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Impact of Yoga Awareness on Transmission Dynamics of Communicable Diseases: *SIR* Model Analysis

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Abstract
Communicable diseases are major health problems that affect the whole economy of the na-tion. So it becomes the prime agenda of developed and developing countries to educate peo-ple about disease dynamics and control strategies. Due to changes in people's living styles, disease treatment modality has been changed. So, we have introduced Yoga awareness as control strategy which includes Ahar, Vihar, Achar and Vichar. They are means for achiev-ing the physical, mental, social and spiritual well-being. Our main aim is to model the role of Yoga awareness in controlling the disease dynamics. It maintains the physical fitness of practitioners and improve the whole metabolism system of the human body. In this work, the previous SIR model is modified by incorporating a new awareness transmission rate β 1 to the existing model. Real life situation data of Yoga aware and aware infected individuals were col-lected from different Yoga centers of Sudurpashchim province, Nepal and analyzed by using mathematical techniques. Stability analysis of governing by ordinary differential equations showed that model is stable. Yoga awareness reproduction number Ra was calculated by next-generation matrix method. Sensitivity analysis of Ra and concerning parameters indicate that Ra decreases with an increase in Yoga awareness coverage level. Also, the recovery rate has the opposite relation with Ra which indicates that the recovery period also decreases with an increase in awareness coverage. Local and global stability analysis showed that disease-free equilibrium exists when Ra < 1 and endemic equilibrium exists when Ra > 1. Numerical simulations also support the analytical results and suggest that Yoga awareness has a positive influence on controlling disease d ynamics. The increase in the coverage of awareness leads to reduced susceptibility and infectivity. So propagation of disease can be controlled by Yoga awareness.

Keywords: Transmission dynamics, Yoga awareness, transmission rate, immunity, endemic equi-librium.

1 Introduction

People have been suffering from communicable diseases for thousands of years because of viruses present in the environment. They can easily affect people. It is very difficult to control its trans-mission mechanism. So one of the cheapest ways to control its transmission mechanism is to adopt healthy habits that increase our body's immunity power that can prevent the entry of virus in our body. Communicable diseases like COVID-19, Influenza, Ebola, SARS, Avian and Swine in-fluenza etc are spreading day by day. They become major health problems which affect the whole economy of the nation. So it has become the prime agenda of developed and developing countries to control such diseases. It seems necessary to educate people about the dynamics of communica-ble diseases and to develop a new control strategy. The spread of a communicable disease is often accompanied by a rise in awareness of those in the social vicinity of infected individuals. Yoga class or any other health-related classes or camp also spreads awareness and consequently, people change their behaviors. Such reactions can incorporate themselves in reducing susceptibility as people try to prevent themselves from catching the disease. Awareness can play a positive role re-sulting in the containment or eradication of a disease [1, 2]. Mathematical modeling is a tool used to study the situation of disease quantitatively. It plays an important role in designing appropriate policies for their (communicable diseases) prevention and prediction. It has a long history in med-ical science which has been using such modeling techniques to analyze the spread and eradication of a particular communicable disease. However, modeling in the field of epidemiology was prop-erly extended, improved and developed at the beginning of the twentieth century when William Hamer and Ronald Ross applied the "law of mass action" to explain epidemic behavior. Modern epidemiology has its theoretical approach to the transmission dynamics of a disease that a disease will not appear in society if certain situations or conditions are met for its prevention [3].

Zewdie et al. [4] formulated and analyzed a $SWEIQR$ mathematical model, where W represent aware mass, in transport-related infection with entry-departure screening. The analytic computations showed that the disease-free equilibrium in the absence of travel was globally asymptotically stable when reproduction numbers is less than one and unstable otherwise. The numerical simulations showed that disseminating awareness through the population reduces the spread of disease. A model with awareness such as closing schools, using face masks, and keeping infected persons away from those susceptible (known as social distancing) also minimized the effects of pandemics like influenza [5]. Rwezaura et al. [6] constructed a deterministic mathematical model with vaccination and treatment to analyze their joint effect in curtailing an influenza epidemic. Their results were interpreted in terms of the vaccination, treatment, vaccination and treatment-induced reproduction numbers. They found that vaccinating and treating individuals concurrently is more effective in slowing down the epidemic than concentrating on a cohort vaccination campaign or treatment campaign only. Raimundo et al. [7] did the same task developing a mathematical model to describe the dynamics of reinfection. They assumed that immune protection wanes over time and reinfection is possible in the dynamics of communicable disease. It is found that eradication depends on vaccination coverage as well as on vaccine efficacy. Singh et al. [8] proposed and analyzed a $SIRS$ epidemic model incorporating disease-induced immunity and media awareness to control epidemics. It is found that disease transmission is reduced by media awareness.

Today, mathematical modeling is used to guide public health policies for public health researchers, policymakers, scientists, medical statisticians, health economists, applied mathematicians, and those interested in society's health improvement movements. It will give an idea of the fact how many people will be suffering in future so that proper medication, preventive measures and facilities can be provided to them and research can be done in this area to control the continuously increasing infected population [9, 10]. This model analyzes an awareness effect in disease dynamics and it is based on authentic data, which were collected from different Yoga 20 centres of Sudurpashchim province of Nepal during the period of COVID-19 (from February 2020 to May 2021).

This model considers the contributions to the overall awareness of *Yoga Sadhaka* individuals, Yoga awareness information campaign, direct contacts between unaware and aware individuals and reported cases of infection. *Yoga Pranayama* develops awareness to unaware individuals during

Yoga classes. We have formulated the SIRS epidemic model with Yoga awareness which includes Ahar (Food), Vihar (Exercise), Achar (Good conduct) and Vichar (Good thought) (AVAV) as control strategy. In this paper, we have utilized our Eastern philosophical knowledge to control the transmission dynamics of disease and break the chain between host, environment, and agent. It contains non-pharmaceutical intervention, a variety of systems of spiritual beliefs and practices as used by ancient seers to explore the exterior and interior world and to achieve wisdom and knowledge of great teachers, or gurus. They did not equate Yoga with religion but considered Yoga as an art of living at the highest level of life–reality, and personal verification rather than on belief. Yoga comes from an oral tradition in which teaching was transmitted from a teacher to a student. The sage Patanjali organized this oral tradition in his classic work *The Yoga Sutras*, a 2000-year-old treatise on yogic philosophy. He defined Yoga as a way to inner joy and outer harmony and it restrains the thought process making the mind calm [11–14]. Yoga awareness is a code of conduct of the highest human virtues like *ahimsa* (noninjury) and *satya* (truth), and the promotion of the noblest feelings like amity and compassion [1]. Today it is also alive in Western society, too [15]. Pranayama, a fourth limb of Yoga, helps in our physical development, body metabolism and improvement of physiological functions [14]. It makes us self-aware and increases our understanding of our thoughts, feelings, values, beliefs, and actions. It develops self-awareness about AVAV which is termed as Yoga awareness.

