Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

# A New Efficient Difference-Type Estimator for Estimating Population Mean using Dual Auxiliary Information under Non-Response

**Udita Gupta, Prof. Manoj Kumar Srivastava, Prof. Namita Srivastava, Anjali Bhardwaj**

Research Scholar, Department of Statistics, I.S.S., Dr. Bhimrao Ambedkar University, Agra. Mail Id: drudita0108@gmail.com

Vice-Chancellor, Shahid Mahendra Karma Vishwavidalaya, Bastar, Mail Id: mksiss87@gmail.com

Professor and Head of Department of Statistics, St. John's College, Agra, Mail Id: drnamita.sjc@gmail.com

Research Scholar, Department of Statistics, I.S.S., Dr. Bhimrao Ambedkar University, Agra, Mail Id: anjalibharadwaj32@gmail.com

Department of Statistics, Dr. Bhimrao Ambedkar University, Agra

***Article History:***

***Received: 12-01-2025***

***Revised: 15-02-2025***

***Accepted: 01-03-2025***

**Abstract:**

In this paper, the problem of estimating the finite population mean by using dual auxiliary information under non-response. This paper proposed a difference-type estimator of population mean under different cases. The expressions of the bias and mean square error (MSE) of the proposed estimator have been obtained up to the first-degree approximation. The proposed estimator has been compared with usual unbiased estimator, ratio estimator, product estimator, difference-cum-ratio estimator, difference-cum-product estimator and other existing estimator and the cases obtained to show the efficiency of the proposed estimator over other considered estimators. Numerical Illustration is carried out to support the theoretical findings.

**Keywords:** Study Variable, Auxiliary Variable, Bias, MSE, Ranked Auxiliary Variable, Non-Response.

## 1. INTRODUCTION

One of the sample survey objectives is to estimate the unknown population parameters of the study variable such as population total, mean, proportion, ratio and variances etc. A procedure is desirable that provides a precise estimator of the parameter of interest by surveying a suitably chosen sample of individuals. Supplementary/ additional information provided by an auxiliary variable which is correlated with the study variable enhances the precision of the estimators. Survey statisticians take advantage of this information whenever it is available to explore the efficient estimators. Ratio, product, regression and their modified estimators are best examples in this regard.

In practice almost all surveys suffer from non-response. Problems with no response are often caused by rejection of topics, absences and sometimes due to the lack of information. The pioneering work of Hansen and Hurwitz (1946), states that "A sub-sample of initially non-responsive individuals is recontacted using a more expensive method, suggesting the first attempt by mail-based questionnaire and the second attempt by a face-to-face interview". When estimating population parameters such as the mean, total or ratio, sampling experts sometimes use auxiliary information to improve efficiency of the estimates.

It is known that the efficiency of an estimator of the population mean of the study variable y can be increased by using auxiliary information which is highly correlated with the study variable y. Rao (1986), Khare and Srivastava (1995, 1997), Okafor and Lee (2000) and Singh and Kumar (2008, 2009, 2010) have proposed some estimator for population mean of the study variable y using auxiliary information in presence of non-response.

Recently, Haq et al. used an additional information of the auxiliary variable called ranked auxiliary variable to develop efficient estimators for the estimation of mean. These estimators are developed only to cope with the simple random sampling scheme.

Here, a new challenge/idea arises to search for a more optimal estimator using dual auxiliary information to deal with non-response scheme. This challenge is successfully accomplished and new optimal estimators for finite population mean are developed under non-response scheme in this paper.

The remaining part of the paper is organized as follows: In section 2, notations under non-response are introduced. In section 3, existing estimator under non-response. In section 4, proposed estimator for estimating finite population mean using the original and ranked auxiliary information are defined. In section 5, theoretical comparison is done between existing and proposed estimator. An empirical study is carried out to evaluate the performance of the proposed estimators which validate the theoretical results in section 6. Conclusions are enclosed in the last section.

## 2. NOTATION

For a finite population $U = (U_1, U_2, U_3 \dots U_N)$ of size N and a random sample of size $n$ is drawn without replacement. Let the characteristics under study, $y$ (say) takes value $y_i$ on the unit $U_i, (i = 1, 2, 3 \dots N)$. In survey on human population, it is often the case that $n_i$ unit respond on the first attempt while $n_2\ (= n - n_1)$ units do not provide any response. In the case of non-response at the initial stage, Hansen and Hurwitz (1946) proposed a double sampling plan for estimating the population mean having the steps given below:

**a)** A simple random sample of size $n$ is drawn and the questionnaire is mailed to the sample units

**b)** A sub-sample of size $r = (n_2/k), (k \geq 1)$ from the $n_2$ non-responding units in the initial attempt is contacted through personal interviews.

In the Hansen and Hurwitz method the population is supposed to be consisting of response stratum of size $N_1$ and the non-response stratum of size $N_2 = (N - N_1)$. Let $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ denote the population mean and variance of the study variable y. Let $\bar{Y}_1 = \frac{1}{N_1}\sum_{i=1}^{N_1} y_i$ and $S_{y(1)}^2 = \frac{1}{N_1-1}\sum_{i=1}^{N_1}(y_i - \bar{Y}_1)^2$ denote the mean and variance of the respondent group (or strata). Similarly, let $\bar{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} y_i$ and $S_{y(2)}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i - \bar{Y}_2)^2$ denote the mean and variance of the non-respondent group (or strata). The population mean can be written as

$$\bar{Y} = W_1\bar{Y}_1 + W_2\bar{Y}_2$$

where $W_1 = \left(\frac{N_1}{N}\right)$ and $W_2 = \left(\frac{N_2}{N}\right)$. The sample mean $\bar{y}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i$ denote the mean of the $n_1$ responding units and $\bar{y}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} y_i$ denote the mean of the $n_2$ non-responding units.

Let $\bar{y}_{2r} = \frac{1}{r}\sum_{i=1}^{r} y_i$ denote the mean of the $r$ sub-sampled units where $r = \frac{n_2}{k}$.Hansen and Hurwitz (1946) suggested an unbiased estimator for the population mean $\bar{Y}$ of the study variable $y$ is given as:

$$\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_{2r} \tag{2.1}$$

where $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$ are responding proportions and non-responding proportions of the sample. The variance $\bar{y}^*$ is given below:

$$V(\bar{y}^*) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2\right] \tag{2.2}$$

where $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ and $C_{y(2)}^2 = \frac{S_{y(2)}^2}{\bar{Y}^2}$.

