{r}'_{12} + 6\mathfrak{r}'_{21} + 9\mathfrak{r}'_{22})$;
$(1\mathfrak{p}''_{11} + 2\mathfrak{p}''_{12} + 1\mathfrak{p}''_{21} + 3\mathfrak{p}''_{22}),), (6\mathfrak{q}'_{11} +7\mathfrak{q}'_{12} + 4\mathfrak{q}'_{21} + 8\mathfrak{q}'_{22}),$
$(11\mathfrak{r}''_{11} + 14\mathfrak{r}''_{12} + 8\mathfrak{r}''_{21} + 13\mathfrak{r}''_{22})\}$

Subject to

$\{(\mathfrak{p}'_{11} + \mathfrak{p}'_{12}, \mathfrak{q}'_{11} +, \mathfrak{q}'_{12}, \mathfrak{r}'_{11} + \mathfrak{r}'_{12})\ ;\ (\mathfrak{p}''_{11} + \mathfrak{p}''_{12}, \mathfrak{q}'_{11} +, \mathfrak{q}'_{12}, \mathfrak{r}''_{11} + \mathfrak{r}''_{12})\} = 1$
$\{(\mathfrak{p}'_{21} + \mathfrak{p}'_{22}, \mathfrak{q}'_{21} + \mathfrak{q}'_{22}, \mathfrak{r}'_{21} + \mathfrak{r}'_{22})\ ;\ (\mathfrak{p}''_{21} + \mathfrak{p}''_{22}, \mathfrak{q}'_{21} + \mathfrak{q}'_{22}, \mathfrak{r}''_{21} + \mathfrak{r}''_{22})\} = 1$
$\{(\mathfrak{p}'_{11} + \mathfrak{p}'_{21}, \mathfrak{q}'_{11} + \mathfrak{q}'_{21}, \mathfrak{r}'_{11} + \mathfrak{r}'_{21})\ ;\ (\mathfrak{p}''_{11} + \mathfrak{p}''_{21},\ \mathfrak{q}'_{11} + \mathfrak{q}'_{21},\ \mathfrak{r}''_{11} + \mathfrak{r}''_{21})\}= 1$
$\{(\mathfrak{p}'_{12} + \mathfrak{p}'_{22}, \mathfrak{q}'_{12} + \mathfrak{q}'_{22}, \mathfrak{r}'_{12} + \mathfrak{r}'_{22})\ ;\ (\mathfrak{p}''_{12} + \mathfrak{p}''_{22}, \mathfrak{q}'_{12} + \mathfrak{q}'_{22},\ \mathfrak{r}''_{12}+\mathfrak{r}''_{22})\} = 1$

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 2 (2024)

4 By using arithmetic operations, the problem converts into crisp assignment problem,
Maximize $\mathcal{Z}^{\sim}=$

$$\frac{4p'_{11}+4p'_{12}+3p'_{21}+5p'_{22}+1p''_{11}+2p''_{12}+1p''_{21}+3p''_{22}+24q'_{11}+28q'_{12}+16q'_{21}+32q'_{22}+8r'_{11}+9r'_{12}+6r'_{21}+9r'_{22}+11r''_{11}+14r''_{12}+8r''_{21}+13r''_{22}}{8}$$

Subject to

$p'_{11}+p'_{12}=1$ $q'_{11}+,q'_{12}=1$ $r'_{11}+r'_{12}=1$ $p''_{11}+p''_{12}=1$ $r''_{11}+r''_{12}=1$
$p'_{21}+p'_{22}=1$ $q'_{21}+q'_{22}=1$ $r'_{11}+r'_{12}=1$ $p''_{21}+p''_{22}=1$ $r''_{21}+r''_{22}=1$
$p'_{11}+p'_{21}=1$ $q'_{11}+q'_{21}=1$ $r'_{11}+r'_{21}=1$ $p''_{11}+p''_{21}=1$ $r''_{11}+r''_{21}=1$
$p'_{12}+p'_{22}=1$ $q'_{12}+q'_{22}=1$ $r'_{12}+r'_{22}=1$ $p''_{12}+p''_{22}=1$ $r''_{12}+r''_{22}=1$