2 Model Formulation

2.1 Assumptions and Model Framework

We use standard $SIRS$ compartmental model with different epidemiological status: susceptible (S), infectious (I), and recovered (R). It is based on the models in the previous studies [16–19]. Since the disease-induced death is negligible and birth rate is nearly equal to death rate, so total population size, $N(t)$, remains constant and homogeneously mixed with each other, where $N(t) = S(t) + I(t) + R(t)$. Let β be the baseline transmission rate and infectious individuals are assumed to be recovered at the rate of γ. Recovered individuals are assumed to gain immunity for certain duration and again they have a chance of being infected. They return to susceptible class at the rate λ. We consider following assumptions

- The population is mixed and interacting homogeneously.
- Yoga awareness includes Pranayama as well as awareness Ahar, Vihar, Aachar, and Vichar. It is assumed that this package of ”Yoga awareness” reduces the disease burden and the transmission rate.
- Recovered individuals develop disease acquired temporary immunity that wanes at rate λ.
- It is assumed that disease transmission rate in aware susceptible β_1 is function of time and rate of change of disease transmission with respect to Yoga aware infected mass is proportional to disease transmission rate and given by $\frac{d\beta_1}{dM} = -c\beta_1$, where time and Yoga aware infected mass $M(t)$ has same scale, where c is non-negative proportionality constant (known as level of awareness coverage). Negative sign indicates that transmission rate decreases as Yoga awareness increases.

  Therefore, Yoga awareness induced effective transmission rate is $\beta_1 = \beta e^{-cM(t)}$, where β is transmission rate of disease in absence of Yoga awareness. It is assumed that individuals adopt behaviors that may reduce the probability of being infected $e^{-cM(t)}$ in β.

  Yoga awareness effect incorporated in our transmission model is $M(t) = p \times m(t)$, where $m(t)$ is the Yoga aware coverage mass at time t which includes the number of Yoga Sadhak (Yoga teachers who conduct classes to aware people during *Yoga Pranayama* classes) individuals and other participants in the Yoga class and p is a scale constant, ($m(t)$ and p are non-negative). This Yoga aware infected term $M(t)$ is based on the assumption that people will pay attention to apply healthy habits along with Pranayama and get themselves safe from disease [16]. The flow diagram of the model is as shown in the following diagram in the Figure 1.

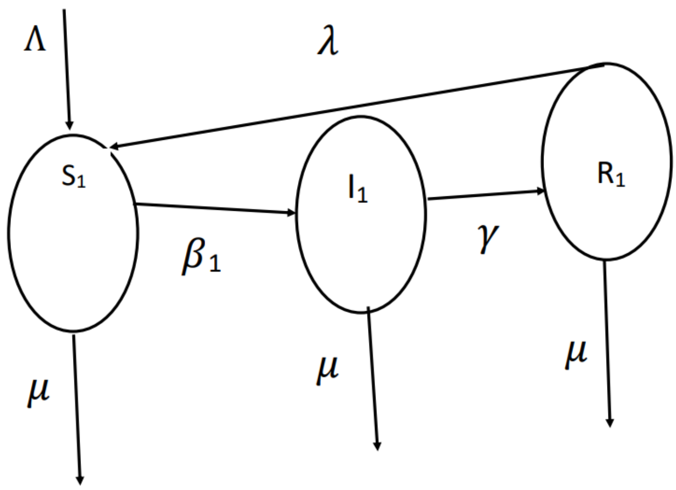

Figure 1: Schematic diagram of $SIRS$ model with Yoga awareness transmission rate $\beta_1 = \beta e^{-cM(t)}$.

Under these assumptions, the system of differential equations that describes disease dynamics is given as follows

$$\begin{aligned} \frac{dS_1}{dt} &= \wedge N - \beta e^{-cM(t)} \frac{S_1 I_1}{N} + \lambda R_1 - \mu S_1 \\ \frac{dI_1}{dt} &= \beta e^{-cM(t)} \frac{S_1 I_1}{N} - \mu I_1 - \gamma I_1 \\ \frac{dR_1}{dt} &= \gamma I_1 - \mu R_1 - \lambda R_1 \end{aligned} \tag{1}$$

With initial conditions

$$S_1(0) = S_0 > 0, I_1(0) = I_0 > 0, R_1(0) = R_0 > 0 \tag{2}$$

Also,

$$N(t) = S_1(t) + I_1(t) + R_1(t)$$

Let us consider the three dimensional region

$$\Omega = [(S_1, I_1, R_!) : 0 \leq S_1, I_1, R_1 \leq \frac{\wedge}{\mu}]$$

Table 1 description of parameters for the system (1).

Parameter	Description	Unit
$\wedge$	Recruitment Rate	$days^-1$
$\frac{1}{\mu}$	Average life-span	day
β	disease transmission rate in absence of Yoga awareness	day^-1
β_1	disease transmission rate with Yoga awareness	day^-1
c	Level of Yoga Awareness coverage	assumed
γ	Rate of recovery from infection	day^-1
λ	Rate at which disease-induced immunity wanes	day^-1

Table 1: Description of parameters.

2.2 Positivity and Boundedness

Since the model (1) with given initial conditions (2) represent the dynamics of population, it is essential to show that its solutions are positive and bounded.

Theorem 2.1. *[20] The solution of system* (1) *with initial conditions (2) are positive and bounded, i.e., all the trajectories of system (1) initiating inside* Ω*, will stay within the interior of* Ω*.*

Proof. Let $R^3_+ = \{(S_1, I_1, R_1) \in R^3_+ : S_1 \geq 0, I_1 \geq 0, R_1 \geq 0\}$ be the three dimentional space.
From (1), we observed that $\frac{dS_1}{dt} = \wedge N + \lambda R_1 > 0$ when $S_1 = 0$
$\frac{dI_1}{dt} = 0$ when $I_1 = 0$
$\frac{dR}{dt} = \gamma_1 I_1 \geq$ when $R_1 = 0$
and $S_1(t), I_1(t), R_1(t)$ are continuous function of t. Thus, the vector field initiating in R^3_+ will remain inside R^3_+ for all the time. Also, the total population, $N(t) = S_1(t) + I_1(t) + R_1(t)$ is constant satisfies $\frac{dN}{dt} = 0$.
Therefore, system (1) is bounded and its any solution originates from Ω remains in Ω. □