Let $x_i(i = 1,2, \ldots N)$ denote the auxiliary variable correlated with the study variable $y_i(i = 1,2, \ldots, N)$. Let $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ and $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ denote the population mean and variance of the auxiliary variable y. Let $\bar{X}_1 = \frac{1}{N_1}\sum_{i=1}^{N_1} x_i$ and $S_{x(1)}^2 = \frac{1}{N_1-1}\sum_{i=1}^{N_1}(x_i - \bar{X}_1)^2$ denote the mean and variance of the respondent group (or strata). Similarly, let $\bar{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} y_i$ and $S_{x(2)}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(x_i - \bar{X}_2)^2$ denote the mean and variance of the non-respondent group (or strata). Let $\bar{x} =$

$\frac{1}{n}\sum_{i=1}^{n} x_i$ denote the mean of all the n units. Let $\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ denote the mean of the $n_1$ responding units and $\bar{x}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_i$ denote the mean of the $n_2$ non-responding units.

Let $\bar{x}_{2r} = \frac{1}{r}\sum_{i=1}^{r} x_i$ denote the mean of the $r$ sub-sampled units where $r = \frac{n_2}{k}$. With this background, define an unbiased estimator of population mean $\bar{X}$ is given as:

$$\bar{x}^* = w_1\bar{x}_1 + w_2\bar{x}_{2r} \quad (2.3)$$

The variance of $\bar{x}^*$ is given below:

$$V(\bar{x}^*) = \bar{X}^2\left[\left(\frac{1-f}{n}\right)C_x^2 + \frac{W_2(k-1)}{n}C_{x(2)}^2\right] \quad (2.4)$$

where $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ and $C_{x(2)}^2 = \frac{S_{x(2)}^2}{\bar{X}^2}$.

2.1 Bias and Mean Square Error (MSE) of the Proposed Estimators

Let us define the following terms:

$\bar{y}^* = \bar{Y}(1+\varepsilon_0)$

$\bar{x}^* = \bar{X}(1+\varepsilon_1)$

$\bar{r}_x^* = \bar{R}_x(1+\varepsilon_2)$

$$E(\varepsilon_0) = E(\varepsilon_1) = \mathrm{E}(\varepsilon_2) = 0 \quad (2.5)$$

$$E(\varepsilon_0^2) = \left[\left(\frac{1-f}{n}\right)C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2\right] = B\ (say) \quad (2.6)$$

$$E(\varepsilon_1^2) = \left[\left(\frac{1-f}{n}\right)C_x^2 + \frac{W_2(k-1)}{n}C_{x(2)}^2\right] = A\ (say) \quad (2.7)$$

$$E(\varepsilon_0\varepsilon_1) = \left[\left(\frac{1-f}{n}\right)\rho_{yx}C_yC_x + \frac{W_2(k-1)}{n}\rho_{yx(2)}C_{y(2)}C_{x(2)}\right] = C\ (say) \quad (2.8)$$

$$E(\varepsilon_2^2) = \left[\left(\frac{1-f}{n}\right)C_{r_x}^2 + \frac{W_2(k-1)}{n}C_{r_x(2)}^2\right] = D\ (say)$$

$$E(\varepsilon_0\varepsilon_2) = \left[\left(\frac{1-f}{n}\right)\rho_{yr_x}C_yC_{r_x} + \frac{W_2(k-1)}{n}\rho_{yr_x(2)}C_{y(2)}C_{r_x(2)}\right] = \mathrm{E}\ (say)$$

$$E(\varepsilon_1\varepsilon_2) = \left[\left(\frac{1-f}{n}\right)\rho_{xr_x}C_xC_{r_x} + \frac{W_2(k-1)}{n}\rho_{xr_x(2)}C_{x(2)}C_{r_x(2)}\right] = \mathrm{F}\ (say)$$

where

$\rho_{yx} = S_{yx}/S_xS_y$

$\rho_{yx(2)} = \frac{S_{yx(2)}}{S_{x(2)}S_{y(2)}}$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$$

$$S_{yx(2)} = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i - \bar{Y}_2)(x_i - \bar{X}_2)$$

$\rho_{yr_x} = S_{yr_x}/S_{r_x}S_y$

$\rho_{yr_x(2)} = \frac{S_{yr_x(2)}}{S_{r_x(2)}S_{y(2)}}$

$$S_{yr_x} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})\left(r_{x(i)} - \bar{R}_x\right)$$

$$S_{yr_x(2)} = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i - \bar{Y}_2)(r_{x(i)} - \bar{R}_{x(2)})$$

$\rho_{xr_x} = S_{xr_x}/S_{r_x}S_x$

$$\rho_{xr_x(2)} = \frac{S_{xr_x(2)}}{S_{r_x(2)}S_{x(2)}}$$

$$S_{xr_x} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})\left(r_{x(i)} - \bar{R}_x\right)$$

$$S_{xr_x(2)} = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(x_i - \bar{X}_2)(r_{x(i)} - \bar{R}_{x(2)})$$

$$R = \frac{\bar{Y}}{\bar{X}}$$

$$\boldsymbol{R}^* = \frac{\bar{\boldsymbol{Y}}}{\bar{\boldsymbol{R}}_{\boldsymbol{x}}}$$

## 3. EXISTING ESTIMATOR

When few observations are missing in the sample. Initial, estimators for estimating population mean $\bar{Y}$ was suggested by Hansen and Hurwitz (1946). Many other authors work for the similar situation. Here, we are giving few of the existing estimators for population mean $\bar{Y}$ when some of the observations are missing.