5 By using software computations we get the following values,

$p'_{11}=1$	$q'_{11}=1$	$r'_{11}=1$	$p''_{11}=1$	$r''_{11}=1$
$p'_{12}=0$	$q'_{12}=0$	$r'_{12}=0$	$p''_{12}=0$	$r''_{12}=0$
$p'_{21}=0$	$q'_{21}=0$	$r'_{21}=0$	$p''_{21}=0$	$r''_{21}=0$
$p'_{22}=1$	$q'_{22}=1$	$r'_{22}=1$	$p''_{22}=1$	$r''_{22}=1$

6 Substitute the above values in (I)
we get $\mathcal{X}^{\sim}{}_{11}=\{(1,1,1)(1,1,1)\}$, $\mathcal{X}^{\sim}{}_{12}=\{(0,0,0)(0,0,0)\}$, $\mathcal{X}^{\sim}{}_{21}=\{(0,0,0)(0,0,0)\}$,
$\mathcal{X}^{\sim}{}_{22}=\{(1,1,1)(1,1,1)\}$ $\rightarrow$ (II)

7 Substitute (II) in the objective function, we get the optimum solution of the Pythagorean fuzzy assignment problem is {(9,14,17);(4,14,24)}
and the job allocations are, $project\ 1 \rightarrow Team\ 1, project\ 2 \rightarrow Team\ 2$

4.3 Unbalanced Pythagorean Fuzzy Assignment Problem

Calculate the optimum solution of the Pythagorean fuzzy assignment problem from the following table values.

	$\mathcal{P}_1$	$\mathcal{P}_2$
$\mathcal{W}$	(2,4,8)(1,4,10)	(2,3,5)(1,3,7)

Solution

The problem that has been presented is an example of an unbalanced Pythagorean fuzzy assignment problem. Add dummy row/column with zero cost entries, for converting it into the balanced problem. Hence the problem becomes,

	$\mathcal{P}_1$	$\mathcal{P}_2$
$\mathcal{W}_1$	(2,4,8)(1,4,10)	(2,3,5)(1,3,7)
$\mathcal{W}_2$	(0,0,0)(0,0,0)	(0,0,0)(0,0,0)

1 Let $\mathcal{X}^{\sim}{}_{11}=\{(p'_{11},q'_{11},r'_{11})(p''_{11},q'_{11},r''_{11})\}$
$\mathcal{X}^{\sim}{}_{12}=\{(p'_{12},q'_{12},r'_{12})(p''_{12},q'_{12},r'_{12})\}$
$\mathcal{X}^{\sim}{}_{21}=\{(p'_{21},q'_{21},r'_{21})(p''_{21},q'_{21},r''_{21})\}$ $\rightarrow$ (I)

$\mathcal{X}^{\sim}{}_{22}= \{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})\}$

Then, modify the above problem as follows,

Minimize $\mathcal{Z}^{\sim} =$ $\{(2, 4, 8)\ (1,4,10)\}\ \{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}+$
$\{\ (2,3,5)\ (1,3,7)\}\ \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r''{}_{12})\}+$
$\{(0, 0, 0)\ (0,0,0)\}\ \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}+$
$\{(0, 0, 0)\ (0,0,0)\}\ \{(p'_{22}, q'_{22}, r'_{22})(p''{}_{22}, q'_{22}, r''{}_{22})\}$

Subject to

$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}+ \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'{}_{12})\}=1$
$\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}+ \{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})\}= 1$
$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}+\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}= 1$
$\{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'{}_{12})\}+ \{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})\} = 1$

Where
$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}, \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'{}_{12})\}, \{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\},$
$\{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})$ are all ≥ 0.