2.3 Normalization of the Model

We can normalize the above system (1) with initial conditions (2) as follow

$$S = \frac{S_1}{N}, I = \frac{I_1}{N}, R_1 = \frac{R_1}{N}.$$

The rescaled equations are

$$\begin{aligned}
\frac{dS}{dt} &= \wedge - \beta e^{-cM(t)} SI + \lambda R - \mu S \\
\frac{dI}{dt} &= \beta e^{-cM(t)} SI - \mu I - \gamma I \\
\frac{dR}{dt} &= \gamma I - \mu R - \lambda R
\end{aligned} \tag{3}$$

with initial conditions

$$S(0) = S_0 > 0, I(0) = I_0 > 0, R(0) = R_0 > 0.$$

2.4 Parameter Estimation

In this section, we estimate parameters for the Yoga awareness effect term under the model considered which includes the term $M(t)$. This Yoga awareness term is incorporated into the incidence rate, $\beta e^{-cM(t)}SI$. This model utilized the real world Yoga awareness coverage data in 20 Yoga centers of Sudurpashchhim province, Nepal as shown in appendix (5) and $M(t) = p \times m(t)$ where, $m(t)$ is the amount of Yoga awareness coverage data at time t, M(t) is Yoga aware infected individuals, and p is the scale constant. The observed data $m(t) = (x_1, x_2, \ldots, x_n)$ and $M(t) = (y_1, y_2, y_3, \ldots, y_n)$ were taken, where x_i represents Yoga aware individuals per center and y_i represents Yoga aware infected individuals per center. The scale constant p is estimated through least-square fitting of total infected Yoga aware individuals $M(t)$ and Yoga aware mass data $m(t)$. The model is fitted to the actual data collected from Yoga c enters. The scale constant p in the Yoga awareness term is estimated by least square method using Mathematica software. *Yoga Sadhaka* individuals conduct Yoga classes and aware other participants present in the class. Besides Yoga Pranayama, they change habits applying awareness Aahar, Vihar, Aachar and Vichar which we have already considered as Yoga awareness. The rest of the parameter values were fixed as given in the Table 2.

Parameter	Value	Reference
R_a	[0.5 - 1.6]	Calculated
β	[0.05 , 1.6]	[17]
λ	0.02	[17]
c	[0, 1]	Assumed
γ	[0.05,0.99]	[17]
p	0.0231	estimmated
μ	0.05	[17]

Table 2: Estimated parametric values

3 Dynamic Behavior of the Model

In this section, we calculate the awareness reproduction number, feasible steady states and analyze the stability of equilibria for the proposed system. The biologically feasible region for the non-dimensional system is

$$\Omega = \{(S, I, R) : 0 \leq S, I, R \leq 1\}.$$

3.1 Equilibrium States and Awareness Reproduction Number

For equilibrium point of the model (3), we have

$$\begin{aligned} \wedge - \beta e^{-cM(t)} SI + \lambda R - \mu S = 0 \\ \beta e^{-cM(t)} SI - \mu I - \gamma I = 0 \\ \gamma I - \mu R - \lambda R = 0. \end{aligned} \tag{4}$$

The system (3) has two equilibrium steady states: disease free equilibrium (DFE) $E^0 = (1, 0, 0)$ and endemic equilibrium(EE) $E^* = (S^*, I^*, R^*)$. Now, we calculate the reproduction number by defining the threshold R_a as the average number of secondary infections produced when one primarily infected person is entered into susceptible aware population. The number R_a is called the awareness reproduction number. Here, we are using the term ‘awareness reproduction number’ for the threshold, because in the model awareness process is used to control the disease. It is one of the most useful parameters that tells us about infection situation (eliminated or survived) in the population. We will find the expression for R_a by using next generation matrix. Let $X = (S, I, R)$. Therefore, $\frac{dX}{dt} = \mathcal{F} - \mathcal{V}$ where

$$\mathcal{F} = \begin{bmatrix} 0 \\ \beta_1 SI \\ 0 \end{bmatrix}$$

$$\mathcal{V} = \begin{bmatrix} -\Lambda + \beta_1 SI - \lambda R + \mu S \\ \mu I + \gamma I \\ -\gamma I + \mu R + \lambda R \end{bmatrix}$$

At disease free equilibrium, variation matrices of $\mathcal{F}$ and $\mathcal{V}$ are given by

$$F_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_1 S & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_0 = \begin{bmatrix} \mu & \beta_1 S & -\lambda \\ 0 & \mu + \gamma & 0 \\ 0 & -\gamma & \lambda \end{bmatrix}$$

The next generation matrix for the model equation (3) is

$$F_0 V_0^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\beta_1 S}{\mu + \gamma} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The spectral radius of the matrix $F_0 V_0^{-1}$ gives expression R_a. Awareness reproduction number is $R_a = \frac{\beta_1}{(\gamma+\mu)} = \frac{\beta e^{-cM}}{(\gamma+\mu)} = \frac{R_0}{e^{cM}}$

Now, solving (4), we obtain a unique positive equilibrium known as endemic equilibrium point, $E_0 = (S^*, I^*, R^*)$. From system (4), we get,

$R^* = \frac{\gamma I^*}{\lambda+\mu}$. $\quad S^* = \frac{(\gamma+\mu)e^{cM}}{\beta} = \frac{e^{cM}}{R_0} \quad M^* = \frac{ln(\frac{S^*\beta}{\gamma+\mu})}{c} = \frac{ln(S^* R_0)}{c}$

Also, $S^* = 1 - I^* - R^*$ and the value of I^* is given by

$$\frac{e^{cM^*}}{R_0} = 1 - \left(1 + \frac{\gamma}{\lambda+\mu}\right)I^* \tag{5}$$

$$I^* = \frac{\lambda+\mu}{\gamma+\lambda+\mu}\left(1 - \frac{\gamma+\mu}{\beta}e^{cM}\right) = \frac{\lambda+\mu}{\lambda+\mu+\gamma}\left(1 - \frac{e^{cM}}{R_0}\right) = \frac{\lambda+\mu}{\lambda+\mu+\gamma}\left(1 - \frac{1}{R_a}\right)$$

If there is no yoga awareness effect, i.e., $c = 0$, then

$I^* = \frac{\lambda+\mu}{\gamma+\lambda+\mu}\left(1 - \frac{\gamma+\mu}{\beta}\right) = \left(\frac{\lambda+\mu}{\lambda+\mu+\gamma}\right)\left(1 - \frac{1}{R_0}\right)$

Clearly, I^* exists if and only if $R_a > 1$.

The endemic equilibrium does not exist for $R_a \leq 1$ and exists for $R_a > 1$.

Also, in presence of Yoga awareness, endemic equilibrium does not exist for $R_a < 1$ which is shown in the Figure 2. But endemic equilibrium exists for $R_a > 1$ which is shown in the Figure 3.