(i) Rao (1986) suggested a ratio estimator for the population mean $\bar{Y}$ of the study variable $y$ is given as

$$t_r^* = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*}\right) \quad (3.1)$$

$$B(t_r^*) = \bar{Y}(A - C)$$

$$MSE(t_r^*) = \bar{Y}^2(B + A - 2C)$$

$$MSE(t_r^*) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\} + \frac{W_2(k-1)}{n}\left[C_{y(2)}^2 + C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}\right]\right]$$

(ii) Khare and Srivastava (1993) suggested a product estimator for the population mean $\boldsymbol{\bar{Y}}$ of the study variable $\boldsymbol{y}$ is given as

$$t_{ks}^* = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{X}}\right) \quad (3.2)$$

$$B(t_{ks}^*) = \bar{Y}C$$

$$MSE(t_{ks}^*) = \bar{Y}^2(A + B + 2C)$$

$$MSE(t_{ks}^*) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)\{C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\} + \frac{W_2(k-1)}{n}\left[C_{y(2)}^2 + C_{x(2)}^2 + 2\rho_{yx(2)}C_{y(2)}C_{x(2)}\right]\right]$$

(iii) Singh et. al. (2008) suggested an exponential ratio type estimators for the population mean $\bar{Y}$ of the study variable $y$ are

$$t_{er}^* = \bar{y}^* exp\left[\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right] \quad (3.3)$$

$$B(t_{er}^*) = \frac{\bar{Y}}{8}(3A - 4C)$$

$$MSE(t_{er}^*) = \bar{Y}^2\left(B + \frac{A}{4} - C\right)$$

$$MSE(t_{er}^*) = \bar{Y}^2\left[\left(\frac{1-f}{n}\right)\left\{C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_yC_x\right\} + \frac{W_2(k-1)}{n}\left[C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} - \rho_{yx(2)}C_{y(2)}C_{x(2)}\right]\right]$$

(iv) Singh et. al. (2008) suggested an exponential product type estimators for the population mean $\bar{Y}$ of the study variable $y$ are

$$t_{ep}^{*} = \bar{y}^{*} exp\left[\frac{\bar{x}^{*} - \bar{X}}{\bar{x}^{*} + \bar{X}}\right] \tag{3.4}$$

$$B\left(t_{ep}^{*}\right) = \frac{\bar{Y}}{8}(4C - A)$$

$$MSE\left(t_{ep}^{*}\right) = \bar{Y}^{2}\left(B + \frac{A}{4} + C\right)$$

$$MSE\left(t_{ep}^{*}\right) = \bar{Y}^{2}\left[\left(\frac{1-f}{n}\right)\left\{C_y^2 + \frac{C_x^2}{4} + \rho_{yx}C_yC_x\right\} + \frac{W_2(k-1)}{n}\left[C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} + \rho_{yx(2)}C_{y(2)}C_{x(2)}\right]\right]$$

(v) Sunil Kumar and Sandeep Bhougal (2011) suggested a modified ratio-product type exponential estimator for the population mean $\bar{Y}$ of the study variable $y$

$$t_{ss}^{*} = \bar{y}^{*}\left\{\alpha exp\left(\frac{\bar{X} - \bar{x}^{*}}{\bar{X} + \bar{x}^{*}}\right) + (1-\alpha) exp\left(\frac{\bar{x}^{*} - \bar{X}}{\bar{x}^{*} + \bar{X}}\right)\right\}$$

The corrected bias of this estimator is

$$B(t_{ss}^{*}) = \left(\frac{A}{8} - \frac{C}{2}\right)(4\alpha - 1)$$

$$MSE(t_{ss}^{*}) = \bar{Y}^{2}\left[B - \frac{C^{2}}{A}\right]$$

## 4. PROPOSED ESTIMATOR

$\bar{y}^{*}, \bar{x}^{*}, \bar{X}$ *and* $\bar{R}_x$ are used. The non-response occurs on both study variable $y$ as well as auxiliary variable $x$, the population mean $\bar{X}$ of the auxiliary variable and the rank of the auxiliary variable $x$ are known. In this, we proposed a difference type estimator and the estimator is

$$t_{rp}^{*} = \omega_1\bar{y}^{*} + \omega_2(\bar{X} - \bar{x}^{*}) + \omega_3(\bar{R}_x - \bar{r}_x^{*}) \tag{4.1}$$

where $\omega_1, \omega_2$ & $\omega_3$ is a real constant to be determined such that the MSE of $t_{rp}^{*}$ is minimum.

Now, expressing $t_{rp}^{*}$ in terms of $\varepsilon's$ we have

$$t_{rp}^{*} = \omega_1\bar{Y}(1+\varepsilon_0) + \omega_2\left(\bar{X} - \bar{X}(1+\varepsilon_1)\right) + \omega_3(\bar{R}_x - \bar{R}_x(1+\varepsilon_2))$$

$$= \omega_1\bar{Y}(1+\varepsilon_0) - \omega_2\bar{X}\varepsilon_1 - \omega_3\bar{R}_x\varepsilon_2 \tag{4.2}$$

Subtracting $\bar{Y}$ on both sides of (4.2), we get

$$t_{rp}^{*} - \bar{Y} = (\omega_1 - 1)\bar{Y} + \omega_1\bar{Y}\varepsilon_0 - \omega_2\bar{X}\varepsilon_1 - \omega_3\bar{R}_x\varepsilon_2 \tag{4.3}$$

Taking Expectation on both sides of (4.3), we get the bias of the estimator $t_{rp}^*$ as

$$E(t_{rp}^* - \bar{Y}) = (\omega_1 - 1)\bar{Y} + \omega_1 \bar{Y} E(\varepsilon_0) - \omega_2 \bar{X} E(\varepsilon_1) - \omega_3 \bar{R}_x E(\varepsilon_2)$$

$$B\left(t_{rp}^*\right) = (\omega_1 - 1)\bar{Y} \tag{4.4}$$

Squaring of (4.3) on both sides, we have

$$\begin{aligned}(t_{rp}^* - \bar{Y})^2 = {} & (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2\varepsilon_0^2 + \omega_2^2\bar{X}^2\varepsilon_1^2 + \omega_3^2\bar{R}_x^2\varepsilon_2^2 + 2\omega_1(\omega_1 - 1)\bar{Y}^2\varepsilon_0 \\ & - 2\omega_2(\omega_1 - 1)\bar{Y}\bar{X}\varepsilon_1 - 2\omega_3(\omega_1 - 1)\bar{Y}\bar{R}_x\varepsilon_2 - 2\omega_1\omega_2\bar{Y}\bar{X}\varepsilon_0\varepsilon_1 - 2\omega_1\omega_3\bar{Y}\bar{R}_x\varepsilon_0\varepsilon_2 \\ & + 2\omega_2\omega_3\bar{Y}\bar{R}_x\varepsilon_1\varepsilon_2 \qquad (4.5)\end{aligned}$$