2 Minimize $\mathcal{Z}^{\sim}= \{(2p'_{11}, 4q'_{11}, 8r'_{11})\ (1p''_{11}, 4q'_{11}, 10r''{}_{11})\} +$
$\{(2p'_{12}, , 3q'_{12}, 5r'_{12})\ (1p''_{12},\ 3q'_{12}, 7r''_{12})\}+$
$\{(0p'_{21}, 0q'_{21}, 0r'_{21})\ (0p''_{21}, 0q'_{21}, 0r''_{21}) +$
$\{(0p'_{22}, 0q'_{22}, 0r'_{22})\ (0p''_{22}, 0q'_{22}, 0r''_{22})\}$

Subject to

$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}+ \{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'{}_{12})\}=1$
$\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}+ \{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})\}= 1$
$\{(p'_{11}, q'_{11}, r'_{11})(p''_{11}, q'_{11}, r''_{11})\}+\{(p'_{21}, q'_{21}, r'_{21})(p''_{21}, q'_{21}, r''_{21})\}= 1$
$\{(p'_{12}, q'_{12}, r'_{12})(p''_{12}, q'_{12}, r'{}_{12})\}+ \{(p'_{22}, q'_{22}, r'_{22})(p'{}_{22}, q'_{22}, r'{}_{22})\} = 1$

3 Applying ranking function,

Minimize $\mathcal{Z}^{\sim}= \mathbb{R}\ \{\{(2p'_{11}+2p'_{12} + 0p'_{21} + 0p'_{22}), (4q'_{11} +3q'_{12} + 0q'_{21} + 0q'_{22}),$
$(8r'_{11}+5r'_{12} + 0r'_{21} + 0r'_{22})\ ;$
$(1p''_{11} + 1p''_{12} + 0p''_{21} + 0p''_{22}),), (4q'_{11} +3q'_{12} + 0q'_{21} + 0q'_{22}),$
$(10r''_{11} + 7r''_{12} + 0r''{}_{21} + 0r''_{22})\}\}$

Subject to

$\{(p'_{11} + p'_{12}, q'_{11} +, q'_{12}, r'_{11} + r'_{12})\ ; (p''{}_{11} + p''_{12}, q'_{11} +, q'_{12}, r''_{11} + r''{}_{12})\} = 1$
$\{(p'_{21} + p'_{22}, q'_{21} + q'_{22}, r'_{21} + r'_{22})\ ; (p''_{21} + p''{}_{22}, q'_{21} + q'_{22}, r''{}_{21} + r''{}_{22})\} = 1$
$\{(p'_{11} + p'_{21}, q'_{11} + q'_{21}, r'_{11} + r'_{21})\ ; (p''{}_{11} + p''_{21},\ q'_{11} + q'_{21},\ r''{}_{11} + r''_{21})\}= 1$
$\{(p'_{12} + p'_{22}, q'_{12} + q'_{22}, r'_{12} + r'_{22})\ ; (p''{}_{12} + p''{}_{22}, q'_{12} + q'_{22},\ r''{}_{12}+r''{}_{22})\} = 1$

4 By using arithmetic operations, then the problem converts into crisp assignment problem,

Minimize $\mathcal{Z}^{\sim} =$

$$\frac{2p'_{11}+2p'_{12}+0p'_{21}+0p'_{22}+1p''_{11}+1p''_{12}+0p''_{21}+0p''_{22}+16q'_{11}+12q'_{12}+0q'_{21}+0q'_{22}+8r'_{11}+5r'_{12}+0r'_{21}+0r'_{22}+10r''_{11}+7r''_{12}+0r''_{21}+0r''_{22}}{8}$$

Subject to

$\mathfrak{p}'_{11} + \mathfrak{p}'_{12} = 1$ $\mathfrak{q}'_{11} +, \mathfrak{q}'_{12}=1$ $\mathfrak{r}'_{11} + \mathfrak{r}'_{12} = 1$ $\mathfrak{p}''_{11} + \mathfrak{p}''_{12} = 1$ $\mathfrak{r}''_{11} + \mathfrak{r}''_{12} = 1$