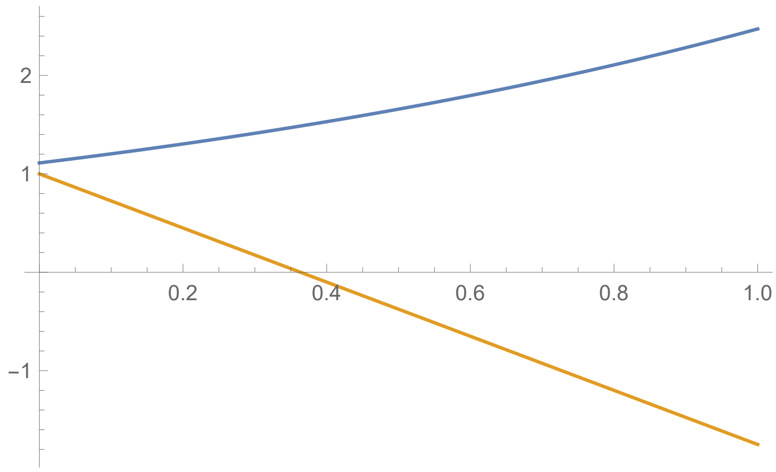

Figure 2: Non-existence of endemic equilibrium for parametric values $\beta = 0.08$, $\gamma = 0.3$, $\lambda = 0.02$, $\mu = 0.05$, $R_a < 1$, where blue color represents $\frac{e^{cM}}{R_0}$ and pink color represents the curve $1 - (1 + \frac{\gamma}{\lambda+\mu})I^*$.

3.2 Stability Analysis

In this section, local and global stability analysis of the model is established.

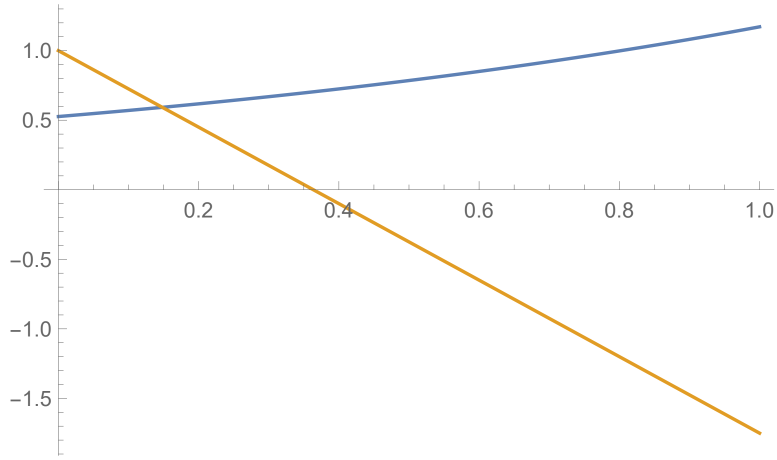

Figure 3: Existence of endemic equilibrium for parametric values $\beta = 0.8$, $\gamma = 0.3$, $\lambda = 0.02$, $\mu = 0.05$, $R_a > 1$, where blue color represents $\frac{e^{cM}}{R_0}$ and pink color represents the curve $1 - (1 + \frac{\gamma}{\lambda+\mu})I^*$.

3.2.1 Local Stability Analysis

Theorem 3.1. *The disease free equilibrium (DFE) E^0 is*

(i) locally asymptotically stable, if $R_a < 1$ and

(ii) unstable, if $R_a > 1$

(iii) population is disease free when $R_a = 1$.

Proof. The variation matrix for DFE is given by

$$V(E_0) = \begin{bmatrix} -\mu & -\beta & \lambda \\ 0 & \beta - \gamma - \mu & 0 \\ 0 & \gamma & -\lambda - \mu \end{bmatrix}$$

If I is identity matrix of order three and K is scalar, then the characteristic equation of $V(E_0)$ is

$$|V(E_0)| = \begin{vmatrix} -\mu - K & -\beta & \lambda \\ 0 & \beta - \gamma - \mu - K & 0 \\ 0 & \gamma & -\lambda - \mu - K \end{vmatrix}$$

$(-\mu - K)(\beta - \gamma - \mu - K)(-\lambda - \mu - K) = 0$

It gives

$K_1 = -\mu, K_2 = -\lambda - \mu\ K_3 = \beta - \gamma - \mu.$

Clearly, the two eigenvalues are negative and third will also be negative if $(\beta - \gamma - \mu) < 0$, i.e., if $\beta \leq \gamma + \mu$

$\frac{\beta}{\gamma+\mu} < 1$

$R_a < 1$

All the values are negative if $R_a < 1$, so system is locally asymptotically stable in this case and unstable when $R_a > 1$.

When $R_a = 1$, from expression (5) $I^* = 0$. So population is disease free. □

For disease free equilibrium $R_a = R_0$.

Theorem 3.2. *The endemic equilibrium (EE) is locally asymptotically stable for* $R_a > 1$.

Proof. We have from endemic equilibrium (5)

$$I^* = \Big(\frac{\lambda + \mu}{\lambda + \mu + \gamma}\Big)\Big(1 - \frac{1}{R_a}\Big).$$

Since, parameters λ, μ, and γ are all positive, if $R_a < 1$, then $I^* < 0$ which is contradiction because we suppose $I^* > 0$ for endemic equilibrium. So, endemic equilibrium is locally asymptotically stable for $R_a > 1$. □

Theorem 3.3. *The Endemic equilibrium E_1 is locally asymptotically stable if the coefficients of the characteristic equation of system* (3) *at E_1 satisfy the Routh-Hurwitz criterion.*

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Proof. The variational matrix E_1 at the endemic equilibrium is given by

$$E_1 = \begin{bmatrix} -\beta e^{-cM} I^* - \mu & \beta e^{-cM} S^* & \lambda \\ \beta e^{-cM} I^* & \beta e^{-cM} S^* - \gamma - \mu & 0 \\ 0 & \gamma & -\mu - \lambda \end{bmatrix}$$

$$E_1 = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & 0 \\ 0 & x_{32} & x_{33} \end{bmatrix}$$

where,

$$x_{11} = -\beta e^{-cM} I^* - \mu, \qquad x_{12} = -\beta e^{-cM} S^*, \qquad x_{13} = \lambda,$$
$$x_{21} = \beta e^{-cM} I^*, \qquad x_{22} = \beta e^{-cM} S^* - \gamma - \mu, \qquad x_{32} = \gamma, \qquad x_{33} = -\lambda - \mu.$$

The characteristic equation at E_1 is $|(KI - E_1)| = 0$, where I is the identity matrix of order 3 and K is the eigenvalue of matrix E_1. By simple computation, the characteristic equation can be expressed in the following form

$$K^3 + AK^2 + BK + C = 0 \tag{6}$$

where,

$$A = -(x_{11} + x_{22} + x_{33}) = \beta_1(I^* - S^*) + 3\mu + \gamma + \lambda$$
$$B = x_{11}x_{22} + x_{11}x_{33} + x_{22}x_{33} - x_{12}x_{21}$$
$$= \beta_1[I^*(\gamma + \lambda + 2\mu) - S^*(2\mu + \lambda)] + (2\mu\gamma + 2\lambda\mu + 3\mu^2 + \gamma\lambda)$$

$$
\begin{aligned}
C &= x_{12}x_{21}x_{33} - x_{13}x_{21}x_{32} - x_{11}x_{22}x_{33} \\
&= (\lambda+\mu)[\beta_1 I^*\gamma + \beta_1 I^*\mu - \beta_1 S^*\mu + \gamma\mu + \mu^2] - \beta_1 I^*\lambda\gamma \\
&= \beta_1[\mu\{\lambda(I^* - S^*) + I^*\gamma + I^*\mu - \mu S^*\}] + \mu(\lambda+\mu)(\gamma+\mu) \qquad (7)
\end{aligned}
$$

Here, we suppose M as a constant coefficient and $\beta_1 = \beta e^{-cM}$.