Taking Expectation on both sides of (4.5), we get the exact mean square error (MSE) of $t_{rp}^*$, as

$$\begin{aligned}E(t_{rp}^* - \bar{Y})^2 = {} & (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2E(\varepsilon_0^2) + \omega_2^2\bar{X}^2E(\varepsilon_1^2) + \omega_3^2\bar{R}_x^2E(\varepsilon_2^2) + 2\omega_1(\omega_1 - 1)\bar{Y}^2E(\varepsilon_0) \\ & - 2\omega_2(\omega_1 - 1)\bar{Y}\bar{X}E(\varepsilon_1) - 2\omega_3(\omega_1 - 1)\bar{Y}\bar{R}_xE(\varepsilon_2) - 2\omega_1\omega_2\bar{Y}\bar{X}E(\varepsilon_0\varepsilon_1) \\ & - 2\omega_1\omega_3\bar{Y}\bar{R}_xE(\varepsilon_0\varepsilon_2) + 2\omega_2\omega_3\bar{Y}\bar{R}_xE(\varepsilon_1\varepsilon_2)\end{aligned}$$

$$\begin{aligned}MSE\left(t_{rp}^*\right) = {} & (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2B + \omega_2^2\bar{X}^2A + \omega_3^2\bar{R}_x^2D - 2\omega_1\omega_2\bar{Y}\bar{X}C - 2\omega_1\omega_3\bar{Y}\bar{R}_xE \\ & + 2\omega_2\omega_3\bar{Y}\bar{R}_xF \qquad (4.6)\end{aligned}$$

### 4.1 Optimum Choice of $\boldsymbol{\omega_1, \omega_2}$ & $\boldsymbol{\omega_3}$ and the Minimum MSE of Proposed Class of Estimator '$\boldsymbol{t_{rp}^*}$'

Differentiating equation (4.6) partially w.r.to $\omega_1, \omega_2$ & $\omega_3$ and equating to zero for obtaining the optimum value of $\omega_1, \omega_2$ & $\omega_3$. The optimum value of $\omega_1, \omega_2$ & $\omega_3$ which makes the MSE minimum of equation (4.6) is given by

$$\begin{bmatrix} 1+B & -C/R & -E/R^* \\ C & -A/R & -F/R^* \\ E & -F/R & -D/R^* \end{bmatrix}\begin{bmatrix}\omega_1 \\ \omega_2 \\ \omega_3\end{bmatrix} = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix} \tag{4.7}$$

Solving (4.7), we get the optimum values of $\omega_1, \omega_2$ & $\omega_3$ as

$$\omega_1 = \frac{AD - F^2}{(1+B)(AD - F^2) + C(EF - CD) - E(AE - CF)} = V_1(say) \qquad (4.8a)$$

$$\omega_2 = \frac{R(CD - EF)}{(1+B)(AD - F^2) + C(EF - CD) - E(AE - CF)} = V_2(say) \qquad (4.8b)$$

$$\omega_3 = R^*\left(\frac{AE - CF}{(1+B)(AD - F^2) + C(EF - CD) - E(AE - CF)}\right) = V_3(say) \qquad (4.8c)$$

Thus, the resulting minimum MSE of the proposed estimator '$t_{rp}^*$' is given by

$$min.MSE\left(t_{rp}^*\right) = (V_1 - 1)^2\bar{Y}^2 + V_1^2\bar{Y}^2B + V_2^2\bar{X}^2A + V_3^2\bar{R}_x^2D - 2V_1V_2\bar{Y}\bar{X}C - 2V_1V_3\bar{Y}\bar{R}_xE + 2V_2V_3\bar{Y}\bar{R}_xF \quad (\mathbf{4.9})$$

**<u>Case 1:</u>** If $\omega_2 = 0$, the proposed estimator '$t_{rp}^*$' will be reduced as

$$t_{rp1}^{*} = \omega_1 \bar{y}^{*} + \omega_3(\bar{R}_x - \bar{r}_x^{*}) \tag{4.10}$$

where $\omega_1$ & $\omega_3$ is a real constant to be determined such that the MSE of $t_{rp1}^{*}$ is minimum.

Now, expressing $t_{rp1}^{*}$ in terms of $\varepsilon's$ we have

$$t_{rp1}^{*} = \omega_1 \bar{Y}(1+\varepsilon_0) + \omega_3(\bar{R}_x - \bar{R}_x(1+\varepsilon_2))$$

$$= \omega_1 \bar{Y}(1+\varepsilon_0) - \omega_3 \bar{R}_x \varepsilon_2 \tag{4.11}$$

$$t_{rp1}^{*} - \bar{Y} = (\omega_1 - 1)\bar{Y} + \omega_1 \bar{Y}\varepsilon_0 - \omega_3 \bar{R}_x \varepsilon_2 \tag{4.12}$$

Taking Expectation on both sides of (4.12), we get the bias of the estimator $t_{rp1}^{*}$ as

$$E(t_{rp1}^{*} - \bar{Y}) = (\omega_1 - 1)\bar{Y} + \omega_1 \bar{Y}E(\varepsilon_0) - \omega_3 \bar{R}_x E(\varepsilon_2)$$

$$B\left(t_{rp1}^{*}\right) = (\omega_1 - 1)\bar{Y} \tag{4.13}$$

$$(t_{rp1}^{*} - \bar{Y})^2 = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2\varepsilon_0^2 + \omega_3^2\bar{R}_x^2\varepsilon_2^2 + 2\omega_1(\omega_1 - 1)\bar{Y}^2\varepsilon_0 - 2\omega_3(\omega_1 - 1)\bar{Y}\bar{R}_x\varepsilon_2 - 2\omega_1\omega_3\bar{Y}\bar{R}_x\varepsilon_0\varepsilon_2 \tag{4.14}$$

Taking Expectation on both sides of (4.14), we get the exact mean square error (MSE) of $t_{rp1}^{*}$, as

$$E(t_{rp1}^{*} - \bar{Y})^2 = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2E(\varepsilon_0^2) + \omega_3^2\bar{R}_x^2E(\varepsilon_2^2) + 2\omega_1(\omega_1 - 1)\bar{Y}^2E(\varepsilon_0) - 2\omega_3(\omega_1 - 1)\bar{Y}\bar{R}_xE(\varepsilon_2) - 2\omega_1\omega_3\bar{Y}\bar{R}_xE(\varepsilon_0\varepsilon_2)$$

$$MSE\left(t_{rp1}^{*}\right) = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2B + \omega_3^2\bar{R}_x^2D - 2\omega_1\omega_3\bar{Y}\bar{R}_xE \tag{4.15}$$