$\mathfrak{p}'_{21} + \mathfrak{p}'_{22} = 1$ $\mathfrak{q}'_{21} + \mathfrak{q}'_{22} = 1$ $\mathfrak{r}'_{11} + \mathfrak{r}'_{12} = 1$ $\mathfrak{p}''_{21} + \mathfrak{p}''_{22} = 1$ $\mathfrak{r}''_{21} + \mathfrak{r}''_{22} = 1$

$\mathfrak{p}'_{11} + \mathfrak{p}'_{21} = 1$ $\mathfrak{q}'_{11} + \mathfrak{q}'_{21} = 1$ $\mathfrak{r}'_{11} + \mathfrak{r}'_{21} =1$ $\mathfrak{p}''_{11} + \mathfrak{p}''_{21} = 1$ $\mathfrak{r}''_{11} + \mathfrak{r}''_{21} = 1$

$\mathfrak{p}'_{12} + \mathfrak{p}'_{22} = 1$ $\mathfrak{q}'_{12} + \mathfrak{q}'_{22} = 1$ $\mathfrak{r}'_{12} + \mathfrak{r}'_{22} = 1$ $\mathfrak{p}''_{12} + \mathfrak{p}''_{22} = 1$ $\mathfrak{r}''_{12}+\mathfrak{r}''_{22} = 1$

5 By using software computations we get the following values

$\mathfrak{p}'_{11} = 0$	$\mathfrak{q}'_{11} = 0$	$\mathfrak{r}'_{11} = 0$	$\mathfrak{p}''_{11} = 0$	$\mathfrak{r}''_{11} = 0$
$\mathfrak{p}'_{12} = 1$	$\mathfrak{q}'_{12} = 1$	$\mathfrak{r}'_{12} = 1$	$\mathfrak{p}''_{12} = 1$	$\mathfrak{r}''_{12} = 1$
$\mathfrak{p}'_{21} = 1$	$\mathfrak{q}'_{21} = 1$	$\mathfrak{r}'_{21} = 1$	$\mathfrak{p}''_{21} = 1$	$\mathfrak{r}''_{21} = 1$
$\mathfrak{p}'_{22} = 0$	$\mathfrak{q}'_{22} = 0$	$\mathfrak{r}'_{22} = 0$	$\mathfrak{p}''_{22} = 0$	$\mathfrak{r}''_{22} = 0$

6 Putting the above values in (I)

we get $\mathcal{X}^{\sim}{}_{11}= \{(0,0,0)(0,0,0)\}$, $\mathcal{X}^{\sim}{}_{12} = \{(1,1,1)(1,1,1)\}$, $\mathcal{X}^{\sim}{}_{21} = \{(1,1,1)(1,1,1)\}$, $\mathcal{X}^{\sim}{}_{22} = \{(0,0,0)(0,0,0)\} \quad \rightarrow \quad (II)$

7 Substitute (I) into the objective function , we get the optimum solution of the pythagorean fuzzy assignment problem is {(2,3,5);(1,3,7)}

and the job allocations are, $\mathcal{W}_1 \rightarrow \mathcal{P}_2, \mathcal{W}_2 \rightarrow No\ job$

5 Conclusion

In an intuitionistic fuzzy sets some of the conditions are give up, it will form Pythagorean fuzzy sets. It will cover more areas comparing with intuitionistic fuzzy sets. Hence it gives an extra opportunity to improving the knowledge, creating it feasible to narrate numerous actual complication in a extra fabulous path. We have solved Pythagorean fuzzy task problem primarily based on triangular Pythagorean fuzzy numbers. We have introduced some functions of the proposed approach to actual world problem. The benefits of the current find out about are, (i) it is very effortless to practice in selection making and (ii) It solves the problem of the task when all of the variables and parameters are positive non-negative triangular Pythagorean integers. (iii) consider giant quantity of data's arise, we can without difficulty calculate the most fulfilling cost by using this software. In future we are designing to prolong our exertion to (i) Interval Pythagorean fuzzy assignment problem (ii) Fractional pythagoean fuzzy assignment problem.