Now, when $R_a > 1$, the endemic equilibrium point exists. Furthermore, benefiting the Routh-Hurwitz criterion [21], all the eigenvalues of characteristic equation have negative real parts if the conditions given below hold: $A > 0, B > 0, C > 0$, $AB - C > 0$ and $ABC - C^2 > 0$. Therefore, the endemic equilibrium E_1 is locally asymptotically stable if $R_a > 1$ and above conditions are satisfied.

Hence, the endemic equilibrium E_1 is locally asymptotically stable if $R_a > 1$ and above conditions are satisfied. □

3.2.2 Global Stability Analysis

Theorem 3.4. *If $R_a < 1$, then the disease-free equilibrium E_0 of the system (3) is globally asymptotically stable in the region Ω. If $R_a > 1$, then the endemic equilibrium E^* is globally asymptotically stable in the region $\Omega = (S, I, R)$.*

Proof. First we prove the global stability at the disease-free equilibrium E_0 when $R_a < 1$. Consider a Lyapunov function $L = I$. Then, the Lyapunov derivative will be

$$
\begin{aligned}
\frac{dL}{dt} &= \frac{dI}{dt} \\
\frac{dL}{dt} &= [\beta e^{-cM(t)}S - \mu - \gamma]I \\
&= [\beta - \mu - \gamma]I \\
&\leq 0, \quad sinceR_a < 1.
\end{aligned}
$$

Thus, if $R_a < 1$, then $\frac{dL}{dt} \leq 0$. Therefore the largest positive invariant set in $(S, I, R) \in \Omega$

is the singleton set E_0, where E_0 is the disease-free equilibrium. Thus, by Lasalle's invariant principle E_0 is globally asymptotically stable in Ω.

In order to prove the global stability of E^* when $R_a > 1$ the system (3) is

$$\begin{aligned}\frac{dI}{dt} &= [\beta e^{-cM(t)}(1 - I - R) - \mu - \gamma]I \\ \frac{dR}{dt} &= \gamma I - \mu R - \lambda R\end{aligned}$$

Now, we discuss in the first quadrant of IR-Plane. Using Dulac's criteria with multipliers $D_1 = \frac{1}{I}$

Let,

$$F_1 = [\beta e^{-cM}(1 - I - R) - \mu - \gamma]I, F_2 = \gamma I - \mu R - \lambda R,$$

$$\begin{aligned}D_1F_1 &= [\beta e^{-cM}(1 - I - R) - \mu - \gamma] \\ D_1F_2 &= \gamma - (\frac{\mu + \lambda}{I})R\end{aligned}$$

We have,

$$\frac{\partial D_1F_1}{\partial I} + \frac{\partial D_1F_2}{\partial R} = -\beta e^{-cM} - \left(\frac{\mu + \lambda}{I}\right) < 0$$

Thus, there is no limit cycle, i.e., no periodic solutions exist in the region. Hence by Poincare-Bendixson theory, endemic equilibrium E^* is globally asymptotically stable in the region Ω for the system (3) and hence, for the original system (1). □

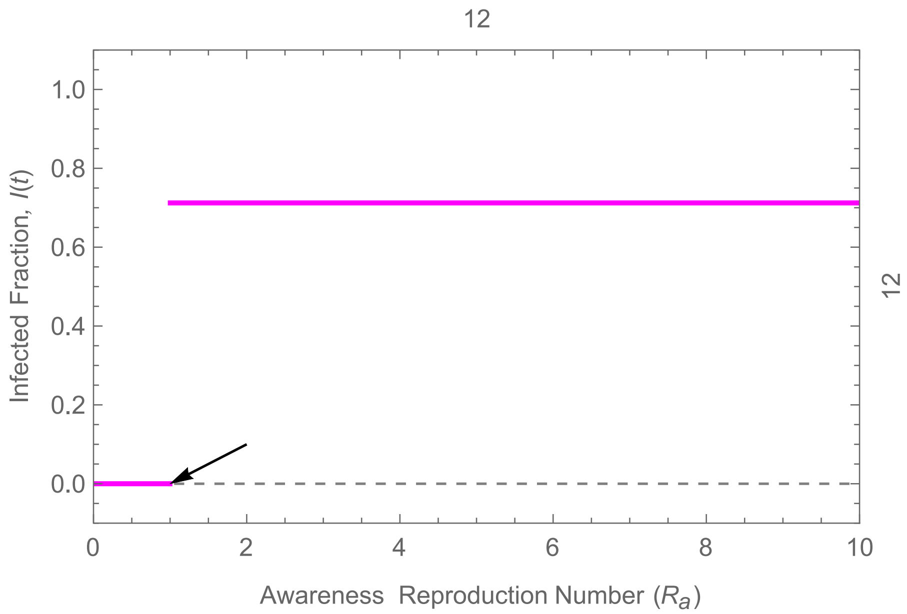


Figure 4: Graph showing the disease free equilibrium state when $R_a < 1$ and endemic equilibrium state when $R_a > 1$.

4 Sensitivity Analysis and Numerical Simulations

4.1 Sensitivity Analysis

In order to get sensitivity analysis, we use parametric values given in the Table 2. We perform the sensitivity analysis of the reproduction number R_a and endemic equilibrium with respect to model parameters. The sensitivity index is given by $\aleph_y^P = \frac{\partial P}{\partial y}\frac{y}{P}$. Sensitivity of R_a to the parameter values y_i are given in the Table 3. We use numerical values from Table 2.

Parameter (y_i)	Sensitivity index of R_a w.r.t parameter y_i	Numerical value
β	1	1
λ	0	0
c	$-cM$	- 0.0231
γ	- $\frac{\gamma}{\gamma+\mu}$	- 0.945
M_0	$-cM$	- 0.0231

Table 3: Sensitivity index of R_a for parametric values.