Differentiating equation (4.15) partially w.r.to $\omega_1$ & $\omega_3$ and equating to zero for obtaining the optimum value of $\omega_1$ & $\omega_3$. The optimum value of $\omega_1$ & $\omega_3$ which makes the MSE minimum of equation (4.15) is given by

$$\omega_1 = \frac{D}{D + BD - E^2} = V_{11}(say) \tag{4.16a}$$

$$\omega_3 = R^{*}\left(\frac{E}{D + BD - E^2}\right) = V_{13}(say) \tag{4.16b}$$

Thus, the resulting minimum MSE of the proposed estimator '$t_{rp1}^{*}$' is given by

$$min.MSE\left(t_{rp1}^{*}\right) = (V_{11} - 1)^2\bar{Y}^2 + V_{11}^2\bar{Y}^2B + V_{13}^2\bar{R}_x^2D - 2V_{11}V_{13}\bar{Y}\bar{R}_xE \tag{4.17}$$

**Case 2:** If $\omega_3 = 0$, the proposed estimator '$t_{rp}^{*}$' will be reduced as

$$t_{rp2}^{*} = \omega_1 \bar{y}^{*} + \omega_2(\bar{X} - \bar{x}^{*}) \tag{4.18}$$

where $\omega_1$ & $\omega_2$ is a real constant to be determined such that the MSE of $t_{rp2}^{*}$ is minimum.

Now, expressing $t_{rp2}^{*}$ in terms of $\varepsilon's$ we have

$$t_{rp2}^{*} = \omega_1 \bar{Y}(1+\varepsilon_0) + \omega_2\left(\bar{X} - \bar{X}(1+\varepsilon_1)\right)$$

$$= \omega_1 \bar{Y}(1+\varepsilon_0) - \omega_2 \bar{X}\varepsilon_1 \quad (4.19)$$

$$t_{rp2}^{*} - \bar{Y} = (\omega_1 - 1)\bar{Y} + \omega_1 \bar{Y}\varepsilon_0 - \omega_2 \bar{X}\varepsilon_1 \quad (4.20)$$

Taking Expectation on both sides of (4.20), we get the bias of the estimator $t_{rp2}^{*}$ as

$$E\left(t_{rp2}^{*} - \bar{Y}\right) = (\omega_1 - 1)\bar{Y} + \omega_1 \bar{Y}E(\varepsilon_0) - \omega_2 \bar{X}E(\varepsilon_1)$$

$$B\left(t_{rp2}^{*}\right) = (\omega_1 - 1)\bar{Y} \quad (4.21)$$

Squaring of (4.20) on both sides, we have

$$(t_{rp2}^{*} - \bar{Y})^2 = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2\varepsilon_0^2 + \omega_2^2\bar{X}^2\varepsilon_1^2 + 2\omega_1(\omega_1 - 1)\bar{Y}^2\varepsilon_0 - 2\omega_2(\omega_1 - 1)\bar{Y}\bar{X}\varepsilon_1 - 2\omega_1\omega_2\bar{Y}\bar{X}\varepsilon_0\varepsilon_1 \quad (4.22)$$

Taking Expectation on both sides of (4.22), we get the exact mean square error (MSE) of $t_{rp2}^{*}$, as

$$E(t_{rp2}^{*} - \bar{Y})^2 = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2E(\varepsilon_0^2) + \omega_2^2\bar{X}^2E(\varepsilon_1^2) + 2\omega_1(\omega_1 - 1)\bar{Y}^2E(\varepsilon_0) - 2\omega_2(\omega_1 - 1)\bar{Y}\bar{X}E(\varepsilon_1) - 2\omega_1\omega_2\bar{Y}\bar{X}E(\varepsilon_0\varepsilon_1)$$

$$MSE\left(t_{rp2}^{*}\right) = (\omega_1 - 1)^2\bar{Y}^2 + \omega_1^2\bar{Y}^2B + \omega_2^2\bar{X}^2A - 2\omega_1\omega_2\bar{Y}\bar{X}C \quad (4.23)$$

Differentiating equation (4.23) partially w.r.to $\omega_1$ & $\omega_2$ and equating to zero for obtaining the optimum value of $\omega_1$ & $\omega_2$. The optimum value of $\omega_1$ & $\omega_2$ which makes the MSE minimum of equation (4.23) is given by

$$\omega_1 = -\frac{A}{C^2 - A(1+B)} = V_{21}(say) \quad (4.24a)$$

$$\omega_2 = -\frac{CR}{C^2 - A(1+B)} = V_{22}(say) \quad (4.24b)$$

Thus, the resulting minimum MSE of the proposed estimator '$t_{rp2}^{*}$' is given by

$$min.\,MSE\left(t_{rp2}^{*}\right) = (V_{21} - 1)^2\bar{Y}^2 + V_{21}^2\bar{Y}^2B + V_{22}^2\bar{X}^2A - 2V_{21}V_{22}\bar{Y}\bar{X}C \quad (4.25)$$

## 5. THEORETICAL EFFICIENCY COMPARISON

The MSE's of the existing estimators to the first degree of approximation are derived as:

$$Var\,(\bar{y}^{*}) = \bar{Y}^2B \quad (5.1)$$

$$MSE(t_r^{*}) = \bar{Y}^2(B + A - 2C) \quad (5.2)$$

$$MSE(t_{ks}^{*}) = \bar{Y}^2(A + B + 2C) \tag{5.3}$$

$$MSE(t_{er}^{*}) = \bar{Y}^2\left(B + \frac{A}{4} - C\right) \tag{5.4}$$

$$MSE\left(t_{ep}^{*}\right) = \bar{Y}^2\left(B + \frac{A}{4} + C\right) \tag{5.5}$$

$$MSE(t_{ss}^{*}) = \bar{Y}^2\left[B - \frac{C^2}{A}\right] \tag{5.6}$$

Below is the comparison between proposed estimators with another existing estimator. Efficiency condition over some related existing estimators.