References

[1] Angelov. PP , Optimization in an intuitionistic fuzzy environment. Fuzzy sets and systems. (86) (1997) 299 - 306.

[2] Atanassav. K, Intuitionistic Fuzzy Sets .Fuzzy sets and systems. 20(1) (1986) 87 - 96.

[3] Atanassav. K, More on Intuitionistic Fuzzy sets. Fuzzy sets and systems.(33)(1989) 37- 46.

[4] Atanassav. K, Intuitionistic Fuzzy sets. physica - verlag , Heidelberg, Newyork (1999) .

[5] Kuhn. H.W, The Hungarian method for the Assignment Problem. Naval Re.Logistics Quarterly.(1955) 83 - 97.

[6] Kumar. R , Edalatpanah . S.A etal., A Pythagorean fuzzy approach to the transportation problem. Complex and intelligent systems.(2019),255 – 263.

[7] Laxminarayan Sahoo (2021), A new score function based Fermatean fuzzy transportation problem,Results in Control and Optimization, Vol 4, https://doi.org/10.1016/j.rico.2021.100040

[8] Luqman A , Akram .M , Alcantud JCR (2020) Digraph and Matrix approach for risk evaluations under Pythagorean fuzzy information. Expert Syst. Appl 170 : 114518.

[9] Mahapatra G.S , Roy TK (2009) Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations. World Acad Sci Eng Technol 50 : 574 -581.

[10] Muhammad Akram, Inayat Ullah et al (2021) Fully Pythagorean fuzzy linear programming problems with equality constraints. Computational and applied Mathematics 40 : 120. https://doi.org/10.1007/s40314-021-01503-9.

[11] Nagoorgani A, Ponnalagu K (2012) A new approach on solving intuitionistic fuzzy linear programming problem. Appl Math Sci 6(70) : 3467 – 3474..

[12] Nagoor Gani and V.N. Mohamed Mohamed (2019) The signature method for solving Intuitionistic fuzzy Assignment Prroblems. Journal of Physical Sciences, Vol 24, 2019,21 – 30. ISSN : 2350 -0352.

[13] Priyanka Nagar, Pankaj Kumar Srivasta et al (2021) Optimization of Fuzzy species Pythagorean Transportation problem under Preserved Uncertainties. Int. J. Of Mathematical , Engineering and Management Sciences 6 (6) : 1629 – 1645.

[14] DOI : 10.33889/IJMEMS.2021.6.6.097.

[15] Ronald R. Yager (2016) Properties and Applications of Pythagorean Fuzzy sets . Springer international Publishing, Switzerland. DOI 10.1007/978-3-319-26302-1_9

[16] Ting – Yu Chen (2018) , An interval –valued Pythagorean Fuzzy outranking method with a closeness – based assignment model for multiple criteria decision making, International journal of intelligent systems, Volume 33, Issue 1 , 126 – 168.

[17] https://doi.org/10.1002/int.21943

[18] Yager R.R, Pythagorean fuzzy subsets. In: 2013joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS).(2013),57 – 61.

[19] Yager RR , Abbasov AM (2013) Pyhagorean membership grades , complex numbers and decision making. Int. J. Intell Syst 28(5) : 436 – 452.

[20] Yager RR (2014) Pythagorean membership grades in multi-criteria decision making.IEEE Trans Fuzzy syst 22(4) : 958 -965.

[21] Zadeh. L. A, Fuzzy sets .Information and control. 8(3) (1965) 338 - 353.

[22] Zhang X , XuZ (2014) Extension of TOPSIS to multiple – criteria decision making with Pythagorean fuzzy sets. Int J. Intell syst 29 (12) : 1061 – 1078.