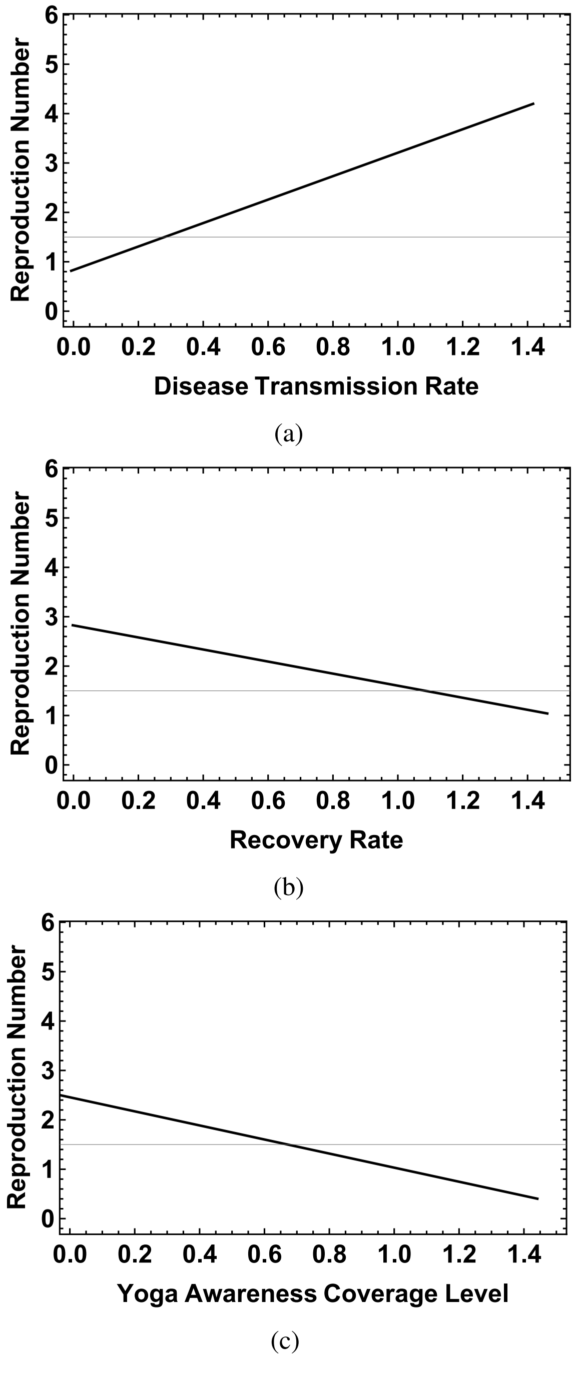
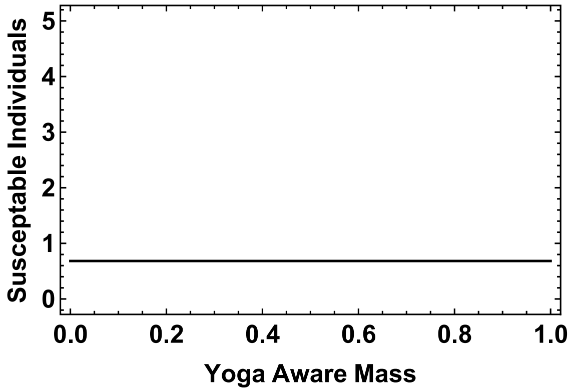
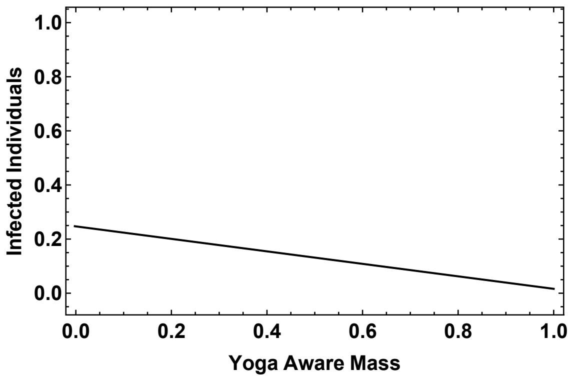


Figure 5: Sensitivity indices of the reproduction number to the parameter values.

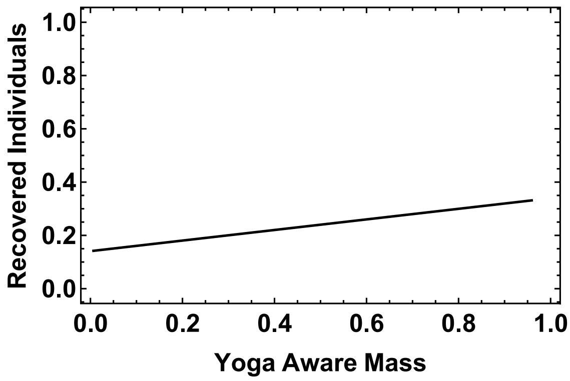
Sensitivity index of the state variables $S(t)$, $I(t)$, $R(t)$ to the parameter y_i where y_i are parameter values mentioned in the Table 2, are given in the Table 4.



(a)



(b)



(c)

Figure 6: $S(t)$, $I(t)$, $R(t)$ to the parameter Yoga awareness mass (Awareness coverage level)

4.2 Numerical Simulations

In this section, we investigate the impacts of the Yoga awareness on disease transmission dynamics. We carry out numerical simulations using the estimated parameter p and the rest of the baseline parameter values in the Table 2. Figures 5 and 6 illustrate the sensi-

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Parameter (y_i)	$\aleph_y^{S^*}$	$\aleph_y^{I^*}$	$\aleph_y^{R^*}$
β	-1	0.00417	0.028166
λ	0	0.2506	-1.5325
c	0.01155	0.03967	0.01072
γ	0.909	-0.000172	-0.00642

Table 4: Sensitivity index of state variables S^*, I^*,R^* w.r.t parameters β, λ, c, γ.