1. $Var\,(\bar{y}^{*}) - MSE\left(t_{rp}^{*}\right) = \bar{Y}^2B - (V_1 - 1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

2. $MSE(t_r^{*}) - MSE\left(t_{rp}^{*}\right) = \bar{Y}^2(B + A - 2C) - (V_1 - 1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

3. $MSE(t_{ks}^{*}) - MSE\left(t_{rp}^{*}\right) = \bar{Y}^2(B + A + 2C) - (V_1 - 1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

4. $MSE(t_{er}^{*}) - MSE\left(t_{rp}^{*}\right) = \bar{Y}^2\left(B + \frac{A}{4} - C\right) - (V_1 - 1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

5. $MSE\left(t_{ep}^{*}\right) - MSE\left(t_{rp}^{*}\right) = \bar{Y}^2\left(B + \frac{A}{4} + C\right) - (V_1 - 1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

6. $Var(\bar{y}^{*}) - MSE\left(t_{rp1}^{*}\right) = \bar{Y}^2B - (V_{11} - 1)^2\bar{Y}^2 - V_{11}^2\bar{Y}^2B - V_{13}^2\bar{R}_x^2D + 2V_{11}V_{13}\bar{Y}\bar{R}_xE \geq 0$

7. $MSE(t_r^{*}) - MSE\left(t_{rp1}^{*}\right) = \bar{Y}^2(B + A - 2C) - (V_{11} - 1)^2\bar{Y}^2 - V_{11}^2\bar{Y}^2B - V_{13}^2\bar{R}_x^2D + 2V_{11}V_{13}\bar{Y}\bar{R}_xE \geq 0$

8. $MSE(t_{ks}^{*}) - MSE\left(t_{rp1}^{*}\right) = \bar{Y}^2(B + A + 2C) - (V_{11} - 1)^2\bar{Y}^2 - V_{11}^2\bar{Y}^2B - V_{13}^2\bar{R}_x^2D + 2V_{11}V_{13}\bar{Y}\bar{R}_xE \geq 0$

9. $MSE(t_{er}^{*}) - MSE\left(t_{rp1}^{*}\right) = \bar{Y}^2\left(B + \frac{A}{4} - C\right) - (V_{11} - 1)^2\bar{Y}^2 - V_{11}^2\bar{Y}^2B - V_{13}^2\bar{R}_x^2D + 2V_{11}V_{13}\bar{Y}\bar{R}_xE \geq 0$

10. $MSE\left(t_{ep}^{*}\right) - MSE\left(t_{rp1}^{*}\right) = \bar{Y}^2\left(B + \frac{A}{4} + C\right) - (V_{11} - 1)^2\bar{Y}^2 - V_{11}^2\bar{Y}^2B - V_{13}^2\bar{R}_x^2D + 2V_{11}V_{13}\bar{Y}\bar{R}_xE \geq 0$

11. $Var(\bar{y}^*) - MSE(t_{rp2}^*) = \bar{Y}^2B - (V_{21}-1)^2\bar{Y}^2 - V_{21}^2\bar{Y}^2B - V_{22}^2\bar{X}^2A + 2V_{21}V_{22}\bar{Y}\bar{X}C \geq 0$

12. $MSE(t_r^*) - MSE\left(t_{rp2}^*\right) = \bar{Y}^2(B+A-2C) - (V_{21}-1)^2\bar{Y}^2 - V_{21}^2\bar{Y}^2B - V_{22}^2\bar{X}^2A + 2V_{21}V_{22}\bar{Y}\bar{X}C \geq 0$

13. $MSE(t_{ks}^*) - MSE\left(t_{rp2}^*\right) = \bar{Y}^2(B+A+2C) - (V_{21}-1)^2\bar{Y}^2 - V_{21}^2\bar{Y}^2B - V_{22}^2\bar{X}^2A + 2V_{21}V_{22}\bar{Y}\bar{X}C \geq 0$

14. $MSE(t_{er}^*) - MSE\left(t_{rp2}^*\right) = \bar{Y}^2\left(B+\frac{A}{4}-C\right) - (V_{21}-1)^2\bar{Y}^2 - V_{21}^2\bar{Y}^2B - V_{22}^2\bar{X}^2A + 2V_{21}V_{22}\bar{Y}\bar{X}C \geq 0$

15. $MSE\left(t_{ep}^*\right) - MSE\left(t_{rp2}^*\right) = \bar{Y}^2\left(B+\frac{A}{4}+C\right) - (V_{21}-1)^2\bar{Y}^2 - V_{21}^2\bar{Y}^2B - V_{22}^2\bar{X}^2A + 2V_{21}V_{22}\bar{Y}\bar{X}C \geq 0$

**<u>COMPARISON WITHIN CASES</u>**

Below is the comparison between proposed estimator and its cases:

a) $MSE\left(t_{rp1}^*\right) - MSE\left(t_{rp}^*\right) = (V_{11}-1)^2\bar{Y}^2 + V_{11}^2\bar{Y}^2B + V_{13}^2\bar{R}_x^2D - 2V_{11}V_{13}\bar{Y}\bar{R}_xE - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

b) $MSE\left(t_{rp2}^*\right) - MSE\left(t_{rp}^*\right) = (V_{21}-1)^2\bar{Y}^2 + V_{21}^2\bar{Y}^2B + V_{22}^2\bar{X}^2A - 2V_{21}V_{22}\bar{Y}\bar{X}C - (V_1-1)^2\bar{Y}^2 - V_1^2\bar{Y}^2B - V_2^2\bar{X}^2A - V_3^2\bar{R}_x^2D + 2V_1V_2\bar{Y}\bar{X}C + 2V_1V_3\bar{Y}\bar{R}_xE - 2V_2V_3\bar{Y}\bar{R}_xF \geq 0$

## 6. <u>NUMERICAL ILLUSTRATION</u>

To illustrate numerical meaning of the theoretical results, consider a real dataset given in Sample Survey by Daroga Singh and F.S. Chaudhary. The description of the dataset is given below:

A list of 70 villages in India along their population in 1981 and cultivated area (in acres) in the same year is considered (Singh and Choudhary, 1986). Here, the cultivated area (in acres) is taken as the main study variable and the population of the village is taken as the auxiliary variable. We treat first 25% values as non-response units. The parameters of the population are as follows (using R Software):

$$\bar{Y} = 982.71, \quad \bar{X} = 1755.53, \quad \bar{R}_x = 35.5, \quad C_y = 0.6235, \quad C_x = 0.8035,$$

$$C_{r_x} = 0.5732, \quad C_{y(2)} = 0.3723, \quad C_{x(2)} = 0.7824, \quad C_{r_x(2)} = 0.5605,$$

$$\rho_{yx} = 0.7776, \quad \rho_{yx(2)} = 0.8223, \quad \rho_{xr_x} = 0.8498, \quad \rho_{xr_x(2)} = 0.8937,$$

$$\rho_{yr_x} = 0.7578, \quad \rho_{yr_x(2)} = 0.9012, \quad W_2 = 0.25, \quad N = 70, \quad n = 17$$