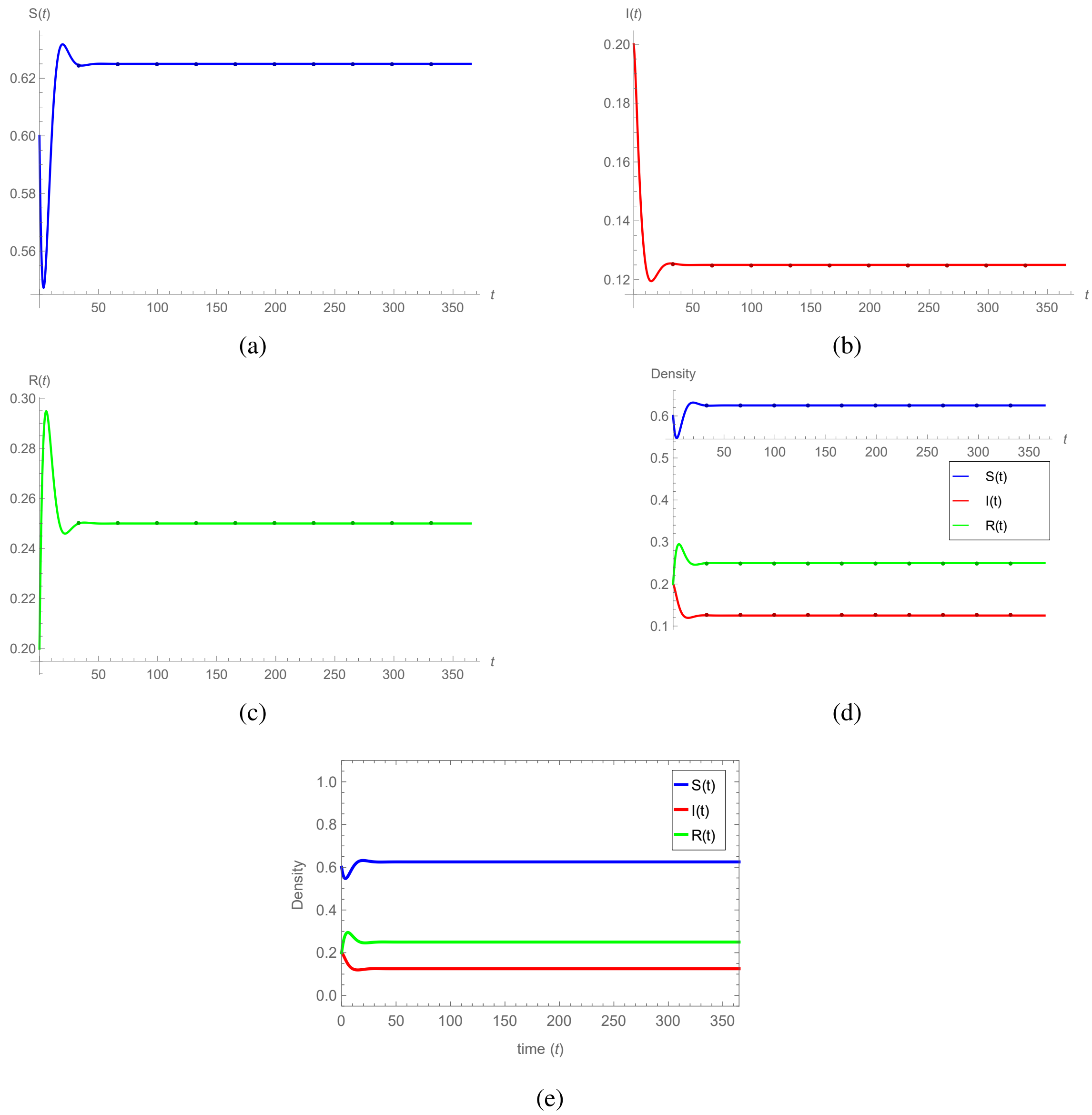
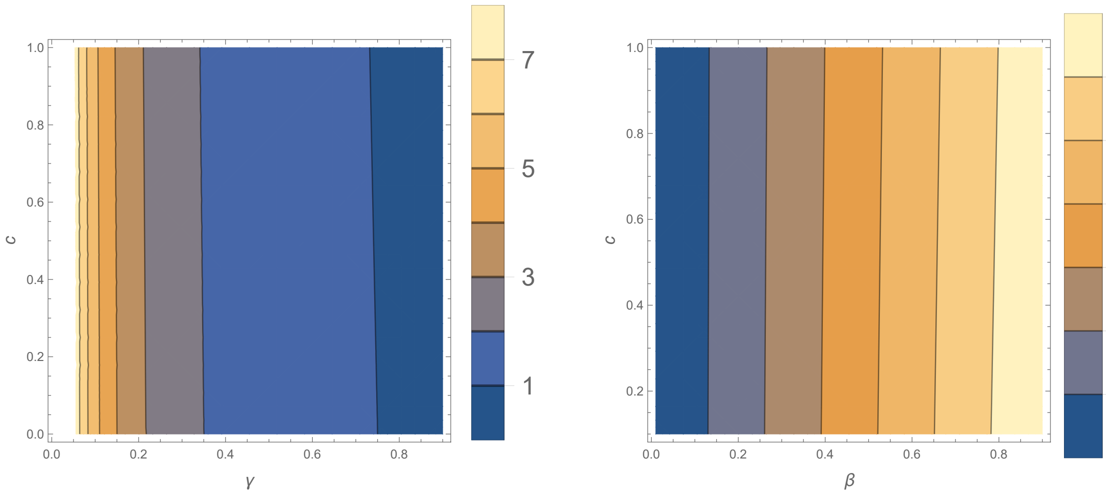


Figure 7: Plot of state variables with time.

tivity of reproduction number to parameters resulting dynamics of Yoga awareness effect term is based on theory. The graphs in the Figure 7 illustrate the dynamics of susceptible,



(a) (b)

Figure 8: Contour plot of reproduction number with parameters.

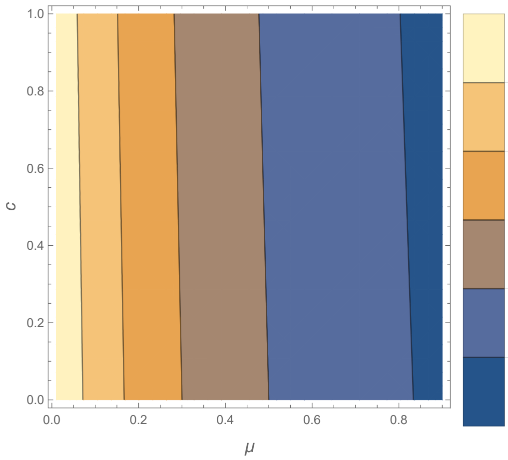


Figure 9: Contour plot of reproduction number with parameters.

infected and recovered individuals and their rate of change. Figure 7 (d), and (e) both illustrates density plot of susceptible, infected, and recovered individuals which shows that Yoga awareness coverage mass reduces susceptibility and infectivity. We use $c = 0.5$ and 1 on disease dynamics to illustrate incidence and cumulative incidence of disease dynamics. The higher the Yoga awareness coverage, the smaller the slopes of epidemic curves in the results of the model. The Yoga awareness coverage term is displayed in contour diagram in the Figure 9. The overall results are almost identical to the results in the Figure

7, which means that the Yoga awareness effect term in the model effectively captures the Yoga aware effect that manifests in real world data.

5 Discussion and Conclusion

5.1 Discussion

Awareness plays a vital role in modern societies, which is also the case when it comes to communicable disease. As societies become larger and more diversified, an individual is hardly able to obtain enough correct information about the communicable disease and art of living. In modern medicine system, the medical physician treats the body of patients, the social worker attends to their emotions and social relations, while the *Yoga Sadhaka* counselor provides guidance for meaningful and purposeful life. Body, mind, cognition, emotion and spirituality are seen as discrete entities. Eastern philosophies of Buddhism, Hinduism and traditional Chinese medicine adopt a holistic conceptualization of an individual and his or her environment. In this view, health is perceived as a harmonious equilibrium that exists between individuals and external environment. Basically, *Ashthanga Yoga* acts as guidelines on how to live self-disciplined life [22]. In the *Yoga Sutra*, Sag Patanjali described *Pranayama* as life force extension. It gains mastery over the respiratory process recognizing the connection between the breath, the mind, and the emotions. It is believed that Yoga not only refreshes the body but also extends life itself [1]. Nowadays, people practice *Pranayama* not only as an isolated technique (i.e., simply sitting and performing a number of breathing exercises), but also as an integration daily life routine. In this article, we have mentioned Yoga awareness as control strategy that considers Yoga is not only *asana* and *Pranayama*, but a way of living healthy life. Good health depends on the long term commitment and the foundation for that needs to be built on four important pillars (Yoga awareness) Ahar (Food), Vihar (Relaxation), Achar (Routines), Vichar (Thoughts) which is supported by WHO's definition that health is a state of physical, mental, and social well-being. So, we have integrated the model which significantly improves physical, mental, and social health.

Yoga Sadhak (Yoga Guru) provides information in Yoga centers and individuals alter their behaviors based on that information. In this regard, it is proper to assume that Yoga awareness coverage might have an influence on infectious disease dynamics. Concerning the Yoga effect, previous studies have suggested a variety of results [23–25]. In these studies, relationship between Yoga practice and health (subjective well-being, diet, smoking, alcohol/caffeine consumption, sleep, stress, social support, mindfulness) were studied and found that Yoga practice had changed health better. Different physical poses and Yoga techniques may have unique health benefits. These research motivated us in this research. Nowadays, though the medical hypotheses and underlying mechanisms of Yoga are infrequently discussed, the participation in Yoga related activities are increasing worldwide. Yoga's effect on prevention of disease was qualitatively studied in these previous researches. Empirical evidences and theories for Yoga mechanisms were studied qualitatively in the areas of hormonal regulation, sympathetic activity in the nervous system and the betterment of physical health attributes such as strength and cardio respiratory health, flexibility, improved balance. Hypothetical effects of Yoga on metabolism, circulation, behavior change, oxidative stress, inflammation and psychological thought processes were also examined [26–28]. Katiyar, V. K., and Pradhan, P. [29] studied on modeling of the breath which we all intake (Pranic bodies) using a mathematical model for different breathing pattern and there steady state equation. They have discussed a compartment model of breath function from lungs to tissues analytically. Similar to that we have considered effect of Yoga on transmission rate and one of the key results is that the Yoga awareness affects the transmission rate β. It influences on reproduction number R_a. When R_a is less than 1, the disease-free equilibrium is stable and when R_a is greater than 1, in contrast, the endemic equilibrium is stable. Yoga awareness leads to decelerate the spread of a disease. In line the these studies, we have modeled real phenomenon of disease transmission mathematically. This present study investigates how the Yoga awareness affects disease transmission dynamics by incorporating the Yoga awareness effect term into the mathematical model. We estimated Yoga awareness coverage term via least-square fitting of the model to the cumulative number of Yoga aware people and Yoga awareness infected mass from Yoga centers. What makes our study distinct is that the incorporated Yoga awareness effect term comes from the awareness coverage data as well as from the theory based on previous studies. Results of numerical simulation suggested that the Yoga awareness can have a positive influence on disease dynamics, more Yoga awareness coverage may lead to a reduced infectivity and epidemic size of disease. Our results highlight that the theory-based and data-based Yoga awareness effect terms have almost the same influence on the disease dynamics under the parameters that we have considered. However, the results may be different for highly transmissible diseases. This suggests that further modeling efforts need to be grounded in real-world knowledge.

It becomes critical to understand the complex interplay between Yoga awareness attention, risk perception, behavior changes and the transmission dynamics of infectious diseases. More work can be done regarding the effect of Yoga awareness on infectious diseases: First, the individual diversity of Yoga awareness credibility can be explored. Since it might be the case that individuals do not believe Yoga awareness coverage to the same degree, incorporating diverse Yoga Pranayama credibility of individuals might make the model more elaborate. Next, influence of the Yoga Guru's opinion may be another topic worth studying. Online Yoga class, Yoga Chautari, Yoga camps etc. exert their influences not only directly on individuals, but also through the opinion leaders who are more intelligent than the *Yoga Sadhaka* persons and can deliver the content of the Patanjali's Yoga philosophy to the population. Thus, it can be assumed that how cautious the *Yoga Guru* is regarding his opinion on the control of infectious disease may have an influence on the behavior of other individuals who are under the influence of the *Yoga Guru*. Furthermore, there are other important factors that can be considered and incorporated into a mathematical model such as various forms of mass (social) media which describes benefits of Yoga and the characteristics of communicable diseases (size or location). In addition, to investigate the type of awareness which work best at reducing transmission dynamics, further research will be needed. Further study need on sensitivity of Yoga awareness dependent perception, change in recovery rate and behavior changes.

5.2 Conclusion

In this paper, we proposed an $SIRS$ epidemic model incorporating Yoga awareness and investigate the asymptotic stability of the model in both disease free and endemic equilibrium states. The disease free equilibrium state is locally asymptotically stable for reproduction number $R_a < 1$. The disease transmission bifurcates at $R_a = 1$. A locally asymptotically stable endemic equilibrium exists for $R_a > 1$. We have observed that Yoga awareness coverage c affect R_a and R_0. We calculate sensitivity indices of the reproduction number on identified respective sensitive parameters. It is found that reproduction number decreases with increase in aware mass. Increased Yoga awareness mass reduces susceptibility and infectivity. Contour plot of Yoga awareness coverage level and model parameters along with reproduction number also shows that Yoga awareness mass coverage level reduces susceptibility and infectivity. It makes faster recovery and longer preservation of immunity. The Figures from 7 to 9 show that Yoga awareness has positive effect on controlling communicable diseases. From these results shown in figures (Figure 5 to Figure 9, we conclude that Yoga awareness reduce susceptibility, reduce infectivity, increase recovery rate and preserved for longer immunity.

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A Appendix

Yoga Awareness data collected from 20 Yoga centers of Sudurpashchim province, Nepal Yoga Centers

Yoga centers codes	Yoga Sadhak Individuals	Infected Yoga Sadhak Individuals
1SK	20	2
2NK	30	3
GK	50	4
TK	60	5
OGK	80	9
M1K	70	6
M2K	60	5
M3K	20	1
D1	30	3
B1	35	3
B2	35	3
Da1	15	1
Do1	20	1
A1	35	2
Ba1	30	2
Kailali	100	12
Online	1005	20
R1	250	8
R2	12	1
R3	25	3

Table 5: Number of Yoga aware people and aware infected people per center.