We computed the percent-relative efficiency (PRE's) of various existing estimators with respect to the usual unbiased estimator $\bar{y}^*$ for different values of $k$, by using the formulae

$$PRE(t^*, \bar{y}^*) = \frac{Var\ (\bar{y}^*)}{MSE\ (*)} \times 100$$

where $t^* = t_r^*, t_{ks}^*, t_{er}^*, t_{ep}^*, t_{ss}^*, t_{rp}^*, t_{rp1}^*, t_{rp2}^*$

**Table 1: Percent-relative efficiency (PRE) of the various estimators**

| $\boldsymbol{PRE(t^*, \bar{y}^*)}$ | $\boldsymbol{(1/k)}$ | | | |
|---|---|---|---|---|
| | $\boldsymbol{(1/2)}$ | $\boldsymbol{(1/3)}$ | $\boldsymbol{(1/4)}$ | $\boldsymbol{(1/5)}$ |
| $PRE(t_r^*, \bar{y}^*)$ | 123.077 | 105.797 | 95.35 | 89.32 |
| $PRE(t_{ks}^*, \bar{y}^*)$ | 21.62 | 21.92 | 22.16 | 22.60 |
| $PRE(t_{er}^*, \bar{y}^*)$ | 204.8 | 182.50 | 169.95 | 162.12 |
| $PRE(t_{ep}^*, \bar{y}^*)$ | 41.83 | 42.44 | 43.16 | 44.072 |
| $PRE(t_{ss}^*, \bar{y}^*)$ | 214.32 | 183.73 | 173.602 | 162.65 |
| $\boldsymbol{PRE(t_{rp}^*, \bar{y}^*)}$ | **277.093** | **281.785** | **288.356** | **295.5968** |
| $\boldsymbol{PRE(t_{rp1}^*, \bar{y}^*)}$ | **245.202** | **255.389** | **265.3895** | **275.097** |
| $\boldsymbol{PRE(t_{rp2}^*, \bar{y}^*)}$ | **242.714** | **248.095** | **249.239** | **251.117** |

## 7. CONCLUSION

In this paper, we have suggested an estimator of the finite population mean that use ranks of the auxiliary variable. Based on both theoretical and numerical findings, it turns out that the proposed estimator $t_{rp}^*$ is more efficient than the usual mean, ratio, product, exponential-ratio and exponential-product estimators. Thus, the suggested estimator $t_{rp}^*$ is to be recommended for efficiently estimating the finite population mean.

## CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest to report regarding the present study.

## ACKNOWLEDGEMENT

The authors are grateful to the experienced referees for their useful comments and suggestions.

## ORCID ID

Udita Gupta https://orcid.org/0009-0000-1754-1418

**REFERENCES**

[1] Azeem M, Hanif M. (2017). Joint influence of measurement error and non-response on estimation of population mean. *Commun Statist. Theory Methods*.14(1):12.

[2] Azeem M. (2014) On estimation of population mean in the presence of measurement error and non-response. Unpublished Ph.D. thesis. National College of Business Administration and Economics. Lahore.

[3] Bahl, S., Tuteja, R.K. (1991). Ratio and product type exponential estimators. *J. Inf.Optim. Sci.* 12(1):159–164.

[4] Bedi, P.K. (1996). Efficient utilization of auxiliary information at estimation stage. *Biometrical J.*38(8):973–976.

[5] Cochran,W.G. (1940). The estimation of the yields of cereal experiments by sampling for the ratio gain to total produce. *J. Agric. Soc.* 30:262–275.

[6] Grover, L.K., Kaur, P. (2011). An improved estimator of the finite population mean in simple random sampling. *Model Assisted Stat. Appl.* 6(1):47–55.

[7] Grover, L.K., Kaur, P. (2014). A generalized class of ratio type exponential estimators of population mean under linear transformation of auxiliary variable. *Commun. Stat. Simul. Comput.* 43(7):1552–1574.

[8] Gupta, S., Shabbir, J. (2008). On improvement in estimating the population mean in simple random sampling. *J. Appl. Stat.* 35(5):559–566.

[9] Haq, A., Shabbir, J. (2013). Improved family of ratio estimators in simple and stratified random sampling. *Commun. Stat. Theory Methods* 42(5):782–799.

[10] Kadilar, C., Cingi, H. (2004). Ratio estimators in simple random sampling. *Appl. Math. Comput.*151(3):893–902.

[11] Kadilar, C., Cingi, H. (2006a). An improvement in estimating the population mean by using the correlation coefficient. *Hacettepe J. Math. Stat.* 35(1):103–109.

[12] Kadilar, C., Cingi,H. (2006b). Improvement in estimating the population mean in simple random sampling. *Appl. Math. Lett.* 19(1):75–79.

[13] Kadilar, C., Cingi, H. (2006c). Ratio estimators for the population variance in simple and stratified random sampling. *Appl. Math. Comput.* 173(2):1047–1059.

[14] Khoshnevisan,M., Singh, R., Chauhan, P., Sawan, N., Smarandache, F. (2007). A general family of estimators for estimating population mean using known value of some population parameter(s). *Far East J. Theor. Stat.* 22(9):181–191.

[15] Koyuncu, N.,Gupta, S., Sousa, R. (2014). Exponential-type estimators of the mean of a sensitive variable in the presence of nonsensitive auxiliary information. *Commun. Stat. Simul. Comput.* 43(7):1583–1594.

[16] Kreuter F, Olson K, Wagner J, et al. Using proxy measure and corrletaes of survey outcomes to adjust for non-response-examples from multiple surveys. J Royal Statist Soc Ser A. 2010;173(Part 3):1–21.

[17] Murthy, M.N. (1964). Product method of estimation. *Sankhya Indian J. Stat. Ser. A (1961–2002)*26(1):69–74.

[18] Murthy, M.N. (1967). *Sampling: theory and method*s. Statistical Publication Society.

[19] Platt, W.J., Evans, G.W., Rathbun, S.L. (1988). The population dynamics of a long-lived conifer (pinus palustris). *Am. Nat.* 131(4):491–525.

[20] Rao, T.J. (1991). On certail methods of improving ration and regression estimators. *Commun. Stat. Theory Methods.* 20(10):3325–3340.

[21] Sahoo J, Sahoo LN, Mohanty S. (1993). A regression approach to estimation using two auxiliary variables. *Curr Sci*. 65(1):73–75.

[22] Shabbir, J., Gupta, S. (2010). On estimating finite population mean in simple and stratified random sampling. *Commun. Stat. Theory Methods* 40(2):199–212.

[23] Shabbir, J., Haq, A., Gupta, S. (2014). A new difference-cum-exponential type estimator of finite population mean in simple random sampling. *Colomb. J. Stat.* 37(1):199–211.

[24] Singh, G.N. (2003a). On the improvement of product method of estimation in sample surveys. *J. Indian Soc. Agric. Stat.* 56(3):267–275.

[25] Singh, S. (2003b). *Advanced sampling theory with applications: how Michael "Selected" Amy*. The Netherlands: Springer.

[26] Singh, H.P., Solanki, R.S. (2013). An efficient class of estimators for the population mean using auxiliary information. *Commun. Stat. Theory Methods* 42(1):145–163.

[27] Singh, H.P., Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean. *Stat. Transition* 6(4):555–560.

[28] Singh, R., Chauhan, P., Sawan, N., Smarandache, F. (2009). Improvement in estimating the population mean using exponential estimator in simple random sampling. *Int. J. Stat. Econ.* 3(A09):13–18.

[29] Sisodia, B.V.S., Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *J. Indian Soc. Agric. Stat.* 33(2):13–18.

[30] Upadhyaya, L.N., Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical J.* 41(5):627–636.

[31] Zahid E, Shabbir J. Estimation of population mean in the presence of measurement error and non-response under stratified random sampling. PLoS One. 2018;13(2):e0191572.

## APPENDIX

```
install.packages("xlsx")
install.packages("readxl")
library(xlsx)
library(readxl)
mydata<-read_excel("C:/Users/DELL/OneDrive/Desktop/daroga singh.xlsx")
head(mydata)
Rx<-rank(mydata$X)
mydata11<-cbind(mydata$Y,mydata$X,Rx)
mydata1<-as.data.frame(mydata11)
head(mydata1)
colnames(mydata1)<-c("Y","X","Rx")
write.csv(mydata1,"mydata1.csv")
getwd()
N<-70
N1<-0.25*N
nonres<-mydata1[1:N1,]
res<-mydata1[N1+1:N,]
#======Population=======
My<-mean(mydata1$Y)
Mx<-mean(mydata1$X)
Mr<-mean(mydata1$Rx)
Vy<-var(mydata1$Y)
Vx<-var(mydata1$X)
Vr<-var(mydata1$Rx)
Cyx<-cor(mydata1$X,mydata1$Y)
Cxr<-cor(mydata1$X,mydata1$Rx)
Cyr<-cor(mydata1$Y,mydata1$Rx)
```

```
#======Non-response=======
my2<-mean(nonres$Y)
mx2<-mean(nonres$X)
mr2<-mean(nonres$Rx)
Vy2<-var(nonres$Y)
Vx2<-var(nonres$X)
Vr2<-var(nonres$Rx)
Cyx2<-cor(nonres$X,nonres$Y)
Cxr2<-cor(nonres$X,nonres$Rx)
Cyr2<-cor(nonres$Y,nonres$Rx)
Cy2<-(Vy/(My^2))
Cx2<-(Vx/(Mx^2))
Cr2<-(Vr/(Mr^2))
Cy22<-(Vy2/(My^2))
Cx22<-(Vx2/(Mx^2))
Cr22<-(Vr2/(Mr^2))
#========k=2/3/4/5=======
k<-2/3/4/5
N<-70
N2<-0.25*N
n<-17
f<-n/N
A11<-(1-f)/n
W2<-N2/N
A12<-W2*(k-1)/n
A<-(A11*Cx2)+(A12*Cx22)
B<-(A11*Cy2)+(A12*Cy22)
C<-(A11*(sqrt(Cy2))*(sqrt(Cx2))*Cyx)+(A12*(sqrt(Cy22))*(sqrt(Cx22))*Cyx2)
D<-(A11*Cr2)+(A12*Cr22)
E<-(A11*(sqrt(Cy2))*(sqrt(Cr2))*Cyr)+(A12*(sqrt(Cy22))*(sqrt(Cr22))*Cyr2)
F<-(A11*(sqrt(Cx2))*(sqrt(Cr2))*Cxr)+(A12*(sqrt(Cx22))*(sqrt(Cr22))*Cxr2)
Rat<-My/Mx
a11<-(Mr/(Mx*(Rat^2)))*((A*D)-(F^2))
a12<-(Mr/(Mx*Rat))*((C*D)-(E*F))
```

```
a13<-(1/Rat)*((A*E)-(C*F))
V01<-(Mr/(Mx*(Rat^2)))
V021<-((1+B)*((A*D)-(F^2)))
V022<-(C*((E*F)-(C*D)))
V023<-(E*((A*E)-(C*F)))
V02<-V021+V022-V023
V0<-V01*V02
W1<-a11/V0
W2<-a12/V0
W3<-a13/V0
Mt1<-(((W1-1)^2)* (My^2)) + ((W1^2)*(My^2)*B) + ((W2^2)*(Mx^2)*A) + ((W3^2)*(Mr^2)*D) -
(2*W1*W2*My*Mx*C) -(2*W1*W3*My*Mr*E) + (2*W2*W3*Mx*Mr*F)
Vay<-((Y^2)*B)
PRE<-((Vay*100)/Mt1)
W11<-(D/(D+(B*D)-(E^2)))
W13<-((Mx*Rat)/Mr)*(E/(D+(B*D)-(E^2)))
Mt2<-(((W11-1)^2)*(My^2))+((W11^2)*(My^2)*B)+((W13^2)*(Mr^2)*D)-
(2*W11*W13*My*Mr*E)
PRE2<-((Vay*100)/Mt2)
V1<-(1/Rat)*((C^2)-(A*(1+B)))
W31<-(-A/(Rat*V1))
W32<-(-C/V1)
Mt3<-(((W31-1)^2)*(My^2))+((W31^2)*(My^2)*B)+((W32^2)*(Mx^2)*A)-
(2*W31*W32*My*Mx*C)
PRE3<-((Vay*100)/Mt3)
```